TeNPy

Release 0.8.3

TeNPy Developers

USER GUIDE

Ι	User guide	1
1	How do I get set up?	5
2	How to read the documentation	7
3	Help - I looked at the documentation, but I don't understand how?	9
4	I found a bug	11
5	Citing TeNPy	13
6	Acknowledgment	15
7	License	17
8	Installation instructions 8.1 Installation with conda from conda-forge 8.2 Installation from PyPi with pip 8.3 Updating to a new version 8.4 Installation from source 8.5 Extra requirements 8.6 Checking the installation 8.7 TeNPy developer team 8.8 License	22 25 26
9	Release Notes 9.1 [latest] 9.2 [v0.8.3] - 2021-03-29 9.3 [v0.8.1] - 2021-03-23 9.4 [v0.8.0] - 2021-03-19 9.5 [0.7.2] - 2020-10-09 9.6 [0.7.1] - 2020-09-04 9.7 [0.7.0] - 2020-09-04 9.8 [0.6.1] - 2020-05-16 9.9 [0.6.0] - 2020-05-16 9.10 [0.5.0] - 2019-12-18 9.11 [0.4.1] - 2019-08-14 9.12 [0.4.0] - 2019-04-28 9.13 [0.3.0] - 2018-02-19 9.14 [0.2.0] - 2017-02-24 9.15 Changes compared to previous TeNPy	

			61
	10.1		61
	10.2		68
	10.3		81
	10.4		91
			92
			97 99
	10.7	Parameters and options	
		Fermions and the Jordan-Wigner transformation	
		Protocol for using (i)DMRG	
	10,10		0,
11	Exan		11
		Toycodes	
		Python scripts	
	11.3	Jupyter Notebooks	.42
12	Troul	bleshooting and FAQ	173
		I get an error when	_
		I get a warning about	
13			175
	13.1	TeNPy related sources	
	13.2	Software-related	
	13.3	General reading	
		References	
	13.3	References	, 0
14	Pape	rs using TeNPy	77
15	Cont	ributing 1	81
	15.1	Coding Guidelines	81
	15.2	Bulding the documentation	.83
	15.3	To-Do list	.84
II	Ref	ference 1	87
	Tenp	y main module	89
	Tenp ; 16.1	y main module run_simulation	1 89
	Tenp ; 16.1 16.2	y main module 1 run_simulation 1 console_main 1	1 89 190
16	Tenp ; 16.1 16.2 16.3	y main module run_simulation	1 89 190 190
16	Tenp 16.1 16.2 16.3 algor	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1	1 89 190 192
16	Tenp: 16.1 16.2 16.3 algor 17.1	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1	1 89 190 192 1 93
16	Tenp; 16.1 16.2 16.3 algor 17.1 17.2	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1 truncation 1	1 89 190 190 192 1 93
16	Tenp. 16.1 16.2 16.3 algor 17.1 17.2 17.3	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1 truncation 1 tebd 1	.90 .90 .92 .93 .94
16	Tenp: 16.1 16.2 16.3 algor 17.1 17.2 17.3 17.4	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1 truncation 1 tebd 1 mps_common 2	1 89 190 190 192 193 194 198
16	Tenp 16.1 16.2 16.3 algor 17.1 17.2 17.3 17.4 17.5	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1 truncation 1 tebd 1 mps_common 2 dmrg 2	189 190 190 192 193 194 198 205 213
16	Tenpy 16.1 16.2 16.3 algor 17.1 17.2 17.3 17.4 17.5 17.6	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1 truncation 1 tebd 1 mps_common 2 dmrg 2 tdvp 2	189 190 192 193 194 194 205 213
16	Tenp: 16.1 16.2 16.3 algor 17.1 17.2 17.3 17.4 17.5 17.6 17.7	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1 truncation 1 tebd 1 mps_common 2 dmrg 2 tdvp 2 purification 2	189 190 192 193 193 194 198 205 213 240
16	Tenpy 16.1 16.2 16.3 algor 17.1 17.2 17.3 17.4 17.5 17.6	y main module 1 run_simulation 1 console_main 1 show_config 1 ithms 1 algorithm 1 truncation 1 tebd 1 mps_common 2 dmrg 2 tdvp 2 purification 2 mpo_evolution 2	189 190 192 193 194 194 205 213

18	linalg		269			
	18.1	np_conserved				
	18.2	charges				
	18.3	svd_robust				
		random_matrix				
	18.5	sparse				
	18.6	lanczos	348			
19	mode	els	353			
		lattice				
			476			
	19.3	tf_ising	507			
	19.4	xxz_chain				
	19.5	spins	535			
	19.6	spins_nnn	549			
	19.7	fermions_spinless				
	19.8	hubbard	565			
		hofstadter				
		haldane				
	19.11	toric_code	596			
20	4	andra	609			
20	netwo 20.1		609			
		mps				
			659			
	20.3	•	670			
	20.5	purification_mps				
	20.5	parmeation_inpo	000			
21	simul	lations	691			
	21.1	simulation	691			
	21.2	measurement				
	21.3	ground_state_search				
	21.4	time_evolution	698			
22	tools		701			
44	22.1		701			
	22.1	params				
	22.3	events				
	22.4	misc	725			
	22.5		736			
	22.6	fit	740			
	22.7	string	744			
	22.8	process	745			
	22.9	optimization	748			
23	versio	on	755			
Bib	liogra	aphy	757			
		Module Index	761			
	Config Option Index					
	Config Index					
Ind			799 801			

Part I

User guide

TeNPy (short for 'Tensor Network Python') is a Python library for the simulation of strongly correlated quantum systems with tensor networks.

The philosophy of this library is to get a new balance of a good readability and usability for new-comers, and at the same time powerful algorithms and fast development of new algorithms for experts. For good readability, we include an extensive documentation next to the code, both in Python doc strings and separately as *user guides*, as well as simple example codes and even toy codes, which just demonstrate various algorithms (like TEBD and DMRG) in ~100 lines per file.

ONE

HOW DO I GET SET UP?

If you have the *conda* package manager, you can install the latest released version of TeNPy with:

conda install --channel=conda-forge physics-tenpy

Further details and alternative methods can be found the file doc/INSTALL.rst. The latest version of the source code can be obtained from https://github.com/tenpy/tenpy.

TWO

HOW TO READ THE DOCUMENTATION

The **documentation is available online** at https://tenpy.readthedocs.io/. The documentation is roughly split in two parts: on one hand the full "reference" containing the documentation of all functions, classes, methods, etc., and on the other hand the "user guide" containing some introductions with additional explanations and examples.

The documentation is based on Python's docstrings, and some additional *.rst files located in the folder *doc/* of the repository. All documentation is formated as reStructuredText, which means it is quite readable in the source plain text, but can also be converted to other formats. If you like it simple, you can just use intective python *help()*, Python IDEs of your choice or jupyter notebooks, or just read the source. Moreover, the documentation gets converted into HTML using Sphinx, and is made available online at https://tenpy.readthedocs.io/. The big advantages of the (online) HTML documentation are a lot of cross-links between different functions, and even a search function. If you prefer yet another format, you can try to build the documentation yourself, as described in doc/contr/build_doc.rst.

THREE

HELP - I LOOKED AT THE DOCUMENTATION, BUT I DON'T UNDERSTAND HOW ...?

We have set up a **community forum** at https://tenpy.johannes-hauschild.de/, where you can post questions and hopefully find answers. Once you got some experience with TeNPy, you might also be able to contribute to the community and answer some questions yourself;-) We also use this forum for official annoucements, for example when we release a new version.

FOUR

I FOUND A BUG

You might want to check the github issues, if someone else already reported the same problem. To report a new bug, just open a new issue on github. If you already know how to fix it, you can just create a pull request:) If you are not sure whether your problem is a bug or a feature, you can also ask for help in the TeNPy forum.

FIVE

CITING TENPY

When you use TeNPy for a work published in an academic journal, you can cite this paper to acknowledge the work put into the development of TeNPy. (The license of TeNPy does not force you, however.) For example, you could add the sentence "Calculations were performed using the TeNPy Library (version X.X. X)\cite{tenpy}." in the acknowledgements or in the main text.

The corresponding BibTex Entry would be the following (the $\url{...}$ requires $\url{...}$ requires $\url{...}$ in the LaTeX preamble.):

To keep us motivated, you can also include your work into the list of papers using TeNPy.

SIX

ACKNOWLEDGMENT

This work was funded by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, Materials Sciences and Engineering Division under Contract No. DE-AC02-05- CH11231 through the Scientific Discovery through Advanced Computing (SciDAC) program (KC23DAC Topological and Correlated Matter via Tensor Networks and Quantum Monte Carlo).

CHAPTER SEVEN

LICENSE

The code is licensed under GPL-v3.0 given in the file LICENSE of the repository, in the online documentation readable at https://tenpy.readthedocs.io/en/latest/install/license.html.

18 Chapter 7. License

EIGHT

INSTALLATION INSTRUCTIONS

With the [conda] package manager you can install python with:

```
conda install --channel=conda-forge physics-tenpy
```

More details and tricks in Installation with conda from conda-forge.

If you don't have conda, but you have [pip], you can:

```
pip install physics-tenpy
```

More details for this method can be found in *Installation from PyPi with pip*.

We also have a bunch of optional *Extra requirements*, which you don't have to install to use TeNPy, but you might want to.

The method with the minimal requirements is to just download the source and adjust the *PYTHONPATH*, as described in *Installation from source*. This is also the recommended way if you plan to modify parts of the source.

8.1 Installation with conda from conda-forge

We provide a package for the [conda] package manager in the *conda-forge* channel, so you can install TeNPy as:

```
conda install --channel=conda-forge physics-tenpy
```

Following the recommondation of conda-forge, you can also make conda-forge the default channel as follows:

```
conda config --add channels conda-forge conda config --set channel_priority strict
```

If you have done this, you don't need to specify the --channel=conda-forge explicitly.

Note: The *numpy* package provided by the *conda-forge* channel by default uses openblas on linux. As outlined in the conda forge docs, you can switch to MKL using:

```
conda install "libblas=*=*mkl"
```

Warning: If you use the *conda-forge* channe and don't pin BLAS to the MKL version as outlined in the above version, but nevertheless have mkl-devel installed during compilation of TeNPy, this can have *crazy* effects on the number of threads used: *numpy* will call openblas and open up \$OMP_NUM_THREADS - 1 new threads, while

MKL called from tenpy will open another \$MKL_NUM_THREADS - 1 threads, making it very hard to control the number of threads used!

Moreover, it is actually recommended to create a separate environment. To create a conda environment with the name *tenpy*, where the TeNPy package (called *physics-tenpy*) is installed:

```
conda create --name tenpy --channel=conda-forge physics-tenpy
```

In that case, you need to activate the environment each time you want to use the package with:

```
conda activate tenpy
```

The big advantage of this approach is that it allows multiple version of software to be installed in parallel, e.g., if one of your projects requires python>=3.8 and another one requires an old library which doesn't support that. Further info can be found in the conda documentation.

8.2 Installation from PyPi with pip

8.2.1 Preparation: install requirements

If you have the [conda] package manager from anaconda, you can just download the environment.yml file (using the *conda-forge* channel, or the environment_other.yml for all other channels) out of the repository and create a new environment (called tenpy, if you don't speficy another name) for TeNPy with all the required packages:

```
conda env create -f environment.yml conda activate tenpy
```

Further information on conda environments can be found in the conda documentation. Note that installing conda also installs a version of [pip].

Alternatively, if you only have [pip] (and not [conda]), install the required packages with the following command (after downloading the requirements.txt file from the repository):

```
pip install -r requirements.txt
```

Note: Make sure that the pip you call corresponds to the python version you want to use. (One way to ensure this is to use python -m pip instead of a simple pip.) Also, you might need to use the argument --user to install the packages to your home directory, if you don't have sudo rights. (Using --user with conda's pip is discouraged, though.)

Warning: It might just be a temporary problem, but I found that the *pip* version of numpy is incompatible with the python distribution of anaconda. If you have installed the intelpython or anaconda distribution, use the *conda* packagemanager instead of *pip* for updating the packages whenever possible!

8.2.2 Installing the latest stable TeNPy package

Now we are ready to install TeNPy. It should be as easy as (note the different package name - 'tenpy' was taken!)

```
pip install physics-tenpy
```

Note: If the installation fails, don't give up yet. In the minimal version, tenpy requires only pure Python with somewhat up-to-date NumPy and SciPy. See *Installation from source*.

8.2.3 Installation of the latest version from Github

To get the latest development version from the github main branch, you can use:

```
pip install git+git://github.com/tenpy/tenpy.git
```

This should already have the lastest features described in *[latest]*. Disclaimer: this might sometimes be broken, although we do our best to keep to keep it stable as well.

8.2.4 Installation from the downloaded source folder

Finally, if you downloaded the source and want to **modify parts of the source**, You can also install TeNPy with in development version with --editable:

```
cd \theta after downloading the source, got to the repository pip install --editable .
```

8.2.5 Uninstalling a pip-installed version

In all of the above cases, you can uninstall tenpy with:

```
pip uninstall physics-tenpy
```

8.3 Updating to a new version

Before you update, take a look at the *Release Notes*, which lists the changes, fixes, and new stuff. Most importantly, it has a section on *backwards incompatible changes* (i.e., changes which may break your existing code) along with information how to fix it. Of course, we try to avoid introducing such incompatible changes, but sometimes, there's no way around them. If you skip some intermediate version(s) for the update, read also the release notes of those!

How to update depends a little bit on the way you installed TeNPy. Of course, you have always the option to just remove the TeNPy files (possibly with a pip uninstall physics—tenpy or conda uninstall physics—tenpy), and to start over with downloading and installing the newest version.

8.3.1 When installed with conda

When you installed TeNPy with [conda], you just need to activate the corresponding environment (e.g. conda activate tenpy) and do a:

```
conda update physics-tenpy
```

8.3.2 When installed with pip

When you installed TeNPy with [pip], you just need to do a:

```
pip install --upgrade physics-tenpy
```

8.3.3 When installed from source

If you used git clone ... to download the repository, you can update to the newest version using [git]. First, briefly check that you didn't change anything you need to keep with git status. Then, do a git pull to download (and possibly merge) the newest commit from the repository.

Note: If some Cython file (ending in .pyx) got renamed/removed (e.g., when updating from v0.3.0 to v0.4.0), you first need to remove the corresponding binary files. You can do so with the command bash cleanup.sh.

Furthermore, whenever one of the cython files (ending in .pyx) changed, you need to re-compile it. To do that, simply call the command bash ./compile again. If you are unsure whether a cython file changed, compiling again doesn't hurt.

To summarize, you need to execute the following bash commands in the repository:

```
# 0) make a backup of the whole folder
git status # check the output whether you modified some files
git pull
bash ./cleanup.sh # (confirm with 'y')
bash ./compile.sh
```

8.4 Installation from source

8.4.1 Minimal Requirements

This code works with a minimal requirement of pure Python>=3.6 and somewhat recent versions of NumPy and SciPy.

8.4.2 Getting the source

The following instructions are for (some kind of) Linux, and tested on Ubuntu. However, the code itself should work on other operating systems as well (in particular MacOS and Windows).

The offical repository is at https://github.com/tenpy/tenpy.git. To get the latest version of the code, you can clone it with [git] using the following commands:

```
git clone https://github.com/tenpy/tenpy.git $HOME/TeNPy
cd $HOME/TeNPy
```

Note: Adjust \$HOME/TeNPy to the path wherever you want to save the library.

Optionally, if you don't want to contribute, you can checkout the latest stable release:

```
git tag \# this prints the available version tags git checkout v0.3.0 \# or whatever is the lastest stable version
```

Note: In case you don't have [git] installed, you can download the repository as a ZIP archive. You can find it under releases, or the latest development version.

8.4.3 Minimal installation: Including tenpy into PYTHONPATH

The python source is in the directory *tenpy/* of the repository. This folder *tenpy/* should be placed in (one of the folders of) the environment variable PYTHONPATH. On Linux, you can simply do this with the following line in the terminal:

```
export PYTHONPATH=$HOME/TeNPy
```

(If you have already a path in this variable, separate the paths with a colon :.) However, if you enter this in the terminal, it will only be temporary for the terminal session where you entered it. To make it permanently, you can add the above line to the file \$HOME/.bashrc. You might need to restart the terminal session or need to relogin to force a reload of the ~/.bashrc.

Whenever the path is set, you should be able to use the library from within python:

tenpy.show_config() prints the current version of the used TeNPy library as well as the versions of the used python, numpy and scipy libraries, which might be different on your computer. It is a good idea to save this data (given as string in tenpy.version.version_summary along with your data to allow to reproduce your results exactly.

If you got a similar output as above: congratulations! You can now run the codes:)

8.4.4 Compilation of np_conserved

At the heart of the TeNPy library is the module tenpy.linalg.np_conseved, which provides an Array class to exploit the conservation of abelian charges. The data model of python is not ideal for the required book-keeping, thus we have implemented the same np_conserved module in Cython. This allows to compile (and thereby optimize) the corresponding python module, thereby speeding up the execution of the code. While this might give a significant speed-up for code with small matrix dimensions, don't expect the same speed-up in cases where most of the CPU-time is already spent in matrix multiplications (i.e. if the bond dimension of your MPS is huge).

To compile the code, you first need to install Cython

```
conda install cython # when using anaconda, or pip install --upgrade Cython # when using pip
```

Moreover, you need a C++ compiler. For example, on Ubuntu you can install sudo apt-get install build_essential, or on Windows you can download MS Visual Studio 2015. If you use anaconda, you can also use conda install -c conda-forge cxx-compiler.

After that, go to the root directory of TeNPy (\$HOME/TeNPy) and simply run

```
bash ./compile.sh
```

Note: There is no need to compile if you installed TeNPy directly with conda or pip. (You can verify this with *tenpy.show_config()* as illustrated below.)

Note that it is not required to separately download (and install) Intel MKL: the compilation just obtains the includes from numpy. In other words, if your current numpy version uses MKL (as the one provided by anaconda), the compiled TeNPy code will also use it.

After a successful compilation, the warning that TeNPy was not compiled should go away:

```
>>> import tenpy
>>> tenpy.show_config()
tenpy 0.7.2.dev130+76c5b7f (compiled without HAVE_MKL),
git revision 76c5b7fe46df3e2241d85c47cbced3400caad05a using
python 3.9.1 | packaged by conda-forge | (default, Jan 10 2021, 02:55:42)
[GCC 9.3.0]
numpy 1.19.5, scipy 1.6.0
```

Note: For further optimization options, e.g. how to link against MKL, look at *Extra requirements* and tenpy. tools.optimization.

8.4.5 Quick-setup of a development environment with conda

You can use the following bash commands to setup a new conda environment called *tenpy_dev* (call it whatever you want!) and install TeNPy in there in a way which allows editing TeNPy's python code and still have it available everywhere in the conda environment:

```
git clone https://github.com/tenpy/tenpy TeNPy cd TeNPy conda env create -f environment.yml -n tenpy_dev conda activate tenpy_dev pip install -e .
```

8.5 Extra requirements

We have some extra requirements that you don't need to install to use TeNPy, but that you might find usefull to work with. TeNPy does not import the following libraries (at least not globally), but some functions might expect arguments behaving like objects from these libraries.

Note: If you created a [conda] environment with conda env create -f environment.yml, all the extra requirements below should already be installed:) (However, a pip install -r requirements.txt does not install all of them.)

8.5.1 Matplotlib

The first extra requirement is the [matplotlib] plotting library. Some functions expect a matplotlib.axes.Axes instance as argument to plot some data for visualization.

8.5.2 Intel's Math Kernel Library (MKL)

If you want to run larger simulations, we recommend the use of Intel's MKL. It ships with a Lapack library, and uses optimization for Intel CPUs. Moreover, it uses parallelization of the LAPACK/BLAS routines, which makes execution much faster. As of now, the library itself supports no other way of parallelization.

If you don't have a python version which is built against MKL, we recommend using [conda] or directly intelpython. Conda has the advantage that it allows to use different environments for different projects. Both are available for Linux, Mac and Windows; note that you don't even need administrator rights to install it on linux. Simply follow the (straight-forward) instructions of the web page for the installation. After a successfull installation, if you run python interactively, the first output line should state the python version and contain Anaconda or Intel Corporation, respectively.

If you have a working conda package manager, you can install the numpy build against MKL with:

```
conda install mkl mkl-devel numpy scipy
```

The mkl-devel package is required for linking against MKL, i.e. for compiling the Cython code. As outlined in /doc/install/conda, on Linux/Mac you also need to pin blas to use MKL with the following line, if you use the `conda-forge` channel:

```
conda install "libblas=*=*mkl"
```

Note: MKL uses different threads to parallelize various BLAS and LAPACK routines. If you run the code on a cluster, make sure that you specify the number of used cores/threads correctly. By default, MKL uses all the available CPUs, which might be in stark contrast than what you required from the cluster. The easiest way to set the used threads is using the environment variable *MKL_NUM_THREADS* (or *OMP_NUM_THREADS*). For a dynamic change of the used threads, you might want to look at *process*.

Compile linking agains MKL

When you compile the Cython files of TeNPy, you have the option to explicitly link against MKL, such that e.g. <code>tenpy.linalg.np_conserved.tensordot()</code> is guaranteed to call the corresponding <code>dgemm</code> or <code>zgemm</code> function in the BLAS from MKL. To link against MKL, the MKL library <code>including the headers</code> must be available during the compilation of TeNPy's Cython files. If you have the MKL library installed, you can export the environemnt variable <code>MKLROOT</code> to point to the root folder. Alternatively, TeNPy will recognise if you are in an active conda environment and have the <code>mkl and mkl-devel</code> conda packages installed during compilation. In this case, it will link against the MKL provided as conda package.

tenpy.show_config() indicates whether you linked successfully against MKL:

```
>>> import tenpy
>>> tenpy.show_config()
tenpy 0.7.2.dev130+76c5b7f (compiled with HAVE_MKL),
git revision 76c5b7fe46df3e2241d85c47cbced3400caad05a using
python 3.9.1 | packaged by conda-forge | (default, Jan 10 2021, 02:55:42)
[GCC 9.3.0]
numpy 1.19.5, scipy 1.6.0
```

8.5.3 HDF5 file format support

We support exporting data to files in the [HDF5] format through the python interface of the h5py <https://docs.h5py.org/en/stable/> package, see Saving to disk: input/output for more information. However, that requires the installation of the HDF5 library and h5py.

8.5.4 YAML parameter files

The tenpy.tools.params.Config class supports reading and writing YAML files, which requires the package *pyyaml*; pip install pyyaml.

8.5.5 Tests

To run the tests, you need to install pytest, which you can for example do with pip install pytest. For information how to run the tests, see *Checking the installation*.

8.6 Checking the installation

The first check of whether tenpy is installed successfully, is to try to import it from within python:

```
>>> import tenpy
```

Note: If this raises a warning Couldn't load compiled cython code. Code will run a bit slower., something went wrong with the compilation of the Cython parts (or you didn't compile at all). While the code might run slower, the results should still be the same.

The function <code>tenpy.show_config()</code> prints information about the used versions of tenpy, numpy and scipy, as well on the fact whether the Cython parts were compiled and could be imported.

As a further check of the installation you can try to run (one of) the python files in the *examples*/ subfolder; hopefully all of them should run without error.

You can also run the automated testsuite with pytest to make sure everything works fine. If you have pytest installed, you can go to the *tests* folder of the repository, and run the tests with:

```
cd tests
pytest
```

In case of errors or failures it gives a detailed traceback and possibly some output of the test. At least the stable releases should run these tests without any failures.

If you can run the examples but not the tests, check whether *pytest* actually uses the correct python version.

The test suite is also run automatically by github actions and with travis-ci, results can be inspected here.

8.7 TeNPy developer team

```
The following people are part of the TeNPy developer team and contributed a.
⇒significant amount.
The full list of contributors can be obtained from the git repository with ``git_
⇒shortlog -sn``.
Johannes Hauschild
                          tenpy@johannes-hauschild.de
Frank Pollmann
Michael P. Zaletel
Maximilian Schulz
Leon Schoonderwoerd
Kévin Hémery
Samuel Scalet
Markus Drescher
Wilhelm Kadow
Gunnar Moeller
Jakob Unfried
Further, the code is based on an earlier version of the library, mainly developed by
Frank Pollmann, Michael P. Zaletel and Roger S. K. Mong.
```

8.8 License

The source code documented here is published under a GPL v3 license, which we include below.

```
GNU GENERAL PUBLIC LICENSE

Version 3, 29 June 2007

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Preamble

The GNU General Public License is a free, copyleft license for software and other kinds of works.
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8.8. License 39

CHAPTER

NINE

RELEASE NOTES

The project adheres semantic versioning.

All notable changes to the project should be documented in the changelog. The most important things should be summarized in the release notes.

The changes in [latest] are implemented in the latest development version on github, but not yet released.

Changes compared to previous TeNPy highlights the most important changes compared to the other, previously developed (closed source) TeNPy version.

9.1 [latest]

9.1.1 Release Notes

TODO: Summarize the most important changes

9.1.2 Changelog

Backwards incompatible changes

• nothing yet

Added

• nothing yet

Changed

• nothing yet

Fixed

• nothing yet

9.2 [v0.8.3] - 2021-03-29

This is another very minor fix-up of /doc/changelog/v0.8.0. Like v0.8.1, it just tries to fix the tests to make building the conda packages work again.

9.3 [v0.8.1] - 2021-03-23

This is only a very minor fix-up of /doc/changelog/v0.8.0.

Despite adding tenpy.networks.mps.MPS.term_list_correlation_function_right(), it only contains minor fixes: - the Simulation class didn't function under windows; some tests were failing. - dtype issues for tenpy.models.model.CouplingModel.add_coupling()

9.4 [v0.8.0] - 2021-03-19

9.4.1 Release Notes

First of all: We have optimized the cython parts such that they can now link directly against MKL and have been optimized for the case of small blocks inside charge-conserved tensors. During compilation, TeNPy now checks whether MKL is available, and then directly links against it. This **changed the depencies**: in particular, when you created a conda environment for TeNPy, it is highly recommended to start off with a new one based on the environment.yml file. If you want to continue using the existing conda environment, you need to conda install mkl-devel before compilation. Additionally, when you use the conda-forge channel of conda, you should pin blas to use MKL by conda install libblas=*=*mkl.

Another great reason to update are **simulation classes** and a console script *tenpy-run* to allow running and even resuming a simulation when it aborted! See /intro/simulation for details.

Further, there is a big change in **verbosity**: we switched to using Python's default logging mechanism. This implies that by default you don't get any output besides error messages and warning any more, at least not in pre-simulation setups. See *Logging and terminal output* on how to get the output back, and what to change in your code.

Finally, note that the default (stable) git branch was renamed from master to main.

9.4.2 Changelog

Backwards incompatible changes

- Drop official support for Python 3.5.
- tenpy.linalg.np_conserved.from_ndarray(): raise *ValueError* instead of just a warning in case of the wrong non-zero blocks. This behaviour can be switched back with the new argument *raise_wrong_sector*.
- Argument v0 of tenpy.networks.mps.MPS.TransferMatrix.eigenvectors() is renamed to $v0_npc$; v0 now serves for non-np_conserved guess.

- Default parameters for lattice initialization in the following classes changed. In particular, the bc_MPS parameter now defaults to 'finite'.
 - tenpy.models.hofstadter.HofstadterFermions
 - tenpy.models.hofstadter.HofstadterBosons
 - tenpy.models.toric_code.ToricCode
- Renamed *tenpy.algorithms.tebd.Engine* to tenpy.algorithms.tebd.TEBDEngine and *tenpy.algorithms.tdvp.Engine* to tenpy.algorithms.tdvp.TDVPEngine to have unique algorithm class-names.
- When running, no longer print stuff by default. Instead, we use Python's *logging* mechanism. To enable printing again, you need to configure the logging to print on "INFO" level (which is the default when running from command line)

As part of this big change in the way verbosity is handled, there were many minor changes: - rename *Config.print_if_verbose* to log() - deprecate the *verbose* class argument of the Config - deprecate the *verbose* class attribute of all classes (if they had it). - change argument names of params().

Added

- Simulation class Simulation and subclasses as a new extra layer for handling the general setup.
- Command line script tenpy-run and run_simulation() for setting up a simulation.
- entanglement_entropy_segment2()
- apply_product_op()
- tenpy.linalg.sparse.FlatLinearOperator.eigenvectors() and eigenvectors() to unify code from tenpy.networks.mps.TransferMatrix.eigenvectors() and tenpy.linalg.lanczos.lanczos_arpack().
- tenpy.tools.misc.group_by_degeneracy()
- tenpy.tools.fit.entropy_profile_from_CFT() and tenpy.tools.fit. central_charge_from_S_profile()
- tenpy.networks.site.Site.multiply_operators() as a variant of multiply_op_names() accepting both string and npc arrays.
- tenpy.tools.events.EventHandler() to simplify call-backs e.g. for measurement codes during an algorithms.
- tenpy.tools.misc.find_subclass() to recursively find subclasses of a given base class by the name. This function is now used e.g. to find lattice classes given the name, hence supporting user-defined lattices defined outside of TeNPy.
- tenpy.tools.misc.get_recursive() and set_recursive() for nested data strucutres, e.g., parameters.
- tenpy.tools.misc.flatten() to turn a nested data structure into a flat one.
- tenpy.networks.mps.InitialStateBuilder to simplify building various initial states.
- Common base class tenpy.algorithms.Algorithm for all algorithms.
- Common base class tenpy.algorithms. Time Evolution Algorithm for time evolution algorithms.
- tenpy.models.lattice.Lattice.Lu as a class attribute.

- tenpy.models.lattice.Lattice.find_coupling_pairs() to automatically find coupling pairs of 'nearest_neighbors' etc..
- tenpy.models.lattice.HelicalLattice allowing to have a much smaller MPS unit cell by shifting the boundary conditions around the cylinder.
- tenpy.networks.purification_mps.PurificationMPS.from_infiniteT_canonical() for a canonical ensemble.

Changed

- For finite DMRG, DMRGEngine. N_sweeps_check now defaults to 1 instead of 10 (which is still the default for infinite MPS).
- Merge tenpy.linalg.sparse.FlatLinearOperator.npc_to_flat_all_sectors() into npc_to_flat(), merge tenpy.linalg.sparse.FlatLinearOperator. flat_to_npc_all_sectors() into flat_to_npc().
- Change the chinfo.names of the specific Site classes to be more consistent and clear.
- Add the more powerful tenpy.networks.site.set_common_charges() to replace tenpy.networks.site.multi_sites_combine_charges().
- Allow swap_op='autoInv' for tenpy.networks.mps.MPS.swap_sites() and explain the idea of the swap_op.
- The tenpy.models.model.CouplingMPOModel.init_lattice() now respects new class attributes default_lattice and force_default_lattice.
- Support additional *priority* argument for get_order_grouped(), issue #122.
- Warn if one of the *add_** methods of the CouplingMPOModel gets called after initialization.

Fixed

- Sign error for the couplings of the tenpy.models.toric_code.ToricCode.
- The form of the eigenvectors returned by tenpy.networks.mps.TransferMatrix. eigenvectors() was dependent on the charge_sector given in the initialization; we try to avoid this now (if possible).
- The charge conserved by SpinHalfFermionSite (cons_Sz='parity') was weird.
- Allow to pass npc Arrays as Arguments to expectation_value_multi_sites() and other correlation functions (issue #116).
- tenpy.tools.hdf5_io did not work with h5py version >= (3,0) due to a change in string encoding (issue #117).
- The overall phase for the returned W from compute K() was undefined.
- tenpy.networks.mpo.MPO.expectation_value() didn't work with max_range=0
- The default *trunc_par* for tenpy.networks.mps.MPS.swap_sites(), permute_sites() and compute_K() was leading to too small chi for intial MPS with small chi.
- issue #120 Lattice with different sites in the unit cell.
- Index offset in tenpy.networks.mps.MPS.expectation_value_term() for the sites to be used.
- issue #121 tenpy.networks.mps.MPS.correlation_length() worked with *charge_sector=0*, but included additional divergent value with *charge_sector=[0]*.

- Some MPS methods (correlation function, expectation value, ...) raised an error for negative site indices even for infinite MPS.
- Warn if we add terms to a couplingMPOMOdel after initialization

9.5 [0.7.2] - 2020-10-09

9.5.1 Release Notes

We've added a list of all papers using (and citing) TeNPy, see *Papers using TeNPy*. Feel free to include your own works!

And a slight simplicifation, which might affect your code: using the *MultiCouplingModel* is no longer necessary, just use the <code>tenpy.models.model.CouplingModel</code> directly.

9.5.2 Changelog

Backwards incompatible changes

- Deprecated the tenpy.models.model.MultiCouplingModel. The functionality is fully merged into the CouplingModel, no need to subclass the MultiCouplingModel anymore.
- The *Kagome* lattice did not include all *next_next_nearest_neighbors*. (It had only the ones across the hexagon, missing those maiking up a bow-tie.)
- Combined arguments *onsite_terms* and *coupling_terms* of tenpy.networks.mpo.MPOGraph. from_terms() into a single argument terms.

Added

- Allow to include jupyter notebooks into the documentation; collect example notebooks in [TeNPyNotebooks].
- term_correlation_function_right() and term_correlation_function_left() for correlation functions with more than one operator on each end.
- tenpy.networks.terms.ExponentiallyDecayingTerms for constructing MPOs with exponential decay, and tenpy.networks.model.CouplingModel. add_exponentially_decaying_coupling() for using it. This closes issue #78.

Fixed

- The IrregularLattice used the 'default' order of the regular lattice instead of whatever the order of the regular lattice was.
- charge_variance() did not work for more than 1 charge.

9.6 [0.7.1] - 2020-09-04

9.6.1 Release Notes

This is just a minor fix to allow building the conda package

9.7 [0.7.0] - 2020-09-04

9.7.1 Release Notes

The big new feature is the implementation of the W_I and W_I method for approximating exponentials of an MPO with an MPO, and MPS compression / MPO application to an MPS, to allow time evolution with ExpMPOEvolution().

9.7.2 Changelog

Backwards incompatible changes

- Remove argument *leg0* from *build_MPO*.
- Remove argument leg0 from from_grids, instead optionally give all legs as argument.
- Moved/renamed the module tenpy.algorithms.mps_sweeps to tenpy.algorithms.mps_common. The old mps_sweeps still exists for compatibility, but raises a warning upon import.
- Moved/renamed the module tenpy.algorithms.purification_tebd to tenpy.algorithms.purification (for the PurificationTEBD and PurificationTEBD2) and tenpy.algorithms.disentangler (for the disentanglers).

Added

- VariationalCompression and VariationalApplyMPO for variational compression
- PurificationApplyMPO and PurificationTwoSiteU for variational compression with purifications.
- Argument insert_all_id for tenpy.networks.mpo.MPOGraph.from_terms() and from_term_list()
- implemented the IrregularLattice.
- extended user guide on lattices, Details on the lattice geometry.
- Function to approximate a decaying function by a sum of exponentials.
- spatial inversion () to perform an explicit spatial inversion of the MPS.

Changed

- By default, for an usual MPO define *IdL* and *IdR* on all bonds. This can generate "dead ends" in the MPO graph of finite systems, but it is useful for the *make_WI/make_WII* for MPO-exponentiation.
- tenpy.models.lattice.Lattice.plot_basis() now allows to shade the unit cell and shift the origin of the plotted basis.
- Don't use bc_shift in tenpy.models.lattice.Lattice.plot_couplings() any more it lead to confusing figures. Instead, the new keyword wrap=True allows to directly connect all sites. This is done to avoid confusing in combination with $plot_bc_identified()$.
- Error handling of non-zero qtotal for TransferMatrix.

Fixed

- Removed double counting of chemical potential terms in the BosonicHaldaneModel and FermionicHaldaneModel.
- Wrong results of tenpy.networks.mps.MPS.get_total_charge() with only_physical_legs=True.
- tenpy.models.lattice.Lattice.plot_bc_identified() had a sign error for the bc_shift.
- calc_H_MPO_from_bond() didn't work for charges with blocks > 1.
- TEBD: keep qtotal of the B tensors constant
- order model parameter was read out but not used in tenpy.models.model.CouplingMPOModel. init_lattice() for 1D lattices.

9.8 [0.6.1] - 2020-05-18

9.8.1 Release Notes

This only is a follow-up release to [0.6.0] - 2020-05-16. It fixes a small bug in the examples/c_tebd.py and some roundoff problems in the tests.

It is now possible to install TeNPy with the conda package manager:

```
conda install --channel=conda-forge physics-tenpy
```

9.9 [0.6.0] - 2020-05-16

9.9.1 Release Notes

This release contains a major update of the documentation, which is now hosted by "Read the Docs" at https://tenpy.readthedocs.io/. Update your bookmark :-)

Apart from that, this release introduces a format how to save and load data (in particular TeNPy classes) to HDF5 files. See *Saving to disk: input/output* for more details. To use that feature, you need to **install** the h5py package (and therefore some version of the HDF5 library). This is easy with anaconda, conda install h5py, but might be cumbersome on your local computing cluster. (However, many university computing clusters have some version of HDF5 installed already. Check with your local sysadmin.)

Moreover, we changed how we read out parameter dictionaries - instead of the *get_parameter()* function, we have now a Config class which behaves like a dictionary, you can simpy use options.get(key, default) for model parameters - as you would do for a python dictionary.

9.9.2 Changelog

Backwards incompatible changes

- Created a class Config to replace Python-native parameter dictionaries and add some useful functionality. Old code using <code>tenpy.tools.params.get_parameter()</code> and <code>tenpy.tools.params.unused_parameters()</code> still works as before, but raises a warning, and should be replaced. For example, if you defined your own models, you should replace calls <code>get_parameter(model_params, "key", "default_value", "ModelName")</code> with <code>model_params.get("key", "default_value")</code>, the latter syntax being what you would use for a normal python dictionary as well.
- Renamed the following class parameter dictionaries to simply *options* for more consitency. Old code using the class attributes should still work (since we provide property aliases), but raises warnings. Note that this affects also derived classes (for example the TwoSiteDMRGEngine).
 - tenpy.algorithms.dmrg.DMRGEngine.DMRG_params (was already renamed to engine_params in versin 0.5.0)
 - tenpy.algorithms.mps_common.Sweep.engine_params
 - tenpy.algorithms.tebd.Engine.TEBD_params
 - tenpy.algorithms.tdvp.Engine.TDVP_params
 - tenpy.linalg.lanczos.Lanczos
- Changed the arguments of tenpy.models.model.MultiCouplingModel(): We replaced the three arguments u0, op0 and other_op with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] by single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*lat.dim. Note the changed order inside the tuple! Old code (which specifies opstr and category as keyword argument, if at all) still works as before, but raises a warning, and should be replaced. Since tenpy.lattice.Lattice.possible_multi_couplings() used similar arguments, they were changed as well.
- Don't save *H_MPO_graph* as model attribute anymore this also wasn't documented.
- Renamed the truncation parameter $symmetry_tol$ to $degeneracy_tol$ and make the criterion more reasonable by not checking $log(S_i/S_j) < log(symmetry_tol)$, but simply $log(S_i/S_j) < degeneracy_tol$. The latter makes more sense, as it is equivalent to $(S_i S_j)/S_j < exp(degeneracy_tol) 1 = degeneracy_tol + \mathcal{O}(degeneracy_tol^2)$.
- Deprecated tenpy.networks.mps.MPS.increase_L() in favor of the newly added tenpy. networks.mps.MPS.enlarge_mps_unit_cell() (taking factor instead of new_L=factor*L as argument).
- tenpy.networks.mps.MPS.correlation_function() now auto-determines whether a Jordan-Wigner string is necessary. If any of the given operators is directly an npc Array, it will now raise an error; set autoJW=False in that case.
- Instead of "monkey-patching" matvec of the tenpy.algorithms.mps_common.EffectiveH for the case that ortho_to_envs is not empty, we defined a proper class NpcLinearOperatorWrapper, which serves as baseclass for OrthogonalNpcLinearOperator. The argument ortho_to_envs has been removed from EffectiveH.

- Switch order of the sites in the unit cell for the *DualSquare*, and redefine what the "default" order means. This is a huge optimization of DMRG, reducing the necessary MPS bond dimension for the ground state to the optimal 2^{L-1} on each bond.
- Deprecated the Lanczos funciton/class argument *orthogonal_to* of in LanczosGroundState. Instead, one can use the <code>OrthogonalNpcLinearOperator</code>.
- Deprecation warning for changing the default argument of *shift_ket* for non-zero *shift_bra* of the *TransferMatrix*.

Added

- tenpy.networks.mpo.MPO.variance() to calculate the variance of an MPO against a finite MPS.
- Classmethod tenpy.networks.MPS.from_lat_product_state() to initialize an MPS from a product state given in lattice coordinates (independent of the *order* of the lattice).
- argument plus_hc for tenpy.models.model.CouplingModel.add_onsite(), tenpy.models.model.CouplingModel.add_coupling(), and tenpy.models.model.MultiCouplingModel.add_multi_coupling() to simplify adding the hermitian conjugate terms.
- parameter *explicit_plus_hc* for *MPOMode1*, *CouplingMode1* and MPO, to reduce MPO bond dimension by not storing Hermitian conjugate terms, but computing them at runtime.
- tenpy.models.model.CouplingModel.add_local_term() for adding a single term to the lattice, and still handling Jordan-Wigner strings etc.
- tenpy.networks.site.Site.get_hc_opname() and hc_ops to allow getting the hermitian conjugate operator (name) of the onsite operators.
- tenpy.tools.hdf5_io with convenience functions for import and output with pickle, as well as an implementation allowing to save and load objects to HDF5 files in the format specified in Saving to disk: input/output.
- human-readable *boundary_conditions* property in *Lattice*.
- save_hdf5 and load_hdf5 methods to support saving/loading to HDF5 for the following classes (and their subclasses): - ChargeInfo - LegCharge - LegPipe - Array - MPS - MPO - Lattice
- tenpy.networks.mps.MPSEnvironment.get_initialization_data() for a convenient way of saving the necessary parts of the environment after an DMRG run.
- Method enlarge_mps_unit_cell for the following classes: MPS MPO Lattice Model, MPOModel, NearestNeighborModel
- tenpy.tools.misc.to_iterable_of_len() for convenience of handling arguments.
- tenpy.models.lattice.Lattice.mps2lat_values_masked() as generalization of tenpy. models.lattice.Lattice.mps2lat_values().
- $\bullet \ \ tenpy. \textit{linalg.sparse.} Or thogonal \textit{NpcLinearOperator} \ \ \textbf{to} \ \ \textbf{orthogonalize} \ \ \textbf{against} \ \ \textbf{vectors.}$
- $\bullet \ \ \textit{tenpy.linalg.sparse.ShiftNpcLinearOperator} \ \ \textbf{to} \ \ \textbf{add} \ \ \textbf{a} \ \ \textbf{constant.}$
- tenpy.linalg.sparse.SumNpcLinearOperator which serves e.g. to add the h.c. during the matvec (in combination with the new tenpy.linalg.sparse.NpcLinearOperator.adjoint()).
- tenpy.algorithms.mps_common.make_eff_H() to simplify implementations of prepare_update().
- attribute options for the Model.
- tenpy.networks.mps.MPS.roll_mps_unit_cell().

Changed

- DEFAULT DMRG paramter 'diag_method' from 'lanczos' to 'default', which is the same for large bond dimensions, but performs a full exact diagonalization if the effective Hamiltonian has small dimensions. The threshold introduced is the new DMRG parameter 'max_N_for_ED'.
- DEFAULT parameter charge_sector=None instead of charge_sector=0 in tenpy.networks. mps.MPS.overlap() to look for eigenvalues of the transfer matrix in *all* charge sectors, and not assume that it's the 0 sector.
- Derive the following classes (and their subclasses) from the new Hdf5Exportable to support saving to HDF5: Site Terms OnsiteTerms CouplingTerms Model, i.e., all model classes.
- Instead of just defining to_matrix and adjoint for EffectiveH, define the interface directly for NpcLinearOperator.
- Try to keep the charge block structure as far as possible for add_charge() and drop_charge()

Fixed

- Adjust the default DMRG parameter min_sweeps if chi_list is set.
- Avoid some unnecessary transpositions in MPO environments for MPS sweeps (e.g. in DMRG).
- sort (bunch=True) could return un-bunched Array, but still set the bunched flag.
- LegPipe did not initialize self.bunched correctly.
- issue #98: Error of calling *psi.canonical_form()* directly after disabling the DMRG mixer.
- svd() with full_matrices=True gave wrong charges.
- tenpy.linalg.np_conserved.Array.drop_charge() and tenpy.lina.np_conserved. Array.drop_charge() did not copy over labels.
- wrong pairs for the *fifth_nearest_neighbors* of the *Honeycomb*.
- Continue in tenpy.algorithms.dmrg.full_diag_effH() with a warning instaed of raising an Error, if the effective Hamltonian is zero.
- correlation_length(): check for hermitian Flag might have raised and Error with new numpy warnings
- correlation_function() did not respect argument str_on_first=False.
- tenpy.networks.mps.MPS.get_op() worked unexpected for infinite bc with incomensurate self.L and len(op_list).
- tenpy.networks.mps.MPS.permute sites() did modify the given perm.
- issue #105 Unintended side-effects using lanczos_params.verbose in combination with orthogonal_to
- issue #108 tenpy.linalg.sparse.FlatLinearOperator._matvec() changes self. _charge_sector

9.10 [0.5.0] - 2019-12-18

9.10.1 Backwards incompatible changes

- Major rewriting of the DMRG Engines, see issue #39 and issue #85 for details. The <code>EngineCombine</code> and <code>EngineFracture</code> have been combined into a single <code>TwoSiteDMRGEngine</code> with an The run function works as before. In case you have directly used the <code>EngineCombine</code> or <code>EngineFracture</code>, you should update your code and use the <code>TwoSiteEngine</code> instead.
- Moved init_LP and init_RP method from MPS into MPSEnvironment and MPOEnvironment.

9.10.2 Changed

- Addition/subtraction of Array: check whether the both arrays have the same labels in differnt order, and in that case raise a warning that we will transpose in the future.
- Made tenpy.linalg.np_conserved.Array.get_block() public (previously tenpy.linalg.np_conserved.Array.get_block).
- groundstate() now returns a tuple (E0, psi0) instead of just psi0. Moreover, the argument charge_sector was added.
- Simplification the Lattice: Instead in of having separate arguments/attributes/functions 'nearest_neighbors', 'next_nearest_neighbors', for 'next_next_nearest_neighbors' and possibly (Honeycomb) even 'fourth_nearest_neighbors', 'fifth_nearest_neighbors', collect them in a dictionary called *pairs*. Old call structures still allowed, but deprecated.
- issue #94: Array addition and *inner()* should reflect the order of the labels, if they coincided. Will change the default behaviour in the future, raising *FutureWarning* for now.
- **Default parameter** for DMRG params: increased precision by setting *P_tol_min* down to the maximum of 1.e-30, lanczos_params['svd_min']**2 * P_tol_to_trunc, lanczos params['trunc cut']**2 * P tol to trunc by default.

9.10.3 Added

- tenpy.algorithms.mps_common with the Sweep class and EffectiveH to be a OneSiteH or TwoSiteH.
- Single-Site DMRG with the SingleSiteDMRG.
- Example function in examples/c_tebd.py how to run TEBD with a model originally having next-nearest neighbors.
- increase_L() to allow increasing the unit cell of an MPS.
- Additional option order='folded' for the Chain.
- tenpy.algorithms.exact_diag.ExactDiag.from_H_mpo() wrapper as replacement for tenpy.networks.mpo.MPO.get_full_hamiltonian() and tenpy.networks.mpo.MPO.get_grouped_mpo(). The latter are now deprecated.
- Argument *max_size* to limit the matrix dimension in *ExactDiag*.
- tenpy.linalg.sparse.FlatLinearOperator.from_guess_with_pipe() to allow quickly converting matvec functions acting on multi-dimensional arrays to a FlatLinearOperator by combining the legs into a LegPipe.

- tenpy.tools.math.speigsh() for hermitian variant of speigs()
- Allow for arguments 'LA', 'SA' in argsort ().
- tenpy.linalg.lanczos.lanczos_arpack() as possiple replacement of the self-implemented lanc-zos function.
- tenpy.algorithms.dmrg.full_diag_effH() as another replacement of lanczos().
- The new DMRG parameter 'diag_method' allows to select a method for the diagonalization of the effective Hamiltonian. See tenpy.algorithms.dmrg.DMRGEngine.diag() for details.
- dtype attribute in *EffectiveH*.
- tenpy.linalg.charges.LegCharge.get_qindex_of_charges() to allow selecting a block of an Array from the charges.
- tenpy.algorithms.mps_common.EffectiveH.to_matrix to allow contracting an EffectiveH to a matrix, as well as metadata tenpy.linalg.sparse.NpcLinearOperator.acts_on and tenpy.algorithms.mps_common.EffectiveH.N.
- argument only_physical_legs in tenpy.networks.mps.MPS.get_total_charge()

9.10.4 Fixed

- MPO expectation_value() did not work for finite systems.
- Calling compute_K () repeatedly with default parameters but on states with different *chi* would use the *chi* of the very first call for the truncation parameters.
- allow MPSEnvironment and MPOEnvironment to have MPS/MPO with different length
- group_sites() didn't work correctly in some situations.
- matvec_to_array() returned the transposed of A.
- tenpy.networks.mps.MPS.from_full() messed up the form of the first array.
- issue #95: blowup of errors in DMRG with *update_env > 0*. Turns out to be a problem in the precision of the truncation error: *TruncationError.eps* was set to 0 if it would be smaller than machine precision. To fix it, I added from_S().

9.11 [0.4.1] - 2019-08-14

9.11.1 Backwards incompatible changes

- Switch the sign of the BoseHubbardModel and FermiHubbardModel to hopping and chemical potential having negative prefactors. Of course, the same adjustment happens in the BoseHubbardChain and FermiHubbardChain.
- moved BoseHubbardModel and BoseHubbardChain as well as FermiHubbardModel and FermiHubbardChain into the new module tenpy.models.hubbard.
- Change arguments of <code>coupling_term_handle_JW()</code> and <code>multi_coupling_term_handle_JW()</code> to use <code>strength</code> and <code>sites</code> instead of <code>op needs JW</code>.
- Only accept valid identifiers as operator names in add_op().

9.11.2 Changed

- grid_concat () allows for None entries (representing zero blocks).
- from_full() allows for 'segment' boundary conditions.
- apply_local_op() allows for n-site operators.

9.11.3 Added

- *max_range* attribute in MPO and *MPOGraph*.
- is_hermitian()
- Nearest-neighbor interaction in BoseHubbardModel
- multiply_op_names() to replace ' '.join(op_names) and allow explicit compression/multiplication.
- order_combine_term() to group operators together.
- dagger () of MPO's (and to implement that also flip_charges_gconj()).
- has_label() to check if a label exists
- qr_li() and rq_li()
- · Addition of MPOs
- 3 additional examples for chern insulators in examples/chern_insulators/.
- FermionicHaldaneModel and BosonicHaldaneModel.
- from_MPOModel() for initializing nearest-neighbor models after grouping sites.

9.11.4 Fixed

- issue #36: long-range couplings could give IndexError.
- issue #42: Onsite-terms in Fermi Hubbard Model were wrong for lattices with non-trivial unit cell.
- Missing a factor 0.5 in GUE ().
- Allow TermList to have terms with multiple operators acting on the same site.
- Allow MPS indices outside unit cell in mps21at_idx() and lat2mps_idx().
- expectation_value() did not work for n-site operators.

9.12 [0.4.0] - 2019-04-28

9.12.1 Backwards incompatible changes

- The argument order of tenpy.models.lattice.Lattice could be a tuple (priority, snake_winding) before. This is no longer valid and needs to be replaced by ("standard", snake_winding, priority).
- Moved the boundary conditions $bc_coupling$ from the tenpy.models.model.CouplingModel into the tenpy.models.lattice.Lattice (as bc). Using the parameter $bc_coupling$ will raise a FutureWarning, one should set the boundary conditions directly in the lattice.

- Added parameter permute (True by default) in tenpy.networks.mps.MPS.from_product_state() and tenpy.networks.mps.MPS.from_Bflat(). The resulting state will therefore be independent of the "conserve" parameter of the Sites unlike before, where the meaning of the p_state argument might have changed.
- Generalize and rename tenpy.networks.site.DoubleSite to tenpy.networks.site. GroupedSite, to allow for an arbitrary number of sites to be grouped. Arguments site0, site1, label0, label1 of the __init__ can be replaced with [site0, site1], [label0, label1] and op0, op1 of the kronecker_product with [op0, op1]; this will recover the functionality of the DoubleSite.
- Restructured callstructure of Mixer in DMRG, allowing an implementation of other mixers. To enable the mixer, set the DMRG parameter "mixer" to True or 'DensityMatrixMixer' instead of just 'Mixer'.
- The interaction parameter in the tenpy.models.bose_hubbbard_chain.BoseHubbardModel (and tenpy.models.bose_hubbbard_chain.BoseHubbardChain) did not correspond to U/2N(N-1) as claimed in the Hamiltonian, but to UN^2 . The correcting factor 1/2 and change in the chemical potential have been fixed.
- Major restructuring of <code>tenpy.linalg.np_conserved</code> and <code>tenpy.linalg.charges</code>. This should not break backwards-compatibility, but if you compiled the cython files, you **need** to remove the old binaries in the source directory. Using <code>bash cleanup.sh</code> might be helpful to do that, but also remove other files within the repository, so be careful and make a backup beforehand to be on the save side. Afterwards recompile with <code>bash compile.sh</code>.
- Changed structure of tenpy.models.model.CouplingModel.onsite_terms and tenpy. models.model.CouplingModel.coupling_terms: Each of them is now a dictionary with category strings as keys and the newly introduced tenpy.networks.terms.OnsiteTerms and tenpy.networks.terms.CouplingTerms as values.
- tenpy.models.model.CouplingModel.calc_H_onsite() is deprecated in favor of new methods.
- Argument raise_op2_left of tenpy.models.model.CouplingModel.add_coupling() is deprecated.

9.12.2 Added

- tenpy.networks.mps.MPS.canonical_form_infinite().
- tenpy.networks.mps.MPS.expectation_value_term(), tenpy.networks.mps.MPS.expectation_value_terms_sum() and tenpy.networks.mps.MPS.expectation_value_multi_sites() for expectation values of terms.
- tenpy.networks.mpo.MPO.expectation_value() for an MPO.
- tenpy.linalg.np_conserved.Array.extend() and tenpy.linalg.charges.LegCharge.extend(), allowing to extend an Array with zeros.
- DMRG parameter 'orthogonal_to' allows to calculate excited states for finite systems.
- possibility to change the number of charges after creating LegCharges/Arrays.
- more general way to specify the order of sites in a tenpy.models.lattice.Lattice.
- new tenpy.models.lattice.Triangular, tenpy.models.lattice.Honeycomb and tenpy. models.lattice.Kagome lattice
- a way to specify nearest neighbor couplings in a Lattice, along with methods to count the number of nearest neighbors for sites in the bulk, and a way to plot them (plot_coupling() and friends)
- tenpy.networks.mpo.MPO.from_grids() to generate the MPO from a grid.
- tenpy.models.model.MultiCouplingModel for couplings involving more than 2 sites.

- request #8: Allow shift in boundary conditions of CouplingModel.
- Allow to use state labels in tenpy.networks.mps.MPS.from_product_state().
- tenpy.models.model.CouplingMPOModel structuring the default initialization of most models.
- Allow to force periodic boundary conditions for finite MPS in the CouplingMPOModel. This is not recommended, though.
- tenpy.models.model.NearestNeighborModel.calc_H_MPO_from_bond() and tenpy. models.model.MPOModel.calc_H_bond_from_MPO() for conversion of H_bond into H_MPO and vice versa.
- tenpy.algorithms.tebd.RandomUnitaryEvolution for random unitary circuits
- Allow documentation links to github issues, arXiv, papers by doi and the forum with e.g. :issue:`5`, :arxiv:`1805.00055`, :doi:`10.21468/SciPostPhysLectNotes.5`, :forum:`3`
- tenpy.models.model.CouplingModel.coupling_strength_add_ext_flux() for adding hoppings with external flux.
- tenpy.models.model.CouplingModel.plot_coupling_terms() to visualize the added coupling terms.
- tenpy.networks.terms.OnsiteTerms, tenpy.networks.terms.CouplingTerms, tenpy.networks.terms.MultiCouplingTerm containing the of terms for the CouplingModel and MultiCouplingModel. This allowed to add the category argument to add_onsite, add_coupling and add_multi_coupling.
- tenpy.networks.terms.TermList as another (more human readable) representation of terms with conversion from and to the other *Term classes.
- tenpy.networks.mps.MPS.init_LP() and tenpy.networks.mps.MPS.init_RP() to initialize left and right parts of an Environment.
- tenpy.networks.mpo.MPOGraph.from_terms() and tenpy.networks.mpo.MPOGraph.from_term_list().
- argument charge_sector in tenpy.networks.mps.MPS.correlation_length().

9.12.3 Changed

- moved toycodes from the folder examples / to a new folder toycodes / to separate them clearly.
- major remodelling of the internals of tenpy.linalg.np_conserved and tenpy.linalg.charges.
 - Introduced the new module tenpy/linalg/_npc_helper.pyx which contains all the Cython code, and gets imported by
 - Array now rejects addition/subtraction with other types
 - Array now rejects multiplication/division with non-scalar types
 - By default, make deep copies of npc Arrays.
- Restructured lanczos into a class, added time evolution calculating exp(A*dt) |psi0>
- Warning for poorly conditioned Lanczos; to overcome this enable the new parameter reortho.
- Simplified call strucutre of extend(), and extend().
- Restructured tenpy.algorithms.dmrg:

- run() is now just a wrapper around the new run(), run(psi, model, pars) is roughly equivalent to eng = EngineCombine(psi, model, pars); eng.run().
- Added init_env() and reset_stats() to allow a simple restart of DMRG with slightly different parameters, e.g. for tuning Hamiltonian parameters.
- Call canonical_form() for infinite systems if the final state is not in canonical form.
- Changed **default values** for some parameters:
 - set trunc_params['chi_max'] = 100. Not setting a *chi_max* at all will lead to memory problems. Disable DMRG_params['chi_list'] = None by default to avoid conflicting settings.
 - reduce to mixer_params['amplitude'] = 1.e-5. A too strong mixer screws DMRG up pretty
 bad.
 - increase Lanczos_params['N_cache'] = N_max (i.e., keep all states)
 - set DMRG_params['P_tol_to_trunc'] = 0.05 and provide reasonable ..._min and ..._max
 values.
 - increased (default) DMRG accuracy by setting DMRG_params['max_E_err'] = 1.e-8 and
 DMRG_params['max_S_err'] = 1.e-5.
 - don't check the (absolute) energy for convergence in Lanczos.
 - set DMRG_params ['norm_tol'] = 1.e-5 to check whether the final state is in canonical form.
- Verbosity of get_parameter() reduced: Print parameters only for verbosity >= 1. and default values only for verbosity >= 2.
- Don't print the energy during real-time TEBD evolution it's preserved up to truncation errors.
- Renamed the SquareLattice class to tenpy.models.lattice.Square for better consistency.
- auto-determine whether Jordan-Wigner strings are necessary in add_coupling().
- The way the labels of npc Arrays are stored internally changed to a simple list with None entries. There is a deprecated properly setter yielding a dictionary with the labels.
- renamed first_LP and last_RP arguments of MPSEnvironment and MPOEnvironment to init_LP and init_RP.
- Testing: insetad of the (outdated) nose, we now use pytest https://pytest.org for testing.

9.12.4 Fixed

- issue #22: **Serious bug** in tenpy.linalg.np_conserved.inner(): if do_conj=True is used with non-zero qtotal, it returned 0. instead of non-zero values.
- avoid error in tenpy.networks.mps.MPS.apply_local_op()
- Don't carry around total charge when using DMRG with a mixer
- Corrected couplings of the FermionicHubbardChain
- issue #2: memory leak in cython parts when using intelpython/anaconda
- issue #4: incompatible data types.
- issue #6: the CouplingModel generated wrong Couplings in some cases
- issue #19: Convergence of energy was slow for infinite systems with N_sweeps_check=1
- · more reasonable traceback in case of wrong labels

- wrong dtype of npc.Array when adding/subtracting/... arrays of different data types
- could get wrong H_bond for completely decoupled chains.
- · SVD could return outer indices with different axes
- tenpy.networks.mps.MPS.overlap() works now for MPS with different total charge (e.g. after psi.apply_local_op(i, 'Sp')).
- skip existing graph edges in MPOGraph.add() when building up terms without the strength part.

9.12.5 Removed

• Attribute chinfo of Lattice.

9.13 [0.3.0] - 2018-02-19

This is the first version published on github.

9.13.1 Added

- · Cython modules for np_conserved and charges, which can optionally be compiled for speed-ups
- tools.optimization for dynamical optimization
- · Various models.
- More predefined lattice sites.
- Example toy-codes.
- · Network contractor for general networks

9.13.2 Changed

• Switch to python3

9.13.3 Removed

• Python 2 support.

9.14 [0.2.0] - 2017-02-24

- Compatible with python2 and python3 (using the 2to3 tool).
- Development version.
- · Includes TEBD and DMRG.

9.15 Changes compared to previous TeNPy

This library is based on a previous (closed source) version developed mainly by Frank Pollmann, Michael P. Zaletel and Roger S. K. Mong. While allmost all files are completely rewritten and not backwards compatible, the overall structure is similar. In the following, we list only the most important changes.

9.15.1 Global Changes

- syntax style based on PEP8. Use \$>yapf -r -i ./ to ensure consitent formatting over the whole project. Special comments # yapf: disable and # yapf: enable can be used for manual formatting of some regions in code.
- Following PEP8, we distinguish between 'private' functions, indicated by names starting with an underscore and to be used only within the library, and the public API. The puplic API should be backwards-compatible with different releases, while private functions might change at any time.
- all modules are in the folder tenpy to avoid name conflicts with other libraries.
- withing the library, relative imports are used, e.g., from ..tools.math import (toiterable, tonparray) Exception: the files in tests/ and examples/ run as __main__ and can't use relative imports
 - Files outside of the library (and in *tests*/, *examples*/) should use absolute imports, e.g. import tenpy. algorithms.tebd
- renamed tenpy/mps/ to tenpy/networks, since it containes various tensor networks.
- added Site describing the local physical sites by providing the physical LegCharge and onsite operators.

9.15.2 np_conserved

- pure python, no need to compile!
- in module tenpy.linalg instead of algorithms/linalg.
- moved functionality for charges to charges
- Introduced the classes ChargeInfo (basically the old q_number, and mod_q) and LegCharge (the old qind, qconj).
- Introduced the class LegPipe to replace the old leg_pipe. It is derived from LegCharge and used as a leg in the array class. Thus any inherited array (after tensordot etc still has all the necessary information to split the legs. (The legs are shared between different arrays, so it's saved only once in memory)
- Enhanced indexing of the array class to support slices and 1D index arrays along certain axes
- more functions, e.g. grid outer()

9.15.3 TEBD

- Introduced TruncationError for easy handling of total truncation error.
- some truncation parameters are renamed and may have a different meaning, e.g. svd_max -> svd_min has no 'log' in the definition.

9.15.4 DMRG

- separate Lanczos module in *tenpy/linalg/*. Strangely, the old version orthoganalized against the complex conjugates of *orthogonal_to* (contrary to it's doc string!) (and thus calculated 'theta_o' as bra, not ket).
- cleaned up, provide prototypes for DMRG engine and mixer.

9.15.5 Tools

- added tenpy.tools.misc, which contains 'random stuff' from old tools.math like to_iterable and to_array (renamed to follow PEP8, documented)
- moved stuff for fitting to tenpy.tools.fit
- enhanced tenpy.tools.string.vert_join() for nice formatting
- moved (parts of) old *cluster/omp.py* to tenpy.tools.process
- added tenpy.tools.params for a simplified handling of parameter/arguments for models and/or algorithms. Similar as the old *models.model.set_var*, but use it also for algorithms. Also, it may modify the given dictionary.

CHAPTER

TEN

INTRODUCTIONS

The following documents are meant as introductions to various topics relevant to TeNPy.

If you are new to TeNPy, read the Overview.

10.1 Overview

10.1.1 Repository

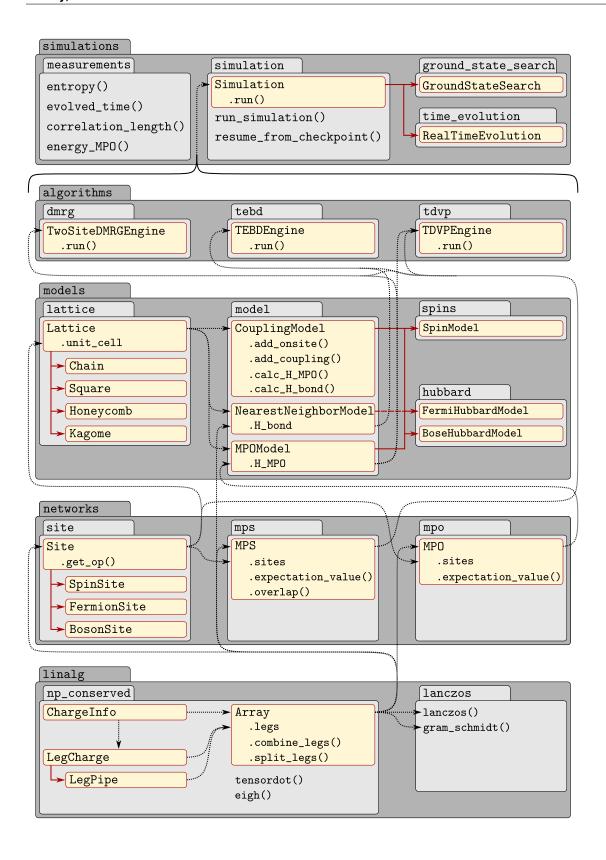
The root directory of the git repository contains the following folders:

- tenpy The actual source code of the library. Every subfolder contains an __init__.py file with a summary what the modules in it are good for. (This file is also necessary to mark the folder as part of the python package. Consequently, other subfolders of the git repo should not include a _init_.py file.)
- **toycodes** Simple toy codes completely independet of the remaining library (i.e., codes in tenpy/). These codes should be quite readable and intend to give a flavor of how (some of) the algorithms work.
- **examples** Some example files demonstrating the usage and interface of the library.
- doc A folder containing the documentation: the user guide is contained in the *.rst files. The online documentation is autogenerated from these files and the docstrings of the library. This folder contains a make file for building the documentation, run make help for the different options. The necessary files for the reference in doc/reference can be auto-generated/updated with make src2html.
- **tests** Contains files with test routines, to be used with *pytest*. If you are set up correctly and have *pytest* installed, you can run the test suite with pytest from within the tests/ folder.
- **build** This folder is not distributed with the code, but is generated by setup.py (or compile.sh, respectively). It contains compiled versions of the Cython files, and can be ignored (and even removed without loosing functionality).

10.1.2 Code structure: getting started

There are several layers of abstraction in TeNPy. While there is a certain hierarchy of how the concepts build up on each other, the user can decide to utilize only some of them. A maximal flexibility is provided by an object oriented style based on classes, which can be inherited and adjusted to individual demands.

The following figure gives an overview of the most important modules, classes and functions in TeNPy. Gray backgrounds indicate (sub)modules, yellow backgrounds indicate classes. Red arrows indicate inheritance relations, dashed black arrows indicate a direct use. (The individual models might be derived from the <code>NearestNeighborModel</code> depending on the geometry of the lattice.) There is a clear hierarchy from high-level algorithms in the <code>tenpy.algorithms</code> module down to basic operations from linear algebra in the <code>tenpy.linalg</code> module.



Most basic level: linear algebra

Note: See *Charge conservation with np_conserved* for more information on defining charges for arrays.

The most basic layer is given by in the <code>linalg</code> module, which provides basic features of linear algebra. In particular, the <code>np_conserved</code> submodule implements an <code>Array</code> class which is used to represent the tensors. The basic interface of <code>np_conserved</code> is very similar to that of the NumPy and SciPy libraries. However, the <code>Array</code> class implements abelian charge conservation. If no charges are to be used, one can use 'trivial' arrays, as shown in the following example code.

```
"""Basic use of the `Array` class with trivial arrays."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3

import tenpy.linalg.np_conserved as npc

M = npc.Array.from_ndarray_trivial([[0., 1.], [1., 0.]])
v = npc.Array.from_ndarray_trivial([2., 4. + 1.j])
v[0] = 3. # set indiviual entries like in numpy
print("|v> =", v.to_ndarray())
# |v> = [ 3.+0.j  4.+1.j]

M_v = npc.tensordot(M, v, axes=[1, 0])
print("M|v> =", M_v.to_ndarray())
# M|v> = [ 4.+1.j  3.+0.j]
print("<v|M|v> =", npc.inner(v.conj(), M_v, axes='range'))
# <v/M/v> = (24+0j)
```

The number and types of symmetries are specified in a *ChargeInfo* class. An *Array* instance represents a tensor satisfying a charge rule specifying which blocks of it are nonzero. Internally, it stores only the non-zero blocks of the tensor, along with one *LegCharge* instance for each leg, which contains the *charges* and sign *qconj* for each leg. We can combine multiple legs into a single larger *LegPipe*, which is derived from the *LegCharge* and stores all the information necessary to later split the pipe.

The following code explicitly defines the spin-1/2 S^+, S^-, S^z operators and uses them to generate and diagonalize the two-site Hamiltonian $H = \vec{S} \cdot \vec{S}$. It prints the charge values (by default sorted ascending) and the eigenvalues of H.

```
"""Explicit definition of charges and spin-1/2 operators."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
# consider spin-1/2 with Sz-conservation
chinfo = npc.ChargeInfo([1]) # just a U(1) charge
# charges for up, down state
p_leg = npc.LegCharge.from_qflat(chinfo, [[1], [-1]])
Sz = npc.Array.from_ndarray([[0.5, 0.], [0., -0.5]], [p_leg, p_leg.conj()])
Sp = npc.Array.from_ndarray([[0., 1.], [0., 0.]], [p_leg, p_leg.conj()])
Sm = npc.Array.from_ndarray([[0., 0.], [1., 0.]], [p_leg, p_leg.conj()])
Hxy = 0.5 * (npc.outer(Sp, Sm) + npc.outer(Sm, Sp))
Hz = npc.outer(Sz, Sz)
H = Hxy + Hz
# here, H has 4 legs
H.iset_leg_labels(["s1", "t1", "s2", "t2"])
H = H.combine_legs([["s1", "s2"], ["t1", "t2"]], qconj=[+1, -1])
```

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10.1. Overview 63

```
# here, H has 2 legs
print(H.legs[0].to_qflat().flatten())
# prints [-2 0 0 2]
E, U = npc.eigh(H) # diagonalize blocks individually
print(E)
# [ 0.25 -0.75 0.25 0.25]
```

Sites for the local Hilbert space and tensor networks

The next basic concept is that of a local Hilbert space, which is represented by a Site in TeNPy. This class does not only label the local states and define the charges, but also provides onsite operators. For example, the SpinHalfSite provides the S^+, S^-, S^z operators under the names 'Sp', 'Sm', 'Sz', defined as Array instances similarly as in the code above. Since the most common sites like for example the SpinSite (for general spin S=0.5, 1, 1.5,...), BosonSite and FermionSite are predefined, a user of TeNPy usually does not need to define the local charges and operators explicitly. The total Hilbert space, i.e, the tensor product of the local Hilbert spaces, is then just given by a list of Site instances. If desired, different kinds of Site can be combined in that list. This list is then given to classes representing tensor networks like the MPS and MPO. The tensor network classes also use Array instances for the tensors of the represented network.

The following example illustrates the initialization of a spin-1/2 site, an MPS representing the Neel state, and an MPO representing the Heisenberg model by explicitly defining the W tensor.

```
"""Initialization of sites, MPS and MPO."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3
from tenpy.networks.site import SpinHalfSite
from tenpy.networks.mps import MPS
from tenpy.networks.mpo import MPO
spin = SpinHalfSite(conserve="Sz")
print(spin.Sz.to_ndarray())
# [[ 0.5 0. ]
  [ 0. -0.511
N = 6 # number of sites
sites = [spin] * N # repeat entry of list N times
pstate = ["up", "down"] * (N // 2) # Neel state
psi = MPS.from_product_state(sites, pstate, bc="finite")
print("<Sz> =", psi.expectation_value("Sz"))
\# \langle Sz \rangle = [0.5 - 0.5 0.5 - 0.5]
print("<Sp_i Sm_j> =", psi.correlation_function("Sp", "Sm"), sep="\n")
\# <Sp_i Sm_j> =
# [[1. 0. 0. 0. 0. 0.]
  [0. 0. 0. 0. 0. 0.]
  [0. 0. 1. 0. 0. 0.]
  [0. 0. 0. 0. 0. 0.]
  [0. 0. 0. 0. 1. 0.]
  [0. 0. 0. 0. 0. 0.]]
# define an MPO
Id, Sp, Sm, Sz = spin.Id, spin.Sp, spin.Sm, spin.Sz
J, Delta, hz = 1., 1., 0.2
W_bulk = [[Id, Sp, Sm, Sz, -hz * Sz], [None, None, None, 0.5 * J * Sm],
          [None, None, None, None, 0.5 * J * Sp], [None, None, None, J * Delta_
\hookrightarrow * Sz],
```

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```
[None, None, None, None, Id]]
W_first = [W_bulk[0]] # first row
W_last = [[row[-1]] for row in W_bulk] # last column
Ws = [W_first] + [W_bulk] * (N - 2) + [W_last]
H = MPO.from_grids([spin] * N, Ws, bc='finite', IdL=0, IdR=-1)
print("<psi|H|psi> =", H.expectation_value(psi))
# <psi|H|psi> = -1.25
```

Models

Note: See *Models* for more information on sites and how to define and extend models on your own.

Technically, the explicit definition of an MPO is already enough to call an algorithm like DMRG in <code>dmrg</code>. However, writing down the *W* tensors is cumbersome especially for more complicated models. Hence, TeNPy provides another layer of abstraction for the definition of models, which we discuss first. Different kinds of algorithms require different representations of the Hamiltonian. Therefore, the library offers to specify the model abstractly by the individual onsite terms and coupling terms of the Hamiltonian. The following example illustrates this, again for the Heisenberg model.

```
"""Definition of a model: the XXZ chain."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3
from tenpy.networks.site import SpinSite
from tenpy.models.lattice import Chain
from tenpy.models.model import CouplingModel, NearestNeighborModel, MPOModel
class XXZChain(CouplingModel, NearestNeighborModel, MPOModel):
   def __init__(self, L=2, S=0.5, J=1., Delta=1., hz=0.):
       spin = SpinSite(S=S, conserve="Sz")
        # the lattice defines the geometry
        lattice = Chain(L, spin, bc="open", bc_MPS="finite")
        CouplingModel.__init__(self, lattice)
        # add terms of the Hamiltonian
        self.add_coupling(J * 0.5, 0, "Sp", 0, "Sm", 1) # Sp_i Sm_{i+1}
        self.add_coupling(J * 0.5, 0, "Sp", 0, "Sm", -1) # Sp_i Sm_{i-1}
        self.add_coupling(J * Delta, 0, "Sz", 0, "Sz", 1)
        # (for site dependent prefactors, the strength can be an array)
        self.add_onsite(-hz, 0, "Sz")
        # finish initialization
        # generate MPO for DMRG
        MPOModel.__init__(self, lat, self.calc_H_MPO())
        # generate H_bond for TEBD
        NearestNeighborModel.__init__(self, lat, self.calc_H_bond())
```

While this generates the same MPO as in the previous code, this example can easily be adjusted and generalized, for example to a higher dimensional lattice by just specifying a different lattice. Internally, the MPO is generated using a finite state machine picture. This allows not only to translate more complicated Hamiltonians into their corresponding MPOs, but also to automate the mapping from a higher dimensional lattice to the 1D chain along which the MPS winds. Note that this mapping introduces longer-range couplings, so the model can no longer be defined to be a <code>NearestNeighborModel</code> suited for TEBD if another lattice than the <code>Chain</code> is to be used. Of course, many

10.1. Overview 65

commonly studied models are also predefined. For example, the following code initializes the Heisenberg model on a kagome lattice; the spin liquid nature of the ground state of this model is highly debated in the current literature.

```
"""Initialization of the Heisenberg model on a kagome lattice."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3

from tenpy.models.spins import SpinModel

model_params = {
    "S": 0.5, # Spin 1/2
    "lattice": "Kagome",
    "bc_MPS": "infinite",
    "bc_y": "cylinder",
    "Ly": 2, # defines cylinder circumference
    "conserve": "Sz", # use Sz conservation
    "Jx": 1.,
    "Jy": 1.,
    "Jz": 1. # Heisenberg coupling
}
model = SpinModel (model_params)
```

Algorithms

Another layer is given by algorithms like DMRG and TEBD. Using the previous concepts, setting up a simulation running those algorithms is a matter of just a few lines of code. The following example runs a DMRG simulation, see <code>dmrg</code>, exemplary for the transverse field Ising model at the critical point.

```
"""Call of (finite) DMRG."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3

from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import dmrg

N = 16  # number of sites
model = TFIChain({"L": N, "J": 1., "g": 1., "bc_MPS": "finite"})
sites = model.lat.mps_sites()
psi = MPS.from_product_state(sites, ['up'] * N, "finite")
dmrg_params = {"trunc_params": {"chi_max": 100, "svd_min": 1.e-10}, "mixer": True}
info = dmrg.run(psi, model, dmrg_params)
print("E =", info['E'])
# E = -20.01638790048513
print("max. bond dimension = ", max(psi.chi))
# max. bond dimension = 27
```

The switch from DMRG to iDMRG in TeNPy is simply accomplished by a change of the parameter "bc_MPS" from "finite" to "infinite", both for the model and the state. The returned E is then the energy density per site. Due to the translation invariance, one can also evaluate the correlation length, here slightly away from the critical point.

```
"""Call of infinite DMRG."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3

from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import dmrg
```

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```
N = 2 # number of sites in unit cell
model = TFIChain({"L": N, "J": 1., "g": 1.1, "bc_MPS": "infinite"})
sites = model.lat.mps_sites()
psi = MPS.from_product_state(sites, ['up'] * N, "infinite")
dmrg_params = {"trunc_params": {"chi_max": 100, "svd_min": 1.e-10}, "mixer": True}
info = dmrg.run(psi, model, dmrg_params)
print("E = ", info['E'])
# E = -1.342864022725017
print("max. bond dimension = ", max(psi.chi))
# max. bond dimension = 56
print("corr. length = ", psi.correlation_length())
# corr. length = 4.915809146764157
```

Running time evolution with TEBD requires an additional loop, during which the desired observables have to be measured. The following code shows this directly for the infinite version of TEBD.

```
"""Call of (infinite) TEBD."""
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3
from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import tebd
M = TFIChain({"L": 2, "J": 1., "g": 1.5, "bc_MPS": "infinite"})
psi = MPS.from_product_state(M.lat.mps_sites(), [0] * 2, "infinite")
tebd_params = {
    "order": 2,
    "delta_tau_list": [0.1, 0.001, 1.e-5],
    "max_error_E": 1.e-6,
    "trunc_params": {
        "chi_max": 30,
        "svd_min": 1.e-10
    }
eng = tebd.TEBDEngine(psi, M, tebd_params)
eng.run_GS() # imaginary time evolution with TEBD
print("E =", sum(psi.expectation_value(M.H_bond)) / psi.L)
print("final bond dimensions: ", psi.chi)
```

Simulations

The top-most layer is given by Simulations. A simulation wraps the whole setup of initializing the Model, MPS and Algorithm classes, running the algorithm, possibly performing measurements, and finally saving results to disk, if desired. It provides some extra functionality like the ability to resume an interrupted simulation, e.g. if your job got killed on the cluster due to runtime limitis.

Ideally, the Simulation (sub) class represents the whole Simulation from start to end, giving re-producable results depending only on the parameters given to it.

10.1. Overview 67

10.2 Charge conservation with np_conserved

The basic idea is quickly summarized: By inspecting the Hamiltonian, you can identify symmetries, which correspond to conserved quantities, called **charges**. These charges divide the tensors into different sectors. This can be used to infer for example a block-diagonal structure of certain matrices, which in turn speeds up SVD or diagonalization a lot. Even for more general (non-square-matrix) tensors, charge conservation imposes restrictions which blocks of a tensor can be non-zero. Only those blocks need to be saved, which ultimately (= for large enough arrays) leads to a speedup of many routines, e.g., tensordot.

This introduction covers our implementation of charges; explaining mathematical details of the underlying symmetry is beyond its scope. We refer you to the corresponding chapter in our [TeNPyNotes] for a more general introduction of the idea (also stating the "charge rule" introduced below). [[singh2010]] explains why it works form a mathematical point of view, [[singh2011]] has the focus on a U(1) symmetry and might be easier to read.

10.2.1 What you really need to know about np_conserved

The good news is: It is not necessary to understand all the details explained in the following sections if you just want to use TeNPy for "standard" simulations like TEBD and DMRG. In praxis, **you will likely not have to define the charges by yourself**. For most simulations using TeNPy, the charges are initially defined in the Site; and there are many pre-defined sites like the :class:SpinHalfSite, which you can just use. The sites in turn are initialized by the Model class you are using (see also Models). From there, all the necessary charge information is automatically propagated along with the tensors.

However, you should definitely know a few basic facts about the usage of charge conservation in TeNPy:

- Instead of using numpy arrays, tensors are represented by the Array class. This class is defined in np_conserved (the name standing for "numpy with charge conservation"). Internally, it stores only non-zero blocks of the tensor, which are "compatible" with the charges of the indices. It has to have a well defined overall charge qtotal. This expludes certain operators (like S^x for Sz conservation) and MPS which are a superpositions of states in different charge sectors.
- There is a class *ChargeInfo* holding the general information what kind of charges we have, and a *LegCharge* for the charge data on a given leg. The leg holds a flag *qconj* which is +1 or -1, depending on whether the leg goes into the tensor (representing a vector space) or out of the tensor (representing the corresponding dual vector space).
- Besides the array class methods, there are a bunch of functions like tensordot(), svd() or eigh() to manipulate tensors. These function have a very similar call structure as the corresponding numpy functions, but they act on our tensor Array class, and preserve the block structure (and exploit it for speed, wherever possible).
- The only allowed "reshaping" operations for those tensors are to combine legs and to split previously combined legs. See the correspoding *section below*.
- It is convenient to use string labels instead of numbers to refer to the various legs of a tensor. The rules how these labels change during the various operations are also described a *section below*.

10.2.2 Introduction to combine_legs, split_legs and LegPipes

Often, it is necessary to "combine" multiple legs into one: for example to perfom a SVD, a tensor needs to be viewed as a matrix. For a flat array, this can be done with np.reshape, e.g., if A has shape (10, 3, 7) then B = np. reshape (A, (30, 7)) will result in a (view of the) array with one less dimension, but a "larger" first leg. By default (order='C'), this results in

```
B[i*3 + j, k] == A[i, j, k] for i in range(10) for j in range(3) for k in range(7)
```

While for a np.array, also a reshaping $(10, 3, 7) \rightarrow (2, 21, 5)$ would be allowed, it does not make sense physically. The only sensible "reshape" operation on an Array are

- 1) to **combine** multiple legs into one **leg pipe** (LegPipe) with combine_legs(), or
- 2) to **split** a pipe of previously combined legs with $split_legs()$.

Each leg has a Hilbert space, and a representation of the symmetry on that Hilbert space. Combining legs corresponds to the tensor product operation, and for abelian groups, the corresponding "fusion" of the representation is the simple addition of charge.

Fusion is not a lossless process, so if we ever want to split the combined leg, we need some additional data to tell us how to reverse the tensor product. This data is saved in the class <code>LegPipe</code>, derived from the <code>LegCharge</code> and used as new <code>leg</code>. Details of the information contained in a LegPipe are given in the class doc string.

The rough usage idea is as follows:

- 1) You can call <code>combine_legs()</code> without supplying any LegPipes, <code>combine_legs</code> will then make them for you. Nevertheless, if you plan to perform the combination over and over again on sets of legs you know to be identical [with same charges etc, up to an overall -1 in <code>qconj</code> on all incoming and outgoing Legs] you might make a LegPipe anyway to save on the overhead of computing it each time.
- 2) In any way, the resulting Array will have a LegPipe as a LegCharge on the combined leg. Thus, it and all tensors inheriting the leg (e.g. the results of svd, tensordot etc.) will have the information how to split the LegPipe back to the original legs.
- 3) Once you performed the necessary operations, you can call split_legs(). This uses the information saved in the *LegPipe* to split the legs, recovering the original legs.

For a LegPipe, <code>conj()</code> changes <code>qconj</code> for the outgoing pipe <code>and</code> the incoming legs. If you need a <code>LegPipe</code> with the same incoming <code>qconj</code>, use <code>outer_conj()</code>.

10.2.3 Leg labeling

It's convenient to name the legs of a tensor: for instance, we can name legs 0, 1, 2 to be 'a', 'b', 'c': T_{i_a,i_b,i_c} . That way we don't have to remember the ordering! Under tensordot, we can then call

```
U = npc.tensordot(S, T, axes = [ [...], ['b'] ] )
```

without having to remember where exactly 'b' is. Obviously U should then inherit the name of its legs from the uncontracted legs of S and T. So here is how it works:

- Labels can *only* be strings. The labels should not include the characters . or ?. Internally, the labels are stored as dict a.labels = {label: leg_position, ...}. Not all legs need a label.
- To set the labels, call

```
A.set_labels(['a', 'b', None, 'c', ... ])
```

which will set up the labeling { 'a': 0, 'b': 1, 'c': 3 ...}.

- (Where implemented) the specification of axes can use either the labels **or** the index positions. For instance, the call tensordot (A, B, [['a', 2, 'c'], [...]]) will interpret 'a' and 'c' as labels (calling get_leg_indices() to find their positions using the dict) and 2 as 'the 2nd leg'. That's why we require labels to be strings!
- Labels will be intelligently inherited through the various operations of np_conserved.
 - Under *transpose*, labels are permuted.
 - Under *tensordot*, labels are inherited from uncontracted legs. If there is a collision, both labels are dropped.
 - Under combine_legs, labels get concatenated with a . delimiter and sourrounded by brackets. Example: let a.labels = {'a': 1, 'b': 2, 'c': 3}. Then if b = a. combine_legs([[0, 1], [2]]), it will have b.labels = {'(a.b)': 0, '(c)': 1}. If some sub-leg of a combined leg isn't named, then a '?#' label is inserted (with # the leg index), e.g., 'a.?0.c'.
 - Under *split_legs*, the labels are split using the delimiters (and the '?#' are dropped).
 - Under conj, iconj: take 'a' -> 'a*', 'a*' -> 'a', and '(a.(b*.c))' -> '(a*.(b.c*))'
 - Under *svd*, the outer labels are inherited, and inner labels can be optionally passed.
 - Under *pinv*, the labels are transposed.

10.2.4 Indexing of an Array

Although it is usually not necessary to access single entries of an Array, you can of course do that. In the simplest case, this is something like A[0, 2, 1] for a rank-3 Array A. However, accessing single entries is quite slow and usually not recommended. For small Arrays, it may be convenient to convert them back to flat numpy arrays with to_ndarray().

On top of that very basic indexing, *Array* supports slicing and some kind of advanced indexing, which is however different from the one of numpy arrarys (described here). Unlike numpy arrays, our Array class does not broadcast existing index arrays – this would be terribly slow. Also, *np.newaxis* is not supported, since inserting new axes requires additional information for the charges.

Instead, we allow just indexing of the legs independent of each other, of the form A [i0, i1, ...]. If all indices i0, i1, ... are integers, the single corresponding entry (of type *dtype*) is returned.

However, the individual 'indices' i0 for the individual legs can also be one of what is described in the following list. In that case, a new Array with less data (specified by the indices) is returned.

The 'indices' can be:

- an *int*: fix the index of that axis, return array with one less dimension. See also take_slice().
- a slice (None) or :: keep the complete axis
- an Ellipsis or . . .: shorthand for slice (None) for missing axes to fix the len
- an 1D bool *ndarray* mask: apply a mask to that axis, see *iproject()*.
- a slice(start, stop, step) or start:stop:step: keep only the indices specified by the slice. This is also implemented with *iproject*.
- an 1D int *ndarray* mask: keep only the indices specified by the array. This is also implemented with *iproject*.

For slices and 1D arrays, additional permuations may be perforned with the help of permute().

If the number of indices is less than rank, the remaining axes remain free, so for a rank 4 Array A, A[i0, i1] == A[i0, i1, ...] == A[i0, i1, :, :].

Note that indexing always **copies** the data – even if *int* contains just slices, in which case numpy would return a view. However, assigning with A[:, [3, 5], 3] = B should work as you would expect.

Warning: Due to numpy's advanced indexing, for 1D integer arrays a0 and a1 the following holds

```
A[a0, a1].to_ndarray() == A.to_ndarray()[np.ix_(a0, a1)] != A.to_ndarray()[a0, a1]
```

For a combination of slices and arrays, things get more complicated with numpys advanced indexing. In that case, a simple $np.ix_(...)$ doesn't help any more to emulate our version of indexing.

10.2.5 Details of the *np_conserved* implementation

Notations

Lets fix the notation of certain terms for this introduction and the doc-strings in np_conserved. This might be helpful if you know the basics from a different context. If you're new to the subject, keep reading even if you don't understand each detail, and come back to this section when you encounter the corresponding terms again.

A Array is a multi-dimensional array representing a **tensor** with the entries:

$$T_{a_0,a_1,...a_{rank-1}}$$
 with $a_i \in \{0,...,n_i-1\}$

Each $leg a_i$ corresponds the a vector space of dimension n_i .

An **index** of a leg is a particular value $a_i \in \{0, ..., n_i - 1\}$.

The **rank** is the number of legs, the **shape** is $(n_0, ..., n_{rank-1})$.

We restrict ourselfes to abelian charges with entries in \mathbb{Z} or in \mathbb{Z}_m . The nature of a charge is specified by m; we set m=1 for charges corresponding to \mathbb{Z} . The number of charges is referred to as **qnumber** as a short hand, and the collection of m for each charge is called **qmod**. The qnumber, qmod and possibly descriptive names of the charges are saved in an instance of ChargeInfo.

To each index of each leg, a value of the charge(s) is associated. A **charge block** is a contiguous slice corresponding to the same charge(s) of the leg. A **qindex** is an index in the list of charge blocks for a certain leg. A **charge sector** is for given charge(s) is the set of all qindices of that charge(s). A leg is **blocked** if all charge sectors map one-to-one to qindices. Finally, a leg is **sorted**, if the charges are sorted lexiographically. Note that a *sorted* leg is always *blocked*. We can also speak of the complete array to be **blocked by charges** or **legcharge-sorted**, which means that all of its legs are blocked or sorted, respectively. The charge data for a single leg is collected in the class *LegCharge*. A *LegCharge* has also a flag **qconj**, which tells whether the charges point *inward* (+1) or *outward* (-1). What that means, is explained later in *Which entries of the npc Array can be non-zero?*.

For completeness, let us also summarize also the internal structure of an <code>Array</code> here: The array saves only non-zero **blocks**, collected as a list of <code>np.array</code> in <code>self._data</code>. The qindices necessary to map these blocks to the original leg indices are collected in <code>self._qdata</code> An array is said to be **qdata-sorted** if its <code>self._qdata</code> is lexiographically sorted. More details on this follow <code>later</code>. However, note that you usually shouldn't access <code>_qdata</code> and <code>_data</code> directly this is only necessary from within <code>tensordot</code>, <code>svd</code>, etc. Also, an array has a **total charge**, defining which entries can be non-zero - details in <code>Which entries of the npc Array can be non-zero?</code>.

Finally, a **leg pipe** (implemented in LegPipe) is used to formally combine multiple legs into one leg. Again, more details follow *later*.

Physical Example

For concreteness, you can think of the Hamiltonian $H=-t\sum_{< i,j>}(c_i^{\dagger}c_j+H.c.)+Un_in_j$ with $n_i=c_i^{\dagger}c_i$. This Hamiltonian has the global U(1) gauge symmetry $c_i\to c_ie^{i\phi}$. The corresponding charge is the total number of particles $N=\sum_i n_i$. You would then introduce one charge with m=1.

Note that the total charge is a sum of local terms, living on single sites. Thus, you can infer the charge of a single physical site: it's just the value $q_i = n_i \in \mathbb{N}$ for each of the states.

Note that you can only assign integer charges. Consider for example the spin 1/2 Heisenberg chain. Here, you can naturally identify the magnetization $S^z = \sum_i S_i^z$ as the conserved quantity, with values $S_i^z = \pm \frac{1}{2}$. Obviously, if S^z is conserved, then so is $2S^z$, so you can use the charges $q_i = 2S_i^z \in \{-1, +1\}$ for the *down* and *up* states, respectively. Alternatively, you can also use a shift and define $q_i = S_i^z + \frac{1}{2} \in \{0, 1\}$.

As another example, consider BCS like terms $\sum_k (c_k^{\dagger} c_{-k}^{\dagger} + H.c.)$. These terms break the total particle conservation, but they preserve the total parity, i.e., $N \mod 2$ is conserved. Thus, you would introduce a charge with m=2 in this case.

In the above examples, we had only a single charge conserved at a time, but you might be lucky and have multiple conserved quantities, e.g. if you have two chains coupled only by interactions. TeNPy is designed to handle the general case of multiple charges. When giving examples, we will restrict to one charge, but everything generalizes to multiple charges.

The different formats for LegCharge

As mentioned above, we assign charges to each index of each leg of a tensor. This can be done in three formats: **qflat**, as **qind** and as **qdict**. Let me explain them with examples, for simplicity considering only a single charge (the most inner array has one entry for each charge).

qflat form: simply a list of charges for each index. An example:

```
qflat = [[-2], [-1], [-1], [0], [0], [0], [3], [3]]
```

This tells you that the leg has size 9, the charges for are [-2], [-1], [-1], ..., [3] for the indices 0, 1, 2, 3,..., 8. You can identify four *charge blocks* slice (0, 1), slice (1, 3), slice (3, 7), slice (7, 9) in this example, which have charges [-2], [-1], [0], [3]. In other words, the indices 1, 2 (which are in slice (1, 3)) have the same charge value [-1]. A *qindex* would just enumerate these blocks as 0, 1, 2, 3.

qind form: a 1D array *slices* **and a 2D array** *charges*. This is a more compact version than the *qflat* form: the *slices* give a partition of the indices and the *charges* give the charge values. The same example as above would simply be:

```
slices = [0, 1, 3, 7, 9]
charges = [[-2], [-1], [0], [3]]
```

Note that *slices* includes 0 as first entry and the number of indices (here 9) as last entries. Thus it has len block_number + 1, where block_number (given by block_number) is the number of charge blocks in the leg, i.e. a *qindex* runs from 0 to block_number-1. On the other hand, the 2D array *charges* has shape (block_number, qnumber), where qnumber is the number of charges (given by *qnumber*).

In that way, the qind form maps an qindex, say qi, to the indices slice (slices [qi], slices [qi+1]) and the charge(s) charges [qi].

qdict form: a dictionary in the other direction than qind, taking charge tuples to slices. Again for the same example:

```
{(-2,): slice(0, 1),
(-1,): slice(1, 3),
(0,): slice(3, 7),
(3,): slice(7, 9)}
```

Since the keys of a dictionary are unique, this form is only possible if the leg is *completely blocked*.

The LegCharge saves the charge data of a leg internally in *qind* form, directly in the attribute *slices* and *charges*. However, it also provides convenient functions for conversion between from and to the *qflat* and *qdict* form.

The above example was nice since all charges were sorted and the charge blocks were 'as large as possible'. This is however not required.

The following example is also a valid *qind* form:

```
slices = [0, 1, 3, 5, 7, 9]
charges = [[-2], [-1], [0], [3]]
```

This leads to the *same qflat* form as the above examples, thus representing the same charges on the leg indices. However, regarding our Arrays, this is quite different, since it diveds the leg into 5 (instead of previously 4) charge blocks. We say the latter example is *not bunched*, while the former one is *bunched*.

To make the different notions of *sorted* and *bunched* clearer, consider the following (valid) examples:

charges					bunched	sorted	blocked
[[-2],	[-1],	[0],	[1],	[3]]	True	True	True
[[-2],	[-1],	[0],	[0],	[3]]	False	True	False
[[-2],	[0],	[-1],	[1],	[3]]	True	False	True
[[-2],	[0],	[-1],	[0],	[3]]	True	False	False

If a leg is *bunched* and *sorted*, it is automatically *blocked* (but not vice versa). See also *below* for further comments on that.

Which entries of the npc Array can be non-zero?

The reason for the speedup with np_conserved lies in the fact that it saves only the blocks 'compatible' with the charges. But how is this 'compatible' defined?

Assume you have a tensor, call it T, and the LegCharge for all of its legs, say a, b, c, \dots

Remeber that the LegCharge associates to each index of the leg a charge value (for each of the charges, if *qnumber* > 1). Let a.to_qflat() [ia] denote the charge(s) of index ia for leg a, and similar for other legs.

In addition, the LegCharge has a flag qconj. This flag **qconj** is only a sign, saved as +1 or -1, specifying whether the charges point 'inward' (+1, default) or 'outward' (-1) of the tensor.

Then, the total charge of an entry T[ia, ib, ic, ...] of the tensor is defined as:

The rule which entries of the a Array can be non-zero (i.e., are 'compatible' with the charges), is then very simple:

Rule for non-zero entries

An entry ia, ib, ic, ... of a *Array* can only be non-zero, if qtotal[ia, ib, ic, ...] matches the *unique* qtotal attribute of the class.

In other words, there is a *single* **total charge** .qtotal attribute of a *Array*. All indices ia, ib, ic, ... for which the above defined qtotal[ia, ib, ic, ...] matches this *total charge*, are said to be **compatible with the charges** and can be non-zero. All other indices are **incompatible with the charges** and must be zero.

In case of multiple charges, qnumber > 1, is a straigth-forward generalization: an entry can only be non-zero if it is compatible with each of the defined charges.

The pesky qconj - contraction as an example

Why did we introduce the <code>qconj</code> flag? Remember it's just a sign telling whether the charge points inward or outward. So whats the reasoning?

The short answer is, that LegCharges actually live on bonds (i.e., legs which are to be contracted) rather than individual tensors. Thus, it is convenient to share the LegCharges between different legs and even tensors, and just adjust the sign of the charges with *qconj*.

As an example, consider the contraction of two tensors, $C_{ia,ic} = \sum_{ib} A_{ia,ib} B_{ib,ic}$. For simplicity, say that the total charge of all three tensors is zero. What are the implications of the above rule for non-zero entries? Or rather, how can we ensure that C complies with the above rule? An entry C [ia, ic] will only be non-zero, if there is an ib such that both A [ia, ib] and B [ib, ic] are non-zero, i.e., both of the following equations are fullfilled:

```
A.qtotal == A.legs[0].to_qflat()[ia] * A.legs[0].qconj + A.legs[1].to_qflat()[ib] * A. \( \to \) legs[1].qconj modulo qmod

B.qtotal == B.legs[0].to_qflat()[ib] * B.legs[0].qconj + B.legs[1].to_qflat()[ic] * B. \( \to \) legs[1].qconj modulo qmod
```

(A.legs[0] is the LegCharge saving the charges of the first leg (with index ia) of A.)

For the uncontracted legs, we just keep the charges as they are:

```
C.legs = [A.legs[0], B.legs[1]]
```

It is then straight-forward to check, that the rule is fullfilled for C, if the following condition is met:

```
A.qtotal + B.qtotal - C.qtotal == A.legs[1].to_qflat()[ib] A.b.qconj + B.legs[0].to_

qflat()[ib] B.b.qconj modulo qmod
```

The easiest way to meet this condition is (1) to require that A.b and B.b share the *same* charges b.to_qflat(), but have opposite *qconj*, and (2) to define C.qtotal = A.qtotal + B.qtotal. This justifies the introduction of *qconj*: when you define the tensors, you have to define the *LegCharge* for the *b* only once, say for A.legs[1]. For B.legs[0] you simply use A.legs[1].conj() which creates a copy of the LegCharge with shared *slices* and *charges*, but opposite *qconj*. As a more impressive example, all 'physical' legs of an MPS can usually share the same *LegCharge* (up to different qconj if the local Hilbert space is the same). This leads to the following convention:

Convention

When an npc algorithm makes tensors which share a bond (either with the input tensors, as for tensordot, or amongst the output tensors, as for SVD), the algorithm is free, but not required, to use the **same** LegCharge for the tensors

sharing the bond, *without* making a copy. Thus, if you want to modify a LegCharge, you **must** make a copy first (e.g. by using methods of LegCharge for what you want to acchive).

Assigning charges to non-physical legs

From the above physical examples, it should be clear how you assign charges to physical legs. But what about other legs, e.g, the virtual bond of an MPS (or an MPO)?

The charge of these bonds must be derived by using the 'rule for non-zero entries', as far as they are not arbitrary. As a concrete example, consider an MPS on just two spin 1/2 sites:

The two legs p are the physical legs and share the same charge, as they both describe the same local Hilbert space. For better distinction, let me label the indices of them by $\uparrow = 0$ and $\downarrow = 1$. As noted above, we can associate the charges 1 $(p = \uparrow)$ and -1 $(p = \downarrow)$, respectively, so we define:

```
chinfo = npc.ChargeInfo([1], ['2*Sz'])
p = npc.LegCharge.from_qflat(chinfo, [1, -1], qconj=+1)
```

For the qconj signs, we stick to the convention used in our MPS code and indicated by the arrows in above 'picture': physical legs are incoming (qconj=+1), and from left to right on the virtual bonds. This is acchieved by using [p, x, y.conj()] as legs for A, and [p, y, z.conj()] for B, with the default qconj=+1 for all p, x, y, z: y.conj() has the same charges as y, but opposite qconj=-1.

The legs x and z of an L=2 MPS, are 'dummy' legs with just one index 0. The charge on one of them, as well as the total charge of both A and B is arbitrary (i.e., a gauge freedom), so we make a simple choice: total charge 0 on both arrays, as well as for x = 0, $x = npc.LegCharge.from_qflat(chinfo, [0], qconj=+1)$.

The charges on the bonds y and z then depend on the state the MPS represents. Here, we consider a singlet $\psi = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ as a simple example. A possible MPS representation is given by:

```
A[up, :, :] = [[1/2.**0.5, 0]] B[up, :, :] = [[0], [-1]] A[down, :, :] = [[0, 1/2.**0.5]] B[down, :, :] = [[1], [0]]
```

There are two non-zero entries in A, for the indices $(a, x, y) = (\uparrow, 0, 0)$ and $(\downarrow, 0, 1)$. For $(a, x, y) = (\uparrow, 0, 0)$, we want:

This fixes the charge of y=0 to 1. A similar calculation for $(a,x,y)=(\downarrow,0,1)$ yields the charge -1 for y=1. We have thus all the charges of the leg y and can define y=npc.LegCharge.from_qflat(chinfo, [1, -1], qconj=+1).

Now take a look at the entries of B. For the non-zero entry $(b, y, z) = (\uparrow, 1, 0)$, we want:

This implies the charge 0 for z=0, thus $z=\text{npc.LegCharge.form_qflat}$ (chinfo, [0], qconj=+1). Finally, note that the rule for $(b,y,z)=(\downarrow,0,0)$ is automatically fullfilled! This is an implication of the fact that the singlet has a well defined value for $S^z_a+S^z_b$. For other states without fixed magnetization (e.g., $|\uparrow\uparrow\rangle+|\downarrow\downarrow\rangle$) this would not be the case, and we could not use charge conservation.

As an exercise, you can calculate the charge of z in the case that A.qtotal=5, B.qtotal = -1 and charge 2 for x=0. The result is -2.

Note: This section is meant be an pedagogical introduction. In you program, you can use the functions $detect_legcharge()$ (which does exactly what's described above) or $detect_qtotal()$ (if you know all LegCharges, but not qtotal).

Array creation

Direct creation

Making an new Array requires both the tensor entries (data) and charge data.

The default initialization a = Array(...) creates an empty Array, where all entries are zero (equivalent to zeros()). (Non-zero) data can be provided either as a dense *np.array* to *from_ndarray()*, or by providing a numpy function such as *np.random*, *np.ones* etc. to *from_func()*.

In both cases, the charge data is provided by one Charge Info, and a LegCharge instance for each of the legs.

Note: The charge data instances are not copied, in order to allow it to be shared between different Arrays. Consequently, you *must* make copies of the charge data, if you manipulate it directly. (However, methods like *sort* () do that for you.)

Indirect creation by manipulating existing arrays

Of course, a new Array can also created using the charge data from exisiting Arrays, for example with zeros_like() or creating a (deep or shallow) copy(). Further, there are many higher level functions like tensordot() or svd(), which also return new Arrays.

Complete blocking of Charges

While the code was designed in such a way that each charge sector has a different charge, the code should still run correctly if multiple charge sectors (for different qindex) correspond to the same charge. In this sense <code>Array</code> can act like a sparse array class to selectively store subblocks. Algorithms which need a full blocking should state that explicitly in their doc-strings. (Some functions (like <code>svd</code> and <code>eigh</code>) require complete blocking internally, but if necessary they just work on a temporary copy returned by <code>as_completely_blocked()</code>).

If you expect the tensor to be dense subject to charge constraints (as for MPS), it will be most efficient to fully block by charge, so that work is done on large chunks.

However, if you expect the tensor to be sparser than required by charge (as for an MPO), it may be convenient not to completely block, which forces smaller matrices to be stored, and hence many zeroes to be dropped. Nevertheless,

the algorithms were not designed with this in mind, so it is not recommended in general. (If you want to use it, run a benchmark to check whether it is really faster!)

If you haven't created the array yet, you can call <code>sort()</code> (with bunch=True) on each <code>LegCharge</code> which you want to block. This sorts by charges and thus induces a permution of the indices, which is also returned as an 1D array perm. For consistency, you have to apply this permutation to your flat data as well.

Alternatively, you can simply call <code>sort_legcharge()</code> on an existing <code>Array</code>. It calls <code>sort()</code> internally on the specified legs and performs the necessary permutations directly to (a copy of) <code>self</code>. Yet, you should keep in mind, that the axes are permuted afterwards.

Internal Storage schema of npc Arrays

The actual data of the tensor is stored in _data. Rather than keeping a single np.array (which would have many zeros in it), we store only the non-zero sub blocks. So _data is a python list of *np.array*'s. The order in which they are stored in the list is not physically meaningful, and so not guaranteed (more on this later). So to figure out where the sub block sits in the tensor, we need the _qdata structure (on top of the LegCharges in legs).

Consider a rank 3 tensor T, with the first leg like:

```
legs[0].slices = np.array([0, 1, 4, ...])
legs[0].charges = np.array([[-2], [1], ...])
```

Each row of *charges* gives the charges for a *charge block* of the leg, with the actual indices of the total tensor determined by the *slices*. The *qindex* simply enumerates the charge blocks of a lex. Picking a qindex (and thus a *charge block*) from each leg, we have a subblock of the tensor.

For each (non-zero) subblock of the tensor, we put a (numpy) ndarray entry in the _data list. Since each subblock of the tensor is specified by *rank* qindices, we put a corresponding entry in _qdata, which is a 2D array of shape (#stored_blocks, rank). Each row corresponds to a non-zero subblock, and there are rank columns giving the corresponding qindex for each leg.

Example: for a rank 3 tensor we might have:

The third subblock has an ndarray t3, and qindices [4 2 2] for the three legs.

• To find the position of t3 in the actual tensor you can use get_slice():

```
T.legs[0].get_slice(4), T.legs[1].get_slice(2), T.legs[2].get_slice(2)
```

The function leg.get_charges(qi) simply returns slice(leg.slices[qi], leg.slices[qi+1])

• To find the charges of t3, we an use get_charge():

```
T.legs[0].get_charge(2), T.legs[1].get_charge(2), T.legs[2].get_charge(2)
```

The function leg.get_charge(qi) simply returns leg.charges[qi] *leg.qconj.

Note: Outside of $np_conserved$, you should use the API to access the entries. If you really need to iterate over all blocks of an Array T, try for (block, blockslices, charges, qindices) in T:

```
do_something().
```

The order in which the blocks stored in _data/_qdata is arbitrary (although of course _data and _qdata must be in correspondence). However, for many purposes it is useful to sort them according to some convention. So we include a flag ._qdata_sorted to the array. So, if sorted (with <code>isort_qdata()</code>, the _qdata example above goes to

Note that *np.lexsort* chooses the right-most column to be the dominant key, a convention we follow throughout.

If _qdata_sorted == True, _qdata and _data are guaranteed to be lexsorted. If _qdata_sorted == False, there is no gaurantee. If an algorithm modifies _qdata, it must set _qdata_sorted = False (unless it gaurantees it is still sorted). The routine sort_qdata() brings the data to sorted form.

10.2.6 See also

- The module <code>tenpy.linalg.np_conserved</code> should contain all the API needed from the point of view of the algorithms. It contians the fundamental <code>Array</code> class and functions for working with them (creating and manipulating).
- The module tenpy.linalg.charges contains implementations for the charge structure, for example the classes ChargeInfo, LegCharge, and LegPipe. As noted above, the 'public' API is imported to (and accessible from) np_conserved.

10.2.7 A full example code for spin-1/2

Below follows a full example demonstrating the creation and contraction of Arrays. (It's the file $a_np_conserved.py$ in the examples folder of the tenpy source.)

```
"""An example code to demonstrate the usage of :class:`~tenpy.linalg.np_conserved.
→Array`.
This example includes the following steps:
1) create Arrays for an Neel MPS
2) create an MPO representing the nearest-neighbour AFM Heisenberg Hamiltonian
3) define 'environments' left and right
4) contract MPS and MPO to calculate the energy
5) extract two-site hamiltonian ``H2`` from the MPO
6) calculate ``exp(-1.j*dt*H2)`` by diagonalization of H2
7) apply ``exp(H2)`` to two sites of the MPS and truncate with svd
Note that this example uses only np_conserved, but no other modules.
Compare it to the example `b_mps.py`,
which does the same steps using a few predefined classes like MPS and MPO.
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
import numpy as np
```

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```
# model parameters
Jxx, Jz = 1., 1.
L = 20
dt = 0.1
cutoff = 1.e-10
print("Jxx=\{Jxx\}, Jz=\{Jz\}, L=\{L:d\}".format(Jxx=Jxx, Jz=Jz, L=L))
print("1) create Arrays for an Neel MPS")
  vL ->--B-->- vR
          /
#
#
          p
# create a ChargeInfo to specify the nature of the charge
chinfo = npc.ChargeInfo([1], ['2*Sz']) # the second argument is just a descriptive,
→ name
# create LegCharges on physical leg and even/odd bonds
p_leg = npc.LegCharge.from_qflat(chinfo, [[1], [-1]]) # charges for up, down
v_leg_even = npc.LegCharge.from_qflat(chinfo, [[0]])
v_leg_odd = npc.LegCharge.from_qflat(chinfo, [[1]])
B_even = npc.zeros([v_leg_even, v_leg_odd.conj(), p_leg],
                   labels=['vL', 'vR', 'p']) # virtual left/right, physical
B_odd = npc.zeros([v_leg_odd, v_leg_even.conj(), p_leg], labels=['vL', 'vR', 'p'])
B_{even}[0, 0, 0] = 1. # up
B_odd[0, 0, 1] = 1. \# down
Bs = [B\_even, B\_odd] * (L // 2) + [B\_even] * (L % 2) # (right-canonical)
Ss = [np.ones(1)] * L # Ss[i] are singular values between <math>Bs[i-1] and Bs[i]
# Side remark:
# An MPS is expected to have non-zero entries everywhere compatible with the charges.
# In general, we recommend to use `sort_legcharge` (or `as_completely_blocked`)
# to ensure complete blocking. (But the code will also work, if you don't do it.)
# The drawback is that this might introduce permutations in the indices of single.
# which you have to keep in mind when converting dense numpy arrays to and from npc.
→Arrays.
print("2) create an MPO representing the AFM Heisenberg Hamiltonian")
          p*
#
          1
#
  wT, ->--W-->- wR
          1
#
#
          /
         p
# create physical spin-1/2 operators Sz, S+, S-
Sz = npc.Array.from_ndarray([[0.5, 0.], [0., -0.5]], [p_leg, p_leg.conj()], labels=['p
                                                                          (continues on next page)
→', 'p*'])
```

```
Sp = npc.Array.from_ndarray([[0., 1.], [0., 0.]], [p_leg, p_leg.conj()], labels=['p',
Sm = npc.Array.from_ndarray([[0., 0.], [1., 0.]], [p_leg, p_leg.conj()], labels=['p',

  'p*'l)

Id = npc.eye_like(Sz, labels=Sz.get_leg_labels()) # identity
mpo_leg = npc.LegCharge.from_qflat(chinfo, [[0], [2], [-2], [0], [0]])
W_{grid} = [Id,
                             Sz,
                                  None
                Sp,
                      Sm.
          [None, None, None, None, 0.5 * Jxx * Sm],
          [None, None, None, None, 0.5 * Jxx * Sp],
          [None, None, None, Jz * Sz
                                               1,
          [None, None, None, Id
                                                 ]]
                                                    # yapf:disable
W = npc.grid_outer(W_grid, [mpo_leg, mpo_leg.conj()], grid_labels=['wL', 'wR'])
# wL/wR = virtual left/right of the MPO
Ws = [W] \star L
print("3) define 'environments' left and right")
  .--->- vR
                 vT, ->---.
# /
\# envL->- wR
                 WI, ->-envR
# /
# .--->- VR*
                 VT, *->----
envL = npc.zeros([W.qet_leg('wL').conj(), Bs[0].qet_leg('vL').conj(), Bs[0].qet_leg(
\hookrightarrow '\veeL')],
                labels=['wR', 'vR', 'vR*'])
envL[0, :, :] = npc.diag(1., envL.legs[1])
envR = npc.zeros([W.get_leg('wR').conj(), Bs[-1].get_leg('vR').conj(), Bs[-1].get_leg(
\hookrightarrow 'VR')],
                labels=['wL', 'vL', 'vL*'])
envR[-1, :, :] = npc.diag(1., envR.legs[1])
print("4) contract MPS and MPO to calculate the energy <psi|H|psi>")
contr = envL
for i in range(L):
   # contr labels: wR, vR, vR*
   contr = npc.tensordot(contr, Bs[i], axes=('vR', 'vL'))
   # wR, vR*, vR, p
   contr = npc.tensordot(contr, Ws[i], axes=(['p', 'wR'], ['p*', 'wL']))
   # vR*, vR, wR, p
   contr = npc.tensordot(contr, Bs[i].conj(), axes=(['p', 'vR*'], ['p*', 'vL*']))
    # vR, wR, vR*
    # note that the order of the legs changed, but that's no problem with labels:
    # the arrays are automatically transposed as necessary
E = npc.inner(contr, envR, axes=(['vR', 'wR', 'vR*'], ['vL', 'wL', 'vL*']))
print("E =", E)
print("5) calculate two-site hamiltonian ``H2`` from the MPO")
# label left, right physical legs with p, q
W0 = W.replace_labels(['p', 'p*'], ['p0', 'p0*'])
W1 = W.replace_labels(['p', 'p*'], ['p1', 'p1*'])
H2 = npc.tensordot(W0, W1, axes=('wR', 'wL')).itranspose(['wL', 'wR', 'p0', 'p1', 'p0*
H2 = H2[0, -1] # (If H has single-site terms, it's not that simple anymore)
```

(continues on next page)

```
print("H2 labels:", H2.get_leg_labels())
print("6) calculate exp(H2) by diagonalization of H2")
# diagonalization requires to view H2 as a matrix
H2 = H2.combine_legs([('p0', 'p1'), ('p0*', 'p1*')], qconj=[+1, -1])
print("labels after combine_legs:", H2.get_leg_labels())
E2, U2 = npc.eigh(H2)
print ("Eigenvalues of H2:", E2)
U_expE2 = U2.scale_axis(np.exp(-1.j * dt * E2), axis=1) # scale_axis ~= apply an_

→ diagonal matrix
exp_H2 = npc.tensordot(U_expE2, U2.conj(), axes=(1, 1))
exp_H2.iset_leg_labels(H2.get_leg_labels())
exp_H2 = exp_H2.split_legs() # by default split all legs which are `LegPipe`
# (this restores the originial labels ['p0', 'p1', 'p0*', 'p1*'] of `H2` in `exp_H2`)
print("7) apply exp(H2) to even/odd bonds of the MPS and truncate with svd")
# (this implements one time step of first order TEBD)
for even_odd in [0, 1]:
    for i in range(even_odd, L - 1, 2):
        B_L = Bs[i].scale_axis(Ss[i], 'vL').ireplace_label('p', 'p0')
        B_R = Bs[i + 1].replace_label('p', 'p1')
        theta = npc.tensordot(B_L, B_R, axes=('vR', 'vL'))
        theta = npc.tensordot(exp_H2, theta, axes=(['p0*', 'p1*'], ['p0', 'p1']))
        # view as matrix for SVD
        theta = theta.combine_legs([('vL', 'p0'), ('p1', 'vR')], new_axes=[0, 1],...
\rightarrowqconj=[+1, -1])
        # now theta has labels '(vL.p0)', '(p1.vR)'
        U, S, V = npc.svd(theta, inner_labels=['vR', 'vL'])
        # truncate
        keep = S > cutoff
        S = S[keep]
        invsq = np.linalg.norm(S)
        Ss[i + 1] = S / invsq
        U = U.iscale_axis(S / invsq, 'vR')
        Bs[i] = U.split_legs('(vL.p0)').iscale_axis(Ss[i]**(-1), 'vL').ireplace_label(
→'p0', 'p')
        Bs[i + 1] = V.split_legs('(p1.vR)').ireplace_label('p1', 'p')
print("finished")
```

10.3 Models

10.3.1 What is a model?

Abstractly, a **model** stands for some physical (quantum) system to be described. For tensor networks algorithms, the model is usually specified as a Hamiltonian written in terms of second quantization. For example, let us consider a spin-1/2 Heisenberg model described by the Hamiltonian

$$H = J \sum_{i} S_{i}^{x} S_{i+1}^{x} + S_{i}^{y} S_{i+1}^{y} + S_{i}^{z} S_{i+1}^{z}$$

Note that a few things are defined more or less implicitly.

- The local Hilbert space: it consists of Spin-1/2 degrees of freedom with the usual spin-1/2 operators S^x , S^y , S^z .
- The geometric (lattice) strucuture: above, we spoke of a 1D "chain".

10.3. Models 81

- The boundary conditions: do we have open or periodic boundary conditions? The "chain" suggests open boundaries, which are in most cases preferable for MPS-based methods.
- The range of *i*: How many sites do we consider (for a 2D system: in each direction)?

Obviously, these things need to be specified in TeNPy in one way or another, if we want to define a model.

Ultimately, our goal is to run some algorithm. However, different algorithm requires the model and Hamiltonian to be specified in different forms. We have one class for each such required form. For example <code>dmrg</code> requires an <code>MPOModel</code>, which contains the Hamiltonian written as an MPO. So a new model class suitable for DMRG should have this general structure:

```
class MyNewModel (MPOModel):
    def __init__(self, model_params):
        lattice = somehow_generate_lattice(model_params)
        H_MPO = somehow_generate_MPO(lattice, model_params)
        # initialize MPOModel
        MPOModel.__init__(self, lattice, H_MPO)
```

On the other hand, if we want to evolve a state with tebd we need a NearestNeighborModel, in which the Hamiltonian is written in terms of two-site bond-terms to allow a Suzuki-Trotter decomposition of the time-evolution operator:

```
class MyNewModel2 (NearestNeighborModel):
    """General strucutre for a model suitable for TEBD."""

def __init__(self, model_params):
    lattice = somehow_generate_lattice(model_params)
    H_bond = somehow_generate_H_bond(lattice, model_params)
    # initialize MPOModel
    NearestNeighborModel.__init__(self, lattice, H_bond)
```

Of course, the difficult part in these examples is to generate the H_MPO and H_bond in the required form. If you want to write it down by hand, you can of course do that. But it can be quite tedious to write every model multiple times, just because we need different representations of the same Hamiltonian. Luckily, there is a way out in TeNPy: the CouplingModel. Before we describe this class, let's discuss the background of the Site and Site class.

10.3.2 The Hilbert space

The **local Hilbert** space is represented by a Site (read its doc-string!). In particular, the Site contains the local LegCharge and hence the meaning of each basis state needs to be defined. Beside that, the site contains the local operators - those give the real meaning to the local basis. Having the local operators in the site is very convenient, because it makes them available by name for example when you want to calculate expectation values. The most common sites (e.g. for spins, spin-less or spin-full fermions, or bosons) are predefined in the module tenpy.networks.site, but if necessary you can easily extend them by adding further local operators or completely write your own subclasses of Site.

The full Hilbert space is a tensor product of the local Hilbert space on each site.

Note: The LegCharge of all involved sites need to have a common ChargeInfo in order to allow the contraction of tensors acting on the various sites. This can be ensured with the function set_common_charges().

An example where $set_common_charges()$ is needed would be a coupling of different types of sites, e.g., when a tight binding chain of fermions is coupled to some local spin degrees of freedom. Another use case of this function would be a model with a \$U(1)\$ symmetry involving only half the sites, say $\sum_{i=0}^{L/2} n_{2i}$.

Note: If you don't know about the charges and $np_conserved$ yet, but want to get started with models right away, you can set conserve=None in the existing sites or use $leg = tenpy.linalg.np_conserved.LegCharge.$ from_trivial(d) for an implementation of your custom site, where d is the dimension of the local Hilbert space. Alternatively, you can find some introduction to the charges in the *Charge conservation with np_conserved*.

10.3.3 The geometry: lattice class

The geometry is usually given by some kind of **lattice** structure how the sites are arranged, e.g. implicitly with the sum over nearest neighbours $\sum_{\langle i,j \rangle}$. In TeNPy, this is specified by a Lattice class, which contains a unit cell of a few Site which are shifted periodically by its basis vectors to form a regular lattice. Again, we have pre-defined some basic lattices like a Chain, two chains coupled as a Ladder or 2D lattices like the Square, Honeycomb and Kagome lattices; but you are also free to define your own generalizations.

MPS based algorithms like DMRG always work on purely 1D systems. Even if our model "lives" on a 2D lattice, these algorithms require to map it onto a 1D chain (probably at the cost of longer-range interactions). This mapping is also done by the lattice by defining the **order** (order) of the sites.

Note: Further details on the lattice geometry can be found in *Details on the lattice geometry*.

10.3.4 The CouplingModel: general structure

The CouplingModel provides a general, quite abstract way to specify a Hamiltonian of couplings on a given lattice. Once initialized, its methods add_onsite() and add_coupling() allow to add onsite and coupling terms repeated over the different unit cells of the lattice. In that way, it basically allows a straight-forward translation of the Hamiltonian given as a math forumla $H = \sum_i A_i B_{i+dx} + ...$ with onsite operators A, B,... into a model class.

The general structure for a new model based on the CouplingModel is then:

In the initialization method __init__ (self, ...) of this class you can then follow these basic steps:

- 0. Read out the parameters.
- 1. Given the parameters, determine the charges to be conserved. Initialize the *LegCharge* of the local sites accordingly.
- 2. Define (additional) local operators needed.
- 3. Initialize the needed *Site*.

Note: Using pre-defined sites like the SpinHalfSite is recommended and can replace steps 1-3.

- 4. Initialize the lattice (or if you got the lattice as a parameter, set the sites in the unit cell).
- 5. Initialize the CouplingModel with CouplingModel.__init__(self, lat).
- 6. Use add_onsite() and add_coupling() to add all terms of the Hamiltonian. Here, the pairs of the lattice can come in handy, for example:

10.3. Models 83

```
self.add_onsite(-np.asarray(h), 0, 'Sz')
for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
    self.add_coupling(0.5*J, u1, 'Sp', u2, 'Sm', dx, plus_hc=True)
    self.add_coupling( J, u1, 'Sz', u2, 'Sz', dx)
```

Note: The method $add_coupling()$ adds the coupling only in one direction, i.e. not switching i and j in a $\sum_{\langle i,j\rangle}$. If you have terms like $c_i^{\dagger}c_j$ or $S_i^+S_j^-$ in your Hamiltonian, you need to add it in both directions to get a Hermitian Hamiltonian! The easiest way to do that is to use the $plus_hc$ option of $add_onsite()$ and $add_coupling()$, as we did for the $J/2(S_i^+S_j^-+h.c.)$ terms of the Heisenberg model above. Alternatively, you can add the hermitian conjugate terms explicitly, see the examples in $add_coupling()$ for more details.

Note that the *strength* arguments of these functions can be (numpy) arrays for site-dependent couplings. If you need to add or multipliy some parameters of the model for the *strength* of certain terms, it is recommended use np.asarray beforehand – in that way lists will also work fine.

- 7. Finally, if you derived from the MPOModel, you can call calc_H_MPO() to build the MPO and use it for the initialization as MPOModel.__init__(self, lat, self.calc_H_MPO()).
- 8. Similarly, if you derived from the <code>NearestNeighborModel</code>, you can call <code>calc_H_bond()</code> to initialze it as <code>NearestNeighborModel.__init__(self, lat, self.calc_H_bond())</code>. Calling self. <code>calc_H_bond()</code> will fail for models which are not nearest-neighbors (with respect to the MPS ordering), so you should only subclass the <code>NearestNeighborModel</code> if the lattice is a simple <code>Chain</code>.

Note: The method <code>add_coupling()</code> works only for terms involving operators on 2 sites. If you have couplings involving more than two sites, you can use the <code>add_multi_coupling()</code> instead. A prototypical example is the exactly solvable <code>ToricCode</code>.

The code of the module <code>tenpy.models.xxz_chain</code> is included below as an illustrative example how to implement a Model. The implementation of the <code>XXZChain</code> directly follows the steps outline above. The <code>XXZChain2</code> implements the very same model, but based on the <code>CouplingMPOModel</code> explained in the next section.

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```
The Hamiltonian reads:
   .. math ::
       H = \sum_{i=1}^{n} \frac{1}{2x}/2 (S^{+}_i S^{-}_{i+1} + S^{-}_i S^{+}_{i+1})
                + \mathtt{Jz} S^z_i S^z_{i+1} \\
            - \sum_i \mathtt{hz} S^z_i
   All parameters are collected in a single dictionary `model_params`, which
   is turned into a :class:`~tenpy.tools.params.Config` object.
   Parameters
   model_params : :class:`~tenpy.tools.params.Config`
       Parameters for the model. See :cfg:config:`XXZChain` below.
   Options
   .. cfg:config :: XXZChain
       :include: CouplingMPOModel
       L : int
           Length of the chain.
       Jxx, Jz, hz : float | array
           Coupling as defined for the Hamiltonian above.
       bc_MPS : {'finite' | 'infinte'}
           MPS boundary conditions. Coupling boundary conditions are chosen.
→appropriately.
   11 11 11
   def __init__(self, model_params):
       # 0) read out/set default parameters
       model_params = asConfig(model_params, "XXZChain")
       L = model_params.get('L', 2)
       Jxx = model_params.get('Jxx', 1.)
       Jz = model_params.get('Jz', 1.)
       hz = model_params.get('hz', 0.)
       bc_MPS = model_params.get('bc_MPS', 'finite')
       # 1-3):
       USE_PREDEFINED_SITE = False
       if not USE_PREDEFINED_SITE:
            # 1) charges of the physical leg. The only time that we actually define,
⇔charges!
           leg = npc.LegCharge.from_qflat(npc.ChargeInfo([1], ['2*Sz']), [1, -1])
           # 2) onsite operators
           Sp = [[0., 1.], [0., 0.]]
           Sm = [[0., 0.], [1., 0.]]
           Sz = [[0.5, 0.], [0., -0.5]]
            # (Can't define Sx and Sy as onsite operators: they are incompatible with,
\hookrightarrow Sz charges.)
           # 3) local physical site
           site = Site(leg, ['up', 'down'], Sp=Sp, Sm=Sm, Sz=Sz)
       else:
           # there is a site for spin-1/2 defined in TeNPy, so just we can just use.
⇔it
            # replacing steps 1-3)
           site = SpinHalfSite(conserve='Sz')
        # 4) lattice
```

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10.3. Models 85

```
bc = 'periodic' if bc_MPS == 'infinite' else 'open'
       lat = Chain(L, site, bc=bc, bc_MPS=bc_MPS)
        # 5) initialize CouplingModel
       CouplingModel.__init__(self, lat)
        # 6) add terms of the Hamiltonian
        # (u is always 0 as we have only one site in the unit cell)
        self.add_onsite(-hz, 0, 'Sz')
        self.add_coupling(Jxx * 0.5, 0, 'Sp', 0, 'Sm', 1, plus_hc=True)
        # the `plus_hc=True` adds the h.c. term
        # see also the examples tenpy.models.model.CouplingModel.add_coupling
       self.add_coupling(Jz, 0, 'Sz', 0, 'Sz', 1)
        # 7) initialize H_MPO
       MPOModel.__init__(self, lat, self.calc_H_MPO())
        # 8) initialize H_bond (the order of 7/8 doesn't matter)
        NearestNeighborModel.__init__(self, lat, self.calc_H_bond())
class XXZChain2(CouplingMPOModel, NearestNeighborModel):
    """Another implementation of the Spin-1/2 XXZ chain with Sz conservation.
    This implementation takes the same parameters as the :class:`XXZChain`, but is.
→ implemented
   based on the :class:`~tenpy.models.model.CouplingMPOModel`.
    Parameters
   model_params : dict | :class:`~tenpy.tools.params.Config`
       See :cfg:config:`XXZChain`
   default_lattice = "Chain"
   force_default_lattice = True
   def init_sites(self, model_params):
       return SpinHalfSite(conserve='Sz') # use predefined Site
   def init_terms(self, model_params):
        # read out parameters
       Jxx = model_params.get('Jxx', 1.)
       Jz = model_params.get('Jz', 1.)
       hz = model_params.get('hz', 0.)
        # add terms
        for u in range(len(self.lat.unit_cell)):
            self.add_onsite(-hz, u, 'Sz')
        for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
            self.add_coupling(Jxx * 0.5, u1, 'Sp', u2, 'Sm', dx, plus_hc=True)
            self.add_coupling(Jz, u1, 'Sz', u2, 'Sz', dx)
```

10.3.5 The easiest way: the CouplingMPOModel

Since many of the basic steps above are always the same, we don't need to repeat them all the time. So we have yet another class helping to structure the initialization of models: the CouplingMPOModel. The general structure of this class is like this:

```
class CouplingMPOModel(CouplingModel, MPOModel):
   default_lattice = "Chain"
   def __init__(self, model_param):
        \# ... follows the basic steps 1-8 using the methods
       lat = self.init_lattice(self, model_param) # for step 4
       self.init_terms(self, model_param) # for step 6
   def init_sites(self, model_param):
        # You should overwrite this in most cases to ensure
        # getting the site(s) and charge conservation you want
       site = SpinSite(...) # or FermionSite, BosonSite, ...
       return site # (or tuple of sites)
   def init_lattice(self, model_param):
       sites = self.init_sites(self, model_param) # for steps 1-3
        # and then read out the class attribute `default_lattice`,
        # initialize an arbitrary pre-defined lattice
        # using model_params['lattice']
        # and enure it's the default lattice if the class attribute
        # `force default lattice` is True.
   def init_terms(self, model_param):
        # does nothing.
        # You should overwrite this
```

The XXZChain2 included above illustrates, how it can be used. You need to implement steps 1-3) by overwriting the method init_sites() Step 4) is performed in the method init_lattice(), which initializes arbitrary 1D or 2D lattices; by default a simple 1D chain. If your model only works for specific lattices, you can overwrite this method in your own class. Step 6) should be done by overwriting the method init_terms(). Steps 5,7,8 and calls to the init_... methods for the other steps are done automatically if you just call the CouplingMPOModel. __init__(self, model_param).

The XXZChain and XXZChain2 work only with the Chain as lattice, since they are derived from the NearestNeighborModel. This allows to use them for TEBD in 1D (yeah!), but we can't get the MPO for DMRG on (for example) a Square lattice cylinder - although it's intuitively clear, what the Hamiltonian there should be: just put the nearest-neighbor coupling on each bond of the 2D lattice.

It's not possible to generalize a <code>NearestNeighborModel</code> to an arbitrary lattice where it's no longer nearest Neigbors in the MPS sense, but we can go the other way around: first write the model on an arbitrary 2D lattice and then restrict it to a 1D chain to make it a <code>NearestNeighborModel</code>.

Let me illustrate this with another standard example model: the transverse field Ising model, implemented in the module <code>tenpy.models.tf_ising</code> included below. The <code>TFIModel</code> works for arbitrary 1D or 2D lattices. The <code>TFIChain</code> is then taking the exact same model making a <code>NearestNeighborModel</code>, which only works for the 1D chain.

```
"""Prototypical example of a quantum model: the transverse field Ising model.
```

(continues on next page)

10.3. Models 87

```
Like the :class: `~tenpy.models.xxz_chain.XXZChain`, the transverse field ising chain
:class:`TFIChain` is contained in the more general :class:`~tenpy.models.spins.
→ SpinChain`;
the idea is more to serve as a pedagogical example for a 'model'.
We choose the field along z to allow to conserve the parity, if desired.
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import numpy as np
from .model import CouplingMPOModel, NearestNeighborModel
from .lattice import Chain
from ..tools.params import asConfig
from ..networks.site import SpinHalfSite
__all__ = ['TFIModel', 'TFIChain']
class TFIModel(CouplingMPOModel):
   r"""Transverse field Ising model on a general lattice.
   The Hamiltonian reads:
    .. math ::
       H = - \sum_{\alpha, \beta} \{ (1), \beta, \beta \} 
            - \sum_{i} \mathtt{q} \sigma^z_i
   Here, :math:`\langle i, j \rangle, i < j` denotes nearest neighbor pairs, each pair_
→appearing
   exactly once.
   All parameters are collected in a single dictionary `model_params`, which
   is turned into a :class: `~tenpy.tools.params.Config` object.
   Parameters
   model_params : :class:`~tenpy.tools.params.Config`
       Parameters for the model. See :cfg:config:`TFIModel` below.
   Options
    .. cfg:config :: TFIModel
       :include: CouplingMPOModel
       conserve : None | 'parity'
           What should be conserved. See :class:`~tenpy.networks.Site.SpinHalfSite`.
       J, q : float | array
           Coupling as defined for the Hamiltonian above.
    def init_sites(self, model_params):
       conserve = model_params.get('conserve', 'parity')
       assert conserve != 'Sz'
       if conserve == 'best':
           conserve = 'parity'
           self.logger.info("%s: set conserve to %s", self.name, conserve)
       site = SpinHalfSite(conserve=conserve)
```

(continues on next page)

```
return site

def init_terms(self, model_params):
    J = np.asarray(model_params.get('J', 1.))
    g = np.asarray(model_params.get('g', 1.))
    for u in range(len(self.lat.unit_cell)):
        self.add_onsite(-g, u, 'Sigmaz')
    for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
        self.add_coupling(-J, u1, 'Sigmax', u2, 'Sigmax', dx)
    # done

class TFIChain(TFIModel, NearestNeighborModel):
    """The :class:`TFIModel` on a Chain, suitable for TEBD.

See the :class:`TFIModel` for the documentation of parameters.
    """
    default_lattice = Chain
    force_default_lattice = True
```

10.3.6 Automation of Hermitian conjugation

As most physical Hamiltonians are Hermitian, these Hamiltonians are fully determined when only half of the mutually conjugate terms is defined. For example, a simple Hamiltonian:

$$H = \sum_{\langle i,j \rangle, i < j} - \mathtt{J}(c_i^\dagger c_j + c_j^\dagger c_i)$$

is fully determined by the term $c_i^{\dagger}c_j$ if we demand that Hermitian conjugates are included automatically. In TeNPy, whenever you add a coupling using $add_onsite()$, $add_coupling()$, or $add_multi_coupling()$, you can use the optional argument $plus_hc$ to automatically create and add the Hermitian conjugate of that coupling term - as shown above.

Additionally, in an MPO, explicitly adding both a non-Hermitian term and its conjugate increases the bond dimension of the MPO, which increases the memory requirements of the MPOEnvironment. Instead of adding the conjugate terms explicitly, you can set a flag *explicit_plus_hc* in the MPOCouplingModel parameters, which will ensure two things:

- 1. The model and the MPO will only store half the terms of each Hermitian conjugate pair added, but the flag $explicit_plus_hc$ indicates that they $represent\ self + h.c.$. In the example above, only the term $c_i^{\dagger}c_j$ would be saved.
- 2. At runtime during DMRG, the Hermitian conjugate of the (now non-Hermitian) MPO will be computed and applied along with the MPO, so that the effective Hamiltonian is still Hermitian.

Note: The model flag <code>explicit_plus_hc</code> should be used in conjunction with the flag <code>plus_hc</code> in <code>add_coupling()</code> or <code>add_multi_coupling()</code>. If <code>plus_hc</code> is <code>False</code> while <code>explicit_plus_hc</code> is <code>True</code> the MPO bond dimension will not be reduced, but you will still pay the additional computational cost of computing the Hermitian conjugate at runtime.

Thus, we end up with several use cases, depending on your preferences. Consider the FermionModel. If you do not care about the MPO bond dimension, and want to add Hermitian conjugate terms manually, you would set $model_par[\ 'explicit_plus_hc'] = False$ and write:

10.3. Models 89

```
self.add_coupling(-J, u1, 'Cd', u2, 'C', dx)
self.add_coupling(np.conj(-J), u2, 'Cd', u1, 'C', -dx)
```

If you wanted to save the trouble of the extra line of code (but still did not care about MPO bond dimension), you would keep the *model_par*, but instead write:

```
self.add_coupling(-J, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Finally, if you wanted a reduction in MPO bond dimension, you would need to set *model_par['explicit_plus_hc'] = True*, and write:

```
self.add_coupling(-J, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

10.3.7 Non-uniform terms and couplings

The CouplingModel-methods add_onsite(), add_coupling(), and add_multi_coupling() add a sum over a "couplig" term shifted by lattice vectors. However, some models are not that "uniform" over the whole lattice.

First of all, you might have some local term that gets added only at one specific location in the lattice. You can add such a term for example with add local term().

Second, if you have irregular lattices, take a look at the corresponding section in *Details on the lattice geometry*.

Finally, note that the argument *strength* for the *add_onsite*, *add_coupling*, and *add_multi_coupling* methods can not only be a numpy scalar, but also a (numpy) array. In general, the sum performed by the methods runs over the given term shifted by lattice vectors *as far as possible to still fit the term into the lattice*.

For the add_onsite() case this criterion is simple: there is exactly one site in each lattice unit cell with the u specified as separate argument, so the correct shape for the strength array is simply given by Ls. For example, if you want the defacto standard model studied for many-body localization, a Heisenberg chain with random, uniform onsite field $h_i^z \in [-W, W]$,

$$H = J \sum_{i=0}^{L-1} \vec{S}_i \cdot \vec{S}_{i+1} - \sum_{i=0}^{L} h_i^z S_i^z$$

you can use the SpinChain with the following model parameters:

```
L = 30 # or whatever you like...
W = 5. # MBL transition at W_c ~= 3.5 J
model_params = {
    'L': L,
    'Jx': 1., 'Jy': 1., 'Jz': 1.,
    'hz': 2.*W*(np.random.random(L) - 0.5), # random values in [-W, W], shape (L,)
    'conserve': 'best',
}
M = tenpy.models.spins.SpinChain(model_params)
```

For add_coupling() and add_multi_coupling(), things become a little bit more complicated, and the correct shape of the *strength* array depends not only on the Ls but also on the boundary conditions of the lattice. Given a term, you can call coupling_shape() and multi_coupling_shape() to find out the correct shape for *strength*. To avoid any ambiguity, the shape of the *strength* always has to fit, at least after a tiling performed by to_array().

For example, consider the Su-Schrieffer-Heeger model, a spin-less FermionChain with hopping strength alternating between two values, say t1 and t2. You can generete this model for example like this:

```
L = 30 # or whatever you like...
t1, t2 = 0.5, 1.5
t_array = np.array([(t1 if i % 2 == 0 else t2) for i in range(L-1)])
model_params = {
    'L': L,
    't': t_array,
    'V': 0., 'mu': 0., # just free fermions, but you can generalize...
    'conserve': 'best'
}
M = tenpy.models.fermions.FermionChain(model_params)
```

10.3.8 Some random remarks on models

- Needless to say that we have also various predefined models under tenpy.models.
- Of course, an MPO is all you need to initialize a MPOModel to be used for DMRG; you don't have to use the CouplingModel or CouplingMPOModel. For example an exponentially decaying long-range interactions are not supported by the coupling model but straight-forward to include to an MPO, as demonstrated in the example examples/mpo_exponentially_decaying.py.
- If you want to debug or double check that the you added the correct terms to a CouplingMPOModel, you can print the terms with print (M.all_coupling_terms().to_TermList()). This will
- If the model of your interest contains Fermions, you should read the Fermions and the Jordan-Wigner transformation.
- We suggest writing the model to take a single parameter dictionary for the initialization, as the CouplingMPOModel does. The CouplingMPOModel converts the dictionary to a dict-like Config with some additional features before passing it on to the *init_lattice*, *init_site*, ... methods. It is recommended to read out providing default values with model_params.get("key", default_value), see get().
- When you write a model and want to include a test that it can be at least constructed, take a look at tests/ test_model.py.

10.4 Simulations

Simulations provide the highest-level interface in TeNPy. They represent one simulation from start (initializing the various classes from given parameters) to end (saving the results to a file). The idea is that they contain the full package of code that you run by a job on a computing cluster. (You don't have to stick to that rule, of course.) In fact, any simulation can be run from the command line, given only a parameter file as input, like this:

```
python -m tenpy -c SimulationClassName parameters.yml
# or alternatively, if tenpy is installed correctly:
tenpy-run -c SimulationClassName parameters.yml
```

Of course, you should replace <code>SimulationClassName</code> with the class name of the simulation class you want to use, for example <code>GroundStateSearch</code> or <code>RealTimeEvolution</code>. For more details, see <code>tenpy.run_commandline()</code>.

In some cases, this might not be enough, and you want to do some pre- or post-processing, or just do something a litte bit differently during the simulation. In that case, you can also define your own simulation class (as subclass of one the existing ones).

10.4. Simulations 91

10.5 Details on the lattice geometry

The Lattice class defines the geometry of the system. In the basic form, it represents a unit cell of a few sites repeated in one or multiple directions. Moreover, it maps this higher-dimensional geometry to a one-dimensional chain for MPS-based algorithms.

10.5.1 Visualization

A plot of the lattice can greatly help to understand which sites are connected by what couplings. The methods plot_* of the Lattice can do a good job for a quick illustration. Let's look at the Honeycomb lattice as an example.

```
import matplotlib.pyplot as plt
from tenpy.models import lattice

plt.figure(figsize=(5, 6))
ax = plt.gca()
lat = lattice.Honeycomb(Lx=4, Ly=4, sites=None, bc='periodic')
lat.plot_coupling(ax)
lat.plot_order(ax, linestyle=':')
lat.plot_sites(ax)
lat.plot_basis(ax, origin=-0.5*(lat.basis[0] + lat.basis[1]))
ax.set_aspect('equal')
ax.set_xlim(-1)
ax.set_ylim(-1)
plt.show()
```

In this case, the unit cell (shaded green) consists of two sites, which for the purpose of plotting we just set to sites=None; in general you should specify instances of Site for that. The unit cell gets repeated in the directions given by the lattice basis (green arrows at the unit cell boundary). Hence, we can label each site by a **lattice** index (x, y, u) in this case, where x in range (Lx), y in range (Ly) specify the translation of the unit cell and u in range $(len(unit_cell))$, here u in [0, 1], specifies the index within the unit cell.

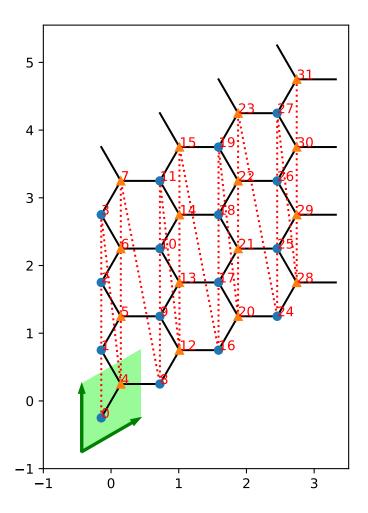
10.5.2 How an MPS winds through the lattice: the *order*

For MPS-based algorithms, we need to map a 2D lattice like the one above to a 1D chain. The red, dashed line in the plot indicates how an MPS winds through the 2D lattice. The **MPS index** i is a simple enumeration of the sites along this line, shown as numbers next to the sites in the plot. The methods $mps2lat_idx()$ and $lat2mps_idx()$ map indices of the MPS to and from indices of the lattice.

The MPS class itself is (mostly) agnostic of the underlying geometry. For example, expectation_value() will return a 1D array of the expectation value on each site indexed by the MPS index i. If you have a two-dimensional lattice, you can use $mps2lat_values()$ to map this result to a 2D array index by the lattice indices.

A suitable order is critical for the efficiency of MPS-based algorithms. On one hand, different orderings can lead to different MPO bond-dimensions, with direct impact on the complexity scaling. On the other hand, it influences how much entanglement needs to go through each bonds of the underlying MPS, e.g., the ground strate to be found in DMRG, and therefore influences the required MPS bond dimensions. For the latter reason, the "optimal" ordering can not be known a priori and might even depend on your coupling parameters (and the phase you are in). In the end, you can just try different orderings and see which one works best.

The simplets way to *change* the order is to use a non-default value for the initialization parameter *order* of the *Lattice* class. This gets passed on to *ordering()*, which you an override in a custom lattice class to define new possible orderings. Alternatively, you can go the most general way and simply set the attribute *order* to be a 2D numpy array with lattice indices as rows, in the order you want.



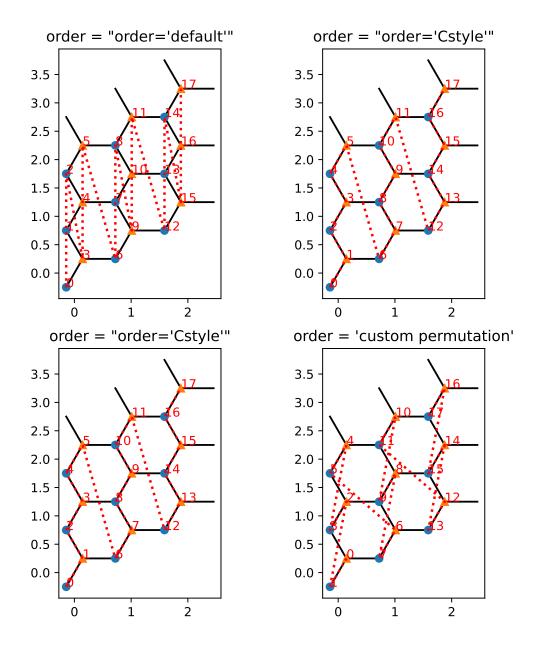
```
import matplotlib.pyplot as plt
from tenpy.models import lattice
Lx, Ly = 3, 3
fig, axes = plt.subplots(2, 2, figsize=(7, 8))
lat1 = lattice.Honeycomb(Lx, Ly, sites=None, bc='periodic') # default order
lat2 = lattice.Honeycomb(Lx, Ly, sites=None, bc='periodic',
                        order="Cstyle") # first method to change order
# alternative: directly set "Cstyle" order
lat3 = lattice.Honeycomb(Lx, Ly, sites=None, bc='periodic')
lat3.order = lat2.ordering("Cstyle") # now equivalent to lat2
# general: can apply arbitrary permutation to the order
lat4 = lattice. Honeycomb (Lx, Ly, sites=None, bc='periodic',
                        order="Cstyle")
old_order = lat4.order
permutation = []
for i in range(0, len(old_order), 2):
   permutation.append(i+1)
   permutation.append(i)
lat4.order = old_order[permutation, :]
for lat, label, ax in zip([lat1, lat2, lat3, lat4],
                          ["order='default'",
                           "order='Cstyle'",
                           "order='Cstyle'",
                           "custom permutation"],
                          axes.flatten()):
   lat.plot_coupling(ax)
   lat.plot_sites(ax)
   lat.plot_order(ax, linestyle=':', linewidth=2.)
   ax.set_aspect('equal')
   ax.set_title('order = ' + repr(label))
plt.show()
```

10.5.3 Boundary conditions

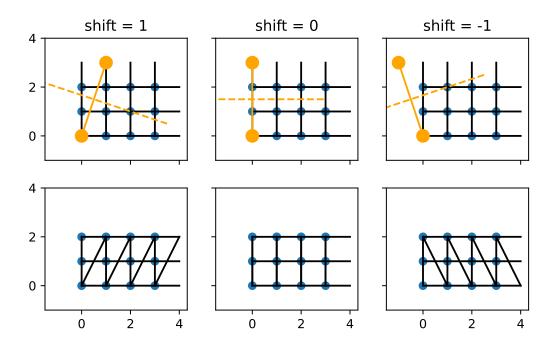
The Lattice defines the **boundary conditions** bc in each direction. It can be one of the usual 'open' or 'periodic' in each direction.

On top of that, there is the *bc_MPS* boundary condition of the MPS, one of 'finite', 'segment', 'infinite'. For an 'infinite' MPS, the whole lattice is repeated in the direction of the *first* basis vector of the lattice. For bc_MPS='infinite', the first direction should always be 'periodic', but you *can* also define a lattice with bc_MPS='finite', bc=['periodic', 'perioid'] for a finite system on the torus. This is discouraged, though, because the ground state MPS will require the *squared* bond dimension for the *same precision* in this case!

For two (or higher) dimensional lattices, e.g for DMRG on an infinite cylinder, you can also specify an integer *shift* instead of just saying 'periodic': Rolling the 2D lattice up into a cylinder, you have a degree of freedom which sites to connect. This is illustrated in the figure below for a *Square* lattice with bc=['periodic', shift] for shift in [-1, 0, 1] (different columns). In the first row, the orange markers indicate a pair of identified sites (see plot_bc_shift()). The dashed orange line indicates the direction of the cylinder axis. The line where the cylinder is "cut open" therefore winds around the the cylinder for a non-zero *shift*. (A similar thing happens even for shift=0 for more complicated lattices with non-orthogonal basis.) In the second row, we directly draw lines between

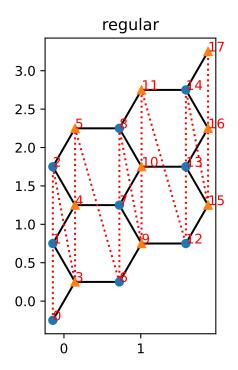


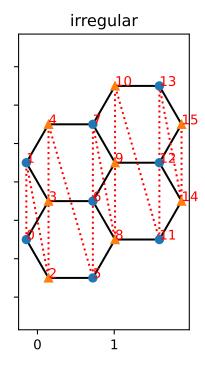
all sites connected by nearest-neighbor couplings, as they appear in the MPO.



10.5.4 Irregular Lattices

The <code>IrregularLattice</code> allows to add or remove sites from/to an existing regular lattice. The doc-string of <code>IrregularLattice</code> contains several examples, let us consider another one here, where we use the <code>IrregularLattice</code> to "fix" the boundary of the Honeycomb lattice: when we use <code>"open"</code> boundary conditions for a finite system, there are two sites (on the lower left, and upper right), wich are not included into any hexagonal. The following example shows how to remove them from the system:





10.6 Logging and terminal output

By default, calling (almost) any function in TeNPy will not print output, appart from error messages, tracebacks, and warnings. Instead, we use Python's logging module to allow fine-grained redirecting of status messages etc.

Of course, when you get an error message, you should be concerned to find out what it is about and how to fix it. (If you believe it is a bug, report it.) Warnings can be reported either using warnings.warn(...) or with the logging mechanism logger.warn(...). The former is used for warnings about things in your setup that you should fix. The latter give you notifications about bad things that can happen in calculations, e.g. bad conditioning of a matrix, but there is not much you can do about it. Those warnings indicate that you should take your results with a grain of salt and carefully double-check them.

10.6.1 Configuring logging

If you also want to see status messages (e.g. during a DMRG run whenever a checkpoint is reached), you can use set the logging level to *logging.INFO* with the following, basic setup:

```
import logging
logging.basicConfig(level=logging.INFO)
```

We use this snippet in our examples to activate the printing of info messages to the standard output stream. For really detailed output, you can even set the level to *logging.DEBUG.* logging.basicConfig() also takes a *filename* argument, which allows to redirect the output to a file instead of stdout. Note that you should call *basicConfig* only once; subsequent calls have no effect.

More detailed configurations can be made through logging.config. For example, the following both prints log messages to stdout and saves them to ouput_filename.log:

```
import logging.config
conf = {
    'version': 1
    'disable_existing_loggers': False,
    'formatters': {'custom': {'format': '%(levelname)-8s: %(message)s'}},
    'handlers': {'to_file': {'class': 'logging.FileHandler',
                             'filename': 'output_filename.log',
                             'formatter': 'custom',
                             'level': 'INFO',
                             'mode': 'a'},
                'to_stdout': {'class': 'logging.StreamHandler',
                               'formatter': 'custom',
                              'level': 'INFO',
                               'stream': 'ext://sys.stdout'}},
    'root': {'handlers': ['to_stdout', 'to_file'], 'level': 'DEBUG'},
logging.config.dictConfig(conf)
```

Note: Whether you use logging.config.fileConfig() or the logging.config.dictConfig(), make sure that you also set disable_existing_loggers=False. Otherwise, it will not work as expected in the case where you import tenpy before setting up the logging.

To also capture warnings, you might also want to call logging.captureWarnings().

In fact, the above is the default configuration used by tenpy.tools.misc.setup_logging(). If you use a Simulation class, it will automatically call setup_logging() for you, saving the log to the same filename as the Simulation.output_filename but with a .log ending. Moreover, you can easily adjust the log levels with simple parameters, for example with the following configuration (using [yaml] notation):

Of course, you can also explicitly call the setup_logging() yourself, if you don't use the Simulation classes:

10.6.2 How to write your own logging (and warning) code

Of course, you can still use simple print (...) statements in your code, and they will just appear on your screen. In fact, this is one of the benefits of logging: you can make sure that you *only* get the print statements you have put yourself, and at the same time redirect the logging messages of tenpy to a file, if you want.

However, these print (...) statements are not re-directed to the log-files. Therefore, if you write your own subclasses like Models, I would recommended that you also use the loggers instead of simple print statements. You can read the official logging tutorial for details, but it's actually straight-forward, and just requires at most two steps.

1. If necessary, import the necessary modules and create a logger at the top of your module:

```
import warnings
import logging
logger = logging.getLogger(__name__)
```

Note: Most TeNPy classes that you might want to subclass, like models, algorithm engines or simulations, provide a Logger as self.logger class attribute. In that case you can even **skip** this step and just use self.logger instead of logger in the snippets below.

2. Inside your funtions/methods/..., make calls like this:

You can use printf-formatting for the arguments of logger.debug(...), logger.info(...), logger.warn(...), as illustrated in the last line.

In summary, instead of just print ("do X") statements, use self.logger.info("do X") inside TeNPy classes, or just logger.info("do X") for the module-wide logger, which you can initialize right at the top of your file with the import statements. If you have non-string arguments, add a formatter string, e.g. replace print (max (psi.chi)) with logger.info("%d", max (psi.chi)), or even better, logger.info("max (chi)=%d", max (psi.chi)). For genereic types, use "%s" or "%r", which converts the other arguments to strings with str(...) or repr(...), respectively.

10.7 Parameters and options

(We use *parameter* and *option* synonymously.)

Standard simulations in TeNPy can be defined by just set of options collected in a dictionary (possibly containing other parameter dictionaries). It can be convenient to represent these options in a [yaml] file, say parameters. yml, which might look like this:

```
output_filename : params_output.h5
overwrite_output : True
model_class : SpinChain
model_params :
    L : 14
    bc_MPS : finite

initial_state_params:
    method : lat_product_state
    product_state : [[up], [down]]
```

(continues on next page)

```
algorithm_class: TwoSiteDMRG
algorithm_params:
    trunc_params:
    chi_max: 120
    svd_min: 1e.-8
    max_sweeps: 10
    mixer: True
```

Note that the default values and even the allowed/used option names often depend on other parameters. For example, the *model_class* parameter above given to a Simulation selects a model class, and diffent model classes might have completely different parameters. This gives you freedom to define your own parameters when you implement a model, but it also makes it a little bit harder to keep track of allowed values.

In the TeNPy documentation, we use the Options sections of doc-strings to define parameters that are read out. Each documented parameter is attributed to one set of parameters, called "config", and managed in a Config class at runtime. The above example represents the config for a *Simulation*, with the *model_params* representing the config given as *options* to the model for initialization. Sometimes, there is also a structure of one *config* including the parameters from another one: For example, the generic parameters for time evolution algorithms, TimeEvolutionAlgorithm are included into the TEBDEngine config, similarly to the sub-classing used.

During runtime, the Config class logs the first use of any parameter (with DEBUG log-level, if the default is used, and with INFO log-level, if it is non-default). Moreover, the default is saved into the parameter dictionary. Hence, it will contain the *full set of all used parameters*, default and non-default, at the end of a simulation, e.g., in the <code>sim_params</code> of the <code>results</code> returned by tenpy.simulations.Simulation.run().

You can find a list of all the different configs in the cfg-config-index, and a list of all parameters in cfg-option-index.

If you add extra options to your configuration that TeNPy doesn't read out by the end of the simulation, it will issue a warning. Getting such a warnings is an indicator for a typo in your configuration, or an option being in the wrong config dictionary.

10.8 Saving to disk: input/output

10.8.1 Using pickle

A simple and pythonic way to store data of TeNPy arrays is to use pickle from the Python standard library. Pickle allows to store (almost) arbitrary python objects, and the Array is no exception (and neither are other TeNPy classes).

Say that you have run DMRG to get a ground state *psi* as an MPS. With pickle, you can save it to disk as follows:

```
import pickle
with open('my_psi_file.pkl', 'wb') as f:
    pickle.dump(psi, f)
```

Here, the with ...: structure ensures that the file gets closed after the pickle dump, and the 'wb' indicates the file opening mode "write binary". Reading the data from disk is as easy as ('rb' for reading binary):

```
with open('my_psi_file.pkl', 'rb') as f:
    psi = pickle.load(f)
```

Note: It is a good (scientific) practice to include meta-data to the file, like the parameters you used to generate that state. Instead of just the *psi*, you can simply store a dictionary containing *psi* and other data, e.g., data = { 'psi':

psi, 'dmrg_params': dmrg_params, 'model_params': model_params}. This can save you a lot of pain, when you come back looking at the files a few month later and forgot what you've done to generate them!

In some cases, compression can significantly reduce the space needed to save the data. This can for example be done with gzip (as well in the Python standard library). However, be warned that it might cause longer loading and saving times, i.e. it comes at the penalty of more CPU usage for the input/output. In Python, this requires only small adjustments:

```
import pickle
import gzip

# to save:
with gzip.open('my_data_file.pkl', 'wb') as f:
    pickle.dump(data, f)
# and to load:
with gzip.open('my_data_file.pkl', 'rb') as f:
    data = pickle.load(data, f)
```

10.8.2 Using HDF5 with h5py

While pickle is great for simple input/output of python objects, it also has disadvantages. The probably most dramatic one is the limited portability: saving data on one PC and loading it on another one might fail! Even exporting data from Python 2 to load them in Python 3 on the same machine can give quite some troubles. Moreover, pickle requires to load the whole file at once, which might be unnecessary if you only need part of the data, or even lead to memory problems if you have more data on disk than fits into RAM.

Hence, we support saving to HDF5 files as an alternative. The h5py package provides a dictionary-like interface for the file/group objects with numpy-like data sets, and is quite easy to use. If you don't know about HDF5, read the quickstart of the h5py documentation (and this guide).

The implementation can be found in the tenpy.tools.hdf5_io module with the Hdf5Saver and Hdf5Loader classes and the wrapper functions save_to_hdf5(), load_from_hdf5().

The usage is very similar to pickle:

Warning: Like loading a pickle file, loading data from a manipulated HDF5 file with the functions described has the potential to cause arbitrary code execution. Only load data from trusted sources!

Note: The hickle package imitates the pickle functionality while saving the data to HDF5 files. However, since it aims to be close to pickle, it results in a more complicated data structure than we want here.

Note: To use the export/import features to HDF5, you need to install the h5py python package (and hence some version of the HDF5 library).

Data format specification for saving to HDF5

This section motivates and defines the format how we save data of TeNPy-defined classes. The goal is to have the <code>save_to_hdf5()</code> function for saving sufficiently simple enough python objects (supported by the format) to disk in an HDF5 file, such that they can be reconstructed with the <code>load_from_hdf5()</code> function, as outlined in the example code above.

Guidelines of the format:

- 0. Store enough data such that <code>load_from_hdf5()</code> can reconstruct a copy of the object (provided that the save did not fail with an error).
- 1. Objects of a type supported by the HDF5 datasets (with the h5py interface) should be directly stored as h5py Dataset. Such objects are for example numpy arrays (of non-object *dtype*), scalars and strings.
- 2. Allow to save (nested) python lists, tuples and dictionaries with values (and keys) which can be saved.
- 3. Allow user-defined classes to implement a well-defined interface which allows to save instances of that class, hence extending what data can be saved. An instance of a class supporting the interface gets saved as an HDF5 Group. Class attributes are stored as entries of the group, metadata like the type should be stored in HDF5 attributes, see attributes.
- 4. Simple and intuitive, human-readable structure for the HDF5 paths. For example, saving a simple dictionary {'a': np.arange(10), 'b': 123.45} should result in an HDF5 file with just the two data sets /a and /b.
- 5. Allow loading only a subset of the data by specifying the *path* of the HDF5 group to be loaded. For the above example, specifying the path /b should result in loading the float 123.45, not the array.
- 6. Avoid unnecessary copies if the same python object is referenced by different names, e.g, for the data { 'c': large_obj, 'd': large_obj} with to references to the same large_obj, save it only once and use HDF5 hard-links such that /c and /d are the same HDF5 dataset/group. Also avoid the copies during the loading, i.e., the loaded dictionary should again have two references to a single object large_obj. This is also necessary to allow saving and loading of objects with cyclic references.

The full format specification is given by the what the code in hdf5_io does... Since this is not trivial to understand, let me summarize it here:

- Following 1), simple scalars, strings and numpy arrays are saved as Dataset. Other objects are saved as a HDF5 Group, with the actual data being saved as group members (as sub-groups and sub-datasets) or as attributes (for metadata or simple data).
- The type of the object is stored in the HDF5 attribute 'type', which is one of the global REPR_* variables in tenpy.tools.hdf5_io. The type determines the format for saving/loading of builtin types (list, ...)
- Userdefined classes which should be possible to export/import need to implement the methods save_hdf5 and from_hdf5 as specified in <code>Hdf5Exportable</code>. When saving such a class, the attribute 'type' is automatically set to 'instance', and the class name and module are saved under the attributes 'module' and 'class'. During loading, this information is used to automatically import the module, get the class and

call the classmethod from_hdf5 for reconstruction. This can only work if the class definition already exists, i.e., you can only save class instances, not classes itself.

- For most (python) classes, simply subclassing <code>Hdf5Exportable</code> should work to make the class exportable. The latter saves the contents of <code>__dict__</code>, with the extra attribute 'format' specifying whether the dictionary is "simple" (see below.).
- The None object is saved as a group with the attribute 'type' being 'None' and no subgroups.
- For iterables (list, tuple and set), we simple enumerate the entries and save entries as group members under the names '0', '1', '2', ..., and a maximum 'len' attribute.
- The format for dictionaries depends on whether all keys are "simple", which we define as being strings which are valid path names in HDF5, see valid_hdf5_path_component (). Following 4), the keys of a simple dictionary are directly used as names for group members, and the values being whatever object the group member represents.
- Partial loading along 5) is possible by directly specifying the subgroup or the path to <code>load_from_hdf5()</code>.
- Guideline 6) is ensured as much as possible. However, there is a bug/exception: tuples with cyclic references are not re-constructed correctly; the inner objects will be lists instead of tuples (but with the same object entries).

Finally, we have to mention that many TeNPy classes are <code>Hdf5Exportable</code>. In particular, the <code>Array</code> supports this. To see what the exact format for those classes is, look at the <code>save_hdf5</code> and <code>from_hdf5</code> methods of those classes.

Note: There can be multiple possible output formats for the same object. The dictionary – with the format for simple keys or general keys – is such an example, but userdefined classes can use the same technique in their *from_hdf5* method. The user might also explicitly choose a "lossy" output format (e.g. "flat" for np_conserved Arrays and LegCharges).

Tip: The above format specification is quite general and not bound to TeNPy. Feel free to use it in your own projects ;-) To separate the development, versions and issues of the format clearly from TeNPy, we maintain the code for it in a separate git repository, https://github.com/tenpy/hdf5_io

10.9 Fermions and the Jordan-Wigner transformation

The Jordan-Wigner tranformation maps fermionic creation- and annihilation operators to (bosonic) spin-operators.

10.9.1 Spinless fermions in 1D

Let's start by explicitly writing down the transformation. With the Pauli matrices $\sigma_j^{x,y,z}$ and $\sigma_j^{\pm} = (\sigma_j^x \pm i\sigma_j^y)/2$ on each site, we can map

$$n_j \leftrightarrow (\sigma_j^z + 1)/2$$

$$c_j \leftrightarrow (-1)^{\sum_{l < j} n_l} \sigma_j^-$$

$$c_j^{\dagger} \leftrightarrow (-1)^{\sum_{l < j} n_l} \sigma_j^+$$

The n_l in the second and third row are defined in terms of Pauli matrices according to the first row. We do not interpret the Pauli matrices as spin-1/2; they have nothing to do with the spin in the spin-full case. If you really want to interpret them physically, you might better think of them as hard-core bosons $(b_j = \sigma_j^-, b_j^\dagger = \sigma_j^+)$, with a spin of the fermions mapping to a spin of the hard-core bosons.

Note that this transformation maps the fermionic operators c_j and c_j^{\dagger} to global operators; although they carry an index j indicating a site, they actually act on all sites 1 <= j! Thus, clearly the operators $\mathbb C$ and $\mathbb C d$ defined in the FermionSite do not directly correspond to c_j and c_j^{\dagger} . The part $(-1)^{\sum_{l < j} n_l}$ is called Jordan-Wigner string and in the FermionSite is given by the local operator $JW := (-1)^{n_l}$ acting all sites 1 < j. Since this important, let me stress it again:

Warning: The fermionic operator c_j (and similar c_j^{\dagger}) maps to a *global* operator consisting of the Jordan-Wigner string built by the local operator JW on sites 1 < j and the local operator C (or Cd, respectively) on site j.

On the sites itself, the onsite operators C and Cd in the FermionSite fulfill the correct anti-commutation relation, without the need to include JW strings. The JW string is necessary to ensure the anti-commutation for operators acting on different sites.

Written in terms of *onsite* operators defined in the FermionSite, with the *i*-th entry entry in the list acting on site *i*, the relations are thus:

```
["JW", ..., "JW", "C", "Id", ..., "Id"] # for the annihilation operator
["JW", ..., "JW", "Cd", "Id", ..., "Id"] # for the creation operator
```

Note that "JW" squares to the identity, "JW JW" == "Id", which is the reason that the Jordan-wigner string completely cancels in $n_j = c_j^{\dagger} c_j$. In the above notation, this can be written as:

```
["JW", ..., "JW", "Cd", "Id", ..., "Id"] * ["JW", ..., "JW", "C", "Id", ..., "Id"]
== ["JW JW", ..., "JW JW", "Cd C", "Id Id", ..., "Id Id"] # by definition of of the tensorproduct
== ["Id", ..., "Id", "N", "Id", ..., "Id"] # by definition of of the local operators
# ("X Y" stands for the local operators X and Y applied on the same site. We assume of that the "Cd" and "C" on the first line act on the same site.)
```

For a pair of operators acting on different sites, JW strings have to be included for every site between the operators. For example, taking i < j, $c_i^{\dagger}c_j \leftrightarrow \sigma_i^+(-1)^{\sum_{i < = l < j} n_l}\sigma_i^-$. More explicitly, for j = i+2 we get:

```
["JW", ..., "JW", "Cd", "Id", "Id", "Id", ..., "Id"] * ["JW", ..., "JW", "JW", "JW", "JW", "C", "Id", ..., "Id"]
== ["JW JW", ..., "JW JW", "Cd JW", "Id JW", "Id C", ..., "Id"]
== ["Id", ..., "Id", "Cd JW", "JW", "C", ..., "Id"]
```

In other words, the Jordan-Wigner string appears only in the range i <= 1 < j, i.e. between the two sites *and* on the smaller/left one of them. (You can easily generalize this rule to cases with more than two c or c^{\dagger} .)

This last line (as well as the last line of the previous example) can be rewritten by changing the order of the operators Cd JW to "JW Cd" == - "Cd". (This is valid because either site i is occupied, yielding a minus sign from the JW, or it is empty, yielding a 0 from the Cd.)

This is also the case for j < i, say j = i-2: $c_i^{\dagger} c_j \leftrightarrow (-1)^{\sum_{j < = l < i} n_l} \sigma_i^+ \sigma_j^-$. As shown in the following, the JW again appears on the left site, but this time acting after C:

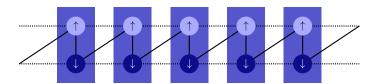
```
["JW", ..., "JW", "JW", "JW", "Cd", "Id", ..., "Id"] * ["JW", ..., "JW", "C", "Id", 

→"Id", "Id", ..., "Id"]
== ["JW JW", ..., "JW JW", "JW C", "JW", "Cd Id", ..., "Id"]
== ["Id", ..., "Id", "JW C", "JW", "Cd", ..., "Id"]
```

10.9.2 Higher dimensions

For an MPO or MPS, you always have to define an ordering of all your sites. This ordering effectifely maps the higher-dimensional lattice to a 1D chain, usually at the expence of long-range hopping/interactions. With this mapping, the Jordan-Wigner transformation generalizes to higher dimensions in a straight-forward way.

10.9.3 Spinful fermions



As illustrated in the above picture, you can think of spin-1/2 fermions on a chain as spinless fermions living on a ladder (and analogous mappings for higher dimensional lattices). Each rung (a blue box in the picture) forms a SpinHalfFermionSite which is composed of two FermionSite (the circles in the picture) for spin-up and spin-down. The mapping of the spin-1/2 fermions onto the ladder induces an ordering of the spins, as the final result must again be a one-dimensional chain, now containing both spin species. The solid line indicates the convention for the ordering, the dashed lines indicate spin-preserving hopping $c_{s,i}^{\dagger}c_{s,i+1} + h.c.$ and visualize the ladder structure. More generally, each species of fermions appearing in your model gets a separate label, and its Jordan-Wigner string includes the signs $(-1)^{n_l}$ of all species of fermions to the 'left' of it (in the sense of the ordering indicated by the solid line in the picture).

In the case of spin-1/2 fermions labeled by \uparrow and \downarrow on each *site*, the complete mapping is given (where j and l are indices of the FermionSite):

$$\begin{split} n_{\uparrow,j} &\leftrightarrow (\sigma_{\uparrow,j}^z+1)/2 \\ n_{\downarrow,j} &\leftrightarrow (\sigma_{\downarrow,j}^z+1)/2 \\ c_{\uparrow,j} &\leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} \sigma_{\uparrow,j}^{-} \\ c_{\uparrow,j}^{\dagger} &\leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} \sigma_{\uparrow,j}^{+} \\ c_{\downarrow,j} &\leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} (-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^{-} \\ c_{\downarrow,j} &\leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} (-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^{+} \\ c_{\downarrow,j}^{\dagger} &\leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} (-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^{+} \end{split}$$

In each of the above mappings the operators on the right hand sides commute; we can rewrite $(-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} = \prod_{l < j} (-1)^{n_{\uparrow,l}} (-1)^{n_{\downarrow,l}}$, which resembles the actual structure in the code more closely. The parts of the operator acting in the same box of the picture, i.e. which have the same index j or l, are the 'onsite' operators in the SpinHalfFermionSite: for example JW on site j is given by $(-1)^{n_{\uparrow,j}} (-1)^{n_{\downarrow,j}}$, Cu is just the $\sigma_{\uparrow,j}^-$, Cdu is $\sigma_{\uparrow,j}^+$, Cd is $(-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^-$, and Cdd is $(-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^-$. Note the asymmetry regarding the spin in the definition of the onsite operators: the spin-down operators include Jordan-Wigner signs for the spin-up fermions on the same site. This asymetry stems from the ordering convention introduced by the solid line in the picture, according to which the spin-up site is "left" of the spin-down site. With the above definition, the operators within the same SpinHalfFermionSite fulfill the expected commutation relations, for example "Cu Cdd" == - "Cdd Cu", but again the JW on sites left of the operator pair is crucial to get the correct commutation relations globally.

Warning: Again, the fermionic operators $c_{\downarrow,j}, c_{\downarrow,j}^{\dagger}, c_{\downarrow,j}, c_{\downarrow,j}^{\dagger}$ correspond to *global* operators consisting of the Jordan-Wigner string built by the local operator JW on sites 1 < j and the local operators 'Cu', 'Cdu', 'Cd', 'Cdd' on site j.

Written explicitly in terms of onsite operators defined in the FermionSite, with the j-th entry entry in the list acting on site j, the relations are:

```
"JW", "Cu",
                         "Id", ...,
                                             # for the annihilation operator spin-up
           "JW", "Cd",
                         "Id", ...,
                                    "Id"]
                                             # for the annihilation operator spin-down
["JW", ...,
           "JW", "Cdu",
                         "Id", ...,
                                    "Id"]
                                             # for the creation operator spin-up
["JW", ...,
           "JW", "Cdd",
                          "Id", ..., "Id"]
                                             # for the creation operator spin-down
```

As you can see, the asymmetry regaring the spins in the definition of the local onsite operators "Cu", "Cdu", "Cdu", "Cdu", "Cdd", "Cdd" lead to a symmetric definition in the global sense. If you look at the definitions very closely, you can see that in terms like ["Id", "Cd JW", "JW", "Cd"] the Jordan-Wigner sign $(-1)^{n_{\uparrow},2}$ appears twice (namely once in the definition of "Cd" and once in the "JW" on site 2) and could in principle be canceled, however in favor of a simplified handling in the code we do not recommend you to cancel it. Similar, within a spinless FermionSite, one can simplify "Cd JW" == "Cd" and "JW C" == "C", but these relations do *not* hold in the SpinHalfSite, and for consistency we recommend to explicitly keep the "JW" operator string even in nearest-neighbor models where it is not strictly necessary.

10.9.4 How to handle Jordan-Wigner strings in practice

There are only a few pitfalls where you have to keep the mapping in mind: When **building a model**, you map the physical fermionic operators to the usual spin/bosonic operators. The algorithms don't care about the mapping, they just use the given Hamiltonian, be it given as MPO for DMRG or as nearest neighbor couplings for TEBD. Only when you do a **measurement** (e.g. by calculating an expectation value or a correlation function), you have to reverse this mapping. Be aware that in certain cases, e.g. when calculating the entanglement entropy on a certain bond, you cannot reverse this mapping (in a straightforward way), and thus your results might depend on how you defined the Jordan-Wigner string.

Whatever you do, you should first think about if (and how much of) the Jordan-Wigner string cancels. For example for many of the onsite operators (like the particle number operator N or the spin operators in the SpinHalfFermionSite) the Jordan-Wigner string cancels completely and you can just ignore it both in onsite-terms and couplings. In case of two operators acting on different sites, you typically have a Jordan-Wigner string inbetween (e.g. for the $c_i^{\dagger}c_j$ examples described above and below) or no Jordan-Wigner strings at all (e.g. for density-density interactions $n_i n_j$). In fact, the case that the Jordan Wigner string on the left of the first non-trivial operator does not cancel is currently not supported for models and expectation values, as it usually doesn't appear in practice. For terms involving more operators, things tend to get more complicated, e.g. $c_i^{\dagger}c_j^{\dagger}c_kc_l$ with i < j < k < l requires a Jordan-Wigner string on sites m with $i \le m < j$ or $k \le m < l$, but not for j < m < k.

Note: TeNPy keeps track of which onsite operators need a Jordan-Wigner string in the Site class, specifically in $need_JW_string$ and $op_needs_JW()$. Hence, when you define custom sites or add extra operators to the sites, make sure that $op_needs_JW()$ returns the expected results.

When **building a model** the Jordan-Wigner strings need to be taken into account. If you just specify the *H_MPO* or *H_bond*, it is *your* responsibility to use the correct mapping. However, if you use the <code>add_coupling()</code> method of the <code>CouplingModel</code>, (or the generalization <code>add_multi_coupling()</code> for more than 2 operators), TeNPy can use the information from the <code>Site</code> class to <code>automatically</code> add <code>Jordan-Wigner</code> strings as needed. Indeed, with the default argument <code>op_string=None</code>, <code>add_coupling</code> will automatically check whether the operators need <code>Jordan-Wigner</code> strings and correspondlingly set <code>op_string='JW'</code>, <code>str_on_first=True</code>, if necessary. For <code>add_multi_coupling</code>, you cann't even explicitly specify the correct <code>Jordan-Wigner</code> strings, but you must use <code>op_string=None</code>, from which it will automatically determine where <code>Jordan-Wigner</code> strings are needed.

Obviously, you should be careful about the convention which of the operators is applied first (in a physical sense as an operator acting on a state), as this corresponds to a sign of the prefactor. Read the doc-strings of add_coupling() add_multi_coupling() for details.

As a concrete example, let us specify a hopping $\sum_i (c_i^{\dagger} c_{i+1} + h.c.) = \sum_i (c_i^{\dagger} c_{i+1} + c_i^{\dagger} c_{i-1})$ in a 1D chain of FermionSite with add_coupling(). The recommended way is just:

```
self.add_coupling(strength, 0, 'Cd', 0, 'C', 1, plus_hc=True)
```

If you want to specify both the Jordan-Wigner string and the h.c. term explicitly, you can use:

```
self.add_coupling(strength, 0, 'Cd', 0, 'C', 1, op_string='JW', str_on_first=True)
self.add_coupling(strength, 0, 'Cd', 0, 'C', -1, op_string='JW', str_on_first=True)
```

Slightly more complicated, to specify the hopping $\sum_{\langle i,j\rangle,s}(c_{s,i}^{\dagger}c_{s,j}+h.c.)$ in the Fermi-Hubbard model on a 2D square lattice, we could use:

```
for (dx, dy) in [(1, 0), (0, 1)]:
    self.add_coupling(strength, 0, 'Cdu', 0, 'Cu', (dx, dy), plus_hc=True) # spin up
    self.add_coupling(strength, 0, 'Cdd', 0, 'Cd', (dx, dy), plus_hc=True) # spin_
    down

# or without `plus_hc`
for (dx, dy) in [(1, 0), (-1, 0), (0, 1), (0, -1)]: # include -dx !
    self.add_coupling(strength, 0, 'Cdu', 0, 'Cu', (dx, dy)) # spin up
    self.add_coupling(strength, 0, 'Cdd', 0, 'Cd', (dx, dy)) # spin down

# or specifying the 'JW' string explicitly
for (dx, dy) in [(1, 0), (-1, 0), (0, 1), (0, -1)]:
    self.add_coupling(strength, 0, 'Cdu', 0, 'Cu', (dx, dy), 'JW', True) # spin up
    self.add_coupling(strength, 0, 'Cdd', 0, 'Cd', (dx, dy), 'JW', True) # spin down
```

The most important functions for doing **measurements** are probably expectation_value() and correlation_function(). Again, if all the Jordan-Wigner strings cancel, you don't have to worry about them at all, e.g. for many onsite operators or correlation functions involving only number operators. If you build multi-site operators to be measured by *expectation value*, take care to include the Jordan-Wigner string correctly.

Some MPS methods like correlation_function(), expectation_value_term() and expectation_value_terms_sum() automatically add Jordan-Wignder strings (at least with default arguments). Other more low-level functions like expectation_value_multi_sites() don't do it. Hence, you should always watch out during measurements, if the function used needs special treatment for Jordan-Wigner strings.

10.10 Protocol for using (i)DMRG

While this documentation contains extensive guidance on how to interact with the tenpy, it is often unclear how to approach a physics question using these methods. This page is an attempt to provide such guidance, describing a protocol on how to go from a model implementation to an answered question.

The basic workflow for an (i)DMRG project is as follows, with individual steps expanded on later where necessary.

- 1. Confirm the correctness of the model implementation.
- 2. Run some low-effort tests to see whether the question seems answerable.
- 3. If the tests are successful, run production-quality simulations. This will be entirely particular to the project you're working on.
- 4. Confirm that your results are converged.

10.10.1 Confirming the model is correct

Although TeNPy makes model implementation much easier than constructing the MPO by hand, one should still ensure that the MPO represents the intended model faithfully. There are several possible ways to do this. Firstly, for sufficiently small system sizes, one can contract the entire MPO into a matrix, and inspect the matrix elements. In TeNPy, this can be done using get_full_hamiltonian(). These should reproduce the analytical Hamiltonian up to machine precision, or any other necessary cut-off (e.g., long-range interactions may be truncated at some finite distance).

Secondly, if the model basis allows it, one can construct (product state) MPSs for known eigenstates of the model and evaluate whether these reproduce the correct eigenvalues upon contraction with the MPO.

Finally, one can sometimes construct a basis of single- or even two-particle MPSs in some basis, and evaluate the MPO on this basis to get a representation of the single- and two-particle Hamiltonian. If the model contains only single- and two-body terms, this latter approach should reproduce all terms in the Hamiltonian.

10.10.2 Low-effort tests

As not every state can be accurately represented by an MPS, some results are outside the reach of (i)DMRG. To prevent wasting considerable numerical resources on a fruitless project, it is recommended to run some low-effort trials first, and see whether any indication of the desired result can be found. If so, one can then go on to more computationally expensive simulations. If not, one should evaluate:

- 1. Whether there is a mistake in the model or simulation set-up,
- 2. Whether a slightly more computationally expensive test would potentially yield a result, or
- 3. Whether your approach is unfortunately out of reach of (i)DMRG.

To set up low-effort trials, one should limit system size, bond dimension and the range of interactions, as well as (if possible) target a non-critical region of phase space. All these measures reduce the size of and/or entanglement entropy needing to be captured by the MPS, which yields both memory and run time advantages. Of course, one introduces a trade-off between computational cost and accuracy, which is why one should be careful to not put too much faith into results obtained at this stage.

10.10.3 Detecting convergence issues

Ensuring that the results of an (i)DMRG simulation are well-converged and thus reliable is a hugely important part of any (i)DMRG study. Possible indications that there might be a convergence issue include:

- 1. The simulation shows a non-monotonous decrease of energy, and/or a non-monotonous increase of entanglement entropy. An increase of energy or decrease of entanglement entropy on subsequent steps within a sweep, or between subsequent sweeps, are particularly suspicious.
- 2. The simulation does not halt because it reached a convergence criterion, but because it reached its maximum number of sweeps.
- 3. Results vary wildly under small changes of parameters. In particular, if a small change in bond dimension yields a big change in results, one should be suspicious of the data.

10.10.4 Combating convergence issues

To combat convergence issues of the (i)DMRG algorithm, several strategies (short of switching to a different method) can be attempted:

- 1. Ensure that there are no errors in the model (see above) or the simulation set-up.
- 2. Increase the maximum bond dimension.
- 3. Ramp up the maximum bond dimension during simulation, rather than starting at the highest value. I.e., define a schedule wherein the first $N_{\rm sweeps}$ sweeps run at some $\chi_1 < \chi_{\rm max}$, the next $N_{\rm sweeps}$ at $\chi_1 < \chi_2 < \chi_{\rm max}$, etc. This can be done through the <code>chi_list</code> option of the <code>DMRGEngine</code>. You should also make sure that the <code>max_hours</code> option is set to sufficiently long runtimes.
- 4. Increase the maximum number of sweeps the algorithm is allowed to make, through the max_sweeps option of the DMRGEngine.
- 5. Change the Mixer settings to in- or decrease the effects of the mixer.
- 6. Change convergence criteria. This will not overcome convergence issues in itself, but can help fine tune the (i)DMRG simulation if it takes a long time to converge (relax the convergence constraints), or if the simulation finishes too soon (tighten the constraints). Criteria to consider are max_E_err and max_S_err, in DMRGEngine.
- 7. Increase the minimum number of sweeps taken by the algorithm. Again, this will not resolve issues due to bad convergence, but might prevent bad results due to premature convergence. This can be done through the min sweeps option of the DMRGEngine.
- 8. Change the size and shape of the MPS unit cell (where possible), in case an artificially enforced translational invariance prevents the algorithm from finding a true ground state which is incommensurate with this periodicity. For example, a chain system which has a true ground state that is periodic in three sites, will not be accurately represented by a two-site MPS unit cell, as the latter enforces two-site periodicity.

In some instances, it is essentially unavoidable to encounter convergence issues. In particular, a simulation of a critical state can cause problems with (i)DMRG convergence, as these states violate the area law underlying an accurate MPS approximation. In these cases, one should acknowledge the difficulties imposed by the method and take care to be very careful in interpreting the data.

CHAPTER

ELEVEN

EXAMPLES

11.1 Toycodes

These toycodes are meant to give you a flavor of the different algorithms, while keeping the codes as readable and simple as possible. The scripts are included in the [TeNPySource] repository in the folder toycodes/, but not part of the basic TeNpy library; the only requirements to run them are Python 3, Numpy, and Scipy.

11.1.1 a mps.py

on github.

```
"""Toy code implementing a matrix product state."""
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import numpy as np
from scipy.linalg import svd
# if you get an error message "LinAlgError: SVD did not converge",
# uncomment the following line. (This requires TeNPy to be installed.)
# from tenpy.linalg.svd_robust import svd # (works like scipy.linalg.svd)
class SimpleMPS:
    """Simple class for a matrix product state.
    We index sites with `i` from 0 to L-1; bond `i` is left of site `i`.
   We *assume* that the state is in right-canonical form.
   Parameters
    Bs, Ss, bc:
        Same as attributes.
   Attributes
    Bs : list of np.Array[ndim=3]
        The 'matrices' in right-canonical form, one for each physical site
        (within the unit-cell for an infinite MPS).
       Each `B[i]` has legs (virtual left, physical, virtual right), in short ``vL i
\hookrightarrow VR``
   Ss : list of np.Array[ndim=1]
       The Schmidt values at each of the bonds, ``Ss[i]`` is left of ``Bs[i]``.
   bc : 'infinite', 'finite'
```

```
Boundary conditions.
   L : int
       Number of sites (in the unit-cell for an infinite MPS).
   nbonds : int
       Number of (non-trivial) bonds: L-1 for 'finite' boundary conditions
   def __init__(self, Bs, Ss, bc='finite'):
       assert bc in ['finite', 'infinite']
       self.Bs = Bs
       self.Ss = Ss
       self.bc = bc
       self.L = len(Bs)
       self.nbonds = self.L - 1 if self.bc == 'finite' else self.L
   def copv(self):
       return SimpleMPS([B.copy() for B in self.Bs], [S.copy() for S in self.Ss],
⇔self.bc)
   def get_theta1(self, i):
        """Calculate effective single-site wave function on sites i in mixed.
→canonical form.
       The returned array has legs ``vL, i, vR`` (as one of the Bs).
       return np.tensordot(np.diag(self.Ss[i]), self.Bs[i], [1, 0]) # vL [vL'],
\hookrightarrow [VL] i VR
   def get_theta2(self, i):
        """Calculate effective two-site wave function on sites i, j=(i+1) in mixed_
⇔canonical form.
       The returned array has legs ``vL, i, j, vR``.
       j = (i + 1) % self.L
       return np.tensordot(self.get_theta1(i), self.Bs[j], [2, 0]) # vL i [vR],_
\hookrightarrow [vL] j vR
   def get_chi(self):
       """Return bond dimensions."""
       return [self.Bs[i].shape[2] for i in range(self.nbonds)]
   def site_expectation_value(self, op):
       """Calculate expectation values of a local operator at each site."""
       result = []
       for i in range(self.L):
           theta = self.get_theta1(i) # vL i vR
           op_theta = np.tensordot(op, theta, axes=[1, 1]) # i [i*], vL [i] vR
           result.append(np.tensordot(theta.conj(), op_theta, [[0, 1, 2], [1, 0, _
→211))
            # [vL*] [i*] [vR*], [i] [vL] [vR]
       return np.real_if_close(result)
   def bond_expectation_value(self, op):
       """Calculate expectation values of a local operator at each bond."""
       result = []
       for i in range(self.nbonds):
           theta = self.get_theta2(i) # vL i j vR
```

```
op_theta = np.tensordot(op[i], theta, axes=[[2, 3], [1, 2]])
            # i j [i*] [j*], vL [i] [j] vR
            result.append(np.tensordot(theta.conj(), op_theta, [[0, 1, 2, 3], [2, 0, ...
\hookrightarrow 1, 3]]))
            # [vL*] [i*] [j*] [vR*], [i] [j] [vL] [vR]
        return np.real_if_close(result)
    def entanglement_entropy(self):
        """Return the (von-Neumann) entanglement entropy for a bipartition at any of ...
→the bonds."""
        bonds = range(1, self.L) if self.bc == 'finite' else range(0, self.L)
        result = []
        for i in bonds:
            S = self.Ss[i].copy()
            S[S < 1.e-20] = 0. # 0*log(0) should give 0; avoid warning or NaN.
            S2 = S * S
            assert abs(np.linalg.norm(S) - 1.) < 1.e-14</pre>
            result.append(-np.sum(S2 * np.log(S2)))
        return np.array(result)
    def correlation_length(self):
        """Diagonalize transfer matrix to obtain the correlation length."""
        import scipy.sparse.linalg.eigen.arpack as arp
        assert self.bc == 'infinite' # works only in the infinite case
        B = self.Bs[0] # vL i vR
        chi = B.shape[0]
        T = \text{np.tensordot}(B, \text{np.conj}(B), \text{axes}=[1, 1]) # vL [i] vR, vL* [i*] vR*
        T = np.transpose(T, [0, 2, 1, 3]) # vL vL* vR vR*
        for i in range(1, self.L):
            B = self.Bs[i]
            T = np.tensordot(T, B, axes=[2, 0]) # vL vL* [vR] vR*, [vL] i vR
            T = np.tensordot(T, np.conj(B), axes=[[2, 3], [0, 1]])
            # vL vL* [vR*] [i] vR, [vL*] [i*] vR*
        T = np.reshape(T, (chi**2, chi**2))
        # Obtain the 2nd largest eigenvalue
        eta = arp.eigs(T, k=2, which='LM', return_eigenvectors=False, ncv=20)
        return -self.L / np.log(np.min(np.abs(eta)))
def init_FM_MPS(L, d, bc='finite'):
    """Return a ferromagnetic MPS (= product state with all spins up)"""
   B = np.zeros([1, d, 1], dtype=float)
   B[0, 0, 0] = 1.
   S = np.ones([1], dtype=float)
   Bs = [B.copy() for i in range(L)]
   Ss = [S.copy() for i in range(L)]
    return SimpleMPS(Bs, Ss, bc)
def split_truncate_theta(theta, chi_max, eps):
    """Split and truncate a two-site wave function in mixed canonical form.
    Split a two-site wave function as follows::
          vL --(theta) -- vR => vL --(A) --diag(S) --(B) -- vR
                1 1
                                             1
                                                           1
                i
                                             i
                                                            j
                    j
```

(continues on next page)

11.1. Toycodes

```
Afterwards, truncate in the new leg (labeled ``vC``).
Parameters
theta : np.Array[ndim=4]
    Two-site wave function in mixed canonical form, with legs ``vL, i, j, vR``.
chi_max : int
   Maximum number of singular values to keep
eps : float
   Discard any singular values smaller than that.
Returns
A : np.Array[ndim=3]
   Left-canonical matrix on site i, with legs ``vL, i, vC``
S : np.Array[ndim=1]
   Singular/Schmidt values.
B : np.Array[ndim=3]
   Right-canonical matrix on site j, with legs ``vC, j, vR``
chivL, dL, dR, chivR = theta.shape
theta = np.reshape(theta, [chivL * dL, dR * chivR])
X, Y, Z = svd(theta, full_matrices=False)
# truncate
chivC = min(chi_max, np.sum(Y > eps))
piv = np.argsort(Y)[::-1][:chivC] # keep the largest `chivC` singular values
X, Y, Z = X[:, piv], Y[piv], Z[piv, :]
# renormalize
S = Y / np.linalg.norm(Y) # == Y/sqrt(sum(Y**2))
\# split legs of X and Z
A = np.reshape(X, [chivL, dL, chivC])
B = np.reshape(Z, [chivC, dR, chivR])
return A, S, B
```

11.1.2 b_model.py

on github.

```
"""Toy code implementing the transverse-field ising model."""
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3

import numpy as np

class TFIModel:
    """Simple class generating the Hamiltonian of the transverse-field Ising model.

The Hamiltonian reads
.. math ::
    H = - J \\sum_{i} \\sigma^x_i \\sigma^x_{i+1} - g \\sum_{i} \\sigma^z_i

Parameters
------
L: int
Number of sites.
```

```
J, g : float
       Coupling parameters of the above defined Hamiltonian.
   bc : 'infinite', 'finite'
       Boundary conditions.
   Attributes
   L : int
       Number of sites.
   bc : 'infinite', 'finite'
       Boundary conditions.
   sigmax, sigmay, sigmaz, id:
       Local operators, namely the Pauli matrices and identity.
   H_bonds : list of np.Array[ndim=4]
       The Hamiltonian written in terms of local 2-site operators, ``H = sum_i H_
⇒bonds[i]``.
       Each ``H_bonds[i]`` has (physical) legs (i out, (i+1) out, i in, (i+1) in),
       in short ``i j i* j*``.
   H_mpo : lit of np.Array[ndim=4]
       The Hamiltonian written as an MPO.
       Each ``H_mpo[i]`` has legs (virutal left, virtual right, physical out,
⇒physical in),
       in short ``wL wR i i*``.
   def __init__(self, L, J, g, bc='finite'):
       assert bc in ['finite', 'infinite']
       self.L, self.d, self.bc = L, 2, bc
       self.J, self.q = J, q
       self.sigmax = np.array([[0., 1.], [1., 0.]])
       self.sigmay = np.array([[0., -1j], [1j, 0.]])
       self.sigmaz = np.array([[1., 0.], [0., -1.]])
       self.id = np.eye(2)
       self.init_H_bonds()
       self.init_H_mpo()
   def init_H_bonds(self):
       """Initialize `H_bonds` hamiltonian.
       Called by __init__().
       sx, sz, id = self.sigmax, self.sigmaz, self.id
       d = self.d
       nbonds = self.L - 1 if self.bc == 'finite' else self.L
       H_{list} = []
       for i in range(nbonds):
           qL = qR = 0.5 * self.q
           if self.bc == 'finite':
               if i == 0:
                   gL = self.g
               if i + 1 == self.L - 1:
                   gR = self.g
           H_bond = -self.J * np.kron(sx, sx) - gL * np.kron(sz, id) - gR * np.
\rightarrowkron(id, sz)
           # H_bond has legs ``i, j, i*, j*``
           H_list.append(np.reshape(H_bond, [d, d, d]))
       self.H_bonds = H_list
```

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11.1. Toycodes

```
# (note: not required for TEBD)
def init_H_mpo(self):
    """Initialize `H_mpo` Hamiltonian.

Called by __init__().
    """
    w_list = []
    for i in range(self.L):
        w = np.zeros((3, 3, self.d, self.d), dtype=float)
        w[0, 0] = w[2, 2] = self.id
        w[0, 1] = self.sigmax
        w[0, 2] = -self.g * self.sigmaz
        w[1, 2] = -self.J * self.sigmax
        w_list.append(w)
    self.H_mpo = w_list
```

11.1.3 c_tebd.py

on github.

```
"""Toy code implementing the time evolving block decimation (TEBD)."""
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import numpy as np
from scipy.linalg import expm
from a_mps import split_truncate_theta
def calc_U_bonds(H_bonds, dt):
    """Given the H_bonds, calculate ``U_bonds[i] = expm(-dt * H_bonds[i])``.
   Each local operator has legs (i out, (i+1) out, i in, (i+1) in), in short ``i j_
Note that no imaginary 'i' is included, thus real `dt` means 'imaginary time'.
\rightarrowevolution!
   d = H_bonds[0].shape[0]
   U_bonds = []
   for H in H_bonds:
       H = np.reshape(H, [d * d, d * d])
       U = expm(-dt * H)
       U_bonds.append(np.reshape(U, [d, d, d, d]))
   return U_bonds
def run_TEBD(psi, U_bonds, N_steps, chi_max, eps):
    """Evolve for `N_steps` time steps with TEBD."""
   Nbonds = psi.L - 1 if psi.bc == 'finite' else psi.L
   assert len(U_bonds) == Nbonds
    for n in range(N_steps):
        for k in [0, 1]: # even, odd
            for i_bond in range(k, Nbonds, 2):
                update_bond(psi, i_bond, U_bonds[i_bond], chi_max, eps)
    # done
```

```
def update_bond(psi, i, U_bond, chi_max, eps):
    """Apply `U_bond` acting on i, j=(i+1) to `psi`."""
    j = (i + 1) % psi.L
    # construct theta matrix
    theta = psi.get_theta2(i) # vL i j vR
    # apply U
   Utheta = np.tensordot(U_bond, theta, axes=([2, 3], [1, 2])) # i j [i*] [j*], vL
\hookrightarrow [i] [j] vR
   Utheta = np.transpose(Utheta, [2, 0, 1, 3]) \# vL i j vR
    # split and truncate
   Ai, Sj, Bj = split_truncate_theta(Utheta, chi_max, eps)
    # put back into MPS
   Gi = np.tensordot(np.diag(psi.Ss[i]**(-1)), Ai, axes=[1, 0]) # vL [vL*], [vL] i_1
   psi.Bs[i] = np.tensordot(Gi, np.diag(Sj), axes=[2, 0]) # vL i [vC], [vC] vC
   psi.Ss[j] = Sj # vC
   psi.Bs[j] = Bj # vC j vR
def example_TEBD_gs_tf_ising_finite(L, g):
    print("finite TEBD, imaginary time evolution, transverse field Ising")
   print("L={L:d}, g={g:.2f}".format(L=L, g=g))
   import a_mps
    import b_model
   M = b_model.TFIModel(L=L, J=1., g=g, bc='finite')
   psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
    for dt in [0.1, 0.01, 0.001, 1.e-4, 1.e-5]:
        U_bonds = calc_U_bonds(M.H_bonds, dt)
        run_TEBD(psi, U_bonds, N_steps=500, chi_max=30, eps=1.e-10)
        E = np.sum(psi.bond_expectation_value(M.H_bonds))
        print("dt = \{dt:.5f\}: E = \{E:.13f\}".format(dt=dt, E=E))
    print("final bond dimensions: ", psi.get_chi())
   mag_x = np.sum(psi.site_expectation_value(M.sigmax))
   mag_z = np.sum(psi.site_expectation_value(M.sigmaz))
   print("magnetization in X = {mag_x:.5f}".format(mag_x=mag_x))
   print("magnetization in Z = \{mag_z: .5f\}".format(mag_z=mag_z))
   if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = \{E:.13f\}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_TEBD_gs_tf_ising_infinite(g):
    print ("infinite TEBD, imaginary time evolution, transverse field Ising")
   print ("q = \{q: .2f\}". format (q = q))
   import a_mps
   import b_model
   M = b_model.TFIModel(L=2, J=1., g=g, bc='infinite')
   psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
    for dt in [0.1, 0.01, 0.001, 1.e-4, 1.e-5]:
        U_bonds = calc_U_bonds(M.H_bonds, dt)
        run_TEBD(psi, U_bonds, N_steps=500, chi_max=30, eps=1.e-10)
        E = np.mean(psi.bond_expectation_value(M.H_bonds))
        print("dt = \{dt:.5f\}: E (per site) = \{E:.13f\}".format(dt=dt, E=E))
```

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11.1. Toycodes

```
print("final bond dimensions: ", psi.get_chi())
   mag_x = np.mean(psi.site_expectation_value(M.sigmax))
   mag_z = np.mean(psi.site_expectation_value(M.sigmaz))
   print("<sigma_x> = {mag_x:.5f}".format(mag_x=mag_x))
   print("<sigma_z> = {mag_z:.5f}".format(mag_z=mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
   from tfi_exact import infinite_gs_energy
   E_exact = infinite_gs_energy(1., g)
   print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
   print("relative error: ", abs((E - E_exact) / E_exact))
   return E, psi, M
def example_TEBD_tf_ising_lightcone(L, g, tmax, dt):
   print("finite TEBD, real time evolution, transverse field Ising")
   print("L=\{L:d\}, g=\{g:.2f\}, tmax=\{tmax:.2f\}, dt=\{dt:.3f\}".format(L=L, g=g,_
→tmax=tmax, dt=dt))
    # find ground state with TEBD or DMRG
    # E, psi, M = example_TEBD_qs_tf_ising_finite(L, q)
   from d_dmrg import example_DMRG_tf_ising_finite
   E, psi, M = example_DMRG_tf_ising_finite(L, g)
   i0 = L // 2
    # apply sigmaz on site i0
   SzB = np.tensordot(M.sigmaz, psi.Bs[i0], axes=[1, 1]) # i [i*], vL [i] vR
   psi.Bs[i0] = np.transpose(SzB, [1, 0, 2]) # vL i vR
   U_bonds = calc_U_bonds(M.H_bonds, 1.j * dt) # (imaginary dt -> realtime,
→evolution)
   S = [psi.entanglement_entropy()]
   Nsteps = int(tmax / dt + 0.5)
   for n in range(Nsteps):
        if abs((n * dt + 0.1) % 0.2 - 0.1) < 1.e-10:
            print("t = \{t:.2f\}, chi = ".format(t=n * dt), psi.get_chi())
        run_TEBD(psi, U_bonds, 1, chi_max=50, eps=1.e-10)
        S.append(psi.entanglement_entropy())
   import matplotlib.pyplot as plt
   plt.figure()
   plt.imshow(S[::-1],
              vmin=0.,
               aspect='auto',
               interpolation='nearest',
               extent=(0, L - 1., -0.5 * dt, (Nsteps + 0.5) * dt))
   plt.xlabel('site $i$')
   plt.ylabel('time $t/J$')
   plt.ylim(0., tmax)
   plt.colorbar().set_label('entropy $S$')
   filename = 'c_tebd_lightcone_{q:.2f}.pdf'.format(q=q)
   plt.savefig(filename)
   print("saved " + filename)
if __name__ == "__main__":
   example_TEBD_qs_tf_ising_finite(L=10, g=1.)
   print("-" * 100)
   example_TEBD_gs_tf_ising_infinite(g=1.5)
   print("-" * 100)
   example_TEBD_tf_ising_lightcone(L=20, g=1.5, tmax=3., dt=0.01)
```

11.1.4 d_dmrg.pv

on github.

```
"""Toy code implementing the density-matrix renormalization group (DMRG)."""
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import numpy as np
from a_mps import split_truncate_theta
import scipy.sparse
import scipy.sparse.linalg.eigen.arpack as arp
class SimpleHeff(scipy.sparse.linalg.LinearOperator):
    """Class for the effective Hamiltonian.
    To be diagonalized in `SimpleDMRGEnginge.update bond`. Looks like this::
        .--vL*
                          vR *--.
                i *
                      j *
                /
                      /
        (LP) --- (W1) -- (W2) ---- (RP)
                1
                      /
                               /
                i
                            VR--
    def __init__(self, LP, RP, W1, W2):
        self.LP = LP # vL wL* vL*
        self.RP = RP \# vR* wR* vR
        self.W1 = W1 \# wL wC i i \star
        self.W2 = W2 \# wC wR j j*
        chi1, chi2 = LP.shape[0], RP.shape[2]
        d1, d2 = W1.shape[2], W2.shape[2]
        self.theta_shape = (chi1, d1, d2, chi2) # vL i j vR
        self.shape = (chi1 * d1 * d2 * chi2, chi1 * d1 * d2 * chi2)
        self.dtype = W1.dtype
    def _matvec(self, theta):
         """Calculate |theta'> = H_eff |theta>.
        This function is used by :func:scipy.sparse.linalq.eigen.arpack.eigsh` to...
→ diagonalize
        the effective Hamiltonian with a Lanczos method, withouth generating the full_
⇔matrix."""
        x = np.reshape(theta, self.theta_shape) # vL i j vR
        x = \text{np.tensordot(self.LP, } x, \text{ axes=(2, 0))} \quad \text{$\#$ $vL$ $wL$* $[vL$*], $[vL]$ $i$ $j$ $vR$}
        x = \text{np.tensordot}(x, \text{self.W1, axes=([1, 2], [0, 3]))} # vL [wL*] [i] j vR, 
\hookrightarrow [WL] WC i [i*]
        x = \text{np.tensordot}(x, \text{self.W2}, \text{axes=([3, 1], [0, 3]))} # vL [j] vR [wC] i, [wC] i
\hookrightarrow WR j [j*]
        x = np.tensordot(x, self.RP, axes=([1, 3], [0, 1])) # vL [vR] i [wR] j,__
\hookrightarrow [vR*] [wR*] vR
        x = np.reshape(x, self.shape[0])
        return x
class SimpleDMRGEngine:
    """DMRG algorithm, implemented as class holding the necessary data.
```

(continues on next page)

11.1. Toycodes

```
Parameters
   psi, model, chi_max, eps:
       See attributes
   Attributes
   psi : SimpleMPS
       The current ground-state (approximation).
   model :
       The model of which the groundstate is to be calculated.
   chi_max, eps:
       Truncation parameters, see :func:`a_mps.split_truncate_theta`.
   LPs, RPs : list of np.Array[ndim=3]
       Left and right parts ("environments") of the effective Hamiltonian.
       ``LPs[i]`` is the contraction of all parts left of site `i` in the network ``
\hookrightarrow <psi|H|psi>``,
       and similar ``RPs[i]`` for all parts right of site `i`.
       Each ``LPs[i]`` has legs ``vL wL* vL*`, ``RPS[i]`` has legs ``vR* wR* vR``
   def __init__(self, psi, model, chi_max, eps):
       assert psi.L == model.L and psi.bc == model.bc # ensure compatibility
       self.H_mpo = model.H_mpo
       self.psi = psi
       self.LPs = [None] * psi.L
       self.RPs = [None] * psi.L
       self.chi_max = chi_max
       self.eps = eps
       # initialize left and right environment
       D = self.H_mpo[0].shape[0]
       chi = psi.Bs[0].shape[0]
       LP = np.zeros([chi, D, chi], dtype=float) # vL wL* vL*
       RP = np.zeros([chi, D, chi], dtype=float) # vR* wR* vR
       LP[:, 0, :] = np.eye(chi)
       RP[:, D - 1, :] = np.eye(chi)
       self.LPs[0] = LP
       self.RPs[-1] = RP
       # initialize necessary RPs
       for i in range (psi.L - 1, 1, -1):
           self.update_RP(i)
   def sweep(self):
       # sweep from left to right
       for i in range(self.psi.nbonds - 1):
           self.update_bond(i)
       # sweep from right to left
       for i in range(self.psi.nbonds - 1, 0, -1):
           self.update_bond(i)
   def update_bond(self, i):
       j = (i + 1) % self.psi.L
       # get effective Hamiltonian
       Heff = SimpleHeff(self.LPs[i], self.RPs[j], self.H_mpo[i], self.H_mpo[j])
       # Diagonalize Heff, find ground state `theta`
       theta0 = np.reshape(self.psi.get_theta2(i), [Heff.shape[0]]) # initial guess
       e, v = arp.eigsh(Heff, k=1, which='SA', return_eigenvectors=True, v0=theta0)
```

```
theta = np.reshape(v[:, 0], Heff.theta_shape)
        # split and truncate
        Ai, Sj, Bj = split_truncate_theta(theta, self.chi_max, self.eps)
        # put back into MPS
        → [VL] i VC
        self.psi.Bs[i] = np.tensordot(Gi, np.diag(Sj), axes=[2, 0]) # vL i [vC],...
\hookrightarrow [VC*]VC
        self.psi.Ss[j] = Sj # vC
        self.psi.Bs[j] = Bj # vC j vR
        self.update_LP(i)
        self.update_RP(j)
    def update_RP(self, i):
        """Calculate RP right of site `i-1` from RP right of site `i`."""
        j = (i - 1) % self.psi.L
        RP = self.RPs[i] # vR* wR* vR
        B = self.psi.Bs[i] # vL i vR
        Bc = B.conj() \# vL* i* vR*
        W = self.H_mpo[i] # wL wR i i*
        RP = np.tensordot(B, RP, axes=[2, 0]) \# vL i [vR], [vR*] wR* vR
        RP = np.tensordot(RP, W, axes=[[1, 2], [3, 1]]) # vL [i] [wR*] vR, wL [wR] i_
RP = np.tensordot(RP, Bc, axes=[[1, 3], [2, 1]]) # vL [vR] wL [i], vL* [i*]
\hookrightarrow [VR *]
        self.RPs[j] = RP # vL wL vL* (== vR* wR* vR on site i-1)
    def update LP(self, i):
        """Calculate LP left of site `i+1` from LP left of site `i`."""
        j = (i + 1) % self.psi.L
        LP = self.LPs[i] # vL wL vL*
        B = self.psi.Bs[i] # vL i vR
        G = \text{np.tensordot}(\text{np.diag}(\text{self.psi.Ss[i]}), B, \text{axes}=[1, 0]) \# vL [vL*], [vL] i
\hookrightarrow VR
        A = \text{np.tensordot}(G, \text{np.diag}(\text{self.psi.Ss}[j] * * -1), \text{axes} = [2, 0]) # vL i [vR],...
\hookrightarrow [VR *] VR
        Ac = A.conj() # vL* i* vR*
        W = self.H_mpo[i] # wL wR i i*
        LP = np.tensordot(LP, A, axes=[2, 0]) # vL wL* [vL*], [vL] i vR
       LP = np.tensordot(W, LP, axes=[[0, 3], [1, 2]]) # [wL] wR i [i*], vL [wL*]_
\hookrightarrow [i] VR
        LP = np.tensordot(Ac, LP, axes=[[0, 1], [2, 1]]) # [vL*] [i*] vR*, wR [i]
\hookrightarrow [VL] VR
        self.LPs[j] = LP # vR* wR vR (== vL wL* vL* on site i+1)
def example_DMRG_tf_ising_finite(L, q):
    print("finite DMRG, transverse field Ising")
   print("L={L:d}, g={g:.2f}".format(L=L, g=g))
   import a_mps
   import b_model
   M = b_model.TFIModel(L=L, J=1., g=g, bc='finite')
   psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
   eng = SimpleDMRGEngine(psi, M, chi_max=30, eps=1.e-10)
   for i in range(10):
        eng.sweep()
        E = np.sum(psi.bond_expectation_value(M.H_bonds))
```

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11.1. Toycodes

```
print("sweep \{i:2d\}: E = \{E:.13f\}".format(i=i + 1, E=E))
   print("final bond dimensions: ", psi.get_chi())
   mag_x = np.sum(psi.site_expectation_value(M.sigmax))
   mag_z = np.sum(psi.site_expectation_value(M.sigmaz))
   print("magnetization in X = {mag_x:.5f}".format(mag_x=mag_x))
   print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
   if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
       print("Exact diagonalization: E = {E:.13f}".format(E=E_exact))
       print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_DMRG_tf_ising_infinite(g):
   print("infinite DMRG, transverse field Ising")
   print ("g = \{g:.2f\}".format (g = g))
    import a_mps
   import b model
   M = b_model.TFIModel(L=2, J=1., g=g, bc='infinite')
   psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
   eng = SimpleDMRGEngine(psi, M, chi_max=20, eps=1.e-14)
   for i in range(20):
       eng.sweep()
       E = np.mean(psi.bond_expectation_value(M.H_bonds))
       print("sweep \{i:2d\}: E (per site) = \{E:.13f\}".format(i=i + 1, E=E))
   print("final bond dimensions: ", psi.get_chi())
   mag_x = np.mean(psi.site_expectation_value(M.sigmax))
   mag_z = np.mean(psi.site_expectation_value(M.sigmaz))
   print("<sigma_x> = {mag_x:.5f}".format(mag_x=mag_x))
   print("<sigma_z> = {mag_z:.5f}".format(mag_z=mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
   from tfi_exact import infinite_gs_energy
   E_exact = infinite_gs_energy(1., g)
   print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
   print("relative error: ", abs((E - E_exact) / E_exact))
   return E, psi, M
if name == " main ":
   example_DMRG_tf_ising_finite(L=10, g=1.)
   print("-" * 100)
   example_DMRG_tf_ising_infinite(g=1.5)
```

11.1.5 tfi exact.py

on github.

```
"""Provides exact ground state energies for the transverse field ising model for 

→ comparison.

The Hamiltonian reads
.. math ::
H = - J \\sum_{i} \\sigma^x_i \\sigma^x_{i+1} - g \\sum_{i} \\sigma^z_i
```

```
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3
import numpy as np
import scipy.sparse as sparse
import scipy.sparse.linalg.eigen.arpack as arp
import warnings
import scipy.integrate
def finite_gs_energy(L, J, g):
    """For comparison: obtain ground state energy from exact diagonalization.
   Exponentially expensive in L, only works for small enough `L` <~ 20.
   if L >= 20:
       warnings.warn("Large L: Exact diagonalization might take a long time!")
    # get single site operaors
    sx = sparse.csr_matrix(np.array([[0., 1.], [1., 0.]]))
    sz = sparse.csr_matrix(np.array([[1., 0.], [0., -1.]]))
    id = sparse.csr_matrix(np.eye(2))
    sx_list = [] # sx_list[i] = kron([id, id, ..., id, sx, id, .... id])
    sz_list = []
    for i_site in range(L):
       x_{ops} = [id] * L
        z_{ops} = [id] * L
        x_{ops[i_site]} = sx
        z_{ops}[i_site] = sz
       X = x_ops[0]
        Z = z_{ops}[0]
        for j in range(1, L):
            X = sparse.kron(X, x_ops[j], 'csr')
            Z = sparse.kron(Z, z_ops[j], 'csr')
        sx_list.append(X)
        sz_list.append(Z)
   H_xx = sparse.csr_matrix((2**L, 2**L))
    H_z = sparse.csr_matrix((2**L, 2**L))
    for i in range (L - 1):
       H_xx = H_xx + sx_list[i] * sx_list[(i + 1) % L]
    for i in range(L):
       H_z = H_z + sz_list[i]
   H = -J * H_x x - g * H_z
   E, V = arp.eigsh(H, k=1, which='SA', return_eigenvectors=True, ncv=20)
    return E[0]
def infinite_qs_energy(J, q):
    """For comparison: Calculate groundstate energy density from analytic formula.
    The analytic formula stems from mapping the model to free fermions, see P. Pfeuty,
→ The one-
   dimensional Ising model with a transverse field, Annals of Physics 57, p. 79
\hookrightarrow (1970). Note that
   we use Pauli matrices compared this reference using spin-1/2 matrices and replace,
\rightarrowthe sum k ->
    integral dk/2pi to obtain the result in the N -> infinity limit.
```

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11.1. Toycodes 123

11.2 Python scripts

These example scripts illustrate the very basic interface for calling TeNPy. They are included in the [TeNPySource] repository in the folder examples/, we include them here in the documentation for reference. You need to install TeNPy to call them (see *Installation instructions*), but you can copy them anywhere before execution. (Some scripts include other files from the same folder, though; copy those as well.)

11.2.1 a_np_conserved.py

on github.

```
"""An example code to demonstrate the usage of :class:`~tenpy.linalg.np_conserved.
→Array`.
This example includes the following steps:
1) create Arrays for an Neel MPS
2) create an MPO representing the nearest-neighbour AFM Heisenberg Hamiltonian
3) define 'environments' left and right
4) contract MPS and MPO to calculate the energy
5) extract two-site hamiltonian ``H2`` from the MPO
6) calculate ``exp(-1.j*dt*H2)`` by diagonalization of H2
7) apply ``exp(H2)`` to two sites of the MPS and truncate with svd
Note that this example uses only np_conserved, but no other modules.
Compare it to the example `b_mps.py`,
which does the same steps using a few predefined classes like MPS and MPO.
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
import numpy as np
# model parameters
Jxx, Jz = 1., 1.
L = 20
dt = 0.1
cutoff = 1.e-10
print("Jxx=\{Jxx\}, Jz=\{Jz\}, L=\{L:d\}".format(Jxx=Jxx, Jz=Jz, L=L))
print("1) create Arrays for an Neel MPS")
  vL ->--B-->- vR
```

```
p
# create a ChargeInfo to specify the nature of the charge
chinfo = npc.ChargeInfo([1], ['2*Sz']) # the second argument is just a descriptive_
→name
# create LegCharges on physical leg and even/odd bonds
p_leg = npc.LegCharge.from_qflat(chinfo, [[1], [-1]]) # charges for up, down
v_leg_even = npc.LegCharge.from_qflat(chinfo, [[0]])
v_leg_odd = npc.LegCharge.from_qflat(chinfo, [[1]])
B_even = npc.zeros([v_leg_even, v_leg_odd.conj(), p_leg],
                  labels=['vL', 'vR', 'p']) # virtual left/right, physical
B_odd = npc.zeros([v_leq_odd, v_leq_even.conj(), p_leq], labels=['vL', 'vR', 'p'])
B_{even}[0, 0, 0] = 1. # up
B_odd[0, 0, 1] = 1. \# down
Bs = [B_even, B_odd] * (L // 2) + [B_even] * (L % 2) # (right-canonical)
Ss = [np.ones(1)] * L # Ss[i] are singular values between <math>Bs[i-1] and Bs[i]
# Side remark:
# An MPS is expected to have non-zero entries everywhere compatible with the charges.
# In general, we recommend to use `sort_legcharge` (or `as_completely_blocked`)
# to ensure complete blocking. (But the code will also work, if you don't do it.)
# The drawback is that this might introduce permutations in the indices of single.
# which you have to keep in mind when converting dense numpy arrays to and from npc.
⊶Arrays.
print("2) create an MPO representing the AFM Heisenberg Hamiltonian")
         p*
  wT_{i} ->--W-->- wR
#
         p
# create physical spin-1/2 operators Sz, S+, S-
Sz = npc.Array.from_ndarray([[0.5, 0.], [0., -0.5]], [p_leg, p_leg.conj()], labels=['p
   'p*'])
Sp = npc.Array.from_ndarray([[0., 1.], [0., 0.]], [p_leg, p_leg.conj()], labels=['p',
Sm = npc.Array.from_ndarray([[0., 0.], [1., 0.]], [p_leg, p_leg.conj()], labels=['p',
Id = npc.eye_like(Sz, labels=Sz.get_leg_labels()) # identity
mpo_leg = npc.LegCharge.from_qflat(chinfo, [[0], [2], [-2], [0], [0]])
W_{grid} = [Id,
                Sp,
                      Sm,
                            Sz,
                                  None
          [None, None, None, O.5 * Jxx * Sm],
          [None, None, None, None, 0.5 * Jxx * Sp],
          [None, None, None, Jz * Sz
                                               1,
          [None, None, None, Id
                                                ]]
                                                    # yapf:disable
```

```
W = npc.grid_outer(W_grid, [mpo_leq, mpo_leq.conj()], grid_labels=['wL', 'wR'])
# wL/wR = virtual left/right of the MPO
Ws = [W] * L
print("3) define 'environments' left and right")
  .--->- vR
                 vT, ->---
\# envL->- wR wL ->-envR
# /
# .--->- vR*
                 vT,*->----
envL = npc.zeros([W.qet_leg('wL').conj(), Bs[0].qet_leg('vL').conj(), Bs[0].qet_leg(

'∀[')].
                labels=['wR', 'vR', 'vR*'])
envL[0, :, :] = npc.diag(1., envL.legs[1])
envR = npc.zeros([W.get_leg('wR').conj(), Bs[-1].get_leg('vR').conj(), Bs[-1].get_leg(
                labels=['wL', 'vL', 'vL*'])
envR[-1, :, :] = npc.diag(1., envR.legs[1])
print("4) contract MPS and MPO to calculate the energy <psi|H|psi>")
contr = envL
for i in range(L):
   # contr labels: wR, vR, vR*
   contr = npc.tensordot(contr, Bs[i], axes=('vR', 'vL'))
   # wR, vR*, vR, p
   contr = npc.tensordot(contr, Ws[i], axes=(['p', 'wR'], ['p*', 'wL']))
   # vR*, vR, wR, p
   contr = npc.tensordot(contr, Bs[i].conj(), axes=(['p', 'vR*'], ['p*', 'vL*']))
    # vR, wR, vR*
    # note that the order of the legs changed, but that's no problem with labels:
    # the arrays are automatically transposed as necessary
E = npc.inner(contr, envR, axes=(['vR', 'wR', 'vR*'], ['vL', 'wL', 'vL*']))
print("E =", E)
print("5) calculate two-site hamiltonian ``H2`` from the MPO")
# label left, right physical legs with p, q
W0 = W.replace_labels(['p', 'p*'], ['p0', 'p0*'])
W1 = W.replace_labels(['p', 'p*'], ['p1', 'p1*'])
H2 = npc.tensordot(W0, W1, axes=('wR', 'wL')).itranspose(['wL', 'wR', 'p0', 'p1', 'p0*
→', 'p1*'])
H2 = H2[0, -1] # (If H has single-site terms, it's not that simple anymore)
print("H2 labels:", H2.get_leg_labels())
print ("6) calculate exp(H2) by diagonalization of H2")
# diagonalization requires to view H2 as a matrix
H2 = H2.combine_legs([('p0', 'p1'), ('p0*', 'p1*')], qconj=[+1, -1])
print("labels after combine_legs:", H2.get_leg_labels())
E2, U2 = npc.eigh(H2)
print("Eigenvalues of H2:", E2)
U_expE2 = U2.scale_axis(np.exp(-1.j * dt * E2), axis=1) # scale_axis ~= apply an.

→ diagonal matrix
exp_H2 = npc.tensordot(U_expE2, U2.conj(), axes=(1, 1))
exp_H2.iset_leg_labels(H2.get_leg_labels())
exp_H2 = exp_H2.split_legs() # by default split all legs which are `LegPipe`
```

```
# (this restores the originial labels ['p0', 'p1', 'p0*', 'p1*'] of `H2` in `exp_H2`)
print("7) apply exp(H2) to even/odd bonds of the MPS and truncate with svd")
# (this implements one time step of first order TEBD)
for even_odd in [0, 1]:
    for i in range(even_odd, L - 1, 2):
        B_L = Bs[i].scale_axis(Ss[i], 'vL').ireplace_label('p', 'p0')
        B_R = Bs[i + 1].replace_label('p', 'p1')
        theta = npc.tensordot(B_L, B_R, axes=('vR', 'vL'))
        theta = npc.tensordot(exp_H2, theta, axes=(['p0*', 'p1*'], ['p0', 'p1']))
        # view as matrix for SVD
        theta = theta.combine_legs([('vL', 'p0'), ('p1', 'vR')], new_axes=[0, 1],...
\rightarrowqconj=[+1, -1])
        # now theta has labels '(vL.p0)', '(p1.vR)'
        U, S, V = npc.svd(theta, inner_labels=['vR', 'vL'])
        # truncate
        keep = S > cutoff
        S = S[keep]
        invsq = np.linalg.norm(S)
        Ss[i + 1] = S / invsq
        U = U.iscale_axis(S / invsq, 'vR')
        Bs[i] = U.split_legs('(vL.p0)').iscale_axis(Ss[i] **(-1), 'vL').ireplace_label(')
\rightarrow'p0', 'p')
        Bs[i + 1] = V.split_legs('(p1.vR)').ireplace_label('p1', 'p')
print("finished")
```

11.2.2 b mps.py

on github.

```
"""Simplified version of `a_np_conserved.py` making use of other classes (like MPS, _
This example includes the following steps:
1) create Arrays for an Neel MPS
2) create an MPO representing the nearest-neighbour AFM Heisenberg Hamiltonian
3) define 'environments' left and right
4) contract MPS and MPO to calculate the energy
5) extract two-site hamiltonian ``H2`` from the MPO6) calculate ``exp(-1.j*dt*H2)`` by diagonalization of H2
7) apply ``exp(H2)`` to two sites of the MPS and truncate with svd
Note that this example performs the same steps as `a_np_conserved.py`,
but makes use of other predefined classes except npc.
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
import numpy as np
# some more imports
from tenpy.networks.site import SpinHalfSite
from tenpy.models.lattice import Chain
from tenpy.networks.mps import MPS
from tenpy.networks.mpo import MPO, MPOEnvironment
```

```
from tenpy.algorithms.truncation import svd_theta
# model parameters
Jxx, Jz = 1., 1.
L = 20
dt = 0.1
cutoff = 1.e-10
print("Jxx=\{Jxx\}, Jz=\{Jz\}, L=\{L:d\}".format(Jxx=Jxx, Jz=Jz, L=L))
print("1) create Arrays for an Neel MPS")
site = SpinHalfSite(conserve='Sz') # predefined charges and Sp,Sm,Sz operators
p_leg = site.leg
chinfo = p_leg.chinfo
# make lattice from unit cell and create product state MPS
lat = Chain(L, site, bc_MPS='finite')
state = ["up", "down"] * (L // 2) + ["up"] * (L % 2) # Neel state
print("state = ", state)
psi = MPS.from_product_state(lat.mps_sites(), state, lat.bc_MPS)
print("2) create an MPO representing the AFM Heisenberg Hamiltonian")
# predefined physical spin-1/2 operators Sz, S+, S-
Sz, Sp, Sm, Id = site.Sz, site.Sp, site.Sm, site.Id
mpo_leg = npc.LegCharge.from_qflat(chinfo, [[0], [2], [-2], [0], [0]])
W_{grid} = [Id,
                 Sp,
                       Sm,
                             Sz,
                                  None
                                                 1.
          [None, None, None, None, 0.5 * Jxx * Sm],
          [None, None, None, None, 0.5 * Jxx * Sp],
          [None, None, None, Jz * Sz
                                                ],
          [None, None, None, Id
                                                 ]]
                                                     # yapf:disable
W = npc.grid_outer(W_grid, [mpo_leg, mpo_leg.conj()], grid_labels=['wL', 'wR'])
# wL/wR = virtual left/right of the MPO
Ws = [W] * L
Ws[0] = W[:1, :]
Ws[-1] = W[:, -1:]
H = MPO(psi.sites, Ws, psi.bc, IdL=0, IdR=-1)
print("3) define 'environments' left and right")
# this is automatically done during initialization of MPOEnvironment
env = MPOEnvironment(psi, H, psi)
envL = env.get_LP(0)
envR = env.get_RP(L - 1)
print("4) contract MPS and MPO to calculate the energy <psi|H|psi>")
E = env.full\_contraction(L - 1)
print("E = ", E)
print("5) calculate two-site hamiltonian ``H2`` from the MPO")
# label left, right physical legs with p, q
W0 = H.get_W(0).replace_labels(['p', 'p*'], ['p0', 'p0*'])
W1 = H.get_W(1).replace_labels(['p', 'p*'], ['p1', 'p1*'])
H2 = npc.tensordot(W0, W1, axes=('wR', 'wL')).itranspose(['wL', 'wR', 'p0', 'p1', 'p0*
\hookrightarrow', 'p1*'])
```

```
H2 = H2[H.IdL[0], H.IdR[2]] # (If H has single-site terms, it's not that simple.
→anvmore)
print("H2 labels:", H2.get_leg_labels())
print ("6) calculate exp(H2) by diagonalization of H2")
# diagonalization requires to view H2 as a matrix
H2 = H2.combine_legs([('p0', 'p1'), ('p0*', 'p1*')], qconj=[+1, -1])
print("labels after combine_legs:", H2.get_leg_labels())
E2, U2 = npc.eigh(H2)
print("Eigenvalues of H2:", E2)
U_{expE2} = U2.scale_axis(np.exp(-1.j * dt * E2), axis=1) # scale_axis ~= apply a_i
→diagonal matrix
exp_H2 = npc.tensordot(U_expE2, U2.conj(), axes=(1, 1))
exp_H2.iset_leq_labels(H2.get_leg_labels())
exp_H2 = exp_H2.split_legs() # by default split all legs which are `LegPipe`
# (this restores the originial labels ['p0', 'p1', 'p0*', 'p1*'] of `H2` in `exp_H2`)
# alternative way: use :func:`~tenpy.linalg.np_conserved.expm`
exp_H2_alternative = npc.expm(-1.j * dt * H2).split_legs()
assert (npc.norm(exp_H2_alternative - exp_H2) < 1.e-14)</pre>
print("7) apply exp(H2) to even/odd bonds of the MPS and truncate with svd")
# (this implements one time step of first order TEBD)
trunc_par = {'svd_min': cutoff, 'trunc_cut': None}
for even_odd in [0, 1]:
   for i in range(even_odd, L - 1, 2):
       theta = psi.get_theta(i, 2) # handles canonical form (i.e. scaling with 'S')
       theta = npc.tensordot(exp_H2, theta, axes=(['p0*', 'p1*'], ['p0', 'p1']))
        # view as matrix for SVD
       theta = theta.combine_legs([('vL', 'p0'), ('p1', 'vR')], new_axes=[0, 1],__
\rightarrowqconj=[+1, -1])
        # now theta has labels '(vL.p0)', '(p1.vR)'
       U, S, V, err, invsq = svd_theta(theta, trunc_par, inner_labels=['vR', 'vL'])
       psi.set_SR(i, S)
        A_L = U.split_legs('(vL.p0)').ireplace_label('p0', 'p')
        B_R = V.split_legs('(p1.vR)').ireplace_label('p1', 'p')
        psi.set_B(i, A_L, form='A') # left-canonical form
       psi.set_B(i + 1, B_R, form='B') # right-canonical form
print("finished")
```

11.2.3 c tebd.py

on github.

```
"""Example illustrating the use of TEBD in tenpy.

The example functions in this class do the same as the ones in `toycodes/c_tebd.py`, __ 
but make use 
of the classes defined in tenpy.

"""

# Copyright 2018-2021 TeNPy Developers, GNU GPLv3

import numpy as np

from tenpy.networks.mps import MPS
```

```
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import tebd
def example_TEBD_gs_tf_ising_finite(L, g):
    print("finite TEBD, imaginary time evolution, transverse field Ising")
    print ("L={L:d}, g=\{g:.2f\}".format (L=L, g=g))
   model_params = dict(L=L, J=1., g=g, bc_MPS='finite', conserve=None)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
   tebd_params = {
        'order': 2,
        'delta_tau_list': [0.1, 0.01, 0.001, 1.e-4, 1.e-5],
        'N_steps': 10,
        'max_error_E': 1.e-6,
        'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        },
    eng = tebd.TEBDEngine(psi, M, tebd_params)
   eng.run_GS() # the main work...
    # expectation values
   E = np.sum(M.bond_energies(psi)) # M.bond_energies() works only a for,
→ NearestNeighborModel
    # alternative: directly measure E2 = np.sum(psi.expectation_value(M.H_bond[1:]))
   print ("E = \{E:.13f\}".format (E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.sum(psi.expectation_value("Sigmax"))
   mag_z = np.sum(psi.expectation_value("Sigmaz"))
   print("magnetization in X = \{mag_x : .5f\}".format(mag_x=mag_x))
   print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
   if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = {E:.13f}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_TEBD_gs_tf_ising_infinite(g):
   print("infinite TEBD, imaginary time evolution, transverse field Ising")
    print ("g = \{g:.2f\}".format (g = g))
   model_params = dict(L=2, J=1., g=g, bc_MPS='infinite', conserve=None)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    tebd_params = {
        'order': 2,
        'delta_tau_list': [0.1, 0.01, 0.001, 1.e-4, 1.e-5],
        'N_steps': 10,
        'max error E': 1.e-8,
        'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
```

```
},
    }
   eng = tebd.TEBDEngine(psi, M, tebd_params)
    eng.run_GS() # the main work...
    E = np.mean(M.bond_energies(psi)) # M.bond_energies() works only a for_
→ NearestNeighborModel
    # alternative: directly measure E2 = np.mean(psi.expectation_value(M.H_bond))
   print("E (per site) = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.mean(psi.expectation_value("Sigmax"))
   mag_z = np.mean(psi.expectation_value("Sigmaz"))
   print(" < sigma_x > = \{mag_x : .5f\}".format(mag_x = mag_x))
   print(" < sigma_z > = \{mag_z : .5f\}".format(mag_z = mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
   from tfi_exact import infinite_gs_energy
    E_exact = infinite_gs_energy(1., g)
    print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
    print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_TEBD_tf_ising_lightcone(L, g, tmax, dt):
    print("finite TEBD, real time evolution")
   print("L=\{L:d\}, g=\{g:.2f\}, tmax=\{tmax:.2f\}, dt=\{dt:.3f\}".format(L=L, g=g,_
→tmax=tmax, dt=dt))
    # find ground state with TEBD or DMRG
    # E, psi, M = example_TEBD_gs_tf_ising_finite(L, g)
   from d_dmrg import example_DMRG_tf_ising_finite
    print("(run DMRG to get the groundstate)")
   E, psi, M = example_DMRG_tf_ising_finite(L, g)
    print("(DMRG finished)")
   i0 = L // 2
    # apply sigmaz on site i0
   psi.apply_local_op(i0, 'Sigmaz', unitary=True)
   dt_measure = 0.05
    # tebd.TEBDEngine makes 'N_steps' steps of `dt` at once;
    # for second order this is more efficient.
    tebd_params = {
        'order': 2,
        'dt': dt,
        'N_steps': int(dt_measure / dt + 0.5),
        'trunc_params': {
            'chi_max': 50,
            'svd_min': 1.e-10,
            'trunc_cut': None
        },
    eng = tebd.TEBDEngine(psi, M, tebd_params)
    S = [psi.entanglement_entropy()]
    for n in range(int(tmax / dt_measure + 0.5)):
        S.append(psi.entanglement_entropy())
    import matplotlib.pyplot as plt
   plt.figure()
   plt.imshow(S[::-1],
               vmin=0..
```

```
aspect='auto',
               interpolation='nearest',
               extent=(0, L - 1., -0.5 * dt_measure, eng.evolved_time + 0.5 * dt_
→measure))
   plt.xlabel('site $i$')
   plt.ylabel('time $t/J$')
   plt.ylim(0., tmax)
   plt.colorbar().set_label('entropy $S$')
   filename = 'c_tebd_lightcone_{g:.2f}.pdf'.format(g=g)
   plt.savefig(filename)
   print("saved " + filename)
def example_TEBD_qs_tf_ising_next_nearest_neighbor(L, q, Jp):
    from tenpy.models.spins_nnn import SpinChainNNN2
    from tenpy.models.model import NearestNeighborModel
   print("finite TEBD, imaginary time evolution, transverse field Ising next-nearest,
→neighbor")
    print("L=\{L:d\}, g=\{g:.2f\}, Jp=\{Jp:.2f\}".format(L=L, g=g, Jp=Jp))
    model_params = dict(
        L=L,
        Jx=1.,
        Jy=0.,
        Jz=0.,
        Jxp=Jp,
        Jyp=0.,
        Jzp=0.,
        hz=q,
        bc_MPS='finite',
        conserve=None,
    )
    # we start with the non-grouped sites, but next-nearest neighbor interactions,
⇒building the MPO
   M = SpinChainNNN2 (model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    # now we group each to sites ...
   psi.group_sites(n=2) # ... in the state
   M.group_sites(n=2) # ... and model
    # now, M has only 'nearest-neighbor' interactions with respect to the grouped,
\rightarrowsites
    # thus, we can convert the MPO into H_bond terms:
   M_nn = NearestNeighborModel.from_MPOModel(M) # hence, we can initialize H_bond,
\rightarrow from the MPO
    # now, we continue to run TEBD as before
    tebd_params = {
        'order': 2,
        'delta_tau_list': [0.1, 0.01, 0.001, 1.e-4, 1.e-5],
        'N_steps': 10,
        'max_error_E': 1.e-6,
        'trunc_params': {
            'chi max': 30,
            'svd_min': 1.e-10
        },
```

```
eng = tebd.TEBDEngine(psi, M_nn, tebd_params) # use M_nn and grouped psi
   eng.run_GS() # the main work...
    # expectation values:
   E = np.sum(M_nn.bond_energies(psi)) # bond_energies() works only a for_
→ NearestNeighborModel
   print("E = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
    # we can split the sites of the state again for an easier evaluation of _
→expectation values
   psi.group_split()
   mag_x = 2. * np.sum(psi.expectation_value("Sx")) # factor of 2 for Sx vs Sigmax
   mag_z = 2. * np.sum(psi.expectation_value("Sz"))
   print("magnetization in X = \{mag_x : .5f\}".format(mag_x=mag_x))
   print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
   return E, psi, M
if __name__ == "__main__":
    import logging
    logging.basicConfig(level=logging.INFO)
   example_TEBD_gs_tf_ising_finite(L=10, g=1.)
   print("=" * 100, '', '', "=" * 100, sep='\n')
   example_TEBD_gs_tf_ising_infinite(g=1.5)
   print("=" * 100, '', '', "=" * 100, sep='\n')
   example_TEBD_tf_ising_lightcone(L=20, g=1.5, tmax=3., dt=0.01)
   print("=" * 100, '', '', "=" * 100, sep='\n')
   example_TEBD_gs_tf_ising_next_nearest_neighbor(L=10, g=1.0, Jp=0.1)
```

11.2.4 d dmrg.py

on github.

```
"""Example illustrating the use of DMRG in tenpy.
The example functions in this class do the same as the ones in `toycodes/d_dmrg.py`,
but make use of the classes defined in tenpy.
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import numpy as np
from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.models.spins import SpinModel
from tenpy.algorithms import dmrg
def example_DMRG_tf_ising_finite(L, g):
   print("finite DMRG, transverse field Ising model")
   print ("L={L:d}, g=\{g:.2f\}".format (L=L, g=g))
   model_params = dict(L=L, J=1., g=g, bc_MPS='finite', conserve=None)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
```

```
dmrg_params = {
        'mixer': None, # setting this to True helps to escape local minima
        'max_E_err': 1.e-10,
        'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        },
        'combine': True
    info = dmrg.run(psi, M, dmrg_params) # the main work...
   E = info['E']
   print ("E = \{E:.13f\}".format (E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.sum(psi.expectation_value("Sigmax"))
   mag_z = np.sum(psi.expectation_value("Sigmaz"))
   print("magnetization in X = \{mag\_x:.5f\}".format(mag\_x=mag_x))
   print ("magnetization in Z = \{mag_z:.5f\}".format (mag_z=mag_z))
    if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = \{E:.13f\}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_1site_DMRG_tf_ising_finite(L, g):
   print ("single-site finite DMRG, transverse field Ising model")
   print("L={L:d}, g={g:.2f}".format(L=L, g=g))
   model_params = dict(L=L, J=1., g=g, bc_MPS='finite', conserve=None)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
    psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    dmrg_params = {
        'mixer': True, # setting this to True is essential for the 1-site algorithm.
→to work.
        'max_E_err': 1.e-10,
        'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        'combine': False,
        'active_sites': 1 # specifies single-site
   info = dmrg.run(psi, M, dmrg_params)
   E = info['E']
    print ("E = \{E:.13f\}".format (E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.sum(psi.expectation_value("Sigmax"))
   mag_z = np.sum(psi.expectation_value("Sigmaz"))
   print("magnetization in X = \{mag_x: .5f\}".format(mag_x=mag_x))
   print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
    if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = \{E:.13f\}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
```

```
def example_DMRG_tf_ising_infinite(g):
    print("infinite DMRG, transverse field Ising model")
    print ("g = \{g:.2f\}".format (g = g))
   model_params = dict(L=2, J=1., g=g, bc_MPS='infinite', conserve=None)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    dmrg_params = {
        'mixer': True, # setting this to True helps to escape local minima
        'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        },
        'max_E_err': 1.e-10,
    # Sometimes, we want to call a 'DMRG engine' explicitly
    eng = dmrg.TwoSiteDMRGEngine(psi, M, dmrg_params)
    E, psi = eng.run() # equivalent to dmrg.run() up to the return parameters.
    print ("E = \{E:.13f\}".format (E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.mean(psi.expectation_value("Sigmax"))
   mag_z = np.mean(psi.expectation_value("Sigmaz"))
   print(" < sigma_x > = \{mag_x : .5f\}".format(mag_x = mag_x))
   print(" < sigma_z > = \{mag_z : .5f\}".format(mag_z = mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
   from tfi_exact import infinite_gs_energy
    E_exact = infinite_gs_energy(1., g)
    print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
    print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_1site_DMRG_tf_ising_infinite(g):
   print("single-site infinite DMRG, transverse field Ising model")
   print ("g = \{g: .2f\}".format (g = g))
   model_params = dict(L=2, J=1., g=g, bc_MPS='infinite', conserve=None)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    dmrg_params = {
        'mixer': True, # setting this to True is essential for the 1-site algorithm,
→to work.
        'trunc_params': {
            'chi_max': 30,
            'svd min': 1.e-10
        },
        'max_E_err': 1.e-10,
        'combine': True
    }
   eng = dmrg.SingleSiteDMRGEngine(psi, M, dmrg_params)
   E, psi = eng.run() # equivalent to dmrg.run() up to the return parameters.
   print("E = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.mean(psi.expectation_value("Sigmax"))
```

```
mag_z = np.mean(psi.expectation_value("Sigmaz"))
   print("<sigma_x> = {mag_x:.5f}".format(mag_x=mag_x))
   print("<sigma_z> = {mag_z:.5f}".format(mag_z=mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
   from tfi_exact import infinite_gs_energy
   E_exact = infinite_gs_energy(1., g)
   print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
   print("relative error: ", abs((E - E_exact) / E_exact))
def example_DMRG_heisenberg_xxz_infinite(Jz, conserve='best'):
   print("infinite DMRG, Heisenberg XXZ chain")
   print("Jz={Jz:.2f}, conserve={conserve!r}".format(Jz=Jz, conserve=conserve))
   model params = dict(
       L=2,
       S=0.5, # spin 1/2
       Jx=1.,
        Jy=1.,
        Jz=Jz,
               # couplings
       bc_MPS='infinite',
       conserve=conserve)
   M = SpinModel(model_params)
   product_state = ["up", "down"] # initial Neel state
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
   dmrg_params = {
        'mixer': True, # setting this to True helps to escape local minima
        'trunc_params': {
            'chi_max': 100,
           'svd_min': 1.e-10,
        'max_E_err': 1.e-10,
   info = dmrg.run(psi, M, dmrg_params)
   E = info['E']
   print("E = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
   Sz = psi.expectation_value("Sz") # Sz instead of Sigma z: spin-1/2 operators!
   mag_z = np.mean(Sz)
   print("<S_z> = [\{Sz0:.5f\}, \{Sz1:.5f\}]; mean = \{maq_z:.5f\}".format(Sz0=Sz[0]),
                                                                      Sz1=Sz[1],
                                                                      mag_z=mag_z))
    # note: it's clear that mean(<Sz>) is 0: the model has Sz conservation!
   print("correlation length:", psi.correlation_length())
   corrs = psi.correlation_function("Sz", "Sz", sites1=range(10))
   print("correlations <Sz_i Sz_j> =")
   print(corrs)
   return E, psi, M
if __name__ == "__main__":
   import logging
   logging.basicConfig(level=logging.INFO)
   example_DMRG_tf_ising_finite(L=10, g=1.)
   print("-" * 100)
   example_1site_DMRG_tf_ising_finite(L=10, g=1.)
   print("-" * 100)
```

```
example_DMRG_tf_ising_infinite(g=1.5)
print("-" * 100)
example_1site_DMRG_tf_ising_infinite(g=1.5)
print("-" * 100)
example_DMRG_heisenberg_xxz_infinite(Jz=1.5)
```

11.2.5 e_tdvp.py

on github.

```
"""Example illustrating the use of TDVP in tenpy.
As of now, we have TDVP only for finite systems. The call structure is quite similar,
⇔to TEBD. A
difference is that we can run one-site TDVP or two-site TDVP. In the former, the bond,
→dimension can
not grow; the latter allows to grow the bond dimension and hence requires a.
→truncation.
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3
import numpy as np
import tenpy.linalg.np_conserved as npc
import tenpy.models.spins
import tenpy.networks.mps as mps
import tenpy.networks.site as site
from tenpy.algorithms import tdvp
from tenpy.networks.mps import MPS
import copy
def run_out_of_equilibrium():
   L = 10
   chi = 5
   delta_t = 0.1
   model_params = {
       'L': L,
        'S': 0.5,
        'conserve': 'Sz',
        'Jz': 1.0,
        'Jy': 1.0,
        'Jx': 1.0,
        'hx': 0.0,
        'hy': 0.0,
        'hz': 0.0,
        'muJ': 0.0,
        'bc_MPS': 'finite',
   }
   heisenberg = tenpy.models.spins.SpinChain(model_params)
   product_state = ["up"] * (L // 2) + ["down"] * (L - L // 2)
    # starting from a domain-wall product state which is not an eigenstate of the
→ Heisenberg model
   psi = MPS.from_product_state(heisenberg.lat.mps_sites(),
                                 product_state,
                                 bc=heisenberg.lat.bc_MPS,
```

```
form='B')
    tdvp_params = {
        'start_time': 0,
        'dt': delta_t,
        'trunc_params': {
            'chi_max': chi,
            'svd_min': 1.e-10,
            'trunc_cut': None
    }
    tdvp_engine = tdvp.TDVPEngine(psi, heisenberg, tdvp_params)
   times = []
   S_mid = []
    for i in range(30):
        tdvp_engine.run_two_sites(N_steps=1)
        times.append(tdvp_engine.evolved_time)
        S_mid.append(psi.entanglement_entropy(bonds=[L // 2])[0])
    for i in range(30):
        tdvp_engine.run_one_site(N_steps=1)
        #psi_2=copy.deepcopy(psi)
        #psi_2.canonical_form()
        times.append(tdvp_engine.evolved_time)
        S_{mid.append} (psi.entanglement_entropy (bonds=[L // 2])[0])
    import matplotlib.pyplot as plt
   plt.figure()
   plt.plot(times, S_mid)
   plt.xlabel('t')
   plt.ylabel('S')
   plt.axvline(x=3.1, color='red')
   plt.text(0.0, 0.0000015, "Two sites update")
    plt.text(3.1, 0.0000015, "One site update")
    plt.show()
if __name__ == "__main__":
    import logging
    logging.basicConfig(level=logging.INFO)
    run_out_of_equilibrium()
```

11.2.6 purification.py

on github.

```
from tenpy.models.tf_ising import TFIChain
from tenpy.networks.purification_mps import PurificationMPS
from tenpy.algorithms.purification import PurificationTEBD, PurificationApplyMPO

def imag_tebd(L=30, beta_max=3., dt=0.05, order=2, bc="finite"):
    model_params = dict(L=L, J=1., g=1.2)
    M = TFIChain(model_params)
    psi = PurificationMPS.from_infiniteT(M.lat.mps_sites(), bc=bc)
    options = {
        'trunc_params': {
```

```
'chi_max': 100,
            'svd min': 1.e-8
        },
        'order': order,
        'dt': dt,
        'N_steps': 1
   beta = 0.
   eng = PurificationTEBD(psi, M, options)
   Szs = [psi.expectation_value("Sz")]
   betas = [0.]
   while beta < beta_max:</pre>
       beta += 2. * dt # factor of 2: |psi> ~= exp^{- dt H}, but rho = |psi><psi|
       betas.append(beta)
       eng.run_imaginary(dt) # cool down by dt
       Szs.append(psi.expectation_value("Sz")) # and further measurements...
   return {'beta': betas, 'Sz': Szs}
def imag_apply_mpo(L=30, beta_max=3., dt=0.05, order=2, bc="finite", approx="II"):
   model_params = dict(L=L, J=1., g=1.2)
   M = TFIChain(model_params)
   psi = PurificationMPS.from_infiniteT(M.lat.mps_sites(), bc=bc)
   options = {'trunc_params': {'chi_max': 100, 'svd_min': 1.e-8}}
   beta = 0.
   if order == 1:
       Us = [M.H_MPO.make_U(-dt, approx)]
   elif order == 2:
       Us = [M.H_MPO.make_U(-d * dt, approx)] for d in [0.5 + 0.5j, 0.5 - 0.5j]]
   eng = PurificationApplyMPO(psi, Us[0], options)
   Szs = [psi.expectation_value("Sz")]
   betas = [0.]
   while beta < beta_max:</pre>
       beta += 2. * dt # factor of 2: |psi> ~= exp^{- dt H}, but rho = |psi><psi|
       betas.append(beta)
        for U in Us:
            eng.init_env(U) # reset environment, initialize new copy of psi
            eng.run() # apply U to psi
       Szs.append(psi.expectation_value("Sz")) # and further measurements...
   return {'beta': betas, 'Sz': Szs}
if __name__ == "__main__":
   import logging
    logging.basicConfig(level=logging.INFO)
   data_tebd = imag_tebd()
   data_mpo = imag_apply_mpo()
   import numpy as np
   from matplotlib.pyplot import plt
   plt.plot(data_mpo['beta'], np.sum(data_mpo['Sz'], axis=1), label='MPO')
   plt.plot(data_tebd['beta'], np.sum(data_tebd['Sz'], axis=1), label='TEBD')
   plt.xlabel(r'$\beta$')
   plt.ylabel(r'total $S^z$')
   plt.show()
```

11.2.7 tfi exact.py

on github.

```
"""Provides exact ground state energies for the transverse field ising model for
⇔comparison.
The Hamiltonian reads
.. math ::
   H = -J \setminus sum_{i} \setminus sigma^x_i \setminus sigma^x_{i+1} - g \setminus sum_{i} \setminus sigma^z_i
# Copyright 2019-2021 TeNPy Developers, GNU GPLv3
import numpy as np
import scipy.sparse as sparse
import scipy.sparse.linalq.eigen.arpack as arp
import warnings
import scipy.integrate
def finite_gs_energy(L, J, g):
    """For comparison: obtain ground state energy from exact diagonalization.
   Exponentially expensive in L, only works for small enough `L` <~ 20.
    if L >= 20:
        warnings.warn("Large L: Exact diagonalization might take a long time!")
    # get single site operaors
    sx = sparse.csr_matrix(np.array([[0., 1.], [1., 0.]]))
   sz = sparse.csr_matrix(np.array([[1., 0.], [0., -1.]]))
   id = sparse.csr_matrix(np.eye(2))
    sx_list = [] # sx_list[i] = kron([id, id, ..., id, sx, id, .... id])
    sz_list = []
    for i_site in range(L):
       x_{ops} = [id] * L
        z_{ops} = [id] * L
       x_{ops[i\_site]} = sx
        z_{ops[i\_site]} = sz
        X = x_ops[0]
        Z = z_{ops}[0]
        for j in range(1, L):
            X = sparse.kron(X, x_ops[j], 'csr')
            Z = sparse.kron(Z, z_ops[j], 'csr')
        sx_list.append(X)
        sz_list.append(Z)
    H_xx = sparse.csr_matrix((2**L, 2**L))
    H_z = sparse.csr_matrix((2**L, 2**L))
    for i in range (L - 1):
        H_xx = H_xx + sx_list[i] * sx_list[(i + 1) % L]
    for i in range(L):
       H_z = H_z + sz_list[i]
   H = -J * H_xx - g * H_z
   E, V = arp.eigsh(H, k=1, which='SA', return_eigenvectors=True, ncv=20)
    return E[0]
def infinite_gs_energy(J, g):
    """For comparison: Calculate groundstate energy density from analytic formula.
```

```
The analytic formula stems from mapping the model to free fermions, see P. Pfeuty,

The one—
dimensional Ising model with a transverse field, Annals of Physics 57, p. 79.

(1970). Note that
we use Pauli matrices compared this reference using spin-1/2 matrices and replace.

the sum_k ->
integral dk/2pi to obtain the result in the N -> infinity limit.

"""

def f(k, lambda_):
    return np.sqrt(1 + lambda_**2 + 2 * lambda_ * np.cos(k))

E0_exact = -g / (J * 2. * np.pi) * scipy.integrate.quad(f, -np.pi, np.pi, args=(J_-
-/ g, ))[0]
return E0_exact
```

11.2.8 z_exact_diag.py

on github.

```
"""A simple example comparing DMRG output with full diagonalization (ED).
Sorry that this is not well documented! ED is meant to be used for debugging only;)
# Copyright 2018-2021 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
from tenpy.models.xxz_chain import XXZChain
from tenpy.networks.mps import MPS
from tenpy.algorithms.exact_diag import ExactDiag
from tenpy.algorithms import dmrq
def example_exact_diagonalization(L, Jz):
   xxz_pars = dict(L=L, Jxx=1., Jz=Jz, hz=0.0, bc_MPS='finite')
   M = XXZChain(xxz_pars)
   product_state = ["up", "down"] * (xxz_pars['L'] // 2) # this selects a charge_
⇔sector!
   psi_DMRG = MPS.from_product_state(M.lat.mps_sites(), product_state)
   charge_sector = psi_DMRG.get_total_charge(True) # ED charge sector should match
   ED = ExactDiag(M, charge_sector=charge_sector, max_size=2.e6)
   ED.build_full_H_from_mpo()
   # ED.build_full_H_from_bonds() # whatever you prefer
   print("start diagonalization")
   ED.full_diagonalization() # the expensive part for large L
   E0_ED, psi_ED = ED.groundstate() # return the ground state
   print("psi_ED =", psi_ED)
   print("run DMRG")
   dmrg.run(psi_DMRG, M, {'verbose': 0}) # modifies psi_DMRG in place!
    # first way to compare ED with DMRG: convert MPS to ED vector
   psi_DMRG_full = ED.mps_to_full(psi_DMRG)
```

```
print("psi_DMRG_full =", psi_DMRG_full)
  ov = npc.inner(psi_ED, psi_DMRG_full, axes='range', do_conj=True)
  print("<psi_ED|psi_DMRG_full> =", ov)
  assert (abs(abs(ov) - 1.) < 1.e-13)

# second way: convert ED vector to MPS
  psi_ED_mps = ED.full_to_mps(psi_ED)
  ov2 = psi_ED_mps.overlap(psi_DMRG)
  print("<psi_ED_mps|psi_DMRG> =", ov2)
  assert (abs(abs(ov2) - 1.) < 1.e-13)
  assert (abs(ov - ov2) < 1.e-13)
  # -> advantage: expectation_value etc. of MPS are available!
  print("<Sz> =", psi_ED_mps.expectation_value('Sz'))

if __name__ == "__main__":
  example_exact_diagonalization(10, 1.)
```

11.3 Jupyter Notebooks

This is a collection of [jupyter] notebooks from the [TeNPyNotebooks] repository. You need to install TeNPy to execute them (see *Installation instructions*), but you can copy them anywhere before execution. Note that some of them might take a while to run, as they contain more extensive examples.

11.3.1 A first TEBD Example

Like examples/c_tebd.py, this notebook shows the basic interface for TEBD. It initalized the transverse field Ising model H = JXX + gZ at the critical point J = g = 1, and an MPS in the all-up state $|\uparrow \cdots \uparrow\rangle$. It then performs a real-time evolution with TEBD and measures a few observables. This setup correspond to a global quench from $g = \infty$ to g = 1.

```
[1]: import numpy as np
    import scipy
    import matplotlib.pyplot as plt
    import matplotlib
[2]: import tenpy
    import tenpy.linalg.np_conserved as npc
    from tenpy.algorithms import tebd
    from tenpy.networks.mps import MPS
    from tenpy.models.tf_ising import TFIChain
[3]: L = 30
[4]: model_params = {
        'J': 1. , 'g': 1., # critical
        'L': L,
        'bc_MPS': 'finite',
    }
    M = TFIChain(model_params)
```

```
Reading 'bc_MPS'='finite' for config TFIChain
    Reading 'L'=30 for config TFIChain
    Reading 'J'=1.0 for config TFIChain
    Reading 'g'=1.0 for config TFIChain
[5]: psi = MPS.from_lat_product_state(M.lat, [['up']])
[6]: tebd_params = {
        'N_steps': 1,
        'dt': 0.1,
        'order': 4,
        'trunc_params': {'chi_max': 100, 'svd_min': 1.e-12}
    eng = tebd.Engine(psi, M, tebd_params)
    Subconfig 'trunc_params'=Config(<3 options>, 'trunc_params') for config TEBD
[7]: def measurement (eng, data):
        keys = ['t', 'entropy', 'Sx', 'Sz', 'corr_XX', 'corr_ZZ', 'trunc_err']
        if data is None:
            data = dict([(k, []) for k in keys])
        data['t'].append(eng.evolved_time)
        data['entropy'].append(eng.psi.entanglement_entropy())
        data['Sx'].append(eng.psi.expectation_value('Sigmax'))
        data['Sz'].append(eng.psi.expectation_value('Sigmaz'))
        data['corr_XX'].append(eng.psi.correlation_function('Sigmax', 'Sigmax'))
        data['trunc_err'].append(eng.trunc_err.eps)
        return data
[8]: data = measurement(eng, None)
[9]: while eng.evolved_time < 5.:
        eng.run()
        measurement (eng, data)
    Reading 'dt'=0.1 for config TEBD
    Reading 'N_steps'=1 for config TEBD
    Reading 'order'=4 for config TEBD
    Calculate U for {'order': 4, 'delta_t': 0.1, 'type_evo': 'real', 'E_offset': None,
     →'tau': 0.1}
    --> time=0.100, max_chi=6, Delta_S=5.5243e-02, S=0.0552432967, since last update: 0.5
    --> time=0.200, max_chi=6, Delta_S=1.0453e-01, S=0.1597705686, since last update: 0.5...
     \hookrightarrow S
    --> time=0.300, max_chi=8, Delta_S=1.1368e-01, S=0.2734471407, since last update: 0.5_
    --> time=0.400, max_chi=10, Delta_S=1.0226e-01, S=0.3757082127, since last update: 0.
     →5 s
    --> time=0.500, max_chi=12, Delta_S=8.2474e-02, S=0.4581821173, since last update: 0.
    --> time=0.600, max_chi=12, Delta_S=6.3393e-02, S=0.5215746922, since last update: 0.
    --> time=0.700, max_chi=15, Delta_S=5.0837e-02, S=0.5724119006, since last update: 0.
    --> time=0.800, max_chi=18, Delta_S=4.7046e-02, S=0.6194580889, since last update: 0.
     \hookrightarrow 5 s
                                                                               (continues on next page)
```

```
--> time=0.900, max_chi=20, Delta_S=5.0606e-02, S=0.6700636890, since last update: 0.
--> time=1.000, max_chi=20, Delta_S=5.7462e-02, S=0.7275254979, since last update: 0.
--> time=1.100, max_chi=24, Delta_S=6.3115e-02, S=0.7906402648, since last update: 0.
--> time=1.200, max_chi=26, Delta_S=6.4779e-02, S=0.8554189184, since last update: 0.
\hookrightarrow5 s
--> time=1.300, max_chi=30, Delta_S=6.2269e-02, S=0.9176882429, since last update: 0.

→ 5 s

--> time=1.400, max_chi=34, Delta_S=5.7451e-02, S=0.9751394599, since last update: 0.
--> time=1.500, max_chi=40, Delta_S=5.2899e-02, S=1.0280386118, since last update: 0.
--> time=1.600, max_chi=40, Delta_S=5.0543e-02, S=1.0785817862, since last update: 0.
--> time=1.700, max_chi=46, Delta_S=5.0852e-02, S=1.1294340516, since last update: 0.
--> time=1.800, max_chi=52, Delta_S=5.2841e-02, S=1.1822748827, since last update: 0.
--> time=1.900, max_chi=57, Delta_S=5.4804e-02, S=1.2370787276, since last update: 0.
\hookrightarrow 9 s
--> time=2.000, max_chi=62, Delta_S=5.5311e-02, S=1.2923895877, since last update: 0.
\rightarrow 9 s
--> time=2.100, max_chi=71, Delta_S=5.3906e-02, S=1.3462960135, since last update: 1.
--> time=2.200, max_chi=80, Delta_S=5.1209e-02, S=1.3975050668, since last update: 1.
--> time=2.300, max_chi=85, Delta_S=4.8446e-02, S=1.4459514729, since last update: 1.
--> time=2.400, max_chi=95, Delta_S=4.6732e-02, S=1.4926837515, since last update: 1.
--> time=2.500, max_chi=100, Delta_S=4.6486e-02, S=1.5391700161, since last update: 1.
\sim 8 s
--> time=2.600, max_chi=100, Delta_S=4.7282e-02, S=1.5864521162, since last update: 1.
\rightarrow 9 s
--> time=2.700, max_chi=100, Delta_S=4.8160e-02, S=1.6346120966, since last update: 1.
--> time=2.800, max_chi=100, Delta_S=4.8202e-02, S=1.6828138289, since last update: 1.
--> time=2.900, max_chi=100, Delta_S=4.7044e-02, S=1.7298576742, since last update: 1.
→7 s
--> time=3.000, max_chi=100, Delta_S=4.5033e-02, S=1.7748910539, since last update: 1.
→6 s
--> time=3.100, max_chi=100, Delta_S=4.2960e-02, S=1.8178514255, since last update: 1.
--> time=3.200, max_chi=100, Delta_S=4.1575e-02, S=1.8594265450, since last update: 1.
→4 s
--> time=3.300, max_chi=100, Delta_S=4.1182e-02, S=1.9006080872, since last update: 1.
--> time=3.400, max_chi=100, Delta_S=4.1503e-02, S=1.9421110677, since last update: 1.
--> time=3.500, max_chi=100, Delta_S=4.1873e-02, S=1.9839839097, since last update: 1.
→6 s
--> time=3.600, max_chi=100, Delta_S=4.1645e-02, S=2.0256293591, since last update: 1.
--> time=3.700, max_chi=100, Delta_S=4.0570e-02, S=2.0661996178, since last update: 1.
                                                                          (continues on next page)
→8 s
```

```
--> time=3.800, max_chi=100, Delta_S=3.8897e-02, S=2.1050963078, since last update: 1.
→6 s
--> time=3.900, max_chi=100, Delta_S=3.7189e-02, S=2.1422855970, since last update: 2.
→0 s
--> time=4.000, max_chi=100, Delta_S=3.5994e-02, S=2.1782792652, since last update: 1.
--> time=4.100, max_chi=100, Delta_S=3.5534e-02, S=2.2138131347, since last update: 1.
→ 6 s
--> time=4.200, max_chi=100, Delta_S=3.5597e-02, S=2.2494096463, since last update: 1.

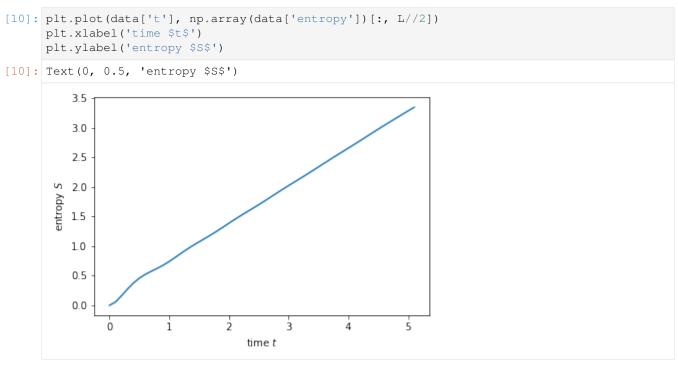
→ 6 s

--> time=4.300, max_chi=100, Delta_S=3.5673e-02, S=2.2850830917, since last update: 1.
→5 s
--> time=4.400, max_chi=100, Delta_S=3.5272e-02, S=2.3203546259, since last update: 1.
--> time=4.500, max_chi=100, Delta_S=3.4188e-02, S=2.3545423452, since last update: 1.
\hookrightarrow 4 s
--> time=4.600, max_chi=100, Delta_S=3.2596e-02, S=2.3871383468, since last update: 1.
→4 s
--> time=4.700, max_chi=100, Delta_S=3.0915e-02, S=2.4180531110, since last update: 1.

→ 6 s

--> time=4.800, max_chi=100, Delta_S=2.9546e-02, S=2.4475988717, since last update: 1.
→7 s
--> time=4.900, max_chi=100, Delta_S=2.8630e-02, S=2.4762292780, since last update: 1.
→ 6 s
--> time=5.000, max_chi=100, Delta_S=2.7962e-02, S=2.5041913364, since last update: 1.
--> time=5.100, max_chi=100, Delta_S=2.7099e-02, S=2.5312904883, since last update: 1.

→ 6 s
```



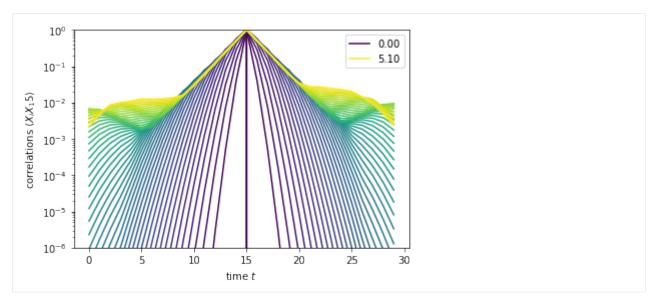
The growth of S linear in time is typical for a global quench and to be expected from the quasi-particle picture

```
[11]: plt.plot(data['t'], np.sum(data['Sx'], axis=1), label="X")
      plt.plot(data['t'], np.sum(data['Sz'], axis=1), label="Z")
      plt.xlabel('time $t$')
      plt.ylabel('magnetization')
      plt.legend(loc='best')
[11]: <matplotlib.legend.Legend at 0x7f1018760e80>
         30
                                                             Х
                                                             Ζ
         25
         20
      magnetization
         15
         10
          5
          0
                                2
                                         3
                                   time t
```

The strict conservation of X being zero is ensured by charge conservation, because X changes the parity sector.

Nevertheless, the XX correlation function can be nontrivial:

```
[12]: corrs = np.array(data['corr_XX'])
                              tmax = data['t'][-1]
                               x = np.arange(L)
                               cmap = matplotlib.cm.viridis
                               for i, t in list(enumerate(data['t'])):
                                                     if i == 0 or i == len(data['t']) - 1:
                                                                          label = '\{t:.2f\}'.format(t=t)
                                                     else:
                                                                           label = None
                                                     plt.plot(x, corrs[i, L//2, :], color=cmap(t/tmax), label=label)
                               plt.xlabel(r'time $t$')
                               \verb|plt.ylabel(r'correlations $\langle X_i X_{j:d} \rangle \rangle | single $(j=L//2)| | single $(j=L/2)| | sin
                               plt.yscale('log')
                              plt.ylim(1.e-6, 1.)
                               plt.legend()
                               plt.show()
```



The output of the run showed that we gradually increased the bond dimension and only reached the maximum chi around t=2.5. At this point we start to truncate significantly, because we cut off the tail whatever the singular values are. This is clearly visible if we plot the truncation error vs. time. Note the log-scale, though: if you are fine with an error of say 1 permille for expectation values, you can still go on for a bit more!

```
[13]: plt.plot(data['t'], data['trunc_err'])
      plt.yscale('log')
       #plt.ylim(1.e-15, 1.)
      plt.xlabel('time $t$')
      plt.ylabel('truncation error')
[13]: Text(0, 0.5, 'truncation error')
           10^{-3}
           10^{-6}
           10^{-9}
       truncation error
           10-12
          10^{-15}
          10-18
          10^{-21}
                                       2
                                                            4
                                                                      5
                                                  3
                                           time t
```

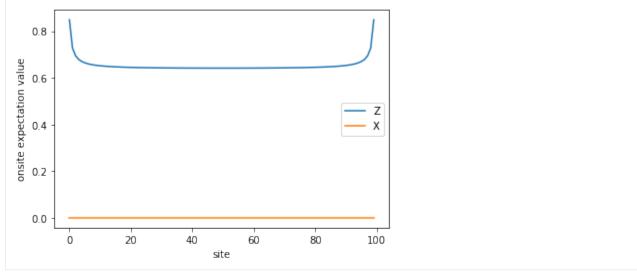
11.3.2 A first finite DMRG Example

Like examples/d_dmrg.py, this notebook shows the basic interface for DMRG. It initalized the transverse field Ising model H = JXX + gZ at the critical point J = g = 1, and a finite MPS in the all-up state $|\uparrow \cdots \uparrow\rangle$. It then runs DMRG to find the ground state. Finally, we look at the profile of the entanglement-cuts.

```
[1]: import numpy as np
    import scipy
    import matplotlib.pyplot as plt
[2]: import tenpy
    import tenpy.linalg.np_conserved as npc
    from tenpy.algorithms import dmrg
    from tenpy.networks.mps import MPS
    from tenpy.models.tf_ising import TFIChain
[3]: L = 100
[4]: model_params = {
        'J': 1. , 'g': 1., # critical
        'L': L,
        'bc_MPS': 'finite',
    }
    M = TFIChain(model_params)
    Reading 'bc_MPS'='finite' for config TFIChain
    Reading 'L'=100 for config TFIChain
    Reading 'J'=1.0 for config TFIChain
    Reading 'g'=1.0 for config TFIChain
[5]: psi = MPS.from_lat_product_state(M.lat, [['up']])
[6]: dmrg_params = {
        'mixer': None, # setting this to True helps to escape local minima
        'max_E_err': 1.e-10,
        'trunc_params': {
            'chi_max': 100,
            'svd_min': 1.e-10,
        },
        'verbose': True,
        'combine': True
    }
    eng = dmrg.TwoSiteDMRGEngine(psi, M, dmrg_params)
    E, psi = eng.run() # the main work; modifies psi in place
    Reading 'combine'=True for config TwoSiteDMRGEngine
    Subconfig 'trunc_params'=Config(<3 options>, 'trunc_params') for config_
    \hookrightarrowTwoSiteDMRGEngine
    Reading 'max_E_err'=1e-10 for config TwoSiteDMRGEngine
    Reading 'mixer'=None for config TwoSiteDMRGEngine
    ______
    sweep 1, age = 100
    Energy = -126.9290280127265333, S = 0.3766760098039985, norm_err = 1.2e-01
    Current memory usage 97.4 MB, time elapsed: 2.3 s
    Delta E = nan, Delta S = nan (per sweep)
    max_trunc_err = 0.0000e+00, max_E_trunc = 1.8474e-13
                                                                            (continues on next page)
```

```
______
sweep 2, age = 100
Energy = -126.9618018107068309, S = 0.5108558254154341, norm_err = 7.0e-03
Current memory usage 99.5 MB, time elapsed: 5.6 s
Delta E = -3.2774e-02, Delta S = 1.3418e-01 (per sweep)
max_trunc_err = 0.0000e+00, max_E_trunc = 1.9895e-13
______
sweep 3, age = 100
Energy = -126.9618767383882130, S = 0.5277643082687588, norm_err = 1.1e-05
Current memory usage 107.0 MB, time elapsed: 10.1 s
Delta E = -7.4928e-05, Delta S = 1.6908e-02 (per sweep)
max_trunc_err = 4.7789e-20, max_E_trunc = 2.8422e-13
MPS bond dimensions: [2, 4, 8, 16, 27, 35, 39, 43, 46, 49, 49, 52, 55, 60, 62, 64, 64,
→ 64, 64, 61, 59, 60, 55, 51, 47, 44, 42, 42, 38, 35, 32, 23, 16, 8, 4, 2]
______
sweep 4, age = 100
Energy = -126.9618767396799086, S = 0.5277994024302479, norm_err = 5.8e-10
Current memory usage 115.4 MB, time elapsed: 16.9 s
Delta E = -1.2917e-09, Delta S = 3.5094e-05 (per sweep)
max_trunc_err = 1.3388e-18, max_E_trunc = 2.8422e-13
MPS bond dimensions: [2, 4, 8, 16, 22, 30, 34, 36, 41, 45, 47, 48, 52, 54, 57, 61, 61,
→ 63, 64, 66, 67, 70, 71, 72, 74, 76, 76, 77, 79, 79, 80, 82, 83, 83, 83, 84, 84, 84,
→ 85, 85, 84, 84, 84, 83, 83, 83, 83, 81, 80, 79, 77, 77, 77, 75, 73, 71, 70, 67, 66,
→ 64, 62, 61, 61, 57, 54, 52, 48, 47, 45, 41, 36, 34, 30, 22, 16, 8, 4, 2]
______
sweep 5, age = 100
Energy = -126.9618767396792691, S = 0.5277994025033066, norm_err = 2.1e-13
Current memory usage 115.4 MB, time elapsed: 20.4 s
Delta E = 6.3949e-13, Delta S = 7.3059e-11 (per sweep)
max_trunc_err = 6.5986e-20, max_E_trunc = 3.1264e-13
MPS bond dimensions: [2, 4, 8, 16, 22, 30, 34, 36, 41, 45, 47, 48, 52, 54, 57, 61, 61,
→ 63, 64, 66, 67, 71, 71, 74, 77, 77, 78, 79, 81, 81, 81, 83, 83, 84, 85, 85, 85, 85,
→ 86, 88, 89, 90, 90, 90, 90, 90, 90, 91, 91, 91, 90, 90, 90, 90, 90, 90, 90, 90, 89,
→ 88, 86, 86, 85, 85, 85, 84, 83, 83, 81, 81, 81, 79, 78, 78, 77, 73, 71, 71, 67, 66,
→ 64, 63, 61, 61, 57, 54, 52, 48, 47, 45, 41, 36, 34, 30, 22, 16, 8, 4, 2]
_____
DMRG finished after 5 sweeps.
total size = 100, maximum chi = 91
______
```

Expectation Values



Chapter 11. Examples

```
plt.figure()
plt.plot(dx, XX_disc, label="X X")
plt.plot(dx, ZZ_disc, label="Z Z")
plt.xlabel(r"distance $|i-j|$")
plt.ylabel(r"disconnected correlations $\langle A_i A_j\rangle - \langle A_i \rangle\
→langle A_j\rangle$")
plt.legend()
plt.loglog()
plt.show()
      10°
 disconnected correlations (A,A,) - (A,)(A,)
     10^{-1}
     10^{-2}
     10^{-3}
     10^{-4}
     10-5
                  ZΖ
     10<sup>-6</sup>
            10°
                                           10<sup>1</sup>
                                    distance |i - j|
```

We find power-law decaying correlations, as expected for a critical model. For a gapped model, we would expect exponentially decaying correlations.

We now look at the entanglement entropy. The transverse-field Ising model is critical at g=J. Conformal field theory, Calabrese, Cardy 2004, predicts an entanglement entropy profile of

where: math: `c = 0.5' is the central charge, : math: `a` is the lattice spacing (we set: math: `a = 1`), : math: `L` is the total size of the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: `a = 1`), : math: `L' is the lattice spacing (we set: math: math

```
plt.legend()
plt.show()
extraced central charge 0.50788 with residuum 1.95e-09
(Expect central charge = 0.5 for the transverse field Ising model.)
    0.55
 entanglement entropy S
    0.50
    0.45
    0.40
    0.35
                                  S
    0.30
                                  fit with c = 0.508
                     20
                                                      80
           0
                                 40
                                           60
                                                                100
                                     bond
```

.]:

11.3.3 Lattices: visualization and further examples

This notebook demonstrates a few ways to visualize lattices and couplings inside the lattice.

```
[1]: import tenpy

[2]: import numpy as np
  import matplotlib.pyplot as plt
  from tenpy.models import lattice
```

In the following, we will focus on the Honeycomb lattice as an example with a non-trivial unit cell. If you want to try it out yourself for a different lattice, simply adjust the following alias and re-run all the cells below:

```
[3]: MyLattice = lattice.Honeycomb

Lu = MyLattice.Lu # = 2 = the number of sites in the unit cell

fig_args = dict(figsize=(7, 5), dpi=150) # make figures a bit larger
```

plotting the lattice itself

To get started, let's recall that a lattice consists of a unit cell that is repeated in the directions of the basis. The following plot visualizes the first unit cell and basis and plots the sites in the whole lattice. For the Honeycomb lattice, we have two different sites in the unit cell, which get visualized by different markers.

```
[4]: lat = MyLattice(5, 4, sites=None, bc='periodic')
```

```
[5]: plt.figure(**fig_args)
    ax = plt.gca()
    lat.plot_sites(ax)
    ax.set_aspect('equal')
    lat.plot_basis(ax, origin=-0.5*(lat.basis[0] + lat.basis[1]))
    ax.set_xlim(-1)
    ax.set_ylim(-1)
[5]: (-1, 5.525)
         5
         4
         3
         2
         1
         0
       -1 ⅓
                                    1
                        0
                                                            3
           -1
                                                2
```

We can also plot the nearest- and next-nearest-neighbor bonds:

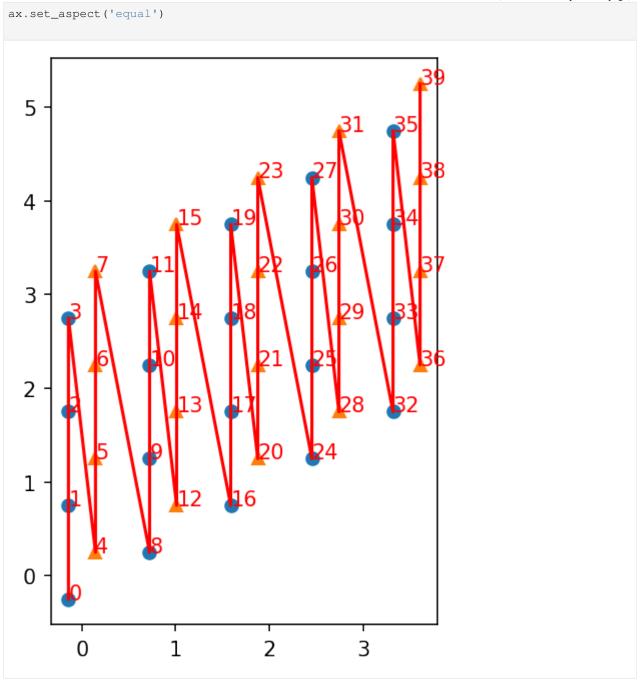
[6]: plt.figure(**fig_args) (continues on next page)

```
ax = plt.gca()
lat.plot_sites(ax)
lat.plot_coupling(ax)
lat.plot_coupling(ax, lat.pairs['next_nearest_neighbors'], linestyle=':', color='r')
ax.set_aspect('equal')
    6
    5
    4
    3
    2
    1
    0
                                2
           0
                                                    4
```

If you have a 1D MPS, it's winding through the lattice is defined by the order (See the userguide with "Details on the lattice geometry"). You can plot it as follows:

```
[7]: plt.figure(**fig_args)
ax = plt.gca()
lat.plot_sites(ax)
lat.plot_order(ax)

(continues on next page)
```

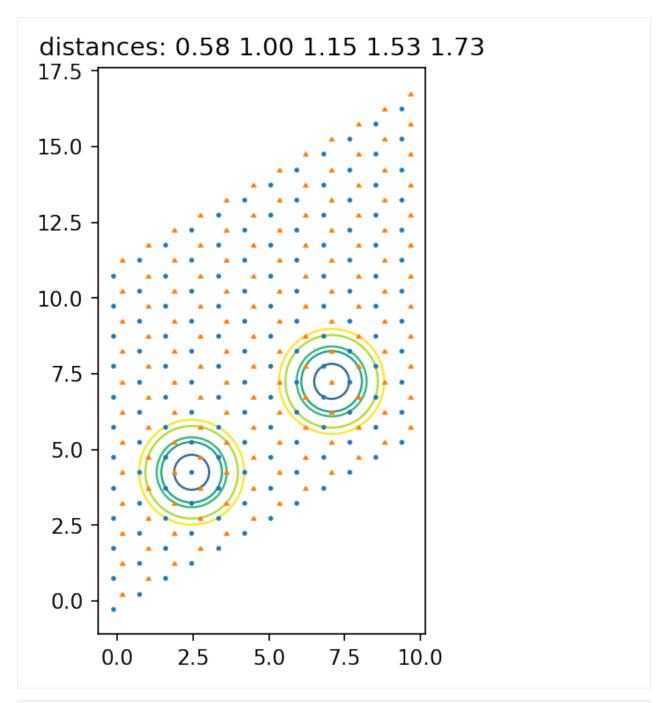


Visually verifying pairs of couplings by plotting them

In this section, we *visually* verify that the lattice pairs like the nearest_neighbors are indeed what they claim to be. To achieve this, we first get all the possible distances of them, plot circles with these distances and lines connecting the points for each distance.

Then, you have to stare at the plots and verify that these couplings include all pairs you want to have.

```
[8]: lat = MyLattice(12, 12, sites=None, bc='periodic') # the lattice to plot
     lat_pairs = lat.pairs # the coupling pairs to plot
 [9]: # get distances of the couplings
     dist_pair = {}
     for pair in lat_pairs:
          #print (pair)
         dist = None
          for u1, u2, dx in lat_pairs[pair]:
             d = lat.distance(u1, u2, dx)
              #print(u1, u2, dx, d)
              if dist is None:
                  dist = d
                  dist_pair[d] = pair
              else:
                  assert abs(dist-d) < 1.e-14</pre>
     dists = sorted(dist_pair.keys())
     if len(dists) != len(lat_pairs):
          raise ValueError("no unique mapping dist -> pair")
[10]: print("(dist) (pairs)")
     for d in dists:
         print("{0:.6f} {1}".format(d, dist_pair[d]))
      (dist)
             (pairs)
     0.577350 nearest_neighbors
     1.000000 next_nearest_neighbors
     1.154701 next_next_nearest_neighbors
     1.527525 fourth_nearest_neighbors
     1.732051 fifth_nearest_neighbors
[11]: colors = [plt.cm.viridis(r/dists[-1]) for r in dists]
     centers = np.array([[3, 3, 0], [8, 3, 1], [3, 8, 2], [8, 8, 3]]) # one center for
      →each site in the unit cell
     us = list(range(Lu))
[12]: fig = plt.figure(**fig_args)
     ax = plt.gca()
     lat.plot_sites(ax, markersize=1.3)
     for u, center in zip(us, centers):
         center = lat.position(center)
          for r, c in zip(dists, colors):
             circ = plt.Circle(center, r, fill=False, color=c)
             ax.add_artist(circ)
     ax.set_aspect(1.)
     t = ax.set\_title("distances: " + ' '.join(['{0:.2f}'.format(d) for d in dists]))
```



```
for dist in dists:
    pair_name = dist_pair[dist]
    print(dist, pair_name)
    pairs = lat.pairs[pair_name]
    fig = plt.figure(**fig_args)
    ax = plt.gca()
    lat.plot_sites(ax, markersize=1.3)
    for u, center in zip(us, centers):
        center = lat.position(center)
        for r, c in zip(dists, colors):
```

```
circ = plt.Circle(center, r, fill=False, color=c)
            ax.add_artist(circ)
   pairs_with_reverse = pairs + [(u2, u1, -np.array(dx)) for u1, u2, dx in pairs]
    for u1, u2, dx in pairs_with_reverse:
        print(u1, u2, dx, lat.distance(u1, u2, dx))
        start = centers[u1]
        end = start.copy()
        end[-1] = u2
        end[:-1] = start[:-1] + dx
       x1, y1 = lat.position(start)
       x2, y2 = lat.position(end)
       ax.arrow(x1, y1, x2-x1, y2 - y1)
    for u in us:
       x, y = lat.position(centers[u])
        number = lat.count_neighbors(u, pair_name)
        ax.text(x, y, str(number), color='r')
    ax.set_aspect(1.)
    ax.set_title(pair_name + ' distance = {0:.3f}'.format(dist))
0.5773502691896258 nearest_neighbors
0 1 [0 0] 0.5773502691896258
1 0 [1 0] 0.5773502691896257
1 0 [0 1] 0.5773502691896258
1 0 [0 0] 0.5773502691896258
0 1 [-1 0] 0.5773502691896257
0 1 [ 0 -1] 0.5773502691896258
0.999999999999999 next_nearest_neighbors
0 0 [1 0] 0.999999999999999
0 0 [0 1] 1.0
0 0 [ 1 -1] 0.999999999999999
1 1 [1 0] 0.999999999999999
1 1 [0 1] 1.0
1 1 [ 1 -1] 0.999999999999999
0 0 [-1 0] 0.999999999999999
0 0 [ 0 -1] 1.0
0 0 [-1 1] 0.999999999999999
1 1 [-1 0] 0.999999999999999
1 1 [ 0 -1] 1.0
1 1 [-1 1] 0.999999999999999
1.1547005383792515 next_next_nearest_neighbors
1 0 [1 1] 1.1547005383792515
0 1 [-1 1] 1.1547005383792515
0 1 [ 1 -1] 1.1547005383792515
0 1 [-1 -1] 1.1547005383792515
1 0 [ 1 -1] 1.1547005383792515
1 0 [-1 1] 1.1547005383792515
1.5275252316519468 fourth_nearest_neighbors
0 1 [0 1] 1.5275252316519468
0 1 [1 0] 1.5275252316519465
0 1 [ 1 -2] 1.5275252316519465
0 1 [ 0 -2] 1.5275252316519468
0 1 [-2 0] 1.5275252316519465
0 1 [-2 1] 1.5275252316519465
1 0 [ 0 -1] 1.5275252316519468
1 0 [-1 0] 1.5275252316519465
1 0 [-1 2] 1.5275252316519465
1 0 [0 2] 1.5275252316519468
```

```
1 0 [2 0] 1.5275252316519465

1 0 [ 2 -1] 1.5275252316519465

1.7320508075688772 fifth_nearest_neighbors

0 0 [1 1] 1.7320508075688772

0 0 [ 2 -1] 1.7320508075688772

0 0 [-1 2] 1.7320508075688772

1 1 [1 1] 1.7320508075688772

1 1 [ 2 -1] 1.7320508075688772

1 1 [ -1 2] 1.7320508075688772

0 0 [-1 -1] 1.7320508075688772

0 0 [-2 1] 1.7320508075688772

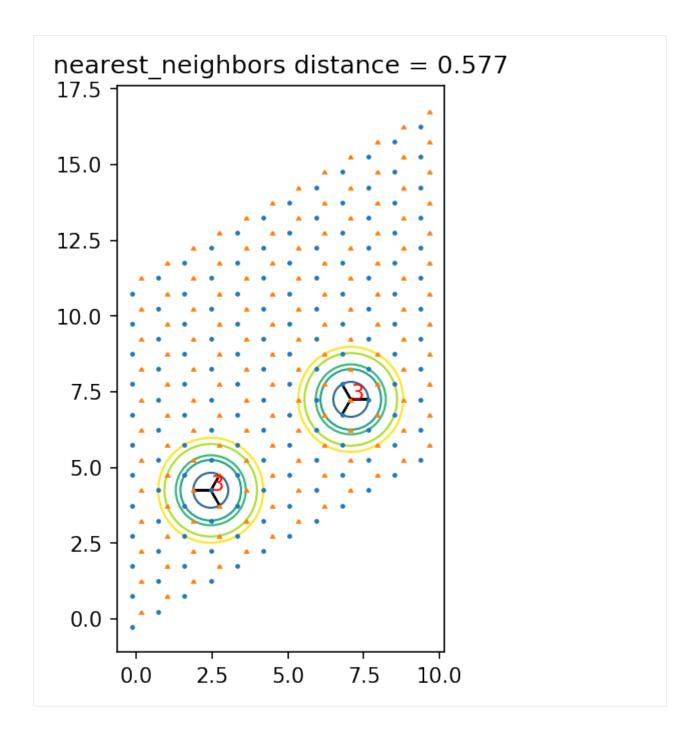
0 0 [ -2 1] 1.7320508075688772

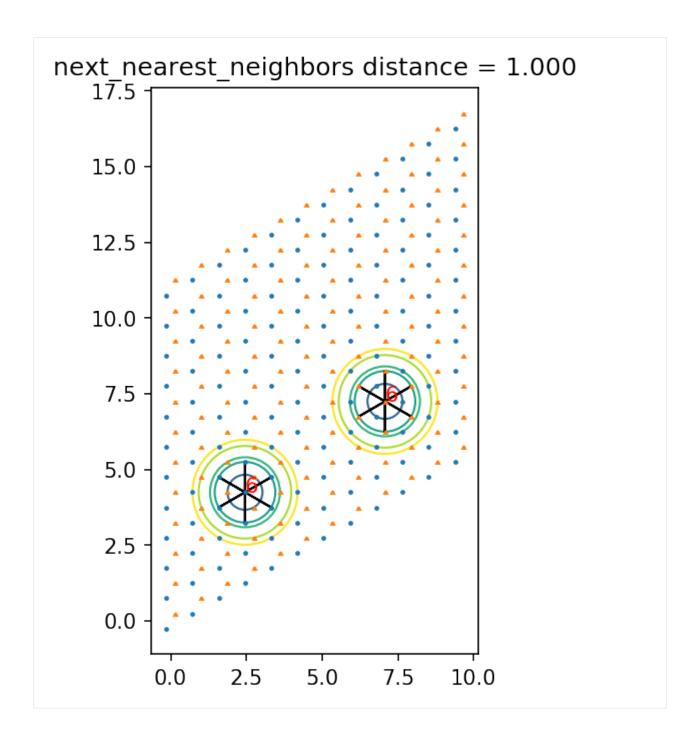
1 1 [ -1 -1] 1.7320508075688772

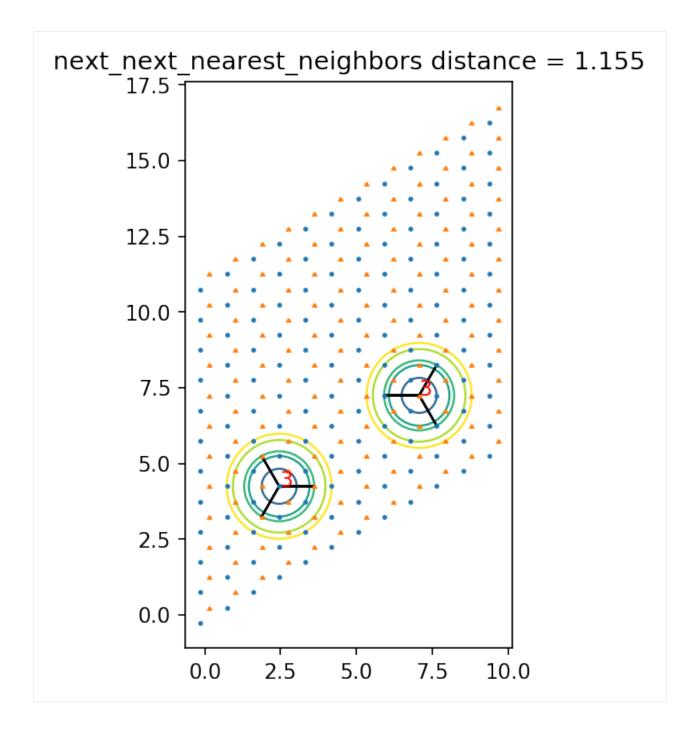
1 1 [ -1 -1] 1.7320508075688772

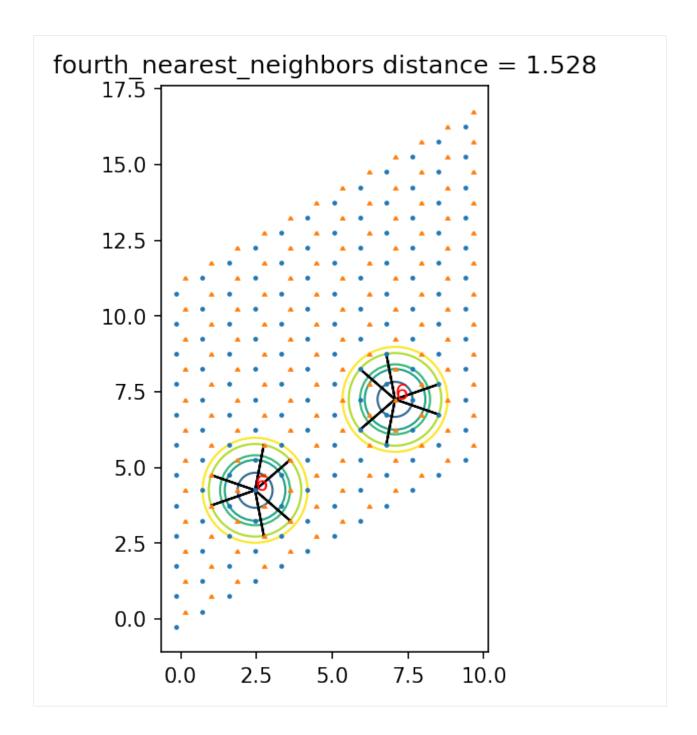
1 1 [ -2 1] 1.7320508075688772

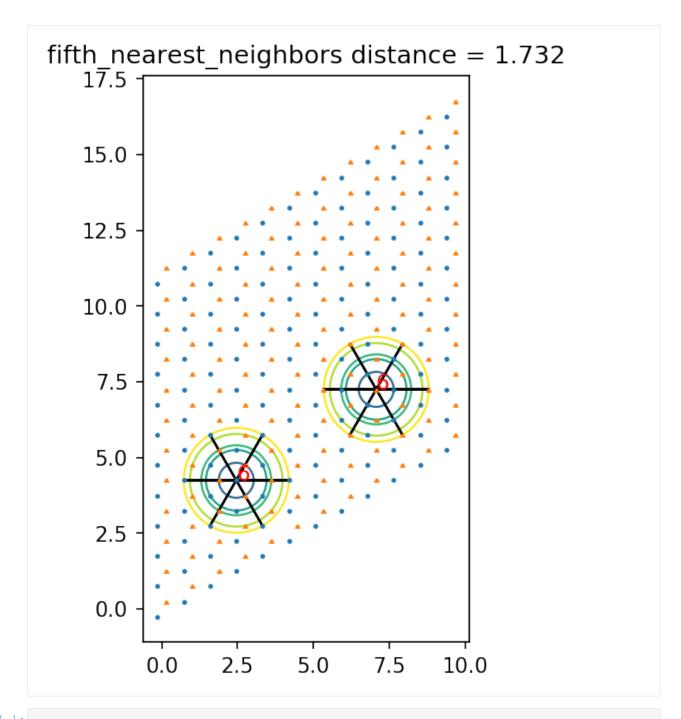
1 1 [ -2 1] 1.7320508075688772
```











11.3.4 Toric Code

This notebook shows how to implement Kitaev's Toric Code model on an infinite cylinder geometry. The Hamiltonian of the Toric Code is:

$$\mathcal{H} = -\sum_{p} B_{p} - \sum_{s} A_{s}$$

where $B_p = \prod_{i \in p} \sigma_i^z$ denotes the product of σ^z around an elementary plaquette and $A_s = \prod_{i \in s} \sigma_i^x$ denotes the product of σ^x on the four links sticking out of a site of the lattice. The B_p and A_s operators commute with each other and have eigenvalues ± 1 , hence the ground state will have eigenvalue +1 for each of them.

On an infinite cylinder, the model exhibits a four-fold degeneracy of the ground state, characterized by the different values ± 1 of the incontractible Wilson and t'Hooft loops. These loop operators are defined by the products

$$W = \prod_{\text{vert links}} \sigma^z \qquad \qquad H = \prod_{\text{hor links}} \sigma^x$$

around the cylinder.

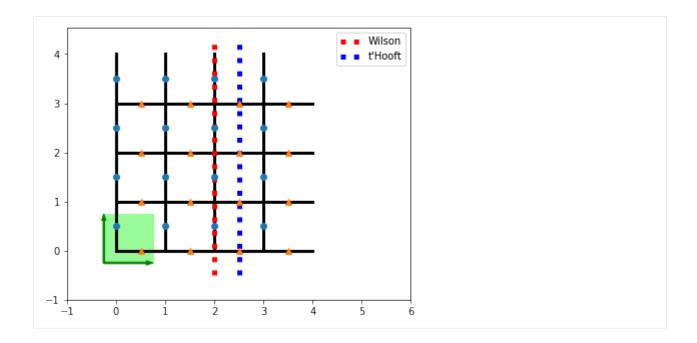
TeNPy already implements this hamiltonian in tenpy.models.toric_code.ToricCode. This notebook demonstrates how to extend the existing model by additinoal terms in the Hamiltonian and convenient functions of the loop operators for simplified measurements. Then we run iDMRG to obtain the four different ground states and check that they are indeed orthogonal and degenerate.

```
[1]: import numpy as np
import scipy
import matplotlib.pyplot as plt
np.set_printoptions(precision=5, suppress=True, linewidth=120)
```

```
import tenpy
import tenpy.linalg.np_conserved as npc
from tenpy.algorithms.dmrg import TwoSiteDMRGEngine
from tenpy.networks.mps import MPS
from tenpy.networks.terms import TermList
from tenpy.models.toric_code import ToricCode, DualSquare
from tenpy.models.lattice import Square
```

Let's plot the DualSquare lattice first to get the geometry clear. The unit cell consists of two sites, the blue one on the vertical links (u=0), and the orange one on the horizontal links (u=1). We also plot lines illustrating where the Wilson and t'Hooft loops go.

```
[3]: plt.figure(figsize=(7, 5))
    ax = plt.gca()
    lat = DualSquare(4, 4, None, bc='periodic')
    sq = Square(4, 4, None, bc='periodic')
    sq.plot_coupling(ax, linewidth=3.)
    ax.plot([2., 2.], [-0.5, 4.3], 'r:', linewidth=5., label="Wilson")
    ax.plot([2.5, 2.5], [-0.5, 4.3], 'b:', linewidth=5., label="t'Hooft")
    lat.plot_sites(ax)
    lat.plot_basis(ax, origin=-0.25*(lat.basis[0] + lat.basis[1]))
    ax.set_aspect('equal')
    ax.set_xlim(-1, 6)
    ax.set_ylim(-1)
    ax.legend()
    plt.show()
```



Extending the existing model

We will implement additional parameters which allow to optionally add terms for the Wilson and t'Hooft loop operators W, H defined above,

$$H o H - J_{WL}W - J_{HL}H$$

Note that we only want to add a single loop for each of them, not one at each possible starting point. Hence, the correct method is add_local_term, not add_multi_coupling_term or add_coupling_term.

The sign of J_{WL} , J_{HL} will allow us to select sectors with $\langle \psi | W | \psi \rangle = \pm 1$ and $\langle \psi | H | \psi \rangle = \pm 1$, respectively. Note that so far the constraints commute with the Hamiltonian and with each other. However, this is no longer the case if we add another global field $H \to H - h \sum_i \sigma_i^z$, which makes the Hamiltonian no longer exactly solvable.

```
[4]: class ExtendedToricCode (ToricCode):
        def init_terms(self, model_params):
            ToricCode.init_terms(self, model_params) # add terms of the original_
     → ToricCode model
            Ly = self.lat.shape[1]
            J_WL = model_params.get('J_WL', 0.)
            J_HL = model_params.get('J_HL', 0.)
             # unit-cell indices:
             # u=0: vertical links
             # u=1: horizontal links
             # Wilson Loop
            x, u = 0, 0 \# vertical links
             self.add_local_term(-J_WL, [('Sigmaz', [x, y, u]) for y in range(Ly)])
             # t'Hooft Loop
            x, u = 0, 1 # vertical links
            self.add_local_term(-J_HL, [('Sigmax', [x, y, u]) for y in range(Ly)])
```

```
h = model_params.get('h', 0.)
       for u in range(2):
           self.add_onsite(-h, u, 'Sigmaz')
   def wilson_loop_y(self, psi):
       """Measure wilson loop around the cylinder."""
       Ly = self.lat.shape[1]
       x, u = 0, 0 \# vertical links
       W = TermList.from_lattice_locations(self.lat, [[("Sigmaz",[x, y, u]) for y in_
→range(Ly)]])
       return psi.expectation_value_terms_sum(W)[0]
   def hooft_loop_v(self, psi):
       """Measure t'Hooft loop around the cylinder."""
       Ly = self.lat.shape[1]
       x, u = 0, 1 # horizontal links
       H = TermList.from_lattice_locations(self.lat, [[("Sigmax",[x, y, u]) for y in_
→range(Ly)]])
       return psi.expectation_value_terms_sum(H)[0]
```

iDMRG

We now run the iDMRG algorithm for the four different sectors to obtain the four degenerate ground states of the system.

To reliably get the 4 different ground states, we will add the energetic constraints for the loops. In this case, the original Hamiltonian will completely commute with the loop operators, such that you directly get the correct ground state. However, this is a very peculiar case - in general, e.g. when you add the external field with $h \neq 0$, this will no longer be the case. We hence demonstrate a more "robust" way of getting the four different ground states (at least if you have an idea which operator distinguishes them): For each sector, we run DMRG twice: first with the energy penalties for the W, W loops to get an initial W in the desired sector. Then we run DMRG again for the clean toric code model, without the W, W added, to make sure we have a ground state of the clean model.

```
[5]: dmrg_params = {
         'mixer': True,
         'trunc_params': {'chi_max': 64,
                          'svd_min': 1.e-8},
         'max_E_err': 1.e-8,
         'max_S_err': 1.e-7,
         'N_sweeps_check': 4,
         'max_sweeps':24,
         'verbose': 0,
    model_params = {
         'Lx': 1, 'Ly': 4, # Ly is set below
         'bc_MPS': "infinite",
         'conserve': None,
         'verbose': 0,
    def run_DMRG(Ly, J_WL, J_HL, h=0.):
        print("=" *80)
         print(f"Start iDMRG for Ly={Ly:d}, J_WL={J_WL:.2f}, J_HL={J_HL:.2f}, h={h:.2f}")
```

```
model_params_clean = model_params.copy()
model_params_clean['Ly'] = Ly
model_params_clean['h'] = h
model_clean = ExtendedToricCode(model_params_clean)
model_params_seed = model_params_clean.copy()
model_params_seed['J_WL'] = J_WL
model_params_seed['J_HL'] = J_HL
model = ExtendedToricCode(model_params_seed)
psi = MPS.from_lat_product_state(model.lat, [[["up"]]])
eng = TwoSiteDMRGEngine(psi, model, dmrg_params)
E0, psi = eng.run()
WL = model.wilson_loop_y(psi)
HL = model.hooft_loop_v(psi)
print(f"after first DMRG run: <psi|W|psi> = {WL: .3f}")
print(f"after first DMRG run: <psi|H|psi> = {HL: .3f}")
print(f"after first DMRG run: E (including W/H loops) = {E0:.10f}")
E0_clean = model_clean.H_MPO.expectation_value(psi)
print(f"after first DMRG run: E (excluding W/H loops) = {E0_clean:.10f}")
# switch to model without Wilson/Hooft loops
eng.init_env(model=model_clean)
E1, psi = eng.run()
WL = model_clean.wilson_loop_y(psi)
HL = model_clean.hooft_loop_y(psi)
print(f"after second DMRG run: <psi|W|psi> = {WL: .3f}")
print(f"after second DMRG run: <psi|H|psi> = {HL: .3f}")
print(f"after second DMRG run: E (excluding W/H loops) = {E1:.10f}")
print("max chi: ", max(psi.chi))
return {'psi': psi,
        'model': model_clean,
        'E0': E0, 'E0_clean': E0_clean, 'E': E1,
        'WL': WL, 'HL': HL}
print("="*80)
```

```
[6]: results_loops = {}
    for J_WL in [-5., 5.]:
       for J_HL in [-5., 5.]:
           results_loops[(J_WL, J_HL)] = run_DMRG(4, J_WL, J_HL)
    ______
    Start iDMRG for Ly=4, J_WL=-5.00, J_HL=-5.00, h=0.00
    after first DMRG run: \langle psi|W|psi \rangle = -1.000
    after first DMRG run: \langle psi|H|psi \rangle = -1.000
    after first DMRG run: E (including W/H loops) = -2.2500000000
    after first DMRG run: E (excluding W/H loops) = -1.0000000000
    after second DMRG run: \langle psi|W|psi \rangle = -1.000
    after second DMRG run: \langle psi|H|psi \rangle = -1.000
    after second DMRG run: E (excluding W/H loops) = -1.0000000000
    max chi: 8
    ______
    Start iDMRG for Ly=4, J_WL=-5.00, J_HL=5.00, h=0.00
    after first DMRG run: \langle psi|W|psi \rangle = -1.000
```

```
after first DMRG run: <psi|H|psi> = 1.000
after first DMRG run: E (including W/H loops) = -2.2500000000
after first DMRG run: E (excluding W/H loops) = -1.0000000000
after second DMRG run: \langle psi|W|psi \rangle = -1.000
after second DMRG run: \langle psi|H|psi \rangle = 1.000
after second DMRG run: E (excluding W/H loops) = -1.0000000000
max chi: 8
______
Start iDMRG for Ly=4, J_WL=5.00, J_HL=-5.00, h=0.00
after first DMRG run: \langle psi|W|psi \rangle = 1.000
after first DMRG run: \langle psi|H|psi \rangle = -1.000
after first DMRG run: E (including W/H loops) = -2.2500000000
after first DMRG run: E (excluding W/H loops) = -1.0000000000
after second DMRG run: <psi|W|psi> = 1.000
after second DMRG run: \langle psi|H|psi \rangle = -1.000
after second DMRG run: E (excluding W/H loops) = -1.0000000000
max chi: 8
______
Start iDMRG for Ly=4, J_WL=5.00, J_HL=5.00, h=0.00
after first DMRG run: \langle psi|W|psi \rangle = 1.000
after first DMRG run: \langle psi|H|psi \rangle = 1.000
after first DMRG run: E (including W/H loops) = -2.2500000000
after first DMRG run: E (excluding W/H loops) = -1.0000000000
after second DMRG run: \langle psi|W|psi \rangle = 1.000
after second DMRG run: \langle psi|H|psi \rangle = 1.000
after second DMRG run: E (excluding W/H loops) = -1.0000000000
max chi: 8
```

As we can see from the output, the idea worked and we indeed get the desired expectation values for the W/H loops. Also, we can directly see that the ground states are degenerate in energy.

To check that we found indeed four orhogonal ground states, let's calculate mutual overlaps $\langle \psi_i | \psi_j \rangle$ for $i, j \in range(4)$:

```
[7]: psi_list = [res['psi'] for res in results_loops.values()]
overlaps= [[psi_i.overlap(psi_j) for psi_j in psi_list] for psi_i in psi_list]
print("overlaps")
print(np.array(overlaps))

overlaps

[[ 1.+0.j  0.+0.j -0.+0.j  0.-0.j]
      [ 0.+0.j  1.+0.j  0.+0.j -0.+0.j]
      [-0.+0.j  0.+0.j  1.+0.j  0.+0.j]
      [ 0.-0.j -0.+0.j  0.+0.j  1.+0.j]]
```

The correlation length is almost vanishing, as it should be:

Intermezzo: Let's quickly check for a single case that this still works for non-zero h=0.1, where the t'Hooft loop no longer commutes with the Hamiltonian:

The t'Hooft loop no longer is exactly 1 in the ground state, since it no longer commutes with the Hamiltonian, but it is still close to one, so it still distinguishes the degenerate states. As expected, the local perturbation of the field can not destry the global, topological nature of the ground state (manifold).

Topological entanglement entropy

Back to h = 0. For the toric code, the half-cylinder entanglement entropy should be $(L_y - 1) \log 2$. Indeed, we get the correct value at bond 0:

```
[10]: print("Entanglement entropies / log(2)")
    print(np.array([psi.entanglement_entropy()[0]/np.log(2) for psi in psi_list]))
    Entanglement entropies / log(2)
    [3. 3. 3. 3.]
```

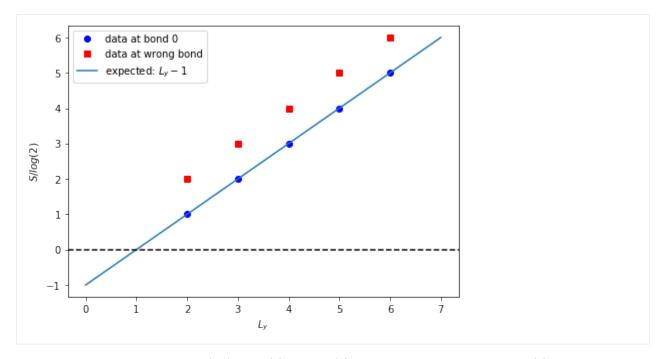
Recall that bond 0 is usually a sensible cut for infinite DMRG on a common lattice. This is especially important if you want to extract the topological entanglement entropy γ from a fit $S(L_y) = \alpha L_y - \gamma$, as corners in the cut can also contribute to γ .

Let us demonstrate the extraction of $\gamma = \log(2)$ for the toric code:

```
[11]: model_params['order'] = 'Cstyle' # The effect doesn't appear with the "default"
     →ordering for the toric code.
     # This is also a hint that you need bond 0: you want something independent of what,
     ⇔order you choose
     # inside the MPS unit cell.
     results_Ly = {}
     for Ly in range (2, 7):
         results_Ly[Ly] = run_DMRG(Ly, 5., 5.)
     ______
     Start iDMRG for Ly=2, J_WL=5.00, J_HL=5.00, h=0.00
     after first DMRG run: \langle psi|W|psi \rangle = 1.000
     after first DMRG run: <psi|H|psi> = 1.000
     after first DMRG run: E (including W/H loops) = -3.5000000000
     after first DMRG run: E (excluding W/H loops) = -1.0000000000
     after second DMRG run: \langle psi|W|psi \rangle = 1.000
     after second DMRG run: \langle psi|H|psi \rangle = 1.000
     after second DMRG run: E (excluding W/H loops) = -1.0000000000
     max chi: 4
     Start iDMRG for Ly=3, J_WL=5.00, J_HL=5.00, h=0.00
```

```
after first DMRG run: \langle psi|H|psi \rangle = 1.000
     after first DMRG run: E (including W/H loops) = -2.66666666667
     after first DMRG run: E (excluding W/H loops) = -1.0000000000
     after second DMRG run: \langle psi|W|psi \rangle = 1.000
     after second DMRG run: \langle psi|H|psi \rangle = 1.000
     after second DMRG run: E (excluding W/H loops) = -1.0000000000
     max chi: 8
     ______
     Start iDMRG for Ly=4, J_WL=5.00, J_HL=5.00, h=0.00
     after first DMRG run: \langle psi|W|psi \rangle = 1.000
     after first DMRG run: \langle psi|H|psi \rangle = 1.000
     after first DMRG run: E (including W/H loops) = -2.2500000000
     after first DMRG run: E (excluding W/H loops) = -1.0000000000
     after second DMRG run: <psi|W|psi> = 1.000
     after second DMRG run: \langle psi|H|psi \rangle = 1.000
     after second DMRG run: E (excluding W/H loops) = -1.0000000000
     max chi: 16
     ______
     Start iDMRG for Ly=5, J_WL=5.00, J_HL=5.00, h=0.00
     after first DMRG run: \langle psi|W|psi \rangle = 1.000
     after first DMRG run: <psi|H|psi> = 1.000
     after first DMRG run: E (including W/H loops) = -2.0000000000
     after first DMRG run: E (excluding W/H loops) = -1.0000000000
     after second DMRG run: \langle psi|W|psi \rangle = 1.000
     after second DMRG run: <psi|H|psi> = 1.000
     after second DMRG run: E (excluding W/H loops) = -1.00000000000
     max chi: 32
     ______
     Start iDMRG for Ly=6, J_WL=5.00, J_HL=5.00, h=0.00
     after first DMRG run: \langle psi|W|psi \rangle = 1.000
     after first DMRG run: <psi|H|psi> = 1.000
     after first DMRG run: E (including W/H loops) = -1.8333333333
     after first DMRG run: E (excluding W/H loops) = -1.0000000000
     after second DMRG run: \langle psi|W|psi \rangle = 1.000
     after second DMRG run: \langle psi|H|psi \rangle = 1.000
     after second DMRG run: E (excluding W/H loops) = -1.0000000000
     max chi: 64
[12]: Lys = np.array(list(results_Ly.keys()))
     S0s = np.array([res['psi'].entanglement_entropy()[0] for res in results_Ly.values()])
     S1s = np.array([res['psi'].entanglement_entropy()[2] for res in results_Ly.values()])
     plt.figure(figsize=(7, 5))
     ax = plt.gca()
     ax.plot(Lys, S0s / np.log(2), 'bo', label='data at bond 0')
     ax.plot(Lys, S1s / np.log(2), 'rs', label='data at wrong bond')
     Lys_all = np.arange(0., max(Lys) + 2.)
     ax.plot(Lys_all, Lys_all - 1., '-', label='expected: $L_y - 1$')
     ax.axhline(0., linestyle='--', color='k')
     ax.set_xlabel('$L_y$')
     ax.set_ylabel('$S / log(2)$')
     ax.legend()
     plt.show()
```

after first DMRG run: $\langle psi|W|psi \rangle = 1.000$



As expected, We find the scaling $S(L_y) = \log(2)L_y - \log(2)$ at bond 0, indicating that $\gamma = \log(2)$. Cutting the MPS at bond 1 (red points) is a "wired" cut of the cylinder and does not show the expected scaling.

[]:

172

CHAPTER

TWELVE

TROUBLESHOOTING AND FAQ

I updated to a new version and now I get and error/warning. Take a look at the section "Backwards incomptatible changes" in /changelog of the corresponding versions since when you updated.

Where did all the output go? Take a look at Logging and terminal output.

What are possible parameters for ...? See Parameters and options.

How can I set the number of threads TeNPy is using? Most algorithms in TeNPy don't use any parallelization besides what the underlying BLAS provides, so that depends on how you installed TeNPy, numpy and scipy! Using for example an export OMP_NUM_THREADS=4 should limit it to 4 threads under usual setups, but you might also want to export MKL_NUM_THREADS=4 instead, if you are sure that you are using MKL.

Why is TeNPy not respecting MKL_NUM_THREADS? It might be that it is not using MKL. On linux, check whether you have installed a pip version of numpy or scipy in \$HOME/.local/lib/python3.* Those packages do not use MKL - you would need to install numpy and scipy from conda. If you use the *conda-forge* channel as recommended in the installation, also make sure that you select the BLAS provided by MKL, see the note in install/conda.rst.

12.1 I get an error when ...

... I try to measure Sx_i Sx_j correlations in a state with Sz consertaion. Right now this is not possible. See the basic facts in Charge conservation with np conserved.

12.2 I get a warning about ...

... an unused parameter. Make sure that you don't have a typo and that it is in the right parameter set! Also, check the logging output whether the parameter was actually used. For further details, see *Parameters and options*

CHAPTER

THIRTEEN

LITERATURE AND REFERENCES

This is a (by far non-exhaustive) list of some references for the various ideas behind the code. They can be cited like this:

- [TeNPyNotes] for TeNPy/software related sources
- [[white1992]] (lowercase first-author + year) for entries from *literature.bib*.

13.1 TeNPy related sources

[TeNPyNotes] are lecture notes, meant as an introduction to tensor networks (focusing on MPS), and introduced TeNPy to the scientific community by giving examples how to call the algorithms in TeNPy. [TeNPySource] is the location of the source code, and the place where you can report bugs. We have split example notebooks into [TeNPyNotebooks] to keep the git history of the original repository clean. [TeNPyDoc] is where the documentation is hosted online. [TeNPyForum] is the place where you can ask questions and look for help when you are stuck with implementing something.

13.2 Software-related

The following links are not physics-related, but are good to know if you want to work with TeNPy (or more generally Python).

13.3 General reading

[[schollwoeck2011]] is an extensive introduction to MPS, DMRG and TEBD with lots of details on the implementations, and a classic read, although a bit lengthy. Our [TeNPyNotes] are a shorter summary of the important concepts, similar as [[orus2014]]. [[paeckel2019]] is a very good, recent review focusing on time evolution with MPS. The lecture notes of [[eisert2013]] explain the area law as motivation for tensor networks very well. PEPS are for example reviewed in [[verstraete2008]], [[eisert2013]] and [[orus2014]]. [[stoudenmire2012]] reviews the use of DMRG for 2D systems. [[cirac2009]] discusses the different groups of tensor network states.

13.4 Algorithm developments

[[white1992], [white1993]] is the invention of DMRG, which started everything. [[vidal2004]] introduced TEBD. [[white2005]] and [[hubig2015]] solved problems for single-site DMRG. [[mcculloch2008]] was a huge step forward to solve convergence problems for infinite DMRG. [[singh2010], [singh2011]] explain how to incorporate Symmetries. [[haegeman2011]] introduced TDVP, again explained more accessible in [[haegeman2016]]. [[zaletel2015]] is another standard method for time-evolution with long-range Hamiltonians. [[karrasch2013]] gives some tricks to do finite-temperature simulations (DMRG), which is a bit extended in [[hauschild2018a]]. [[vidal2007]] introduced MERA. The scaling $S = c/6log(\chi)$ at a 1D critical point is explained in [[pollmann2009]].

13.5 References

FOURTEEN

PAPERS USING TENPY

This page collects papers using (and citing) the TeNPy library, both as an inspiration what can be done, as well as to keep track of the usage, such that we can see how useful our work is to the community. It keeps us motivated!

To include your own work, you can either fill out this template on github, or you can directly add your citation in this Zotero online library (and notify us about it or just wait).

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CHAPTER

FIFTEEN

CONTRIBUTING

There are lots of things where you can help, even if you don't wont to dig deep into the source code. You are welcome to do any of the following things, all of them are very helpful!

- Report bugs and problems, such that they can be fixed.
- Implement new models.
- Update and extend the documentation.
- Provide examples of how to use TeNPy.
- Give feedback on how you like TeNPy and what you would like to see improved.
- · Help fixing bugs.
- Help fixing minor issues.
- Extend the functionality by implementing new functions, methods, and algorithms.

The code is maintained in a git repository, the official repository is on github. Even if you're not yet on the developer team, you can still submit pull requests on github. If you're unsure how or what to do, you can ask for help in the [TeNPyForum]. If you want to become a member of the developer team, just ask;-)

Thank You!

15.1 Coding Guidelines

To keep consistency, we ask you to comply with the following guidelines for contributions. However, these are just guidelines - it still helps if you contribute something, even if doesn't follow these rules ;-)

• Use a code style based on **PEP 8**. The git repo includes a config file .style.yapf for the python package yapf. yapf is a tool to auto-format code, e.g., by the command yapf -i some/file (-i for "in place"). We run yapf on a regular basis on the github main branch. If your branch diverged, it might help to run yapf before merging.

Note: Since no tool is perfect, you can format some regions of code manually and enclose them with the special comments # yapf: disable and # yapf: enable.

• Every function/class/module should be documented by its doc-string, see **PEP 257**. We auto-format the doc-strings with docformatter on a regular basis.

Additional documentation for the user guide is in the folder doc/.

The documentation uses *reStructuredText*. If you are new to *reStructuredText*, read this introduction. We use the *numpy* style for doc-strings (with the napoleon extension to sphinx). You can read abouth them in these

Instructions for the doc strings. In addition, you can take a look at the following example file. Helpful hints on top of that:

```
r"""<- this r makes me a raw string, thus '\' has no special meaning.
Otherwise you would need to escape backslashes, e.g. in math formulas.
You can include cross references to classes, methods, functions, modules like
:class:`~tenpy.linalq.np_conserved.Array`, :meth:`~tenpy.linalq.np_conserved.
→Array.to_ndarray`,
:func:`tenpy.tools.math.toiterable`, :mod:`tenpy.linalq.np_conserved`.
The ~ in the beginning makes only the last part of the name appear in the ...
→generated documentation.
Documents of the userguide can be referenced with :doc: `/intro_npc` even from_
\hookrightarrow inside the doc-strings.
You can also cross-link to other documentations, e.g. :class:`numpy.ndarray`, :
→func`scipy.linalg.svd` and :mod: will work.
Moreover, you can link to github issues, arXiv papers, dois, and topics in the
→community forum with
e.g. :issue:`5`, :arxiv:`1805.00055`, :doi:`10.1000/1` and :forum:`3`.
Citations from the literature list can be cited as :cite:`white1992` using the
⇒bibtex key.
Write inline formulas as :math:`H |\Psi\rangle = E |\Psi\rangle` or displayed_
⇔equations as
.. math ::
  e^{i pi} + 1 = 0
In doc-strings, math can only be used in the Notes section.
To refer to variables within math, use `\mathtt{varname}`.
.. todo ::
  This block can describe things which need to be done and is automatically,
→included in a section of :doc:`todo`.
```

• Use relative imports within TeNPy. Example:

```
from ..linalg import np_conserved as npc
```

• Use the python package pytest for testing. Run it simply with pytest in *tests/*. You should make sure that all tests run through, before you git push back into the public repo. Long-running tests are marked with the attribute *slow*; for a quick check you can also run pytest -m "not slow".

We have set up github actions to automatically run the tests.

- Reversely, if you write new functions, please also include suitable tests!
- During development, you might introduce # TODO comments. But also try to remove them again later! If you're not 100% sure that you will remove it soon, please add a doc-string with a . . todo :: block, such that we can keep track of it.

Unfinished functions should raise NotImplementedError().

- Summarize the changes you have made in the Changelog under [latest].
- If you want to try out new things in temporary files: any folder named playground is ignored by git.

- If you add a new toycode or example: add a reference to include it in the documentation.
- We've created a sphinx extensions for documenting config-option dictionaries. If a class takes a dictionary of options, we usually call it *options*, convert it to a Config at the very beginning of the __init__ with asConfig(), save it as self.options, and document it in the class doc-string with a . . cfg:config :: directive. The name of the *config* should usually be the class-name (if that is sufficiently unique), or for algorithms directly the common name of the algorithm, e.g. "DMRG"; use the same name for the use the same name for the documentation of the . . cfg:config :: directive as for the Config class instance. Attributes which are simply read-out options should be documented by just referencing the options with the :cfg:option:`configname.optionname` role.

15.2 Bulding the documentation

You can use Sphinx to generate the full documentation in various formats (including HTML or PDF) yourself, as described in the following.

First, I will assume that you downloaded the [TeNPySource] repository with:

```
git clone --recursive https://github.com/tenpy/tenpy
```

This includes the [TeNPyNotebooks] as a git submodule; you might need to git submodule update if it is out of date.

Building the documentation requires a few more packages (including Sphinx). The recommended way is to create a separate conda environment for it with:

Alternatively, you can use *pip* and pip install -r doc/requirements.txt, but this will not be able to install all dependencies: some packages like Graphviz are not available from pip alone.

Afterwards, simply go to the folder doc/ and run the following command:

```
make html
```

This should generate the html documentation in the folder *doc/sphinx_build/html*. Open this folder (or to be precise: the file *index.html* in it) in your webbroser and enjoy this and other documentation beautifully rendered, with cross links, math formulas and even a search function. Other output formats are available as other make targets, e.g., make latexpdf.

Note: Building the documentation with sphinx requires loading the TeNPy modules. The *conf.py* adjusts the python *sys.path* to include the */tenpy* folder from root directory of the git repository. It will not use the cython-compiled parts.

15.3 To-Do list

You can check https://github.com/tenpy/tenpy/issues for things to be done.

The following list is auto-generated by sphinx, extracting . . todo :: blocks from doc-strings of the code.

Todo: TDVP is currently not implemented with the sweep class.

(The *original entry* is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/algorithms/mps_common of tenpy.algorithms.mps_common.Sweep, line 6.)

Todo:

- implement or wrap netcon.m, a function to find optimal contractionn sequences (arXiv:1304.6112)
- · improve helpfulness of Warnings
- _do_trace: trace over all pairs of legs at once. need the corresponding npc function first.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/algorithms/network_coft tenpy.algorithms.network_contractor, line 10.)

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/algorithms/tdvp.py:doc

Todo: This is still a beta version, use with care. The interface might still change.

of tenpy.algorithms.tdvp, line 12.)

Todo: long-term: Much of the code is similar as in DMRG. To avoid too much duplicated code, we should have a general way to sweep through an MPS and updated one or two sites, used in both cases.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/algorithms/tdvp.py:docof tenpy.algorithms.tdvp, line 16.)

Todo: add further terms (e.g. c^dagger c^dagger + h.c.) to the Hamiltonian.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/models/fermions_spin of tenpy.models.fermions_spinless, line 3.)

Todo: WARNING: These models are still under development and not yet tested for correctness. Use at your own risk! Replicate known results to confirm models work correctly. Long term: implement different lattices. Long term: implement variable hopping strengths Jx, Jy.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/models/hofstadter.py:coof tenpy.models.hofstadter, line 3.)

Todo: This is a naive, expensive implementation contracting the full network. Try to follow arXiv:1711.01104 for a better estimate; would that even work in the infinite limit?

(The *original entry* is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/networks/mpo.py:docs of tenpy.networks.mpo.MPO.variance, line 5.)

Todo: might be useful to add a "cleanup" function which removes operators cancelling each other and/or unused states. Or better use a 'compress' of the MPO?

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/networks/mpo.py:docs of tenpy.networks.mpo.MPOGraph, line 17.)

Todo: Make more general: it should be possible to specify states as strings.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/networks/mps.py:docs of tenpy.networks.mps.build_initial_state, line 14.)

Todo: test, provide more.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/simulations/measurem of tenpy.simulations.measurement, line 7.)

Todo: provide examples, give user guide

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/simulations/simulation of tenpy.simulations.simulation, line 8.)

Todo: For memory caching with big MPO environments, we need a Hdf5Cacher clearing the memo's every now and then (triggered by what?).

(The original entry is located in /home/docs/checkouts/readthedocs.org/user_builds/tenpy/checkouts/v0.8.3/tenpy/tools/hdf5_io.py:docst of tenpy.tools.hdf5_io, line 65.)

15.3. To-Do list 185

Part II

Reference

CHAPTER

SIXTEEN

TENPY MAIN MODULE

• full name: tenpy

• parent module: tenpy

• type: module

Submodules

algorithms	A collection of algorithms such as TEBD and DMRG.
linalg	Linear-algebra tools for tensor networks.
models	Definition of the various models.
networks	Definitions of tensor networks like MPS and MPO.
tools	A collection of tools: mostly short yet quite useful func-
	tions.
version	Access to version of this library.

Module description

TeNPy - a Python library for Tensor Network Algorithms

TeNPy is a library for algorithms working with tensor networks, e.g., matrix product states and -operators, designed to study the physics of strongly correlated quantum systems. The code is intended to be accessible for newcommers and yet powerful enough for day-to-day research.

run_simulation([simulation_class_name,])	Run the simulation with a simulation class.
console_main()	Command line interface.
show_config()	Print information about the version of tenpy and used
	libraries.

16.1 run_simulation

• full name: tenpy.run_simulation

• parent module: tenpy

· type: function

Run the simulation with a simulation class.

Parameters

- **simulation_class_name** (*str*) The name of a (sub)class of Simulation to be used for running the simulaiton.
- **simulation_class_kwargs** (*dict | None*) A dictionary of keyword-arguments to be used for the initializing the simulation.
- **simulation_params Further keyword arguments as documented in the corresponding simulation class, see :cfg:config`Simulation`.

Returns The results from running the simulation, i.e., what tenpy.simulations. Simulation.run() returned.

Return type results

16.2 console main

• full name: tenpy.console_main

• parent module: tenpy

• type: function

tenpy.console_main()

Command line interface.

See also run_simulation() for the python interface running a simulation.

When tenpy is installed correctly via pip/conda, a command line script called tenpy-run is set up, which calls this function, i.e., you can do the following in the terminal:

```
tenpy-run --help
```

Equivalently, you can also invoke the tenpy module from your python interpreter like this:

```
python -m tenpy --help
```

(continues on next page)

(continued from previous page)

```
A yaml (*.yml) file with the simulation parameters/options.
optional arguments:
    -h, --help
        show this help message and exit
    --import-module MODULE, -i MODULE
        Import the given python MODULE before setting up the simulation. This is,
\hookrightarrowuseful if the
        module contains user-defined subclasses. Use python-style names like_
\hookrightarrow `numpy` without the
        .py ending.
    --sim-class SIM_CLASS, -c SIM_CLASS
        selects the Simulation (sub)class, e.g. 'GroundStateSearch' (default) or
        'RealTimeEvolution'.
    --option KEY VALUE, -o KEY VALUE
        Allows overwriting some options from the yaml files. KEY can be recursive.
        ``/``, e.g. ``algorithm_params/trunc_params/chi``. VALUE is initialized_
→by python's
        ``eval(VALUE)`` with `np`, `scipy` and `tenpy` defined. Thus ``'1.2'`` and ``'2.*np.pi*1.j/6'`` or ``'np.linspace(0., 1., 6)'`` will work if you_
⇒include the quotes
        on the command line to ensure that the VALUE is passed as a single_
\rightarrowargument.
    --version, -v
        show program's version number and exit
Examples
In the simplest case, you just give a single yaml file with all the parameters as,
→argument:
    tenpy-run my_params.yml
If you implemented a custom simulation class called ``MyGreatSimulation`` in a
file ``my_simulations.py``, you can use it like this:
    tenpy-run -i my_simulations -c MyGreatSimulation my_params.yml
Further, you can overwrite one or multiple options of the parameters file:
    tenpy-run my_params.yml -o output_filename '"output.h5"' -o model_params/Jz 2.
Note that string values for the options require double quotes on the command line.
```

16.2. console main 191

16.3 show_config

• full name: tenpy.show_config

• parent module: tenpy

• type: function

tenpy.show_config()

Print information about the version of tenpy and used libraries.

The information printed is tenpy.version.version_summary.

tenpy.__version__ = '0.8.3'

hard-coded version string

tenpy.__full_version__ = '0.8.3'

full version from git description, and numpy/scipy/python versions

Submodules

algorithms	A collection of algorithms such as TEBD and DMRG.
linalg	Linear-algebra tools for tensor networks.
models	Definition of the various models.
networks	Definitions of tensor networks like MPS and MPO.
simulations	Simulation setup.
tools	A collection of tools: mostly short yet quite useful func-
	tions.
version	Access to version of this library.

CHAPTER

SEVENTEEN

ALGORITHMS

• full name: tenpy.algorithms

• parent module: tenpy

• type: module

Module description

A collection of algorithms such as TEBD and DMRG.

Submodules

algorithm	This module contains some base classes for algorithms.
truncation	Truncation of Schmidt values.
tebd	Time evolving block decimation (TEBD).
mps_common	'Sweep' algorithm and effective Hamiltonians.
dmrg	Density Matrix Renormalization Group (DMRG).
tdvp	Time Dependant Variational Principle (TDVP) with
	MPS (finite version only).
purification	Algorithms for using Purification.
mpo_evolution	Time evolution using the WI or WII approximation of
	the time evolution operator.
network_contractor	Network Contractor.
exact_diag	Full diagonalization (ED) of the Hamiltonian.

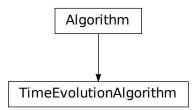
17.1 algorithm

• full name: tenpy.algorithms.algorithm

• parent module: tenpy.algorithms

• type: module

Classes



Algorithm(psi, model, options, *[, resume_data])	Base class and common interface for a tensor-network based algorithm in TeNPy.
<pre>TimeEvolutionAlgorithm(psi, model, options, *)</pre>	Common interface for (real) time evolution algorithms.

Module description

This module contains some base classes for algorithms.

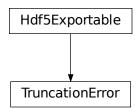
17.2 truncation

• full name: tenpy.algorithms.truncation

• parent module: tenpy.algorithms

• type: module

Classes



TruncationError([eps, ov])	Class representing a truncation error.

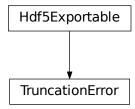
17.2.1 TruncationError

• full name: tenpy.algorithms.truncation.TruncationError

• parent module: tenpy.algorithms.truncation

• type: class

Inheritance Diagram



Methods

TruncationErrorinit([eps, ov])	Initialize self.
TruncationError.copy()	Return a copy of self.
TruncationError.from_S(S_discarded[,	Construct TruncationError from discarded singular val-
norm_old])	ues.
TruncationError.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
h5gr,)	
TruncationError.from_norm(norm_new[,	Construct TruncationError from norm after and before
norm_old])	the truncation.
TruncationError.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	

Class Attributes and Properties

TruncationError.ov_err	Error 1. $-\circ v$ of the overlap with the correct state.

class tenpy.algorithms.truncation.TruncationError(eps=0.0, ov=1.0)
 Bases: tenpy.tools.hdf5_io.Hdf5Exportable

Class representing a truncation error.

The default initialization represents "no truncation".

17.2. truncation 195

Warning: For imaginary time evolution, this is *not* the error you are interested in!

Parameters

- eps (float) See below.
- ov (float) See below.

eps

The total sum of all discared Schmidt values squared. Note that if you keep singular values up to 1.e-14 (= a bit more than machine precision for 64bit floats), *eps* is on the order of 1.e-28 (due to the square)!

Type float

ov

A lower bound for the overlap $|\langle \psi_{trunc} | \psi_{correct} \rangle|^2$ (assuming normalization of both states). This is probably the quantity you are actually interested in. Takes into account the factor 2 explained in the section on Errors in the *TEBD Wikipedia article https://en.wikipedia.org/wiki/Time-evolving_block_decimation*.

Type float

copy()

Return a copy of self.

classmethod from_norm(norm_new, norm_old=1.0)

Construct TruncationError from norm after and before the truncation.

Parameters

- norm_new (float) Norm of Schmidt values kept, $\sqrt{\sum_{akept} \lambda_a^2}$ (before renormalization).
- norm_old (float) Norm of all Schmidt values before truncation, $\sqrt{\sum_a \lambda_a^2}$

classmethod from_S (S_discarded, norm_old=None)

Construct TruncationError from discarded singular values.

Parameters

- S_discarded (1D numpy array) The singular values discarded.
- norm_old (float) Norm of all Schmidt values before truncation, $\sqrt{\sum_a \lambda_a^2}$. Default (None) is 1.

property ov err

Error 1. -ov of the overlap with the correct state.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with <code>save_dict_content()</code>, storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Functions

svd_theta(theta, trunc_par[, qtotal_LR,])	Performs SVD of a matrix <i>theta</i> (= the wavefunction)
	and truncates it.
truncate(S, options)	Given a Schmidt spectrum S, determine which values to
	keep.

17.2.2 svd theta

- full name: tenpy.algorithms.truncation.svd_theta
- parent module: tenpy.algorithms.truncation
- type: function

tenpy.algorithms.truncation.svd_theta(theta, $trunc_par$, $qtotal_LR=[None, None]$, $in-ner_labels=['vR', 'vL']$)

Performs SVD of a matrix *theta* (= the wavefunction) and truncates it.

Perform a singular value decomposition (SVD) with svd() and truncates with truncate(). The result is an approximation theta $\sim=$ tensordot(U.scale_axis(S*renormalization, 1), VH, axes=1)

Parameters

- theta (Array, shape (M, N)) The matrix, on which the singular value decomposition (SVD) is performed. Usually, *theta* represents the wavefunction, such that the SVD is a Schmidt decomposition.
- trunc_par (dict) truncation parameters as described in truncate().
- qtotalLR((charges, charges)) The total charges for the returned *U* and *VH*.
- inner_labels ((string, string)) Labels for the *U* and *VH* on the newlycreated bond.

Returns

- U (Array) Matrix with left singular vectors as columns. Shape (M, M) or (M, K) depending on *full_matrices*.
- **S**(*1D ndarray*) The singluar values of the array. If no *cutoff* is given, it has length min (M, N). Normalized to np.linalg.norm(S) == 1.

17.2. truncation 197

- VH (Array) Matrix with right singular vectors as rows. Shape (N, N) or (K, N) depending on full matrices.
- **err** (*TruncationError*) The truncation error introduced.
- renormalization (*float*) Factor, by which S was renormalized.

Module description

Truncation of Schmidt values.

Often, it is necessary to truncate the number of states on a virtual bond of an MPS, keeping only the state with the largest Schmidt values. The function truncate () picks exactly those from a given Schmidt spectrum λ_a , depending on some parameters explained in the doc-string of the function.

Further, we provide TruncationError for a simple way to keep track of the total truncation error.

The SVD on a virtual bond of an MPS actually gives a Schmidt decomposition $|\psi\rangle = \sum_a \lambda_a |L_a\rangle |R_a\rangle$ where $|L_a\rangle$ and $|R_a\rangle$ form orthonormal bases of the parts left and right of the virtual bond. Let us assume that the state is properly normalized, $\langle \psi | \psi \rangle = \sum_a \lambda_a^2 = 1$. Assume that the singular values are ordered descending, and that we keep the first χ_c of the initially χ Schmidt values.

Then we decompose the untruncated state as $|\psi\rangle = \sqrt{1-\epsilon}|\psi_{tr}\rangle + \sqrt{\epsilon}|\psi_{tr}^{\perp}\rangle$ where $|\psi_{tr}\rangle = \frac{1}{\sqrt{1-\epsilon}}\sum_{a<\chi_c}\lambda_a|L_a\rangle|R_a\rangle$ is the truncated state kept (normalized to 1), $|\psi_{tr}^{\perp}\rangle = \frac{1}{\sqrt{\epsilon}}\sum_{a>=\chi_c}\lambda_a|L_a\rangle|R_a\rangle$ is the discarded part (orthogonal to the kept part) and the *truncation error of a single truncation* is defined as $\epsilon = 1 - |\langle\psi|\psi_{tr}\rangle|^2 = \sum_{a>=\chi_c}\lambda_a^2$.

Warning: For imaginary time evolution (e.g. with TEBD), you try to project out the ground state. Then, looking at the truncation error defined in this module does *not* give you any information how good the found state coincides with the actual ground state! (Instead, the returned truncation error depends on the overlap with the initial state, which is arbitrary > 0)

Warning: This module takes only track of the errors coming from the truncation of Schmidt values. There might be other sources of error as well, for example TEBD has also an discretisation error depending on the chosen time step.

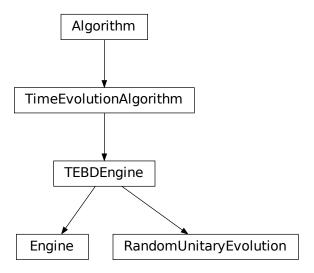
17.3 tebd

• full name: tenpy.algorithms.tebd

• parent module: tenpy.algorithms

• type: module

Classes



Engine(psi, model, options)	Deprecated old name of TEBDEngine.
RandomUnitaryEvolution(psi, options)	Evolution of an MPS with random two-site unitaries in
	a TEBD-like fashion.
TEBDEngine(psi, model, options, **kwargs)	Time Evolving Block Decimation (TEBD) algorithm.

17.3.1 Engine

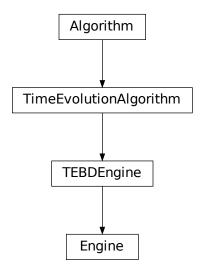
• full name: tenpy.algorithms.tebd.Engine

• parent module: tenpy.algorithms.tebd

• type: class

17.3. tebd 199

Inheritance Diagram



Methods

Engineinit(psi, model, options)	Initialize self.
Engine.calc_U(order, delta_t[, type_evo,])	Calculate self.U_bond from self.
	<pre>bond_eig_{vals,vecs}.</pre>
Engine.get_resume_data()	Return necessary data to resume a run () interrupted at
	a checkpoint.
Engine.resume_run()	Resume a run that was interrupted.
Engine.run()	Run TEBD real time evolution by <i>N_steps</i> `*` <i>dt</i> .
Engine.run_GS()	TEBD algorithm in imaginary time to find the ground
	state.
Engine.suzuki_trotter_decomposition(ord	erReturns list of necessary steps for the suzuki trotter de-
)	composition.
Engine.suzuki_trotter_time_steps(order)	Return time steps of U for the Suzuki Trotter decompo-
	sition of desired order.
Engine.update(N_steps)	Evolve by N_steps * U_param['dt'].
Engine.update_bond(i, U_bond)	Updates the B matrices on a given bond.
Engine.update_bond_imag(i, U_bond)	Update a bond with a (possibly non-unitary) <i>U_bond</i> .
Engine.update_imag(N_steps)	Perform an update suitable for imaginary time evolu-
	tion.
Engine.update_step(U_idx_dt, odd)	Updates either even <i>or</i> odd bonds in unit cell.

Class Attributes and Properties

Engine.TEBD_params

```
Engine.trunc_err_bonds
```

truncation error introduced on each non-trivial bond.

Engine.verbose

```
class tenpy.algorithms.tebd.Engine (psi, model, options)
```

Bases: tenpy.algorithms.tebd.TEBDEngine

Deprecated old name of TEBDEngine.

```
calc_U (order, delta_t, type_evo='real', E_offset=None)
```

Calculate self.U_bond from self.bond_eig_{vals, vecs}.

This function calculates

- U_bond = exp(-i dt (H_bond-E_offset_bond)) for type_evo='real', or
- U_bond = exp(- dt H_bond) for type_evo='imag'.

For first order (in *delta_t*), we need just one dt=delta_t. Higher order requires smaller *dt* steps, as given by *suzuki_trotter_time_steps()*.

Parameters

- order (int) Trotter order calculated U_bond. See update for more information.
- $delta_t(float)$ Size of the time-step used in calculating U_bond
- type_evo('imag' | 'real') Determines whether we perform real or imaginary time-evolution.
- **E_offset** (None | list of float) Possible offset added to H_bond for real-time evolution.

get_resume_data()

Return necessary data to resume a run () interrupted at a checkpoint.

At a checkpoint, you can save psi, model and options along with the data returned by this function. When the simulation aborts, you can resume it using this saved data with:

```
eng = AlgorithmClass(psi, model, options, resume_data=resume_data)
eng.resume_run(resume_data)
```

An algorithm which doesn't support this should override resume_run to raise an Error.

Returns resume_data – Dictionary with necessary data (apart from copies of *psi*, *model*, *options*) that allows to continue the simulation from where we are now.

Return type dict

resume_run()

Resume a run that was interrupted.

In case we saved an intermediate result at a checkpoint, this function allows to resume the run() of the algorithm (after re-initialization with the resume_data). Since most algorithms just have a while loop with break conditions, the default behaviour implemented here is to just call run().

run()

Run TEBD real time evolution by *N_steps*`*`dt.

17.3. tebd 201

run GS()

TEBD algorithm in imaginary time to find the ground state.

Note: It is almost always more efficient (and hence advisable) to use DMRG. This algorithms can nonetheless be used quite well as a benchmark and for comparison.

```
option TEBDEngine.delta_tau_list: list
```

A list of floats: the timesteps to be used. Choosing a large timestep $delta_tau$ introduces large (Trotter) errors, but a too small time step requires a lot of steps to reach $exp(-tau \ H) \ --> |psi0><psi0|$. Therefore, we start with fairly large time steps for a quick time evolution until convergence, and the gradually decrease the time step.

option TEBDEngine.order: int

Order of the Suzuki-Trotter decomposition.

```
option TEBDEngine.N_steps: int
```

Number of steps before measurement can be performed

static suzuki trotter decomposition (order, N steps)

Returns list of necessary steps for the suzuki trotter decomposition.

We split the Hamiltonian as $H = H_{even} + H_{odd} = H[0] + H[1]$. The Suzuki-Trotter decomposition is an approximation $\exp(tH) \approx prod_{(j,k) \in ST} \exp(d[j]tH[k]) + O(t^{order+1})$.

Parameters order (1, 2, 4, '4_opt') – The desired order of the Suzuki-Trotter decomposition. Order 1 approximation is simply $e^A a^B$. Order 2 is the "leapfrog" $e^A A/2 e^B e^A A/2$. Order 4 is the fourth-order from [[suzuki1991]] (also referenced in [[schollwoeck2011]]), and '4_opt' gives the optmized version of Equ. (30a) in [[barthel2020]].

Returns ST_decomposition – Indices j, k of the time-steps d = suzuki_trotter_time_step(order) and the decomposition of H. They are chosen such that a subsequent application of $\exp(d[j] t H[k])$ to a given state |psi> yields (exp(N_steps t H[k]) + O(N_steps t^{order+1})) |psi>.

Return type list of (int, int)

static suzuki_trotter_time_steps(order)

Return time steps of U for the Suzuki Trotter decomposition of desired order.

See suzuki_trotter_decomposition() for details.

Parameters order (int) – The desired order of the Suzuki-Trotter decomposition.

Returns time_steps - We need U = exp(-i H_{even/odd} delta_t * dt) for the dt returned in this list.

Return type list of float

property trunc_err_bonds

truncation error introduced on each non-trivial bond.

```
update(N\_steps)
```

```
Evolve by N_steps * U_param['dt'].
```

Parameters N_steps (*int*) – The number of steps for which the whole lattice should be updated.

Returns trunc_err – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

update_bond(i, U_bond)

Updates the B matrices on a given bond.

Function that updates the B matrices, the bond matrix s between and the bond dimension chi for bond i. The correponding tensor networks look like this:

Parameters

- \mathbf{i} (int) Bond index; we update the matrices at sites i-1, i.
- **U_bond** (*Array*) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0*', 'p1*'.

Returns trunc_err – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

update_bond_imag(i, U_bond)

Update a bond with a (possibly non-unitary) *U_bond*.

Similar as update_bond(); but after the SVD just keep the A, S, B canonical form. In that way, one can sweep left or right without using old singular values, thus preserving the canonical form during imaginary time evolution.

Parameters

- \mathbf{i} (int) Bond index; we update the matrices at sites i-1, i.
- **U_bond** (*Array*) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0*', 'p1*'.

Returns trunc_err – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

update_imag(N_steps)

Perform an update suitable for imaginary time evolution.

Instead of the even/odd brick structure used for ordinary TEBD, we 'sweep' from left to right and right to left, similar as DMRG. Thanks to that, we are actually able to preserve the canonical form.

Parameters N_steps (*int*) – The number of steps for which the whole lattice should be updated.

Returns trunc_err – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

update_step(U_idx_dt, odd)

Updates either even or odd bonds in unit cell.

Depending on the choice of p, this function updates all even $(\mathbb{E}, \text{ odd=False,0})$ or odd (\bigcirc) (odd=True,1) bonds:

17.3. tebd 203

Note that finite boundary conditions are taken care of by having Us [0] = None.

Parameters

- **U_idx_dt** (*int*) Time step index in self._U, evolve with Us[i] = self. U[U_idx_dt][i] at bond (i-1,i).
- **odd** (bool/int) Indication of whether to update even (odd=False, 0) or even (odd=True, 1) sites

Returns trunc_err – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

Module description

Time evolving block decimation (TEBD).

The TEBD algorithm (proposed in [[vidal2004]]) uses a trotter decomposition of the Hamiltonian to perform a time evoltion of an MPS. It works only for nearest-neighbor hamiltonians (in tenpy given by a NearestNeighborModel), which can be written as $H = H^{even} + H^{odd}$, such that H^{even} contains the the terms on even bonds (and similar H^{odd} the terms on odd bonds). In the simplest case, we apply first $U = \exp(-i*dt*H^{even})$, then $U = \exp(-i*dt*H^{odd})$ for each time step dt. This is correct up to errors of $O(dt^2)$, but to evolve until a time T, we need T/dt steps, so in total it is only correct up to error of O(T*dt). Similarly, there are higher order schemata (in dt) (for more details see TEBDEngine.update()).

Remember, that bond i is between sites (i-1, i), so for a finite MPS it looks like:

After each application of a *Ui*, the MPS needs to be truncated - otherwise the bond dimension *chi* would grow indefinitely. A bound for the error introduced by the truncation is returned.

If one chooses imaginary dt, the exponential projects (for sufficiently long 'time' evolution) onto the ground state of the Hamiltonian.

Note: The application of DMRG is typically much more efficient than imaginary TEBD! Yet, imaginary TEBD might

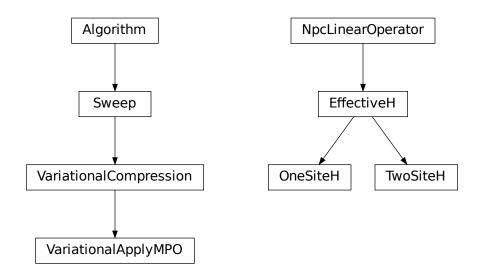
17.4 mps_common

• full name: tenpy.algorithms.mps_common

• parent module: tenpy.algorithms

• type: module

Classes



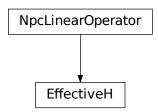
<pre>EffectiveH(env, i0[, combine, move_right])</pre>	Prototype class for local effective Hamiltonians used in
	sweep algorithms.
OneSiteH(env, i0[, combine, move_right])	Class defining the one-site effective Hamiltonian for
	Lanczos.
Sweep(psi, model, options, *[, resume_data])	Prototype class for a 'sweeping' algorithm.
TwoSiteH(env, i0[, combine, move_right])	Class defining the two-site effective Hamiltonian for
	Lanczos.
VariationalApplyMPO(psi, U_MPO, options[,	Variational compression for applying an MPO to an
])	MPS (in place).
VariationalCompression(psi, options[,])	Variational compression of an MPS (in place).

17.4. mps_common 205

17.4.1 EffectiveH

- full name: tenpy.algorithms.mps_common.EffectiveH
- parent module: tenpy.algorithms.mps_common
- type: class

Inheritance Diagram



Methods

EffectiveHinit(env, i0[, combine,])	Initialize self.
EffectiveH.adjoint()	Return the hermitian conjugate of self
EffectiveH.combine_theta(theta)	Combine the legs of theta, such that it fits to how we
	combined the legs of self.
EffectiveH.matvec(vec)	Calculate the action of the operator on a vector <i>vec</i> .
EffectiveH.to_matrix()	Contract self to a matrix.

Class Attributes and Properties

```
EffectiveH.acts_on

EffectiveH.length
```

class tenpy.algorithms.mps_common.**EffectiveH** (env, i0, combine=False, move_right=True)
Bases: tenpy.linalg.sparse.NpcLinearOperator

Prototype class for local effective Hamiltonians used in sweep algorithms.

As an example, the local effective Hamiltonian for a two-site (DMRG) algorithm looks like:

(continues on next page)

(continued from previous page)

.---

where HO and H1 are MPO tensors.

Parameters

- **env** (MPOEnvironment) Environment for contraction <psi|H|psi>.
- i0 (int) Index of the active site if length=1, or of the left-most active site if length>1.
- **combine** (bool, optional) Whether to combine legs into pipes as far as possible. This reduces the overhead of calculating charge combinations in the contractions.
- move_right (bool, optional) Whether the sweeping algorithm that calls for an *EffectiveH* is moving to the right.

length

Number of (MPS) sites the effective hamiltonian covers. NB: Class attribute.

Type int

dtype

The data type of the involved arrays.

Type np.dtype

N

Contracting *self* with as_matrix() will result in an *N*'x'N matrix.

Type int

acts_on

Labels of the state on which self acts. NB: class attribute. Overwritten by normal attribute, if combine.

Type list of str

combine

Whether to combine legs into pipes as far as possible. This reduces the overhead of calculating charge combinations in the contractions.

Type bool

move_right

Whether the sweeping algorithm that calls for an *EffectiveH* is moving to the right.

Type bool

combine_theta(theta)

Combine the legs of *theta*, such that it fits to how we combined the legs of *self*.

Parameters theta (Array) – Wave function to apply the effective Hamiltonian to, with uncombined legs.

Returns theta – Wave function with labels as given by *self.acts_on*.

Return type Array

adjoint()

Return the hermitian conjugate of self

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

matvec(vec)

Calculate the action of the operator on a vector vec.

Note that we don't require vec to be one-dimensional. However, for square operators we require that the result of matvec has the same legs (in the same order) as vec such that they can be added. Note that this excludes a non-trivial qtotal for square operators.

to_matrix()

Contract self to a matrix.

If self represents an operator with very small shape, e.g. because the MPS bond dimension is very small, an algorithm might choose to contract self to a single tensor.

Returns matrix – Contraction of the represented operator.

Return type Array

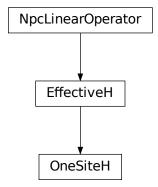
17.4.2 OneSiteH

• full name: tenpy.algorithms.mps_common.OneSiteH

• parent module: tenpy.algorithms.mps_common

· type: class

Inheritance Diagram



Methods

OneSiteHinit(env, i0[,	combine,	Initialize self.
move_right])		
OneSiteH.adjoint()		Return the hermitian conjugate of <i>self</i> .
OneSiteH.combine_Heff()		Combine LP and RP with W to form LHeff and RHeff,
		depending on the direction.
OneSiteH.combine_theta(theta)		Combine the legs of <i>theta</i> , such that it fits to how we
		combined the legs of <i>self</i> .
OneSiteH.matvec(theta)		Apply the effective Hamiltonian to <i>theta</i> .
		continues on next page

Table 13 – continued from previous page

OneSiteH.to matrix()

Contract self to a matrix.

Class Attributes and Properties

```
OneSiteH.acts on
```

OneSiteH.length

class tenpy.algorithms.mps_common.**OneSiteH** (env, i0, combine=False, move_right=True)

Bases: tenpy.algorithms.mps_common.EffectiveH

Class defining the one-site effective Hamiltonian for Lanczos.

The effective one-site Hamiltonian looks like this:

```
| .-- --.
| | | | |
| LP----W0----RP
| | | | |
```

If *combine* is True, we define either *LHeff* as contraction of *LP* with *W* (in the case *move_right* is True) or *RHeff* as contraction of *RP* and *W*.

Parameters

- **env** (MPOEnvironment) Environment for contraction <psi|H|psi>.
- i0 (int) Index of the active site if length=1, or of the left-most active site if length>1.
- **combine** (bool) Whether to combine legs into pipes. This combines the virtual and physical leg for the left site (when moving right) or right side (when moving left) into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive, $O(2d^3\chi^3D)$. Is originally from the wo-site method; unclear if it works well for 1 site.
- move_right (bool) Whether the the sweep is moving right or left for the next update.

length

Number of (MPS) sites the effective hamiltonian covers.

Type int

acts_on

Labels of the state on which self acts. NB: class attribute. Overwritten by normal attribute, if combine.

Type list of str

combine, move_right

See above.

Type bool

LHeff, RHeff

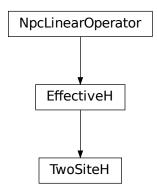
Only set if combine, and only one of them depending on move_right. If $move_right$ was True, LHeff is set with labels ' (vR*.p0)', 'wR', '(vR.p0*)' for bra, MPO, ket; otherwise RHeff is set with labels ' (p0*.vL)', 'wL', ' (p0, vL*)'

```
Type Array
LP, WO, RP
     Tensors making up the network of self.
         Type Array
matvec(theta)
     Apply the effective Hamiltonian to theta.
         Parameters theta (Array) - Labels: vL, p0, vR if combine=False, (vL.p0), vR or
             vL, (p0.vR) if True (depending on the direction of movement)
         Returns Product of theta and the effective Hamiltonian.
         Return type theta Array
combine_Heff()
     Combine LP and RP with W to form LHeff and RHeff, depending on the direction.
     In a move to the right, we need LHeff. In a move to the left, we need RHeff. Both contain the same W.
combine theta(theta)
     Combine the legs of theta, such that it fits to how we combined the legs of self.
         Parameters theta (Array) - Wave function with labels 'vL', 'p0', 'p1', 'vR'
         Returns theta – Wave function with labels 'vL', 'p0', 'p1', 'vR'
         Return type Array
to matrix()
    Contract self to a matrix.
adjoint()
    Return the hermitian conjugate of self.
```

17.4.3 TwoSiteH

- full name: tenpy.algorithms.mps_common.TwoSiteH
- $\bullet \ parent \ module: \ \textit{tenpy.algorithms.mps} \underline{-} \textit{common}$
- · type: class

Inheritance Diagram



Methods

TwoSiteHinit(env, i0[, move_right])	combine,	Initialize self.
TwoSiteH.adjoint()		Return the hermitian conjugate of <i>self</i> .
TwoSiteH.combine_Heff()		Combine LP and RP with W to form LHeff and RHeff.
TwoSiteH.combine_theta(theta)		Combine the legs of <i>theta</i> , such that it fits to how we
		combined the legs of <i>self</i> .
TwoSiteH.matvec(theta)		Apply the effective Hamiltonian to <i>theta</i> .
TwoSiteH.to_matrix()		Contract self to a matrix.

Class Attributes and Properties

```
TwoSiteH.length
```

class tenpy.algorithms.mps_common.**TwoSiteH**(env, i0, combine=False, move_right=True)
Bases: tenpy.algorithms.mps_common.EffectiveH

Class defining the two-site effective Hamiltonian for Lanczos.

The effective two-site Hamiltonian looks like this:

```
| .-- --.
| | | | | |
| LP---W0--W1---RP
| | | | |
```

If *combine* is True, we define *LHeff* and *RHeff*, which are the contractions of *LP* with *W0*, and *RP* with *W1*, respectively.

Parameters

- env (MPOEnvironment) Environment for contraction <psi|H|psi>.
- i0 (int) Index of the active site if length=1, or of the left-most active site if length>1.
- **combine** (bool) Whether to combine legs into pipes. This combines the virtual and physical leg for the left site (when moving right) or right side (when moving left) into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive, $O(2d^3\chi^3D)$.
- move_right (bool) Whether the the sweep is moving right or left for the next update.

combine

Whether to combine legs into pipes. This combines the virtual and physical leg for the left site and right site into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive, $O(2d^3\chi^3D)$.

```
Type bool
```

length

Number of (MPS) sites the effective hamiltonian covers.

```
Type int
```

acts on

Labels of the state on which *self* acts. NB: class attribute. Overwritten by normal attribute, if *combine*.

Type list of str

LHeff

Left part of the effective Hamiltonian. Labels '(vR*.p0)', 'wR', '(vR.p0*)' for bra, MPO, ket.

```
Type Array
```

RHeff

Right part of the effective Hamiltonian. Labels '(p1*.vL)', 'wL', '(p1.vL*)' for ket, MPO, bra.

```
Type Array
```

LP, WO, W1, RP

Tensors making up the network of self.

```
Type Array
```

matvec(theta)

Apply the effective Hamiltonian to theta.

```
Parameters theta (Array) - Labels: vL, p0, p1, vR if combine=False, (vL.p0), (p1.vR) if True
```

Returns Product of *theta* and the effective Hamiltonian.

Return type theta *Array*

combine_Heff()

Combine LP and RP with W to form LHeff and RHeff.

Combine LP with W0 and RP with W1 to get the effective parts of the Hamiltonian with piped legs.

```
combine_theta (theta)
   Combine the legs of theta, such that it fits to how we combined the legs of self.

Parameters theta (Array) - Wave function with labels 'vL', 'p0', 'p1', 'vR'

Returns theta - Wave function with labels 'vL', 'p0', 'p1', 'vR'

Return type Array

to_matrix()
   Contract self to a matrix.

adjoint()
   Return the hermitian conjugate of self.
```

Module description

'Sweep' algorithm and effective Hamiltonians.

Many MPS-based algorithms use a 'sweep' structure, wherein local updates are performed on the MPS tensors sequentially, first from left to right, then from right to left. This procedure is common to DMRG, TDVP, sequential time evolution, etc.

Another common feature of these algorithms is the use of an effective local Hamiltonian to perform the local updates. The most prominent example of this is probably DMRG, where the local MPS object is optimized with respect to the rest of the MPS-MPO-MPS network, the latter forming the effective Hamiltonian.

The Sweep class attempts to generalize as many aspects of 'sweeping' algorithms as possible. *EffectiveH* and its subclasses implement the effective Hamiltonians mentioned above. Currently, effective Hamiltonians for 1-site and 2-site optimization are implemented.

The VariationalCompression and VariationalApplyMPO implemented here also directly use the Sweep class.

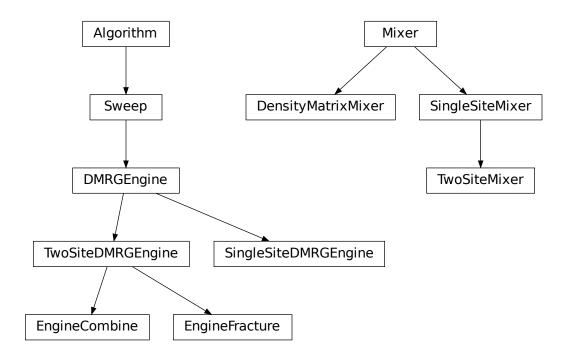
17.5 dmrg

• full name: tenpy.algorithms.dmrg

• parent module: tenpy.algorithms

• type: module

Classes



DMD CEL (DMDC has also with assumed without for the
DMRGEngine(psi, model, options, **kwargs)	DMRG base class with common methods for the
	TwoSiteDMRG and SingleSiteDMRG.
DensityMatrixMixer(options)	Mixer based on density matrices.
EngineCombine(psi, model, DMRG_params)	Engine which combines legs into pipes as far as possi-
	ble.
EngineFracture(psi, model, DMRG_params)	Engine which keeps the legs separate.
Mixer(options)	Base class of a general Mixer.
SingleSiteDMRGEngine(psi, model, options,)	Engine for the single-site DMRG algorithm.
SingleSiteMixer(options)	Mixer for single-site DMRG.
TwoSiteDMRGEngine(psi, model, options,	Engine for the two-site DMRG algorithm.
**kwargs)	
TwoSiteMixer(options)	Mixer for two-site DMRG.

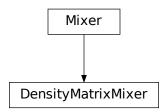
17.5.1 DensityMatrixMixer

• full name: tenpy.algorithms.dmrg.DensityMatrixMixer

• parent module: tenpy.algorithms.dmrg

• type: class

Inheritance Diagram



Methods

D 11 26 1 1 261 (cm/'cm)	T., '4', 1', 10
DensityMatrixMixerinit(options)	Initialize self.
DensityMatrixMixer.get_xL(wL_leg, Id_L,	Generate the coupling of the MPO legs for the reduced
Id_R)	density matrix.
DensityMatrixMixer.get_xR(wR_leg, Id_L,	Generate the coupling of the MPO legs for the reduced
Id_R)	density matrix.
DensityMatrixMixer.mix_rho_L(engine,	Calculated mixed reduced density matrix for left site.
theta,)	
DensityMatrixMixer.mix_rho_R(engine,	Calculated mixed reduced density matrix for left site.
theta,)	
DensityMatrixMixer.perturb_svd(engine,	Mix extra terms to theta and perform an SVD.
)	
DensityMatrixMixer.	Update the amplitude, possibly disable the mixer.
update_amplitude(sweeps)	

class tenpy.algorithms.dmrg.DensityMatrixMixer(options)

Bases: tenpy.algorithms.dmrg.Mixer

Mixer based on density matrices.

This mixer constructs density matrices as described in the original paper [[white2005]].

perturb_svd (engine, theta, i0, update_LP, update_RP)

Mix extra terms to theta and perform an SVD.

We calculate the left and right reduced density using the mixer (which might include applications of H). These density matrices are diagonalized and truncated such that we effectively perform a svd for the case mixer.amplitude=0.

Parameters

- **engine** (SingleSiteDMRGEngine|TwoSiteDMRGEngine) The DMRG engine calling the mixer.
- theta (Array) The optimized wave function, prepared for svd.
- **i0** (*int*) Site index; *theta* lives on i0, i0+1.
- update_LP (bool) Whether to calculate the next env.LP[i0+1].
- update_RP (bool) Whether to calculate the next env.RP [i0].

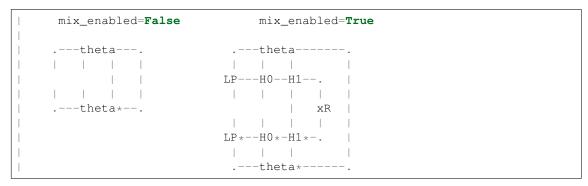
Returns

- U (Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- **S** (1D ndarray | 2D Array) Without mixer just the singluar values of the array; with mixer it might be a general matrix; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(p1.vR)'.
- **err** (*TruncationError*) The truncation error introduced.

mix_rho_L (engine, theta, i0, mix_enabled)

Calculated mixed reduced density matrix for left site.

Pictorially:



Parameters

- engine (DMRGEngine) The DMRG engine calling the mixer.
- **theta** (*Array*) Ground state of the effective Hamiltonian, prepared for svd.
- **i0** (*int*) Site index; theta lives on i0, i0+1.
- mix_enabled (bool) Whether we should perturb the density matrix.

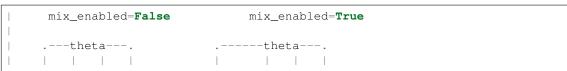
Returns rho_L – A (hermitian) square array with labels '(vL.p0)', '(vL*.p0*)', Mainly the reduced density matrix of the left part, but with some additional mixing.

Return type Array

mix_rho_R (engine, theta, i0, mix_enabled)

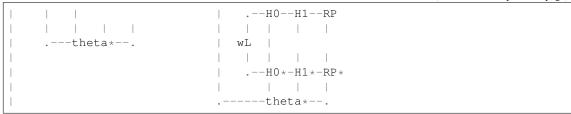
Calculated mixed reduced density matrix for left site.

Pictorially:



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Parameters

- engine (DMRGEngine) The DMRG engine calling the mixer.
- **theta** (*Array*) Ground state of the effective Hamiltonian, prepared for svd.
- **i0** (*int*) Site index; *theta* lives on i0, i0+1.
- mix_enabled (bool) Whether we should perturb the density matrix.

Returns rho_R – A (hermitian) square array with labels '(p1.vR)', '(p1*.vR*)'. Mainly the reduced density matrix of the right part, but with some additional mixing.

Return type Array

get_xR (wR_leg, Id_L, Id_R)

Generate the coupling of the MPO legs for the reduced density matrix.

Parameters

- wR_leg (LegCharge) LegCharge to be connected to.
- IdL (int | None) Index within the leg for which the MPO has only identities to the left.
- IdR (int | None) Index within the leg for which the MPO has only identities to the right.

Returns

- **mixed_xR** (*Array*) Connection of the MPOs on the right for the reduced density matrix *rhoL*. Labels ('wL', 'wL*').
- add_separate_Id (bool) If Id_L is None, we can't include the identity into mixed_xR, so it has to be added directly in mix rho L().

get_xL (wL_leg, Id_L, Id_R)

Generate the coupling of the MPO legs for the reduced density matrix.

Parameters

- wL_leg (LegCharge) LegCharge to be connected to.
- Id_L (int | None) Index within the leg for which the MPO has only identities to the left.
- Id_R (int | None) Index within the leg for which the MPO has only identities to the right.

Returns

- mixed_xL (Array) Connection of the MPOs on the left for the reduced density matrix *rhoR*. Labels ('wR', 'wR*').
- add_separate_Id (bool) If Id_R is None, we can't include the identity into mixed_xL, so it has to be added directly in mix_rho_R().

update_amplitude (sweeps)

Update the amplitude, possibly disable the mixer.

Parameters sweeps (int) – The number of performed sweeps, to check if we need to disable the mixer.

Returns mixer – Returns *self* if we should continue mixing, or None, if the mixer should be disabled.

Return type Mixer | None

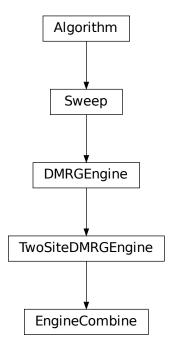
17.5.2 EngineCombine

• full name: tenpy.algorithms.dmrg.EngineCombine

• parent module: tenpy.algorithms.dmrg

• type: class

Inheritance Diagram



Methods

EngineCombineinit(psi, model, DMRG_params)	Initialize self.
EngineCombine.diag(theta_guess)	Diagonalize the effective Hamiltonian represented by self.
EngineCombine.environment_sweeps(N_swee	psperform <i>N_sweeps</i> sweeps without optimization to update the environment.
EngineCombine.get_resume_data()	Return necessary data to resume a run () interrupted at a checkpoint.
EngineCombine.get_sweep_schedule()	Define the schedule of the sweep.
EngineCombine.init_env([model, resume_data])	(Re-)initialize the environment.
EngineCombine.make_eff_H()	Create new instance of <i>self.EffectiveH</i> at <i>self.i0</i> and set it to <i>self.eff_H</i> .
EngineCombine.mixed_svd(theta)	Get (truncated) <i>B</i> from the new theta (as returned by diag).
EngineCombine.mixer_activate()	Set self.mixer to the class specified by options['mixer'].
EngineCombine.mixer_cleanup()	Cleanup the effects of a mixer.
EngineCombine.plot_sweep_stats([axes,])	Plot sweep_stats to display the convergence with the sweeps.
<pre>EngineCombine.plot_update_stats(axes[,])</pre>	Plot update_stats to display the convergence during the sweeps.
EngineCombine.post_update_local(update_date_date_date_date_date_date_date_	
EngineCombine.prepare_svd(theta)	Transform theta into matrix for svd.
EngineCombine.prepare_update()	<pre>Prepare self for calling update_local() on sites i0 : i0+n_optimize.</pre>
EngineCombine.reset_stats([resume_data])	Reset the statistics, useful if you want to start a new sweep run.
EngineCombine.resume_run()	Resume a run that was interrupted.
EngineCombine.run()	Run the DMRG simulation to find the ground state.
EngineCombine.set_B(U, S, VH)	Update the MPS with the U, S, VH returned by self.mixed_svd.
EngineCombine.sweep([optimize, meas_E_trunc])	One 'sweep' of a the algorithm.
EngineCombine.update_LP(U)	Update left part of the environment.
EngineCombine.update_RP(VH)	Update right part of the environment.
EngineCombine.update_local(theta[, optimize])	Perform site-update on the site i0.

Class Attributes and Properties

EngineCombine.DMRG_params	
EngineCombine.engine_params	
EngineCombine.n_optimize	the number of sites to be optimized over at once.
EngineCombine.verbose	

```
class tenpy.algorithms.dmrg.EngineCombine(psi, model, DMRG_params)
```

Bases: tenpy.algorithms.dmrg.TwoSiteDMRGEngine

Engine which combines legs into pipes as far as possible.

This engine combines the virtual and physical leg for the left site and right site into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive, $O(2d^3\chi^3D)$.

Deprecated since version 0.5.0: Directly use the TwoSiteDMRGEngine with the DMRG parameter combine=True.

DefaultMixer

alias of tenpy.algorithms.dmrg.DensityMatrixMixer

EffectiveH

alias of tenpy.algorithms.mps_common.TwoSiteH

diag(theta_guess)

Diagonalize the effective Hamiltonian represented by self.

```
option DMRGEngine.max N for ED: int
```

Maximum matrix dimension of the effective hamiltonian up to which the 'default' diag_method uses ED instead of Lanczos.

```
option DMRGEngine.diag_method: str
```

One of the folloing strings:

'default' Same as 'lanczos' for large bond dimensions, but if the total dimension of the effective Hamiltonian does not exceed the DMRG parameter 'max_N_for_ED' it uses 'ED_block'.

'lanczos' lanczos () Default, the Lanczos implementation in TeNPy.

- 'arpack' lanczos_arpack() Based on scipy.linalg.sparse.eigsh(). Slower than 'lanczos', since it needs to convert the npc arrays to numpy arrays during *each* matvec, and possibly does many more iterations.
- **'ED_block'** full_diag_effH() Contract the effective Hamiltonian to a (large!) matrix and diagonalize the block in the charge sector of the initial state. Preserves the charge sector of the explicitly conserved charges. However, if you don't preserve a charge explicitly, it can break it. For example if you use a SpinChain({'conserve': 'parity'}), it could change the total "Sz", but not the parity of 'Sz'.
- **'ED_all'** full_diag_effH() Contract the effective Hamiltonian to a (large!) matrix and diagonalize it completely. Allows to change the charge sector even for explicitly conserved charges. For example if you use a SpinChain({'conserve': 'Sz'}), it can change the total "Sz".

Parameters theta_guess (Array) – Initial guess for the ground state of the effective Hamiltonian.

Returns

- **E0** (*float*) Energy of the found ground state.
- **theta** (*Array*) Ground state of the effective Hamiltonian.
- **N** (*int*) Number of Lanczos iterations used. –1 if unknown.
- **ov_change** (*float*) Change in the wave function 1. abs(<theta_quess|theta_diaq>)

environment_sweeps (N_sweeps)

Perform *N_sweeps* sweeps without optimization to update the environment.

Parameters N_sweeps (int) - Number of sweeps to run without optimization

get resume data()

Return necessary data to resume a run () interrupted at a checkpoint.

At a checkpoint, you can save psi, model and options along with the data returned by this function. When the simulation aborts, you can resume it using this saved data with:

```
eng = AlgorithmClass(psi, model, options, resume_data=resume_data)
eng.resume_run(resume_data)
```

An algorithm which doesn't support this should override *resume_run* to raise an Error.

Returns resume_data – Dictionary with necessary data (apart from copies of *psi*, *model*, *options*) that allows to continue the simulation from where we are now.

Return type dict

get_sweep_schedule()

Define the schedule of the sweep.

One 'sweep' is a full sequence from the leftmost site to the right and back. Only those *LP* and *RP* that can be used later should be updated.

Returns schedule - Schedule for the sweep. Each entry is (i0, move_right, (update_LP, update_RP)), where i0 is the leftmost of the self.EffectiveH. length sites to be updated in update_local(), move_right indicates whether the next i0 in the schedule is rigth (True) of the current one, and update_LP, update_RP indicate whether it is necessary to update the LP and RP. The latter are chosen such that the environment is growing for infinite systems, but we only keep the minimal number of environment tensors in memory.

Return type iterable of (int, bool, (bool, bool))

```
init env(model=None, resume data=None)
```

(Re-)initialize the environment.

This function is useful to (re-)start a Sweep with a slightly different model or different (engine) parameters. Note that we assume that we still have the same *psi*. Calls <code>reset_stats()</code>.

Parameters

- model (MPOModel) The model representing the Hamiltonian for which we want to find the ground state. If None, keep the model used before.
- resume_data (None | dict) Given when resuming a simulation, as returned by $qet_resume_data()$.

Options

Deprecated since version 0.6.0: Options LP, LP_age, RP and RP_age are now collected in a dictionary init_env_data with different keys init_LP, init_RP, age_LP, age_RP

Deprecated since version 0.8.0: Instead of passing the *init_env_data* as a option, it should be passed as dict entry of *resume_data*.

```
option Sweep.init_env_data: dict
   Dictionary as returned by self.env.get_initialization_data() from
   get_initialization_data().
```

option Sweep.orthogonal_to: list of MPS

List of other matrix product states to orthogonalize against. Works only for finite systems. This parameter can be used to find (a few) excited states as follows. First, run DMRG to find the ground

state and then run DMRG again while orthogonalizing against the ground state, which yields the first excited state (in the same symmetry sector), and so on.

option Sweep.start_env: int

Number of sweeps to be performed without optimization to update the environment.

Raises ValueError – If the engine is re-initialized with a new model, which legs are incompatible with those of hte old model.

make_eff_H()

Create new instance of self. EffectiveH at self. i0 and set it to self. eff_H.

mixed_svd(theta)

Get (truncated) B from the new theta (as returned by diag).

The goal is to split theta and truncate it:

```
| -- theta -- ==> -- U -- S -- VH -
```

Without a mixer, this is done by a simple svd and truncation of Schmidt values.

With a mixer, the state is perturbed before the SVD. The details of the perturbation are defined by the Mixer class.

Note that the returned S is a general (not diagonal) matrix, with labels 'vL', 'vR'.

Parameters theta (Array) – The optimized wave function, prepared for svd.

Returns

- **U** (Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- **S** (1D ndarray | 2D *Array*) Without mixer just the singluar values of the array; with mixer it might be a general matrix with labels 'vL', 'vR'; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(p1.vR)'.
- **err** (*TruncationError*) The truncation error introduced.

mixer activate()

Set *self.mixer* to the class specified by *options['mixer']*.

```
option TwoSiteDMRGEngine.mixer: str | class | bool
```

Chooses the Mixer to be used. A string stands for one of the mixers defined in this module, a class is used as custom mixer. Default (None) uses no mixer, True uses <code>DensityMatrixMixer</code> for the 2-site case and <code>SingleSiteMixer</code> for the 1-site case.

```
option TwoSiteDMRGEngine.mixer_params: dict
    Mixer parameters as described in Mixer.
```

mixer_cleanup()

Cleanup the effects of a mixer.

A sweep () with an enabled Mixer leaves the MPS psi with 2D arrays in S. To recover the originial form, this function simply performs one sweep with disabled mixer.

property n_optimize

the number of sites to be optimized over at once.

Indirectly set by the class attribute *EffectiveH* and it's *length*. For example, TwoSiteDMRGEngine uses the *TwoSiteH* and hence has n_optimize=2, while the SingleSiteDMRGEngine has n_optimize=1.

```
plot_sweep_stats (axes=None, xaxis='time', yaxis='E', y_exact=None, **kwargs)
Plot sweep_stats to display the convergence with the sweeps.
```

Parameters

- axes (matplotlib.axes.Axes) The axes to plot into. Defaults to matplotlib. pyplot.gca()
- **xaxis** (key of sweep_stats) Key of sweep_stats to be used for the x-axis and y-axis of the plots.
- yaxis (key of sweep_stats) Key of sweep_stats to be used for the x-axis and y-axis of the plots.
- **y_exact** (*float*) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y_exact) /y_exact) on a log-scale yaxis.
- **kwargs Further keyword arguments given to axes.plot(...).

```
plot_update_stats (axes, xaxis='time', yaxis='E', y_exact=None, **kwargs)
Plot update_stats to display the convergence during the sweeps.
```

Parameters

- axes (matplotlib.axes.Axes) The axes to plot into. Defaults to matplotlib. pyplot.gca()
- **xaxis** ('N_updates' | 'sweep' | keys of update_stats) Key of update_stats to be used for the x-axis of the plots. 'N_updates' is just enumerating the number of bond updates, and 'sweep' corresponds to the sweep number (including environment sweeps).
- yaxis ('E' | keys of update_stats) Key of update_stats to be used for the y-axis of the plots. For 'E', use the energy (per site for infinite systems).
- **y_exact** (*float*) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y_exact) /y_exact) on a log-scale yaxis.
- **kwargs Further keyword arguments given to axes.plot(...).

post_update_local (update_data)

Perform post-update actions.

Compute truncation energy, remove *LP/RP* that are no longer needed and collect statistics.

Parameters update_data (dict) - What was returned by update_local().

```
prepare_svd(theta)
```

Transform theta into matrix for svd.

prepare_update()

Prepare self for calling update_local() on sites i0: i0+n_optimize.

Returns theta – Current best guess for the ground state, which is to be optimized. Labels are 'vL', 'p0', 'p1', 'vR', or combined versions of it (if *self.combine*). For single-site DMRG, the 'p1' label is missing.

Return type Array

reset_stats (resume_data=None)

Reset the statistics, useful if you want to start a new sweep run.

```
option DMRGEngine.chi_list: dict | None
```

A dictionary to gradually increase the *chi_max* parameter of *trunc_params*. The key defines starting from which sweep *chi_max* is set to the value, e.g. {0: 50, 20: 100} uses chi_max=50 for

the first 20 sweeps and chi_max=100 afterwards. Overwrites *trunc_params['chi_list']*`. By default (None) this feature is disabled.

```
option DMRGEngine.sweep_0: int
```

The number of sweeps already performed. (Useful for re-start).

resume_run()

Resume a run that was interrupted.

In case we saved an intermediate result at a checkpoint, this function allows to resume the run () of the algorithm (after re-initialization with the *resume_data*). Since most algorithms just have a while loop with break conditions, the default behaviour implemented here is to just call run ().

run()

Run the DMRG simulation to find the ground state.

Returns

- E (float) The energy of the resulting ground state MPS.
- **psi** (MPS) The MPS representing the ground state after the simluation, i.e. just a reference to psi.

Options

option DMRGEngine.diag_method: str

Method to be used for diagonalization, default 'default'. For possible arguments see DMRGEngine.diag().

```
option DMRGEngine.E_tol_to_trunc: float
```

It's reasonable to choose the Lanczos convergence criteria 'E_tol' not many magnitudes lower than the current truncation error. Therefore, if $E_tol_to_trunc$ is not None, we update E_tol of $lanczos_params$ to max_E_trunc*E_tol_to_trunc, restricted to the interval $[E_tol_min, E_tol_max]$, where max_E_trunc is the maximal energy difference due to truncation right after each Lanczos optimization during the sweeps.

```
option DMRGEngine.E_tol_max: float
   See E_tol_to_trunc
```

```
option DMRGEngine.E_tol_min: float
    See E_tol_to_trunc
```

```
option DMRGEngine.max_E_err: float
```

Convergence if the change of the energy in each step satisfies $-Delta E / max(|E|, 1) < max_E_err$. Note that this is also satisfied if Delta E > 0, i.e., if the energy increases (due to truncation).

```
option DMRGEngine.max_hours: float
```

If the DMRG took longer (measured in wall-clock time), 'shelve' the simulation, i.e. stop and return with the flag shelve=True.

```
option DMRGEngine.max_S_err: float
```

Convergence if the relative change of the entropy in each step satisfies $|Delta S|/S < max_S_err$

```
option DMRGEngine.max_sweeps: int
```

Maximum number of sweeps to be performed.

option DMRGEngine.min_sweeps: int

Minimum number of sweeps to be performed. Defaults to 1.5*N_sweeps_check.

option DMRGEngine.N sweeps check: int

Number of sweeps to perform between checking convergence criteria and giving a status update.

option DMRGEngine.norm_tol: float

After the DMRG run, update the environment with at most *norm_tol_iter* sweeps until np.linalg.norm(psi.norm_err()) < norm_tol.

option DMRGEngine.norm tol iter: float

Perform at most *norm_tol_iter*`*`*update_env* sweeps to converge the norm error below *norm_tol*. If the state is not converged after that, call canonical_form() instead.

option DMRGEngine.P_tol_to_trunc: float

It's reasonable to choose the Lanczos convergence criteria 'P_tol' not many magnitudes lower than the current truncation error. Therefore, if $P_tol_to_trunc$ is not None, we update P_tol of $lanczos_params$ to max_trunc_err*P_tol_to_trunc, restricted to the interval $[P_tol_min, P_tol_max]$, where max_trunc_err is the maximal truncation error (discarded weight of the Schmidt values) due to truncation right after each Lanczos optimization during the sweeps.

option DMRGEngine.P_tol_max: float

See *P_tol_to_trunc*

option DMRGEngine.P_tol_min: float

See *P_tol_to_trunc*

option DMRGEngine.update_env: int

Number of sweeps without bond optimization to update the environment for infinite boundary conditions, performed every *N_sweeps_check* sweeps.

set B(U, S, VH)

Update the MPS with the U, S, VH returned by self.mixed_svd.

Parameters

- **U** (Array) Left and Right-canonical matrices as returned by the SVD.
- VH (Array) Left and Right-canonical matrices as returned by the SVD.
- **S** (1D array | 2D Array) The middle part returned by the SVD, theta = U S VH. Without a mixer just the singular values, with enabled *mixer* a 2D array.

sweep (optimize=True, meas_E_trunc=False)

One 'sweep' of a the algorithm.

Iteratate over the bond which is optimized, to the right and then back to the left to the starting point.

Parameters

- **optimize** (bool, optional) Whether we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).
- meas_E_trunc (bool, optional) Whether to measure truncation energies.

Returns

- max_trunc_err (float) Maximal truncation error introduced.
- max_E_trunc (None | float) None if meas_E_trunc is False, else the maximal change of the energy due to the truncation.

$\mathtt{update_LP}\left(U\right)$

Update left part of the environment.

We always update the environment at site i0 + 1: this environment then contains the site where we just performed a local update (when sweeping right).

```
Parameters U (Array) – The U as returned by the SVD, with combined legs, labels 'vL. p0', 'vR'.
```

update_RP(VH)

Update right part of the environment.

We always update the environment at site i0: this environment then contains the site where we just performed a local update (when sweeping left).

```
Parameters VH (Array) - The VH as returned by SVD, with combined legs, labels 'vL', '(vR.p1)'.
```

update_local (theta, optimize=True)

Perform site-update on the site 10.

Parameters

- theta (Array) Initial guess for the ground state of the effective Hamiltonian.
- **optimize** (bool) Wheter we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).

Returns

update_data – Data computed during the local update, as described in the following:

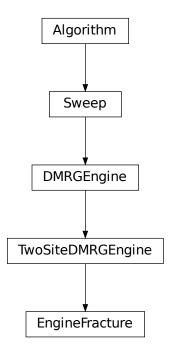
- **E0** [float] Total energy, obtained *before* truncation (if optimize=True), or *after* truncation (if optimize=False) (but never None).
- **N** [int] Dimension of the Krylov space used for optimization in the lanczos algorithm. 0 if optimize=False.
- **age** [int] Current size of the DMRG simulation: number of physical sites involved into the contraction.
- **U, VH:** Array *U* and *VH* returned by mixed_svd().
- ov_change: float Change in the wave function 1. abs(<theta_guess|theta>)
 induced by diag(), not including the truncation!

Return type dict

17.5.3 EngineFracture

- full name: tenpy.algorithms.dmrg.EngineFracture
- parent module: tenpy.algorithms.dmrg
- type: class

Inheritance Diagram



Methods

	T '.' 1' 10
EngineFractureinit(psi, mode	, Initialize self.
DMRG_params)	
EngineFracture.diag(theta_guess)	Diagonalize the effective Hamiltonian represented by
	self.
EngineFracture.environment_sweeps(N_s	weepserform N_sweeps sweeps without optimization to up-
	date the environment.
EngineFracture.get_resume_data()	Return necessary data to resume a run () interrupted at
	a checkpoint.
EngineFracture.get_sweep_schedule()	Define the schedule of the sweep.
EngineFracture.init_env([model, re	- (Re-)initialize the environment.
sume_data])	
EngineFracture.make_eff_H()	Create new instance of self.EffectiveH at self.i0 and set
	it to self.eff_H.
EngineFracture.mixed_svd(theta)	Get (truncated) B from the new theta (as returned by
	diag).
EngineFracture.mixer_activate()	Set <i>self.mixer</i> to the class specified by <i>options['mixer']</i> .
EngineFracture.mixer_cleanup()	Cleanup the effects of a mixer.
EngineFracture.plot_sweep_stats([axes,	Plot sweep_stats to display the convergence with
])	the sweeps.
	continues on next page

Table 21 – continued from previous page

EngineFracture.plot_update_stats(axes[,	Plot update_stats to display the convergence dur-
])	ing the sweeps.
EngineFracture.post_update_local(update_	date) form post-update actions.
EngineFracture.prepare_svd(theta)	Transform theta into matrix for svd.
EngineFracture.prepare_update()	Prepare self for calling update_local() on sites i0
	: i0+n_optimize.
<pre>EngineFracture.reset_stats([resume_data])</pre>	Reset the statistics, useful if you want to start a new
	sweep run.
EngineFracture.resume_run()	Resume a run that was interrupted.
EngineFracture.run()	Run the DMRG simulation to find the ground state.
EngineFracture.set_B(U, S, VH)	Update the MPS with the U, S, VH returned by
	self.mixed_svd.
EngineFracture.sweep([optimize,	One 'sweep' of a the algorithm.
meas_E_trunc])	
EngineFracture.update_LP($f U$)	Update left part of the environment.
EngineFracture.update_RP(VH)	Update right part of the environment.
EngineFracture.update_local(theta[, opti-	Perform site-update on the site i0.
mize])	

Class Attributes and Properties

the number of sites to be optimized over at once.

class tenpy.algorithms.dmrg.EngineFracture(psi, model, DMRG_params)

 $Bases: \verb|tenpy.algorithms.dmrg.TwoSiteDMRGEngine| \\$

Engine which keeps the legs separate.

Due to a different contraction order in matvec (), this engine might be faster than EngineCombine, at least for large physical dimensions and if the MPO is sparse. One matvec () is $O(2\chi^3d^2W + 2\chi^2d^3W^2)$.

Deprecated since version 0.5.0: Directly use the TwoSiteDMRGEngine with the DMRG parameter combine=False.

DefaultMixer

alias of tenpy.algorithms.dmrg.DensityMatrixMixer

EffectiveH

alias of tenpy.algorithms.mps_common.TwoSiteH

diag(theta_guess)

Diagonalize the effective Hamiltonian represented by self.

option DMRGEngine.max_N_for_ED: int

Maximum matrix dimension of the effective hamiltonian up to which the 'default' diag_method uses ED instead of Lanczos.

option DMRGEngine.diag_method: str

One of the folloing strings:

'default' Same as 'lanczos' for large bond dimensions, but if the total dimension of the effective Hamiltonian does not exceed the DMRG parameter 'max_N_for_ED' it uses 'ED_block'.

'lanczos' lanczos () Default, the Lanczos implementation in TeNPy.

- 'arpack' lanczos_arpack() Based on scipy.linalg.sparse.eigsh(). Slower than 'lanczos', since it needs to convert the npc arrays to numpy arrays during *each* matvec, and possibly does many more iterations.
- **'ED_block'** full_diag_effH() Contract the effective Hamiltonian to a (large!) matrix and diagonalize the block in the charge sector of the initial state. Preserves the charge sector of the explicitly conserved charges. However, if you don't preserve a charge explicitly, it can break it. For example if you use a SpinChain({'conserve': 'parity'}), it could change the total "Sz", but not the parity of 'Sz'.
- **'ED_all'** full_diag_effH() Contract the effective Hamiltonian to a (large!) matrix and diagonalize it completely. Allows to change the charge sector even for explicitly conserved charges. For example if you use a SpinChain({'conserve': 'Sz'}), it can change the total "Sz".

Parameters theta_guess (Array) – Initial guess for the ground state of the effective Hamiltonian.

Returns

- **E0** (*float*) Energy of the found ground state.
- theta (Array) Ground state of the effective Hamiltonian.
- N (*int*) Number of Lanczos iterations used. –1 if unknown.
- **ov_change** (*float*) Change in the wave function 1. abs(<theta_guess|theta_diag>)

environment_sweeps (N_sweeps)

Perform *N_sweeps* sweeps without optimization to update the environment.

Parameters N_sweeps (int) - Number of sweeps to run without optimization

get_resume_data()

Return necessary data to resume a run () interrupted at a checkpoint.

At a checkpoint, you can save psi, model and options along with the data returned by this function. When the simulation aborts, you can resume it using this saved data with:

```
eng = AlgorithmClass(psi, model, options, resume_data=resume_data)
eng.resume_run(resume_data)
```

An algorithm which doesn't support this should override *resume_run* to raise an Error.

Returns resume_data – Dictionary with necessary data (apart from copies of *psi*, *model*, *options*) that allows to continue the simulation from where we are now.

Return type dict

get_sweep_schedule()

Define the schedule of the sweep.

One 'sweep' is a full sequence from the leftmost site to the right and back. Only those *LP* and *RP* that can be used later should be updated.

Returns schedule - Schedule for the sweep. Each entry is (i0, move_right, (update_LP, update_RP)), where i0 is the leftmost of the self.EffectiveH. length sites to be updated in update_local(), move_right indicates whether the next i0 in the schedule is rigth (True) of the current one, and update_LP, update_RP indicate

whether it is necessary to update the *LP* and *RP*. The latter are chosen such that the environment is growing for infinite systems, but we only keep the minimal number of environment tensors in memory.

Return type iterable of (int, bool, (bool, bool))

```
init env(model=None, resume data=None)
```

(Re-)initialize the environment.

This function is useful to (re-)start a Sweep with a slightly different model or different (engine) parameters. Note that we assume that we still have the same *psi*. Calls reset_stats().

Parameters

- model (MPOModel) The model representing the Hamiltonian for which we want to find the ground state. If None, keep the model used before.
- resume_data (None / dict) Given when resuming a simulation, as returned by get_resume_data().

Options

Deprecated since version 0.6.0: Options LP, LP_age, RP and RP_age are now collected in a dictionary init_env_data with different keys init_LP, init_RP, age_LP, age_RP

Deprecated since version 0.8.0: Instead of passing the *init_env_data* as a option, it should be passed as dict entry of *resume data*.

```
option Sweep.init_env_data: dict
```

```
Dictionary as returned by self.env.get_initialization_data() from get initialization data().
```

```
option Sweep.orthogonal_to: list of MPS
```

List of other matrix product states to orthogonalize against. Works only for finite systems. This parameter can be used to find (a few) excited states as follows. First, run DMRG to find the ground state and then run DMRG again while orthogonalizing against the ground state, which yields the first excited state (in the same symmetry sector), and so on.

```
option Sweep.start_env: int
```

Number of sweeps to be performed without optimization to update the environment.

Raises ValueError – If the engine is re-initialized with a new model, which legs are incompatible with those of hte old model.

```
make_eff_H()
```

Create new instance of *self.EffectiveH* at *self.i0* and set it to *self.eff_H*.

```
mixed svd(theta)
```

Get (truncated) *B* from the new theta (as returned by diag).

The goal is to split theta and truncate it:

```
| -- theta -- ==> -- U -- S -- VH -
```

Without a mixer, this is done by a simple svd and truncation of Schmidt values.

With a mixer, the state is perturbed before the SVD. The details of the perturbation are defined by the Mixer class.

Note that the returned S is a general (not diagonal) matrix, with labels 'vL', 'vR'.

Parameters theta (Array) – The optimized wave function, prepared for svd.

Returns

- U (Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- **S** (1D ndarray | 2D *Array*) Without mixer just the singluar values of the array; with mixer it might be a general matrix with labels 'vL', 'vR'; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(p1.vR)'.
- **err** (*TruncationError*) The truncation error introduced.

mixer activate()

Set *self.mixer* to the class specified by *options['mixer']*.

```
option TwoSiteDMRGEngine.mixer: str | class | bool
```

Chooses the Mixer to be used. A string stands for one of the mixers defined in this module, a class is used as custom mixer. Default (None) uses no mixer, True uses <code>DensityMatrixMixer</code> for the 2-site case and <code>SingleSiteMixer</code> for the 1-site case.

```
option TwoSiteDMRGEngine.mixer_params: dict
    Mixer parameters as described in Mixer.
```

mixer_cleanup()

Cleanup the effects of a mixer.

A sweep () with an enabled Mixer leaves the MPS psi with 2D arrays in S. To recover the originial form, this function simply performs one sweep with disabled mixer.

property n_optimize

the number of sites to be optimized over at once.

Indirectly set by the class attribute *EffectiveH* and it's *length*. For example, TwoSiteDMRGEngine uses the *TwoSiteH* and hence has n_optimize=2, while the SingleSiteDMRGEngine has n_optimize=1.

```
plot_sweep_stats (axes=None, xaxis='time', yaxis='E', y_exact=None, **kwargs)
Plot sweep_stats to display the convergence with the sweeps.
```

Parameters

- axes (matplotlib.axes.Axes) The axes to plot into. Defaults to matplotlib. pyplot.gca()
- **xaxis** (key of sweep_stats) Key of sweep_stats to be used for the x-axis and y-axis of the plots.
- yaxis (key of sweep_stats) Key of sweep_stats to be used for the x-axis and y-axis of the plots.
- **y_exact** (*float*) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y_exact) /y_exact) on a log-scale yaxis.
- **kwargs Further keyword arguments given to axes.plot(...).

```
plot_update_stats (axes, xaxis='time', yaxis='E', y_exact=None, **kwargs)
```

Plot update_stats to display the convergence during the sweeps.

Parameters

• axes (matplotlib.axes.Axes) - The axes to plot into. Defaults to matplotlib. pyplot.gca()

- **xaxis** ('N_updates' | 'sweep' | keys of update_stats) Key of update_stats to be used for the x-axis of the plots. 'N_updates' is just enumerating the number of bond updates, and 'sweep' corresponds to the sweep number (including environment sweeps).
- yaxis ('E' | keys of update_stats) Key of update_stats to be used for the y-axis of the plots. For 'E', use the energy (per site for infinite systems).
- **y_exact** (float) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y_exact) /y_exact) on a log-scale yaxis.
- **kwargs Further keyword arguments given to axes.plot (...).

post_update_local (update_data)

Perform post-update actions.

Compute truncation energy, remove *LP/RP* that are no longer needed and collect statistics.

Parameters update_data (dict) – What was returned by update_local().

prepare_svd(theta)

Transform theta into matrix for svd.

prepare_update()

Prepare self for calling update_local() on sites i0: i0+n_optimize.

Returns theta – Current best guess for the ground state, which is to be optimized. Labels are 'vL', 'p0', 'p1', 'vR', or combined versions of it (if *self.combine*). For single-site DMRG, the 'p1' label is missing.

Return type Array

reset_stats (resume_data=None)

Reset the statistics, useful if you want to start a new sweep run.

```
option DMRGEngine.chi_list: dict | None
```

A dictionary to gradually increase the *chi_max* parameter of *trunc_params*. The key defines starting from which sweep *chi_max* is set to the value, e.g. {0: 50, 20: 100} uses chi_max=50 for the first 20 sweeps and chi_max=100 afterwards. Overwrites *trunc_params['chi_list']*'. By default (None) this feature is disabled.

```
option DMRGEngine.sweep_0: int
```

The number of sweeps already performed. (Useful for re-start).

resume_run()

Resume a run that was interrupted.

In case we saved an intermediate result at a checkpoint, this function allows to resume the run() of the algorithm (after re-initialization with the *resume_data*). Since most algorithms just have a while loop with break conditions, the default behaviour implemented here is to just call run().

run()

Run the DMRG simulation to find the ground state.

Returns

- E (*float*) The energy of the resulting ground state MPS.
- **psi** (MPS) The MPS representing the ground state after the similation, i.e. just a reference to psi.

Options

option DMRGEngine.diag_method: str

Method to be used for diagonalization, default 'default'. For possible arguments see DMRGEngine.diag().

option DMRGEngine.E_tol_to_trunc: float

It's reasonable to choose the Lanczos convergence criteria 'E_tol' not many magnitudes lower than the current truncation error. Therefore, if $E_tol_to_trunc$ is not None, we update E_tol of $lanczos_params$ to max_E_trunc*E_tol_to_trunc, restricted to the interval $[E_tol_min, E_tol_max]$, where max_E_trunc is the maximal energy difference due to truncation right after each Lanczos optimization during the sweeps.

option DMRGEngine.E_tol_max: float

See *E_tol_to_trunc*

option DMRGEngine.E_tol_min: float

See *E_tol_to_trunc*

option DMRGEngine.max_E_err: float

Convergence if the change of the energy in each step satisfies $-Delta E / max(|E|, 1) < max_E_err$. Note that this is also satisfied if Delta E > 0, i.e., if the energy increases (due to truncation).

option DMRGEngine.max_hours: float

If the DMRG took longer (measured in wall-clock time), 'shelve' the simulation, i.e. stop and return with the flag shelve=True.

option DMRGEngine.max_S_err: float

Convergence if the relative change of the entropy in each step satisfies $|Delta S|/S < max_S_err$

option DMRGEngine.max_sweeps: int

Maximum number of sweeps to be performed.

option DMRGEngine.min_sweeps: int

Minimum number of sweeps to be performed. Defaults to 1.5*N_sweeps_check.

option DMRGEngine.N sweeps check: int

Number of sweeps to perform between checking convergence criteria and giving a status update.

option DMRGEngine.norm_tol: float

After the DMRG run, update the environment with at most *norm_tol_iter* sweeps until np.linalg.norm(psi.norm_err()) < norm_tol.

option DMRGEngine.norm_tol_iter: float

Perform at most *norm_tol_iter*`*`*update_env* sweeps to converge the norm error below *norm_tol*. If the state is not converged after that, call canonical_form() instead.

option DMRGEngine.P_tol_to_trunc: float

It's reasonable to choose the Lanczos convergence criteria 'P_tol' not many magnitudes lower than the current truncation error. Therefore, if $P_tol_to_trunc$ is not None, we update P_tol of $lanczos_params$ to max_trunc_err*P_tol_to_trunc, restricted to the interval $[P_tol_min, P_tol_max]$, where max_trunc_err is the maximal truncation error (discarded weight of the Schmidt values) due to truncation right after each Lanczos optimization during the sweeps.

option DMRGEngine.P_tol_max: float See P tol to trunc

option DMRGEngine.P_tol_min: float
 See P_tol_to_trunc

option DMRGEngine.update env: int

Number of sweeps without bond optimizaiton to update the environment for infinite boundary conditions, performed every *N_sweeps_check* sweeps.

$\mathtt{set_B}\left(U,S,V\!H\right)$

Update the MPS with the U, S, VH returned by self.mixed_svd.

Parameters

- **U** (Array) Left and Right-canonical matrices as returned by the SVD.
- VH (Array) Left and Right-canonical matrices as returned by the SVD.
- **S** (1D array | 2D Array) The middle part returned by the SVD, theta = U S VH. Without a mixer just the singular values, with enabled *mixer* a 2D array.

```
sweep (optimize=True, meas_E_trunc=False)
```

One 'sweep' of a the algorithm.

Iteratate over the bond which is optimized, to the right and then back to the left to the starting point.

Parameters

- **optimize** (bool, optional) Whether we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).
- meas_E_trunc(bool, optional) Whether to measure truncation energies.

Returns

- max_trunc_err (float) Maximal truncation error introduced.
- max_E_trunc (None | float) None if meas_E_trunc is False, else the maximal change of the energy due to the truncation.

${\tt update_LP}\,(U)$

Update left part of the environment.

We always update the environment at site i0 + 1: this environment then contains the site where we just performed a local update (when sweeping right).

```
Parameters U (Array) – The U as returned by the SVD, with combined legs, labels 'vL. p0', 'vR'.
```

$update_RP(VH)$

Update right part of the environment.

We always update the environment at site i0: this environment then contains the site where we just performed a local update (when sweeping left).

```
Parameters VH (Array) – The VH as returned by SVD, with combined legs, labels 'vL', '(vR.p1)'.
```

update_local (theta, optimize=True)

Perform site-update on the site i0.

Parameters

- theta (Array) Initial guess for the ground state of the effective Hamiltonian.
- **optimize** (bool) Wheter we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).

Returns

update_data - Data computed during the local update, as described in the following:

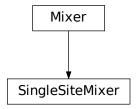
- **E0** [float] Total energy, obtained *before* truncation (if optimize=True), or *after* truncation (if optimize=False) (but never None).
- **N** [int] Dimension of the Krylov space used for optimization in the lanczos algorithm. 0 if optimize=False.
- **age** [int] Current size of the DMRG simulation: number of physical sites involved into the contraction.
- **U, VH:** Array U and VH returned by $mixed_svd()$.
- ov_change: float Change in the wave function 1. abs(<theta_guess|theta>)
 induced by diag(), not including the truncation!

Return type dict

17.5.4 SingleSiteMixer

- full name: tenpy.algorithms.dmrg.SingleSiteMixer
- parent module: tenpy.algorithms.dmrg
- type: class

Inheritance Diagram



Methods

SingleSiteMixerinit(options)	Initialize self.
SingleSiteMixer.perturb_svd(engine, theta,	Mix extra terms to theta and perform an SVD.
)	
SingleSiteMixer.subspace_expand(engine,	Expand the MPS subspace, to allow the bond dimension
)	to increase.
SingleSiteMixer.	Update the amplitude, possibly disable the mixer.
update_amplitude(sweeps)	

class tenpy.algorithms.dmrg.SingleSiteMixer(options)

 $Bases: \verb|tenpy.algorithms.dmrg.Mixer| \\$

Mixer for single-site DMRG.

Performs a subspace expansion following [[hubig2015]].

```
perturb_svd (engine, theta, i0, move_right, next_B)
```

Mix extra terms to theta and perform an SVD.

We calculate the left and right reduced density matrix using the mixer (which might include applications of H). These density matrices are diagonalized and truncated such that we effectively perform a svd for the case mixer.amplitude=0.

Parameters

- engine (DMRGEngine) The DMRG engine calling the mixer.
- theta (Array) The optimized wave function, prepared for svd.
- i0 (int) The site index where theta lives.
- move_right (bool) Whether we move to the right (True) or left (False).
- next_B (Array) The subspace expansion requires to change the tensor on the next site as well. If move_right, it should correspond to engine.psi.get_B(i0+1, form='B'). If not move_right, it should correspond to engine.psi.get_B(i0-1, form='A').

Returns

- U (Array) Left-canonical part of tensordot(theta, next_B). Labels '(vL.p0)',
 'vR'.
- **S** (1D ndarray) (Perturbed) singular values on the new bond (between theta and next_B).
- VH (Array) Right-canonical part of tensordot(theta, next_B). Labels 'vL', '(p1. vR)'.
- **err** (*TruncationError*) The truncation error introduced.

subspace_expand (engine, theta, i0, move_right, next_B)

Expand the MPS subspace, to allow the bond dimension to increase.

This is the subspace expansion following [[hubig2015]].

Parameters

- engine (DMRGEngine) The DMRG engine calling the mixer.
- theta (Array) Optimized guess for the ground state of the effective local Hamiltonian.
- i0 (int) Site index at which the local update has taken place.
- move_right (bool) Whether the next *i0* of the sweep will be right or left of the current one.
- next_B (Array) The subspace expansion requires to change the tensor on the next site as well. If move_right, it should correspond to engine.psi.get_B(i0+1, form='B'). If not move_right, it should correspond to engine.psi.get_B(i0-1, form='A').

Returns

- *theta* Local MPS tensor at site *i0* after subspace expansion.
- next_B MPS tensor at site i0+1 or i0-1 (depending on sweep direction) after subspace expansion.

update_amplitude (sweeps)

Update the amplitude, possibly disable the mixer.

Parameters sweeps (int) – The number of performed sweeps, to check if we need to disable the mixer.

Returns mixer – Returns *self* if we should continue mixing, or None, if the mixer should be disabled.

Return type Mixer | None

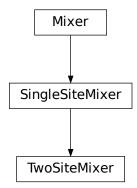
17.5.5 TwoSiteMixer

• full name: tenpy.algorithms.dmrg.TwoSiteMixer

• parent module: tenpy.algorithms.dmrg

• type: class

Inheritance Diagram



Methods

TwoSiteMixerinit(options)	Initialize self.
TwoSiteMixer.perturb_svd(engine, theta, i0,	Mix extra terms to theta and perform an SVD.
)	
TwoSiteMixer.subspace_expand(engine,	Expand the MPS subspace, to allow the bond dimension
theta,)	to increase.
TwoSiteMixer.update_amplitude(sweeps)	Update the amplitude, possibly disable the mixer.

class tenpy.algorithms.dmrg.TwoSiteMixer(options)

Bases: tenpy.algorithms.dmrg.SingleSiteMixer

Mixer for two-site DMRG.

This is the two-site version of the mixer described in [[hubig2015]]. Equivalent to the DensityMatrixMixer, but never construct the full density matrix.

perturb_svd (engine, theta, i0, move_right)

Mix extra terms to theta and perform an SVD.

Parameters

- engine (DMRGEngine) The DMRG engine calling the mixer.
- theta (Array) The optimized wave function, prepared for svd.
- **i0** (*int*) Site index; theta lives on i0, i0+1.
- update_LP (bool) Whether to calculate the next env.LP[i0+1].
- update_RP (bool) Whether to calculate the next env.RP[i0].

Returns

- U (Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- **S** (1D ndarray | 2D Array) Without mixer just the singluar values of the array; with mixer it might be a general matrix; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(vR.p1)'.
- **err** (*TruncationError*) The truncation error introduced.

subspace expand (*engine*, *theta*, *i0*, *move right*, *next B*)

Expand the MPS subspace, to allow the bond dimension to increase.

This is the subspace expansion following [[hubig2015]].

Parameters

- engine (DMRGEngine) The DMRG engine calling the mixer.
- theta (Array) Optimized guess for the ground state of the effective local Hamiltonian.
- i0 (int) Site index at which the local update has taken place.
- **move_right** (bool) Whether the next *i0* of the sweep will be right or left of the current one.
- next_B (Array) The subspace expansion requires to change the tensor on the next site as well. If move_right, it should correspond to engine.psi.get_B(i0+1, form='B'). If not move_right, it should correspond to engine.psi.get_B(i0-1, form='A').

Returns

- *theta* Local MPS tensor at site *i0* after subspace expansion.
- next_B MPS tensor at site i0+1 or i0-1 (depending on sweep direction) after subspace expansion.

update amplitude(sweeps)

Update the amplitude, possibly disable the mixer.

Parameters sweeps (int) – The number of performed sweeps, to check if we need to disable the mixer.

Returns mixer – Returns *self* if we should continue mixing, or None, if the mixer should be disabled.

Return type Mixer | None

Functions

chi_list(chi_max[, dchi, nsweeps])	Compute a 'ramping-up' chi_list.
<pre>full_diag_effH(effH, theta_guess[, keep_sector])</pre>	Perform an exact diagonalization of <i>effH</i> .
run(psi, model, options)	Run the DMRG algorithm to find the ground state of the
	given model.

17.5.6 chi_list

- full name: tenpy.algorithms.dmrg.chi_list
- parent module: tenpy.algorithms.dmrg
- type: function

```
tenpy.algorithms.dmrg.chi_list(chi_max, dchi=20, nsweeps=20)
Compute a 'ramping-up' chi_list.
```

The resulting chi_list allows to increases *chi* by *dchi* every *nsweeps* sweeps up to a given maximal *chi_max*.

Parameters

- **chi_max** (*int*) Final value for the bond dimension.
- dchi (int) Step size how to increase chi
- nsweeps (int) Step size for sweeps

Returns chi_list – To be used as *chi_list* parameter for DMRG, see run(). Keys increase by *nsweeps*, values by *dchi*, until a maximum of *chi_max* is reached.

Return type dict

17.5.7 full_diag_effH

- full name: tenpy.algorithms.dmrg.full_diag_effH
- parent module: tenpy.algorithms.dmrg
- type: function

tenpy.algorithms.dmrg.full_diag_effH(effH, theta_guess, keep_sector=True)
Perform an exact diagonalization of effH.

This function offers an alternative to lanczos().

Parameters

- **effH** (*EffectiveH*) The effective Hamiltonian.
- **theta_guess** (*Array*) Current guess to select the charge sector. Labels as specified by effH.acts_on.

Module description

Density Matrix Renormalization Group (DMRG).

Although it was originally not formulated with tensor networks, the DMRG algorithm (invented by Steven White in 1992 [[white1992]]) opened the whole field with its enormous success in finding ground states in 1D.

We implement DMRG in the modern formulation of matrix product states [[schollwoeck2011]], both for finite systems ('finite' or 'segment' boundary conditions) and in the thermodynamic limit ('infinite' b.c.).

The function run () - well - runs one DMRG simulation. Internally, it generates an instance of an Sweep. This class implements the common functionality like defining a *sweep*, but leaves the details of the contractions to be performed to the derived classes.

Currently, there are two derived classes implementing the contractions: SingleSiteDMRGEngine and TwoSiteDMRGEngine. They differ (as their name implies) in the number of sites which are optimized simultaneously. They should both give the same results (up to rounding errors). However, if started from a product state, SingleSiteDMRGEngine depends critically on the use of a Mixer, while TwoSiteDMRGEngine is in principle more computationally expensive to run and has occasionally displayed some convergence issues.. Which one is preffered in the end is not obvious a priori and might depend on the used model. Just try both of them.

A Mixer should be used initially to avoid that the algorithm gets stuck in local energy minima, and then slowly turned off in the end. For SingleSiteDMRGEngine, using a mixer is crucial, as the one-site algorithm cannot increase the MPS bond dimension by itself.

A generic protocol for approaching a physics question using DMRG is given in /intro/protocol.

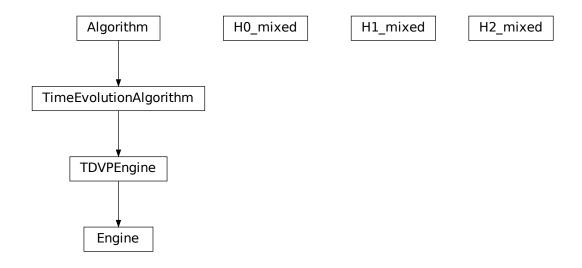
17.6 tdvp

• full name: tenpy.algorithms.tdvp

• parent module: tenpy.algorithms

• type: module

Classes



Engine(psi, model, options)	Deprecated old name of TDVPEngine.
HO_mixed(Lp, Rp)	Class defining the zero site Hamiltonian for Lanczos.
H1_mixed(Lp, Rp, W)	Class defining the one site Hamiltonian for Lanczos.
H2_mixed(Lp, Rp, W0, W1)	Class defining the two sites Hamiltonian for Lanczos.
TDVPEngine(psi, model, options[, environment])	Time dependent variational principle algorithm for
	MPS.

17.6.1 Engine

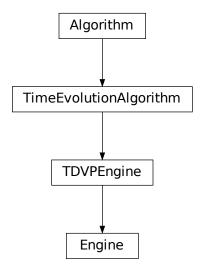
• full name: tenpy.algorithms.tdvp.Engine

 $\bullet \ \ parent \ module: \ \textit{tenpy.algorithms.tdvp}$

• type: class

17.6. tdvp 241

Inheritance Diagram



Methods

Engineinit(psi, model, options)	Initialize self.	
Engine.get_resume_data()	Return necessary data to resume a run () interrupted at	
	a checkpoint.	
Engine.resume_run()	Resume a run that was interrupted.	
Engine.run()	(Real-)time evolution with TDVP.	
<pre>Engine.run_one_site([N_steps])</pre>	Run the TDVP algorithm with the one site algorithm.	
Engine.run_two_sites([N_steps])	Run the TDVP algorithm with two sites update.	
Engine.set_anonymous_svd(U, new_label)	Relabel the svd.	
Engine.sweep_left_right()	Performs the sweep left->right of the second order	
	TDVP scheme with one site update.	
Engine.sweep_left_right_two()	Performs the sweep left->right of the second order	
	TDVP scheme with two sites update.	
Engine.sweep_right_left()	Performs the sweep right->left of the second order	
	TDVP scheme with one site update.	
Engine.sweep_right_left_two()	Performs the sweep left->right of the second order	
	TDVP scheme with two sites update.	
Engine.theta_svd_left_right(theta)	Performs the SVD from left to right.	
Engine.theta_svd_right_left(theta)	Performs the SVD from right to left.	
Engine.update $_s_h0(s, H, dt)$	Update with the zero site Hamiltonian (update of the	
	singular value)	
Engine.update_theta_h1(Lp, Rp, theta, W, dt)	Update with the one site Hamiltonian.	
Engine.update_theta_h2(Lp, Rp, theta, W0,	Update with the two sites Hamiltonian.	
W1, dt)		

Class Attributes and Properties

```
Engine.TDVP_params
```

Engine.verbose

class tenpy.algorithms.tdvp.**Engine**(*psi*, *model*, *options*)

Bases: tenpy.algorithms.tdvp.TDVPEngine

Deprecated old name of TDVPEngine.

```
get_resume_data()
```

Return necessary data to resume a run () interrupted at a checkpoint.

At a checkpoint, you can save psi, model and options along with the data returned by this function. When the simulation aborts, you can resume it using this saved data with:

```
eng = AlgorithmClass(psi, model, options, resume_data=resume_data)
eng.resume_run(resume_data)
```

An algorithm which doesn't support this should override *resume_run* to raise an Error.

Returns resume_data – Dictionary with necessary data (apart from copies of *psi*, *model*, *options*) that allows to continue the simulation from where we are now.

Return type dict

```
resume_run()
```

Resume a run that was interrupted.

In case we saved an intermediate result at a checkpoint, this function allows to resume the run() of the algorithm (after re-initialization with the $resume_data$). Since most algorithms just have a while loop with break conditions, the default behaviour implemented here is to just call run().

run()

(Real-)time evolution with TDVP.

```
run_one_site(N_steps=None)
```

Run the TDVP algorithm with the one site algorithm.

Warning: Be aware that the bond dimension will not increase!

Parameters N_steps(integer. Number of steps)-

```
run_two_sites(N_steps=None)
```

Run the TDVP algorithm with two sites update.

The bond dimension will increase. Truncation happens at every step of the sweep, according to the parameters set in trunc_params.

Parameters N_steps (integer. Number of steps) -

set_anonymous_svd(U, new_label)

Relabel the syd.

Parameters U (tenpy.linalg.np_conserved.Array) - the tensor which lacks a leg label

17.6. tdvp 243

sweep left right()

Performs the sweep left->right of the second order TDVP scheme with one site update.

Evolve from 0.5*dt.

sweep_left_right_two()

Performs the sweep left->right of the second order TDVP scheme with two sites update.

Evolve from 0.5*dt

sweep_right_left()

Performs the sweep right->left of the second order TDVP scheme with one site update.

Evolve from 0.5*dt

sweep_right_left_two()

Performs the sweep left->right of the second order TDVP scheme with two sites update.

Evolve from 0.5*dt

theta_svd_left_right (theta)

Performs the SVD from left to right.

Parameters theta (tenpy.linalg.np_conserved.Array) – the theta tensor on which the SVD is applied

theta svd right left(theta)

Performs the SVD from right to left.

Parameters theta (tenpy.linalg.np_conserved.Array,) - The theta tensor on which the SVD is applied

$update_s_h0(s, H, dt)$

Update with the zero site Hamiltonian (update of the singular value)

Parameters

- **s** (tenpy.linalg.np_conserved.Array) representing the singular value matrix which is updated
- H (HO_mixed) zero site Hamiltonian that we need to apply on the singular value matrix
- dt (complex number) time step of the evolution

$update_theta_h1$ (Lp, Rp, theta, W, dt)

Update with the one site Hamiltonian.

Parameters

- **Lp** (Array) tensor representing the left environment
- **Rp** (*Array*) tensor representing the right environment
- theta (Array) the theta tensor which needs to be updated
- W (Array) MPO which is applied to the 'p' leg of theta

update_theta_h2 (*Lp*, *Rp*, theta, *W0*, *W1*, *dt*)

Update with the two sites Hamiltonian.

Parameters

- Lp (tenpy.linalg.np_conserved.Array) tensor representing the left environment
- Rp (tenpy.linalg.np_conserved.Array) tensor representing the right environment

- theta (tenpy.linalg.np_conserved.Array) the theta tensor which needs to be updated
- W (tenpy.linalg.np_conserved.Array) MPO which is applied to the 'p0' leg of theta
- W1 (tenpy.linalg.np_conserved.Array) MPO which is applied to the 'p1' leg of theta

17.6.2 H0 mixed

• full name: tenpy.algorithms.tdvp.H0_mixed

• parent module: tenpy.algorithms.tdvp

• type: class

Inheritance Diagram

H0 mixed

Methods

```
H0_mixed.__init__(Lp, Rp) Initialize self.
H0_mixed.matvec(x)
```

```
class tenpy.algorithms.tdvp.{\tt H0\_mixed}(Lp,Rp)
```

Bases: object

Class defining the zero site Hamiltonian for Lanczos.

Parameters

- Lp $(tenpy.linalg.np_conserved.Array)$ left part of the environment
- \mathbf{Rp} (tenpy.linalg.np_conserved.Array) right part of the environment

Lр

left part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

Rp

right part of the environment

Type tenpy.linalg.np_conserved.Array

17.6. tdvp 245

17.6.3 H1 mixed

• full name: tenpy.algorithms.tdvp.H1_mixed

• parent module: tenpy.algorithms.tdvp

• type: class

Inheritance Diagram

H1 mixed

Methods

H1_mixedinit(Lp, Rp, W)	Initialize self.	
H1_mixed.matvec(theta)		

```
\textbf{class} \texttt{ tenpy.algorithms.tdvp.H1\_mixed} (\mathit{Lp}, \mathit{Rp}, \mathit{W})
```

Bases: object

Class defining the one site Hamiltonian for Lanczos.

Parameters

- Lp (tenpy.linalg.np_conserved.Array) left part of the environment
- Rp (tenpy.linalg.np_conserved.Array) right part of the environment
- M(tenpy.linalg.np_conserved.Array) MPO which is applied to the 'p' leg of theta

Lр

left part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

Rр

right part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

W

MPO which is applied to the 'p0' leg of theta

```
Type tenpy.linalg.np_conserved.Array
```

17.6.4 H2 mixed

• full name: tenpy.algorithms.tdvp.H2_mixed

• parent module: tenpy.algorithms.tdvp

• type: class

Inheritance Diagram

H2 mixed

Methods

```
H2_mixed.__init__(Lp, Rp, W0, W1) Initialize self.
H2_mixed.matvec(theta)
```

```
class tenpy.algorithms.tdvp.{\tt H2\_mixed}\,(Lp,Rp,W0,W1) Bases: object
```

Class defining the two sites Hamiltonian for Lanczos.

Parameters

- Lp (tenpy.linalg.np_conserved.Array) left part of the environment
- **Rp** (tenpy.linalg.np_conserved.Array) right part of the environment
- W (tenpy.linalg.np_conserved.Array) MPO which is applied to the 'p0' leg of theta

Lр

left part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

Rр

right part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

WΟ

MPO which is applied to the 'p0' leg of theta

```
Type tenpy.linalg.np_conserved.Array
```

W1

MPO which is applied to the 'p1' leg of theta

```
Type tenpy.linalg.np_conserved.Array
```

17.6. tdvp 247

Module description

Time Dependant Variational Principle (TDVP) with MPS (finite version only).

The TDVP MPS algorithm was first proposed by [[haegeman2011]]. However the stability of the algorithm was later improved in [[haegeman2016]], that we are following in this implementation. The general idea of the algorithm is to project the quantum time evolution in the manyfold of MPS with a given bond dimension. Compared to e.g. TEBD, the algorithm has several advantages: e.g. it conserves the unitarity of the time evolution and the energy (for the single-site version), and it is suitable for time evolution of Hamiltonian with arbitrary long range in the form of MPOs. We have implemented the one-site formulation which **does not** allow for growth of the bond dimension, and the two-site algorithm which does allow the bond dimension to grow - but requires truncation as in the TEBD case.

Todo: This is still a beta version, use with care. The interface might still change.

Todo: long-term: Much of the code is similar as in DMRG. To avoid too much duplicated code, we should have a general way to sweep through an MPS and updated one or two sites, used in both cases.

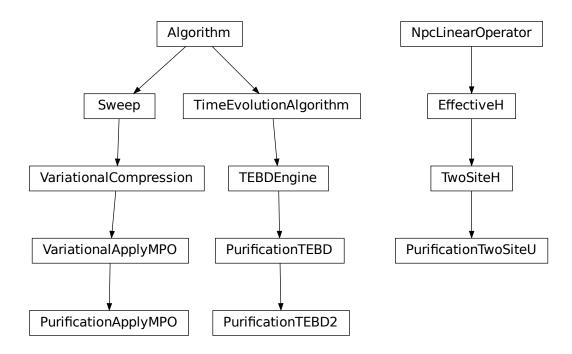
17.7 purification

• full name: tenpy.algorithms.purification

• parent module: tenpy.algorithms

• type: module

Classes



PurificationApplyMPO(psi, U_MPO, options[,	Variant of VariationalApplyMPO suitable for purifica-
])	tion.
PurificationTEBD(psi, model, options)	Time evolving block decimation (TEBD) for purifica-
	tion MPS.
PurificationTEBD2(psi, model, options)	Similar as PurificationTEBD, but perform sweeps in-
	stead of brickwall.
PurificationTwoSiteU(env, iO[, combine,])	Variant of <i>TwoSiteH</i> suitable for purification.

17.7.1 PurificationApplyMPO

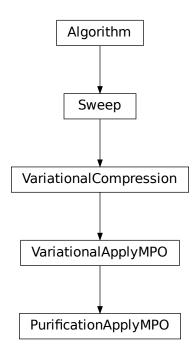
• full name: tenpy.algorithms.purification.PurificationApplyMPO

• parent module: tenpy.algorithms.purification

• type: class

17.7. purification 249

Inheritance Diagram



Methods

PurificationApplyMPOinit(psi,	Initialize self.
U_MPO,)	initialize sen.
PurificationApplyMPO.	Perform <i>N_sweeps</i> sweeps without optimization to up-
environment_sweeps(N_sweeps)	date the environment.
PurificationApplyMPO.	Return necessary data to resume a run () interrupted at
get_resume_data()	a checkpoint.
PurificationApplyMPO.	Define the schedule of the sweep.
get_sweep_schedule()	
PurificationApplyMPO.init_env(U_MPO[,	Initialize the environment.
])	
PurificationApplyMPO.make_eff_H()	Create new instance of self.EffectiveH at self.i0 and set
	it to self.eff_H.
PurificationApplyMPO.	Algorithm-specific actions to be taken after local up-
<pre>post_update_local()</pre>	date.
PurificationApplyMPO.prepare_update()	Prepare everything algorithm-specific to perform a local
	update.
PurificationApplyMPO.	Reset the statistics.
reset_stats([resume_data])	
PurificationApplyMPO.resume_run()	Resume a run that was interrupted.
	continues on next page

Table 33 – continued from previous page

PurificationApplyMPO.run()	Run the compression.
PurificationApplyMPO.sweep([optimize])	One 'sweep' of a sweeper algorithm.
PurificationApplyMPO.update_LP(_)	
PurificationApplyMPO.update_RP(_)	
PurificationApplyMPO.update_local(_[,	Perform local update.
optimize])	•
PurificationApplyMPO.	Given a new two-site wave function theta, split it and
update_new_psi(theta)	save it in psi.

Class Attributes and Properties

PurificationApplyMPO.engine_params	
PurificationApplyMPO.n_optimize	the number of sites to be optimized over at once.
PurificationApplyMPO.verbose	

class tenpy.algorithms.purification.PurificationApplyMPO(psi, U_MPO, options, resume_data=None)

 $Bases: \verb|tenpy.algorithms.mps_common.VariationalApplyMPO| \\$

Variant of Variational ApplyMPO suitable for purification.

EffectiveH

 ${\bf alias\ of\ tenpy.} algorithms. purification. Purification {\it TwoSiteU}$

update_local (_, optimize=True)

Perform local update.

This simply contracts the environments and *theta* from the *ket* to get an updated *theta* for the bra *self.psi* (to be changed in place).

update_new_psi(theta)

Given a new two-site wave function theta, split it and save it in psi.

environment_sweeps (N_sweeps)

Perform *N_sweeps* sweeps without optimization to update the environment.

Parameters N_sweeps (int) – Number of sweeps to run without optimization

get_resume_data()

Return necessary data to resume a run () interrupted at a checkpoint.

At a checkpoint, you can save psi, model and options along with the data returned by this function. When the simulation aborts, you can resume it using this saved data with:

```
eng = AlgorithmClass(psi, model, options, resume_data=resume_data)
eng.resume_run(resume_data)
```

An algorithm which doesn't support this should override *resume_run* to raise an Error.

Returns resume_data – Dictionary with necessary data (apart from copies of *psi*, *model*, *options*) that allows to continue the simulation from where we are now.

Return type dict

17.7. purification 251

get_sweep_schedule()

Define the schedule of the sweep.

One 'sweep' is a full sequence from the leftmost site to the right and back. Only those *LP* and *RP* that can be used later should be updated.

Returns schedule – Schedule for the sweep. Each entry is (i0, move_right, (update_LP, update_RP)), where i0 is the leftmost of the self.EffectiveH. length sites to be updated in update_local(), move_right indicates whether the next i0 in the schedule is rigth (True) of the current one, and update_LP, update_RP indicate whether it is necessary to update the LP and RP. The latter are chosen such that the environment is growing for infinite systems, but we only keep the minimal number of environment tensors in memory.

Return type iterable of (int, bool, (bool, bool))

init_env(U_MPO, resume_data=None)

Initialize the environment.

Parameters

- $U_MPO(MPO)$ The MPO to be applied to the sate.
- resume_data (dict) May contain in

make eff H()

Create new instance of *self.EffectiveH* at *self.i0* and set it to *self.eff_H*.

property n_optimize

the number of sites to be optimized over at once.

Indirectly set by the class attribute *EffectiveH* and it's *length*. For example, TwoSiteDMRGEngine uses the *TwoSiteH* and hence has n_optimize=2, while the SingleSiteDMRGEngine has n_optimize=1.

post_update_local (update_data)

Algorithm-specific actions to be taken after local update.

An example would be to collect statistics.

prepare_update()

Prepare everything algorithm-specific to perform a local update.

reset stats(resume data=None)

Reset the statistics. Useful if you want to start a new Sweep run.

This method is expected to be overwritten by subclass, and should then define self.update_stats and self.sweep stats dicts consistent with the statistics generated by the algorithm particular to that subclass.

option Sweep.sweep_0: int

Number of sweeps that have already been performed.

```
option Sweep.chi_list: None | dict(int -> int)
```

By default (None) this feature is disabled. A dict allows to gradually increase the *chi_max*. An entry *at_sweep*: *chi* states that starting from sweep *at_sweep*, the value *chi* is to be used for trunc_params['chi_max']. For example chi_list={0: 50, 20: 100} uses chi_max=50 for the first 20 sweeps and chi_max=100 afterwards.

resume_run()

Resume a run that was interrupted.

In case we saved an intermediate result at a checkpoint, this function allows to resume the run () of the algorithm (after re-initialization with the resume_data). Since most algorithms just have a while loop with break conditions, the default behaviour implemented here is to just call run ().

run()

Run the compression.

The state psi is compressed in place.

Returns max_trunc_err – The maximal truncation error of a two-site wave function.

Return type TruncationError

sweep (optimize=True)

One 'sweep' of a sweeper algorithm.

Iteratate over the bond which is optimized, to the right and then back to the left to the starting point. If optimize=False, don't actually diagonalize the effective hamiltonian, but only update the environment.

Parameters optimize (bool, optional) – Whether we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).

Returns max_trunc_err – Maximal truncation error introduced.

Return type float

17.7.2 PurificationTEBD2

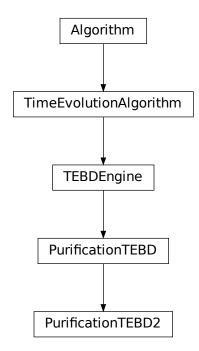
• full name: tenpy.algorithms.purification.PurificationTEBD2

• parent module: tenpy.algorithms.purification

· type: class

17.7. purification 253

Inheritance Diagram



Methods

PurificationTEBD2init(psi, model, op-	Initialize self.
tions)	
PurificationTEBD2.calc_U(order, delta_t[,	see calc_U()
])	
PurificationTEBD2.disentangle(theta)	Disentangle <i>theta</i> before splitting with svd.
PurificationTEBD2.	Try global disentangling by determining the maximally
disentangle_global([pair])	entangled pairs of sites.
PurificationTEBD2.	Perform a sweep through the system and disentangle
disentangle_global_nsite([n])	<pre>with disentangle_n_site().</pre>
PurificationTEBD2.	Generalization of disentangle () to n sites.
disentangle_n_site(i, n, theta)	
PurificationTEBD2.get_resume_data()	Return necessary data to resume a run () interrupted at
	a checkpoint.
PurificationTEBD2.resume_run()	Resume a run that was interrupted.
PurificationTEBD2.run()	Run TEBD real time evolution by <i>N_steps</i> `*` <i>dt</i> .
PurificationTEBD2.run_GS()	TEBD algorithm in imaginary time to find the ground
	state.
PurificationTEBD2.run_imaginary(beta)	Run imaginary time evolution to cool down to the given
	beta.
	continues on next page

Table	35 -	continued	from	previous	page
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PurificationTEBD2.	Returns list of necessary steps for the suzuki trotter de-
$suzuki_trotter_decomposition()$	composition.
PurificationTEBD2.	Return time steps of U for the Suzuki Trotter decompo-
suzuki_trotter_time_steps(order)	sition of desired order.
PurificationTEBD2.update(N_steps)	Evolve by N_steps * U_param['dt'].
PurificationTEBD2.update_bond(i, U_bond)	Updates the B matrices on a given bond.
PurificationTEBD2.update_bond_imag $(i,$	Update a bond with a (possibly non-unitary) <i>U_bond</i> .
U_bond)	
PurificationTEBD2.update_imag(N_steps)	Perform an update suitable for imaginary time evolu-
	tion.
PurificationTEBD2.update_step(U_idx_dt,	Updates bonds in unit cell.
odd)	

Class Attributes and Properties

PurificationTEBD2.TEBD_params	
PurificationTEBD2.disent_iterations	For each bond the total number of iterations performed
	in any Disentangler.
PurificationTEBD2.trunc_err_bonds	truncation error introduced on each non-trivial bond.
PurificationTEBD2.verbose	

class tenpy.algorithms.purification.PurificationTEBD2 (psi, model, options)

Bases: tenpy.algorithms.purification.PurificationTEBD

Similar as PurificationTEBD, but perform sweeps instead of brickwall.

Instead of the A-B pattern of even/odd bonds used in TEBD, perform sweeps similar as in DMRG for real-time evolution (similar as update_imag() does for imaginary time evolution).

update (N_steps)

Evolve by N_steps * U_param['dt'].

Parameters N_steps (int) – The number of steps for which the whole lattice should be updated.

Returns trunc_err – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

update_step(U_idx_dt, odd)

Updates bonds in unit cell.

Depending on the choice of *odd*, perform a sweep to the left or right, updating once per site with a time step given by U_idx_dt.

Parameters

- U_idx_dt (int) Time step index in self._U, evolve with Us[i] = self. U[U_idx_dt][i] at bond (i-1,i).
- **odd** (bool/int) Indication of whether to update even (odd=False, 0) or even (odd=True, 1) sites

17.7. purification 255

Returns trunc_err – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

```
Return type TruncationError
```

```
calc_U (order, delta_t, type_evo='real', E_offset=None)
see calc_U()
```

property disent_iterations

For each bond the total number of iterations performed in any Disentangler.

disentangle (theta)

Disentangle theta before splitting with svd.

For the purification we write $\rho_P = Tr_Q |\psi_{P,Q}> <\psi_{P,Q}|$. Thus, we can actually apply any unitary to the auxiliar Q space of $|\psi>$ without changing the result.

Note: We have to apply the *same* unitary to the 'bra' and 'ket' used for expectation values / correlation functions!

The behaviour of this function is set by used_disentangler, which in turn is obtained from get_disentangler(options['disentangle']), see get_disentangler() for details on the syntax.

```
Parameters theta (Array) - Wave function to disentangle, with legs 'vL', 'vR', 'p0', 'p1', 'q0', 'q1'.
```

Returns

- theta_disentangled (Array) Disentangled theta; npc.tensordot(U, theta, axes=[['q0*', 'q1*'], ['q0', 'q1']]).
- U (Array) The unitary used to disentangle *theta*, with labels 'q0', 'q1', 'q0*', 'q1*'. If no unitary was found/applied, it might also be None.

disentangle_global (pair=None)

Try global disentangling by determining the maximally entangled pairs of sites.

Caclulate the mutual information (in the auxiliar space) between two sites and determine where it is maximal. Disentangle these two sites with <code>disentangle()</code>

disentangle global nsite(n=2)

Perform a sweep through the system and disentangle with <code>disentangle_n_site()</code>.

Parameters n (int) – maximal number of sites to disentangle at once.

disentangle n site (i, n, theta)

Generalization of disentangle () to n sites.

Simply group left and right n/2 physical legs, adjust labels, and apply disentangle() to disentangle the central bond. Recursively proceed to disentangle left and right parts afterwards. Scales (for even n) as $O(\chi^3 d^n d^{n/2})$.

get_resume_data()

Return necessary data to resume a run () interrupted at a checkpoint.

At a checkpoint, you can save psi, model and options along with the data returned by this function. When the simulation aborts, you can resume it using this saved data with:

```
eng = AlgorithmClass(psi, model, options, resume_data=resume_data)
eng.resume_run(resume_data)
```

An algorithm which doesn't support this should override *resume_run* to raise an Error.

Returns resume_data – Dictionary with necessary data (apart from copies of *psi*, *model*, *options*) that allows to continue the simulation from where we are now.

Return type dict

resume_run()

Resume a run that was interrupted.

In case we saved an intermediate result at a checkpoint, this function allows to resume the *run()* of the algorithm (after re-initialization with the *resume_data*). Since most algorithms just have a while loop with break conditions, the default behaviour implemented here is to just call *run()*.

run()

Run TEBD real time evolution by *N_steps*`*`*dt*.

run_GS()

TEBD algorithm in imaginary time to find the ground state.

Note: It is almost always more efficient (and hence advisable) to use DMRG. This algorithms can nonetheless be used quite well as a benchmark and for comparison.

```
option TEBDEngine.delta_tau_list: list
```

A list of floats: the timesteps to be used. Choosing a large timestep *delta_tau* introduces large (Trotter) errors, but a too small time step requires a lot of steps to reach exp(-tau H) --> |psi0><psi0|. Therefore, we start with fairly large time steps for a quick time evolution until convergence, and the gradually decrease the time step.

option TEBDEngine.order: int

Order of the Suzuki-Trotter decomposition.

option TEBDEngine.N_steps: int

Number of steps before measurement can be performed

run_imaginary (beta)

Run imaginary time evolution to cool down to the given beta.

Note that we don't change the *norm* attribute of the MPS, i.e. normalization is preserved.

Parameters beta (float) – The inverse temperature beta = 1/T, by which we should cool down. We evolve to the closest multiple of options ['dt'], see also evolved time.

static suzuki_trotter_decomposition(order, N_steps)

Returns list of necessary steps for the suzuki trotter decomposition.

We split the Hamiltonian as $H = H_{even} + H_{odd} = H[0] + H[1]$. The Suzuki-Trotter decomposition is an approximation $\exp(tH) \approx prod_{(j,k) \in ST} \exp(d[j]tH[k]) + O(t^{order+1})$.

Parameters order (1, 2, 4, '4_opt') – The desired order of the Suzuki-Trotter decomposition. Order 1 approximation is simply $e^A a^B$. Order 2 is the "leapfrog" $e^A A^2 e^B e^A A^2$. Order 4 is the fourth-order from [[suzuki1991]] (also referenced in [[schollwoeck2011]]), and '4_opt' gives the optmized version of Equ. (30a) in [[barthel2020]].

Returns ST_decomposition – Indices j, k of the time-steps d = suzuki_trotter_time_step(order) and the decomposition of H. They are chosen such that a subsequent application of $\exp(d[j] + H[k])$ to a given state |psi> yields (exp(N_steps + H[k]) + O(N_steps + forder+1))) |psi>.

Return type list of (int, int)

17.7. purification 257

static suzuki trotter time steps (order)

Return time steps of U for the Suzuki Trotter decomposition of desired order.

See suzuki_trotter_decomposition() for details.

Parameters order (int) – The desired order of the Suzuki-Trotter decomposition.

Returns time_steps - We need U = exp(-i H_{even/odd} delta_t * dt) for the *dt* returned in this list.

Return type list of float

property trunc_err_bonds

truncation error introduced on each non-trivial bond.

update_bond(i, U_bond)

Updates the B matrices on a given bond.

Function that updates the B matrices, the bond matrix s between and the bond dimension chi for bond i. This would look something like:

Parameters

- \mathbf{i} (int) Bond index; we update the matrices at sites i-1, i.
- **U_bond** (Array) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0*', 'p1*' for U_bond .

Returns trunc_err – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

update_bond_imag(i, U_bond)

Update a bond with a (possibly non-unitary) *U_bond*.

Similar as update_bond(); but after the SVD just keep the A, S, B canonical form. In that way, one can sweep left or right without using old singular values, thus preserving the canonical form during imaginary time evolution.

Parameters

- \mathbf{i} (int) Bond index; we update the matrices at sites i-1, i.
- **U_bond** (*Array*) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0*', 'p1*'.

Returns trunc_err – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

update_imag (N_steps)

Perform an update suitable for imaginary time evolution.

Instead of the even/odd brick structure used for ordinary TEBD, we 'sweep' from left to right and right to left, similar as DMRG. Thanks to that, we are actually able to preserve the canonical form.

Parameters N_steps (int) – The number of steps for which the whole lattice should be updated.

Returns trunc_err – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

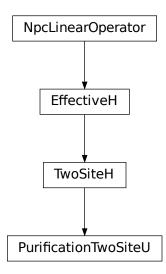
17.7.3 PurificationTwoSiteU

• full name: tenpy.algorithms.purification.PurificationTwoSiteU

• parent module: tenpy.algorithms.purification

• type: class

Inheritance Diagram



Methods

PurificationTwoSiteUinit(env, i0[,	Initialize self.
])	
PurificationTwoSiteU.adjoint()	Return the hermitian conjugate of <i>self</i> .
PurificationTwoSiteU.combine_Heff()	Combine LP and RP with W to form LHeff and RHeff.
PurificationTwoSiteU.	Combine the legs of theta, such that it fits to how we
combine_theta(theta)	combined the legs of <i>self</i> .
	continues on next page

17.7. purification 259

Table 37 – continued from previous page

PurificationTwoSiteU.matvec(theta)	Apply the effective Hamiltonian to <i>theta</i> .
PurificationTwoSiteU.to_matrix()	Contract <i>self</i> to a matrix.

Class Attributes and Properties

PurificationTwoSiteU.acts_on

PurificationTwoSiteU.length

Bases: tenpy.algorithms.mps_common.TwoSiteH

Variant of *TwoSiteH* suitable for purification.

The MPO gets only applied to the physical legs p0, p1, the ancialla legs q0, q1 of theta are ignored.

combine Heff()

Combine LP and RP with W to form LHeff and RHeff.

Combine LP with W0 and RP with W1 to get the effective parts of the Hamiltonian with piped legs.

adjoint()

Return the hermitian conjugate of *self*.

combine_theta(theta)

Combine the legs of theta, such that it fits to how we combined the legs of self.

Parameters theta (Array) - Wave function with labels 'vL', 'p0', 'p1', 'vR'

Returns theta – Wave function with labels 'vL', 'p0', 'p1', 'vR'

Return type Array

matvec(theta)

Apply the effective Hamiltonian to theta.

Parameters theta (Array) - Labels: vL, p0, p1, vR if combine=False, (vL.p0), (p1.vR) if True

Returns Product of *theta* and the effective Hamiltonian.

Return type theta Array

to_matrix()

Contract self to a matrix.

Module description

Algorithms for using Purification.

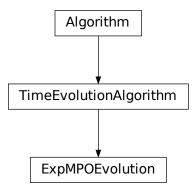
17.8 mpo_evolution

• full name: tenpy.algorithms.mpo_evolution

• parent module: tenpy.algorithms

• type: module

Classes



ExpMPOEvolution(psi, model, options)

Time evolution of an MPS using the W_I or W_II approximation for exp(H dt).

Module description

Time evolution using the WI or WII approximation of the time evolution operator.

17.9 network_contractor

• full name: tenpy.algorithms.network_contractor

• parent module: tenpy.algorithms

• type: module

Functions

contract(tensor_list[, tensor_names,])	Contract a network of tensors.
ncon(tensor_list, leg_links, sequence)	Implementation of ncon.m for TeNPy Arrays.

17.9.1 contract

- full name: tenpy.algorithms.network_contractor.contract
- parent module: tenpy.algorithms.network contractor
- type: function

```
tenpy.algorithms.network_contractor.contract(tensor_list, tensor_names=None, leg_contractions=None, open_legs=None, sequence=None)
```

Contract a network of tensors.

Based on the MatLab function ncon.m as described in arXiv:1402.0939.

Parameters

- tensor_list (list of Array) The tensors to be contracted.
- **leg_contractions** (list of [n1, 11, n2, 12]) A list of contraction instructions. An entry of leg_contractions has the form [n1, 11, n2, 12], where n1, n2 are entries of *tensor_names* and each identify an *Array* in *tensor_list*. 11, 12 are leg labels of the corresponding *Array*. The instruction implies to contract leg 11 of tensor n1 with leg 12 of tensor n2.
- open_legs (list of [n1, 11, 1]) A list of instructions for "open" (uncontracted) legs. [n1, 11, 1] implies that leg 11 of tensor n1 is not contracted and is labelled 1 in the result.
- **tensor_names** (list of str) A list of names for each tensor, to be used in leg_contractions and open_legs. The default value is list(range(len(tensor_list))), so that the tensor "names" are 0, 1, 2,
- **sequence** (*list of int*) The order in which the leg_contractions are to be performed. An entry of network_contractor.outer_product indicates performing an outer product. This corresponds to the zero-in-sequence convention of arXiv:1304.6112

Returns result – The number or tensor resulting from the contraction.

Return type Array | complex

17.9.2 ncon

- full name: tenpy.algorithms.network_contractor.ncon
- parent module: tenpy.algorithms.network_contractor
- type: function

```
tenpy.algorithms.network_contractor.ncon(tensor_list, leg_links, sequence) Implementation of ncon.m for TeNPy Arrays.
```

This function is a python implementation of ncon.m (arXiv:1304.6112) for tenpy Array. contract () is a wrapper that translates from a more python/tenpy input style

Parameters

- tensor_list (list of :class:'Array') Tensors to be contracted.
- **leg_links** (*list of list of int*) Each entry of leg_links describes the connectivity of the corresponding tensor in *tensor_list*. Each entry is a list that has an entry for each leg of the corresponding tensor. Values 0, 1, 2, . . . are labels of contracted legs and should appear exactly twice in *leg_links*. Values -1, -2, -3, . . . are labels of uncontracted legs and indicate the final ordering (-1 is first axis).
- **sequence** (list of int) The order in which the contractions are to be performed. An entry of network_contractor.outer_product indicates performing an outer product. This corresponds to the zero-in-sequence convention of arXiv:1304.6112

Returns result – The number or tensor resulting from the contraction.

Return type Array | complex

Module description

Network Contractor.

A tool to contract a network of multiple tensors.

This is an implementation of 'NCON: A tensor network contractor for MATLAB' by Robert N. C. Pfeifer, Glen Evenbly, Sukhwinder Singh, Guifre Vidal, see arXiv:1402.0939

Todo:

- implement or wrap netcon.m, a function to find optimal contractionn sequences (arXiv:1304.6112)
- improve helpfulness of Warnings
- _do_trace: trace over all pairs of legs at once. need the corresponding npc function first.

17.10 exact_diag

• full name: tenpy.algorithms.exact_diag

• parent module: tenpy.algorithms

• type: module

17.10. exact_diag 263

Classes

ExactDiag

17.10.1 ExactDiag

• full name: tenpy.algorithms.exact_diag.ExactDiag

• parent module: tenpy.algorithms.exact_diag

• type: class

Inheritance Diagram

ExactDiag

Methods

<pre>ExactDiaginit(model[, charge_sector,</pre>	Initialize self.		
])			
<pre>ExactDiag.build_full_H_from_bonds()</pre>	Calculate self.full_H from self.mpo.		
ExactDiag.build_full_H_from_mpo()	Calculate self.full_H from self.mpo.		
ExactDiag.exp_H(dt)	Return $U(dt) := exp(-i H dt)$.		
ExactDiag.from_H_mpo(H_MPO, *args,	Wrapper taking directly an MPO instead of a Model.		
**kwargs)			
ExactDiag.full_diagonalization(*args,	Full diagonalization to obtain all eigenvalues and eigen-		
	ε		
**kwargs)	vectors.		
**kwargs)	vectors.		
**kwargs) ExactDiag.full_to_mps(psi[, canonical_form])	vectors. Convert a full state (with a single leg) to an MPS.		
**kwargs) ExactDiag.full_to_mps(psi[, canonical_form])	vectors. Convert a full state (with a single leg) to an MPS. Pick the ground state energy and ground state from		
**kwargs) ExactDiag.full_to_mps(psi[, canonical_form]) ExactDiag.groundstate([charge_sector])	vectors. Convert a full state (with a single leg) to an MPS. Pick the ground state energy and ground state from self.V.		
**kwargs) ExactDiag.full_to_mps(psi[, canonical_form]) ExactDiag.groundstate([charge_sector]) ExactDiag.matvec(psi)	vectors. Convert a full state (with a single leg) to an MPS. Pick the ground state energy and ground state from self.V. Allow to use <i>self</i> as LinearOperator for lanczos.		

Table 42 – continued from previous page

ExactDiag.sparse_diag(k, *args, **kwargs)

Call speigs ().

Bases: object

(Full) exact diagonalization of the Hamiltonian.

Parameters

- model (MPOmodel | CouplingModel) The model which is to be diagonalized.
- charge_sector (None | charges) If not None, project onto the given charge sector.
- **sparse** (bool) If True, don't sort/bunch the LegPipe used to combine the physical legs. This results in array *blocks* with just one entry, requires much more charge data, and is not what *np_conserved* was designed for, so it's not recommended.
- max_size (int) The build_H_* functions will do nothing (but emit a warning) if the total size of the Hamiltonian would be larger than this.

model

The model which is to be diagonalized.

Type MPOmodel|CouplingModel

chinfo

The nature of the charge (which is the same for all sites).

Type ChargeInfo

charge sector

If not None, we project onto the given charge sector.

Type None I charges

max size

The build_H_* functions will do nothing (but emit a warning) if the total size of the Hamiltonian would be larger than this.

Type int

full H

The full Hamiltonian to be diagonalized with legs '(p0.p1...)', '(p0*,p1*...)' (in that order). None if the build_H_* functions haven't been called yet, or if max_size would have been exceeded.

Type Array | None

E

1D array of eigenvalues.

Type ndarray | None

V

Eigenvectors. First leg 'ps' are physical legs, the second leg 'ps*' corresponds to the eigenvalues.

Type Array | None

_sites

The sites in the given order.

Type list of Site

17.10. exact_diag 265

```
labels p
     The labels use for the physical legs; just ['p0', 'p1', ..., 'p{L-1}'].
         Type list or str
labels pconj
     Just each of labels p with an \star.
         Type list or str
_pipe
     The pipe from the single physical legs to the full combined leg.
         Type LegPipe
_pipe_conj
     Just _pipe.conj().
         Type LegPipe
_{\mathtt{mask}}
     Bool mask, which of the indices of the pipe are in the desired charge_sector.
         Type 1D bool ndarray | None
classmethod from_H_mpo(H_MPO, *args, **kwargs)
     Wrapper taking directly an MPO instead of a Model.
         Parameters
             • H_MPO (MPO) – The MPO representing the Hamiltonian.
             • *args – Further keyword arguments as for the __init__ of the class.
             • **kwargs - Further keyword arguments as for the __init__ of the class.
build full H from mpo()
     Calculate self.full_H from self.mpo.
build_full_H_from_bonds()
     Calculate self.full_H from self.mpo.
full_diagonalization(*args, **kwargs)
     Full diagonalization to obtain all eigenvalues and eigenvectors.
     Arguments are given to eigh.
groundstate (charge_sector=None)
     Pick the ground state energy and ground state from self. V.
         Parameters charge_sector (None | 1D ndarray) - By default (None), consider all
             charge sectors. Alternatively, give the qtotal which the returned state should have.
         Returns
             • E0 (float) – Ground state energy (possibly in the given sector).
             • psi0 (Array) – Ground state (possibly in the given sector).
exp_H(dt)
     Return U(dt) := exp(-i H dt).
mps_to_full (mps)
     Contract an MPS along the virtual bonds and combine its legs.
         Parameters mps (MPS) – The MPS to be contracted.
```

```
Return type Array

full_to_mps (psi, canonical_form='B')
Convert a full state (with a single leg) to an MPS.

Parameters

• psi (Array) – The state (with a single leg) which should be splitted into an MPS.

• canonical_from (Array) – The form in which the MPS will be afterwards.

Returns mps – An normalized MPS representation in canonical form.

Return type MPS

matvec (psi)
Allow to use self as LinearOperator for lanczos.

Just applies full_H to (the first axis of) the given psi.

sparse_diag (k, *args, **kwargs)
Call speigs ().
```

Module description

Full diagonalization (ED) of the Hamiltonian.

The full diagonalization of a small system is a simple approach to test other algorithms. In case you need the full spectrum, a full diagonalization is often the only way. This module provides functionality to quickly diagonalize the Hamiltonian of a given model. This might be used to obtain the spectrum, the ground state or highly excited states.

Note: Good use of symmetries is crucial to increase the treatable system size. While we can simply use the defined *LegCharge* of a model, we don't make use of any other symmetries like translation symmetry, SU(2) symmetry or inversion symmetries. In other words, this code does not aim to provide state-of-the-art exact diagonalization, but just the ability to diagonalize the defined models for small system sizes without addional extra work.

17.10. exact_diag 267

EIGHTEEN

LINALG

full name: tenpy.linalgparent module: tenpy

• type: module

Module description

Linear-algebra tools for tensor networks.

Most notably is the module np_conserved, which contains everything needed to make use of charge conservervation in the context of tensor networks.

Relevant contents of *charges* are imported to *np_conserved*, so you propably won't need to import *charges* directly.

Submodules

np_conserved	A module to handle charge conservation in tensor networks.
charges	Basic definitions of a charge.
svd_robust	(More) robust version of singular value decomposition.
random_matrix	Provide some random matrix ensembles for numpy.
sparse	Providing support for sparse algorithms (using matrix-
	vector products only).
lanczos	Lanczos algorithm for np_conserved arrays.

18.1 np_conserved

• full name: tenpy.linalg.np_conserved

ullet parent module: tenpy.linalg

• type: module

Classes

Array

Array(legcharges[, dtype, qtotal, labels])	A multidimensional array (=tensor) for using charge
	conservation.

18.1.1 Array

• full name: tenpy.linalg.np_conserved.Array

• parent module: tenpy.linalg.np_conserved

• type: class

Inheritance Diagram

Array

Methods

Arrayinit(legcharges[, dtype, qtotal,])	see help(self)		
Array.add_charge(add_legs[, chinfo, qtotal])	Add charges.		
Array.add_leg(leg, i[, axis, label])	Add a leg to self, setting the current array as slice for a		
	given index.		
Array.add_trivial_leg([axis, label, qconj])	Add a trivial leg (with just one entry) to <i>self</i> .		
Array.as_completely_blocked()	Gives a version of self which is completely blocked by		
	charges.		
Array.astype(dtype[, copy])	Return copy with new dtype, upcasting all blocks in		
	_data.		
Array.binary_blockwise(func, other, *args,	Roughly return func(self, other), block-		
)	wise.		
Array.change_charge(charge, new_qmod[,])	Change the <i>qmod</i> of one charge in <i>chinfo</i> .		
Array.combine_legs(combine_legs[, new_axes,	Reshape: combine multiple legs into multiple pipes.		
])			
	continues on next page		

Table	3 – cont	inued	from	pr	eviou	ıs pag	е	
	-		Retu	ırn	copy	which	is	con

Array.complex_conj()	Return copy which is complex conjugated <i>without</i> con-
TITTEY . COMPTER_CONJU	jugating the charge data.
Array.conj([complex_conj, inplace])	Conjugate: complex conjugate data, conjugate charge
	data.
Array.copy([deep])	Return a (deep or shallow) copy of self.
Array.drop_charge([charge, chinfo])	Drop (one of) the charges.
Array.extend(axis, extra)	Increase the dimension of a given axis, filling the values
	with zeros.
Array.from_func(func, legcharges[, dtype,])	Create an Array from a numpy func.
Array.from_func_square(func, leg[, dtype,	Create an Array from a (numpy) function.
])	
Array.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Array.from_ndarray(data_flat, legcharges[,])	convert a flat (numpy) ndarray to an Array.
Array.from_ndarray_trivial(data_flat[,])	convert a flat numpy ndarray to an Array with trivial
	charge conservation.
Array.gauge_total_charge(axis[, newqtotal,	Changes the total charge by adjusting the charge on a
])	certain leg.
Array.get_block(qindices[, insert])	Return the ndarray in _data representing the block
	corresponding to qindices.
Array.get_leg(label)	Return self.legs[self.
	<pre>get_leg_index(label)].</pre>
Array.get_leg_index(label)	translate a leg-index or leg-label to a leg-index.
Array.get_leg_indices(labels)	Translate a list of leg-indices or leg-labels to leg indices.
Array.get_leg_labels()	Return list of the leg labels, with <i>None</i> for anonymous
	legs.
Array.has_label(label)	Check whether a given label exists.
Array.iadd_prefactor_other(prefactor,	self += prefactor * other for scalar prefac-
other)	tor and Array other.
Array.ibinary_blockwise(func, other, *args,	Roughly self = func(self, other), block-
)	wise; in place.
Array.iconj([complex_conj])	Wraper around self.conj() with
7 7 7 7 (5 11 11 1 1)	inplace=True.
Array.idrop_labels([old_labels])	Remove leg labels from self; in place.
Array.iproject(mask, axes)	Applying masks to one or multiple axes; in place.
Array.ipurge_zeros([cutoff, norm_order])	Removes selfdata blocks with <i>norm</i> less than cutoff; in place.
Array.ireplace_label(old_label, new_label)	Replace the leg label <i>old_label</i> with <i>new_label</i> ; in place.
Array.ireplace_labels(old_labels,	Replace leg label old_labels[i] with
new_labels)	new_labels[i]; in place.
Array.is_completely_blocked()	Return bool whether all legs are blocked by charge.
Array.iscale_axis(s[, axis])	Scale with varying values along an axis; in place.
Array.iscale_prefactor(prefactor)	self *= prefactor for scalar prefactor.
Array.iset_leg_labels(labels)	Set labels for the different axes/legs; in place.
Array.isort_qdata()	(Lexiographically) sort selfqdata; in place.
Array.iswapaxes(axis1, axis2)	Similar as np. swapaxes; in place.
Array.itranspose([axes])	Transpose axes like <i>np.transpose</i> ; in place.
Array.iunary_blockwise(func, *args,	Roughly self = f(self), block-wise; in place.
**args, **kwargs)	Roughly Sett - 1 (Sett), block-wise, ill place.
Array.make_pipe(axes, **kwargs)	Generates a LegPipe for specified axes.
Array.matvec(other)	This function is used by the Lanczos algorithm needed
ALLAY . Macvec (outer)	for DMRG.
	continues on next page
	continues on next page

18.1. np_conserved 271

Table	3 –	continued	from	previous	nage
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Array.norm([ord, convert_to_float])	Norm of flattened data.		
Array.permute(perm, axis)	Apply a permutation in the indices of an axis.		
Array.replace_label(old_label, new_label)	Return a shallow copy with the leg label old_label re-		
	placed by new_label.		
Array.replace_labels(old_labels, new_labels)	Return a shallow copy with old_labels[i] re-		
	<pre>placed by new_labels[i].</pre>		
Array.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.		
Array.scale_axis(s[, axis])	Same as iscale_axis(), but return a (deep) copy.		
Array.sort_legcharge([sort, bunch])	Return a copy with one or all legs sorted by charges.		
Array.sparse_stats()	Returns a string detailing the sparse statistics.		
Array.split_legs([axes, cutoff])	Reshape: opposite of combine_legs: split (some) legs		
	which are LegPipes.		
Array.squeeze([axes])	Remove single-dimenisional legs, like np.		
	squeeze().		
Array.take_slice(indices, axes)	Return a copy of self fixing indices along one or multi-		
	ple axes.		
Array.test_sanity()	Sanity check.		
Array.to_ndarray()	Convert self to a dense numpy ndarray.		
Array.transpose([axes])	Like itranspose(), but on a deep copy.		
Array.unary_blockwise(func, *args, **kwargs)	Roughly return func(self), block-wise.		
Array.zeros_like()	Return a copy of self with only zeros as entries, contain-		
	ing no _data.		

Class Attributes and Properties

Array.labels	
Array.ndim	Alias for rank or len(self.shape).
Array.size	The number of dtype-objects stored.
Array.stored_blocks	The number of (non-zero) blocks stored in _data.

class tenpy.linalg.np_conserved.Array(legcharges, dtype=<class 'numpy.float64'>, qtotal=None, labels=None)

Bases: object

A multidimensional array (=tensor) for using charge conservation.

An *Array* represents a multi-dimensional tensor, together with the charge structure of its legs (for abelian charges). Further information can be found in *Charge conservation with np_conserved*.

The default __init__() (i.e. Array(...)) does not insert any data, and thus yields an Array 'full' of zeros, equivalent to zeros(). Further, new arrays can be created with one of from_ndarray_trivial(), from_ndarray(), or from_func(), and of course by copying/tensordot/svd etc.

In-place methods are indicated by a name starting with i. (But is_completely_blocked is not inplace...)

Parameters

- **legcharges** (list of *LegCharge*) The leg charges for each of the legs. The ChargeInfo is read out from it.
- dtype (type or string) The data type of the array entries. Defaults to np.float64.
- **qtotal** (1D array of QTYPE) The total charge of the array. Defaults to 0.

• labels (list of {str | None}) - Labels associated to each leg, None for non-named labels.

rank

The rank or "number of dimensions", equivalent to len (shape).

Type int

shape

The number of indices for each of the legs.

Type tuple(int)

dtype

The data type of the entries.

Type np.dtype

chinfo

The nature of the charge.

Type ChargeInfo

qtotal

The total charge of the tensor.

Type 1D array

legs

The leg charges for each of the legs.

Type list of LegCharge

_labels

Labels for the different legs, None for non-labeled legs.

Type list of { str | None }

data

The actual entries of the tensor.

Type list of arrays

_qdata

For each of the _data entries the qindices of the different legs.

Type 2D array (len(_data), rank), dtype np.intp

_qdata_sorted

Whether self. qdata is lexsorted. Defaults to *True*, but *must* be set to *False* by algorithms changing qdata.

Type Bool

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

copy (deep=True)

Return a (deep or shallow) copy of self.

Both deep and shallow copies will share chinfo and the LegCharges in legs.

In contrast to a deep copy, the shallow copy will also share the tensor entries, namely the *same* instances of gdata and data and labels (and other 'immutable' properties like the shape or dtype).

Note: Shallow copies are *not* recommended unless you know the consequences! See the following examples illustrating some of the pitfalls.

Examples

Be **very** careful when making non-deep copies: In the following example, the original *a* is changed if and only if the corresponding block existed in *a* before.

```
>>> a = npc.Array.from_ndarray_trivial(np.arange(6.).reshape(2, 3))
>>> print(a.to_ndarray())
[[0. 1. 2.]
[3. 4. 5.]]
>>> b = a.copy(deep=False) # shallow copy
>>> b[1, 2] = 8.
>>> a[1, 2] # changed!
8.0
```

Other *inplace* operations **might*** have no effect at all (although we don't guarantee that):

```
>>> a *= 2 # has no effect on `b`
>>> b.iconj() # nor does this change `a`
```

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5*().

Specifically, it saves <code>chinfo</code>, <code>legs</code>, <code>dtype</code> under these names, <code>qtotal</code> as "total_charge", <code>_data</code> as "blocks", <code>_qdata</code> as :block_inds", the labels in the list-form (as returned by <code>get_leg_labels()</code>). Moreover, it saves <code>rank</code>, <code>shape</code> and <code>_qdata_sorted</code> (under the name "block_inds_sorted") as HDF5 attributes.

Parameters

- hdf5 saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group `) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

classmethod from_ndarray_trivial (*data_flat*, *dtype=None*, *labels=None*) convert a flat numpy ndarray to an Array with trivial charge conservation.

Parameters

- **data_flat** (*array_like*) The data to be converted to a Array.
- dtype (np.dtype) The data type of the array entries. Defaults to dtype of data_flat.
- labels (list of {str | None}) Labels associated to each leg, None for nonnamed labels.

Returns res – An Array with data of data flat.

Return type Array

classmethod from_ndarray (data_flat, legcharges, dtype=None, qtotal=None, cutoff=None, labels=None, raise_wrong_sector=True)
convert a flat (numpy) ndarray to an Array.

Parameters

- data_flat (array_like) The flat ndarray which should be converted to a npc Array. The shape has to be compatible with legcharges.
- legcharges (list of LegCharge) The leg charges for each of the legs. The ChargeInfo is read out from it.
- **dtype** (np.dtype) The data type of the array entries. Defaults to dtype of *data_flat*.
- qtotal (None | charges) The total charge of the new array.
- **cutoff** (*float*) Blocks with np.max(np.abs(block)) > cutoff are considered as zero. Defaults to *QCUTOFF*.
- labels (list of {str | None}) Labels associated to each leg, None for nonnamed labels.
- raise_wrong_sector (bool) If True, raise a ValueError in case of non-zero entries (larger than *cutoff*) in the wrong blocks of *data_flat*. If *False*, just raise a warning.

Returns res – An Array with data of *data_flat*.

Return type Array

See also:

detect_qtotal used to detect qtotal if not given.

This function creates an array and fills the blocks *compatible* with the charges using *func*, where *func* is a function returning a *array_like* when given a shape, e.g. one of np.ones or np.random. standard normal.

Parameters

- func (callable) A function-like object which is called to generate the data blocks. We expect that func returns a flat array of the given shape convertible to dtype. If no shape_kw is given, it is called as func(shape, *func_args, **func_kwargs), otherwise as func(*func_args, `shape_kw`=shape, **func_kwargs). shape is a tuple of int.
- **legcharges** (list of LegCharge) The leg charges for each of the legs. The ChargeInfo is read out from it.

- **dtype** (None | type | string) The data type of the output entries. Defaults to np.float64. Defaults to None: obtain it from the return value of the function. Note that this argument is not given to func, but rather a type conversion is performed afterwards. You might want to set a dtype in func_kwargs as well.
- **qtotal** (None / charges) The total charge of the new array. Defaults to charge 0.
- func args (iterable) Additional arguments given to func.
- **func_kwargs** (dict) Additional keyword arguments given to *func*.
- **shape_kw** (*None | str*) If given, the keyword with which shape is given to *func*.
- labels (list of {str | None}) Labels associated to each leg, None for nonnamed labels.

Returns res – An Array with blocks filled using *func*.

Return type Array

 $\begin{tabular}{ll} \textbf{classmethod from_func_square} (func, & leg, & dtype=None, & func_args=(), & func_kwargs=\{\}, \\ & shape_kw=None, labels=None) \end{tabular}$

Create an Array from a (numpy) function.

This function creates an array and fills the blocks *compatible* with the charges using *func*, where *func* is a function returning a *array_like* when given a shape, e.g. one of np.ones or np.random.standard_normal or the functions defined in *random_matrix*.

Parameters

- func (callable) A function-like object which is called to generate the data blocks. We expect that func returns a flat array of the given shape convertible to dtype. If no shape_kw is given, it is called like func (shape, *fargs, **fkwargs), otherwise as func (*fargs, `shape_kw`=shape, **fkwargs). shape is a tuple of int.
- leg (LegCharge) The leg charges for the first leg; the second leg is set to leg. conj(). The ChargeInfo is read out from it.
- **dtype** (*None* / type / string) The data type of the output entries. Defaults to *None*: obtain it from the return value of the function. Note that this argument is not given to func, but rather a type conversion is performed afterwards. You might want to set a *dtype* in *func_kwargs* as well.
- **func_args** (*iterable*) Additional arguments given to *func*.
- **func_kwargs** (dict) Additional keyword arguments given to *func*.
- shape_kw (None | str) If given, the keyword with which shape is given to func.
- labels (list of {str | None}) Labels associated to each leg, None for nonnamed labels.

Returns res – An Array with blocks filled using *func*.

Return type Array

zeros_like()

Return a copy of self with only zeros as entries, containing no _data.

property size

The number of dtype-objects stored.

property stored_blocks

The number of (non-zero) blocks stored in _data.

```
property ndim
     Alias for rank or len (self.shape).
\texttt{get\_leg\_index}(label)
     translate a leg-index or leg-label to a leg-index.
         Parameters label (int | string) - The leg-index directly or a label (string) set before.
         Returns leg index – The index of the label.
         Return type int
     See also:
     get_leg_indices calls get_leg_index for a list of labels.
     iset_leg_labels set the labels of different legs.
get_leg_indices (labels)
     Translate a list of leg-indices or leg-labels to leg indices.
         Parameters labels (iterable of string/int) - The leg-labels (or directly indices)
             to be translated in leg-indices.
         Returns leg_indices – The translated labels.
         Return type list of int
     See also:
     get_leg_index used to translate each of the single entries.
     iset_leg_labels set the labels of different legs.
iset leg labels (labels)
     Set labels for the different axes/legs; in place.
     Introduction to leg labeling can be found in Charge conservation with np_conserved.
         Parameters labels (iterable (strings | None), len=self.rank) - One la-
             bel for each of the legs. An entry can be None for an anonymous leg.
     See also:
     get_leg translate the labels to indices.
get_leg_labels()
     Return list of the leg labels, with None for anonymous legs.
has label (label)
     Check whether a given label exists.
get_leg(label)
     Return self.legs[self.get_leg_index(label)].
     Convenient function returning the leg corresponding to a leg label/index.
ireplace_label (old_label, new_label)
     Replace the leg label old_label with new_label; in place.
replace_label (old_label, new_label)
     Return a shallow copy with the leg label old label replaced by new label.
```

```
ireplace_labels (old_labels, new_labels)
     Replace leg label old_labels[i] with new_labels[i]; in place.
replace_labels (old_labels, new_labels)
     Return a shallow copy with old_labels[i] replaced by new_labels[i].
idrop labels(old labels=None)
     Remove leg labels from self; in place.
         Parameters old labels (list of str/int) – The leg labels/indices for which the label
             should be removed. By default (None), remove all labels.
sparse_stats()
     Returns a string detailing the sparse statistics.
to_ndarray()
     Convert self to a dense numpy ndarray.
get_block (qindices, insert=False)
     Return the ndarray in _data representing the block corresponding to qindices.
         Parameters
             • qindices (1D array of np.intp) - The qindices, for which we need to look in
               _qdata.
             • insert (bool) - If True, insert a new (zero) block, if qindices is not existent in self.
               data. Otherwise just return None.
         Returns block – The block in _data corresponding to qindices. If insert`=False and there is
             not block with gindices, return ``None`.
         Return type ndarray | None
         Raises IndexError – If qindices are incompatible with charge and raise_incomp_q.
take_slice (indices, axes)
     Return a copy of self fixing indices along one or multiple axes.
     For a rank-4 Array A.take_slice([i, j], [1,2]) is equivalent to A[:, i, j, :].
         Parameters
             • indices ((iterable of) int) - The (flat) index for each of the legs specified by
               axes
             • axes ((iterable of) str/int) - Leg labels or indices to specify the legs for
               which the indices are given.
         Returns sliced self – A copy of self, equivalent to taking slices with indices inserted in axes.
         Return type Array
     See also:
     add_leg opposite action of inserting a new leg.
add_trivial_leg (axis=0, label=None, qconj=1)
```

• label (str | None) – If not None, use it as label for the new leg.

• axis (int) – The new leg is inserted before index axis.

Add a trivial leg (with just one entry) to self.

Parameters

• qconj(+1 / -1) – The direction of the new leg.

Returns extended – A (possibly) *shallow* copy of self with an additional leg of ind_len 1 and charge 0.

Return type Array

```
add_leg(leg, i, axis=0, label=None)
```

Add a leg to self, setting the current array as slice for a given index.

Parameters

- leg (LegCharge) The charge data of the leg to be added.
- i (int) Index within the leg for which the data of *self* should be set.
- **axis** (axis) The new leg is inserted before this current axis.
- label (str | None) If not None, use it as label for the new leg.

Returns extended — A copy of self with the new *leg* at axis axis, such that extended. take_slice(i, axis) returns a copy of self.

Return type Array

See also:

take_slice opposite action reducing the number of legs.

```
extend (axis, extra)
```

Increase the dimension of a given axis, filling the values with zeros.

Parameters

- axis (int / str) The axis (or axis-label) to be extended.
- **extra** (LegCharge | int) By what to extend, i.e. the charges to be appended to the leg of *axis*. An int stands for extending the length of the array by a single new block of that size with zero charges.

Returns extended – A copy of self with the specified axis increased.

Return type Array

```
gauge_total_charge (axis, newqtotal=None, new_qconj=None)
```

Changes the total charge by adjusting the charge on a certain leg.

The total charge is given by finding a nonzero entry $[i1,i2,\ldots]$ and calculating:

Thus, the total charge can be changed by redefining (= shifting) the LegCharge of a single given leg. This is exactly what this function does.

Parameters

- axis (int or string) The new leg (index or label), for which the charge is changed.
- newqtotal (charge values, defaults to 0) The new total charge.

• new_qconj ({+1, -1, None}) - Whether the new LegCharge points inward (+1) or outward (-1) afterwards. By default (None) use the previous self.legs[leg]. qconj.

Returns copy - A shallow copy of self with copy.qtotal == newqtotal and new copy.legs[leg]. The new leg will be a :class`LegCharge`, even if the old leg was a LegPipe.

Return type Array

add_charge (add_legs, chinfo=None, qtotal=None)
Add charges.

Parameters

- add_legs (iterable of LegCharge) One *LegCharge* for each axis of *self*, to be added to the one in *legs*.
- chargeinfo (ChargeInfo) The ChargeInfo for all charges; create new if None.
- qtotal (None | charges) The total charge with respect to add_legs. If None, derive it from non-zero entries of self.

Returns charges_added – A copy of *self*, where the LegCharges *add_legs* where added to *self.legs*. Note that the LegCharges are neither bunched or sorted; you might want to use <code>sort_legcharge()</code>.

Return type Array

drop_charge (charge=None, chinfo=None)

Drop (one of) the charges.

Parameters

- **charge** (*int* / *str*) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.
- chinfo (ChargeInfo) The ChargeInfo with charge dropped; create a new one if None.

Returns dropped – A copy of *self*, where the specified *charge* has been removed. Note that the LegCharges are neither bunched or sorted; you might want to use <code>sort_legcharge()</code>.

Return type Array

change_charge (charge, new_qmod, new_name=", chinfo=None)

Change the *qmod* of one charge in *chinfo*.

Parameters

- **charge** (int / str) Number or *name* of the charge (within *chinfo*) which is to be changed. None means dropping all charges.
- new_qmod (int) The new qmod to be set.
- **new_name** (str) The new name of the charge.
- **chinfo** (ChargeInfo) The ChargeInfo with *qmod* of *charge* **changed**; **create** a new one if None.

Returns changed – A copy of *self*, where the *qmod* of the specified *charge* has been changed. Note that the LegCharges are neither bunched or sorted; you might want to use <code>sort_legcharge()</code>.

Return type Array

is_completely_blocked()

Return bool whether all legs are blocked by charge.

sort_legcharge (sort=True, bunch=True)

Return a copy with one or all legs sorted by charges.

Sort/bunch one or multiple of the LegCharges. Legs which are sorted *and* bunched are guaranteed to be blocked by charge.

Parameters

- **sort** (*True* | *False* | *list* of {*True*, *False*, *perm*}) A single bool holds for all legs, default=True. Else, *sort* should contain one entry for each leg, with a bool for sort/don't sort, or a 1D array perm for a given permuation to apply to a leg.
- bunch (True | False | list of {True, False}) A single bool holds for all legs, default=True. Whether or not to bunch at each leg, i.e. combine contiguous blocks with equal charges.

Returns

- **perm** (*tuple of 1D arrays*) The permutation applied to each of the legs, such that cp. to_ndarray() = self.to_ndarray() [np.ix_(*perm)].
- result (Array) A shallow copy of self, with legs sorted/bunched.

isort_qdata()

(Lexiographically) sort self._qdata; in place.

Lexsort self._qdata and self._data and set self._qdata_sorted = True.

make_pipe (axes, **kwargs)

Generates a LegPipe for specified axes.

Parameters

- axes (iterable of str/int) The leg labels for the axes which should be combined. Order matters!
- **kwargs Additional keyword arguments given to LegPipe.

Returns pipe – A pipe of the legs specified by axes.

Return type LegPipe

 $\verb|combine_legs| (combine_legs, new_axes=None, pipes=None, qconj=None)|$

Reshape: combine multiple legs into multiple pipes. If necessary, transpose before.

Parameters

- **combine_legs**((iterable of) iterable of {str/int})-Bundles of leg indices or labels, which should be combined into a new output pipes. If multiple pipes should be created, use a list fore each new pipe.
- new_axes (None | (iterable of) int) The leg-indices, at which the combined legs should appear in the resulting array. Default: for each pipe the position of its first pipe in the original array, (taking into account that some axes are 'removed' by combining). Thus no transposition is performed if combine_legs contains only contiguous ranges.
- pipes (None | (iterable of) {LegPipes | None}) Optional: provide one or multiple of the resulting LegPipes to avoid overhead of computing new leg pipes for the same legs multiple times. The LegPipes are conjugated, if that is necessary for compatibility with the legs.

• **qconj** $((iterable \ of) \ \{+1, -1\})$ - Specify whether new created pipes point inward or outward. Defaults to +1. Ignored for given *pipes*, which are not newly calculated.

Returns reshaped – A copy of self, whith some legs combined into pipes as specified by the arguments.

```
Return type Array
```

See also:

split_legs inverse reshaping splitting LegPipes.

Notes

Labels are inherited from self. New pipe labels are generated as '(' + '.'.join(*leglabels) + ')'. For these new labels, previously unlabeled legs are replaced by '?#', where # is the leg-index in the original tensor *self*.

Examples

Indices of *combine_legs* refer to the original array. If transposing is necessary, it is performed automatically:

split_legs (axes=None, cutoff=0.0)

Reshape: opposite of combine_legs: split (some) legs which are LegPipes.

Reverts <code>combine_legs()</code> (except a possibly performed <code>transpose</code>). The splited legs are replacing the LegPipes at their position, see the examples below. Labels are split reverting what was done in <code>combine_legs()</code>. '?#' labels are replaced with <code>None</code>.

Parameters

• axes ((iterable of) int/str) - Leg labels or indices determining the axes to split. The corresponding entries in self.legs must be LegPipe instances. Defaults to all legs, which are LegPipe instances.

• **cutoff** (*float*) - Splitted data blocks with np.max(np.abs(block)) > cutoff are considered as zero. Defaults to 0.

Returns reshaped – A copy of self where the specified legs are splitted.

```
Return type Array
```

See also:

combine legs this is reversed by split legs.

Examples

Given a rank-5 Array *orig_array*, you can combine it and split it again:

```
>>> orig_array = npc.Array.from_ndarray_trivial(np.arange(60).reshape([2, 3,...
\leftrightarrow 2, 1, 5]),
                                                labels=['a', 'b', 'c', 'd', 'e
>>> orig_array.shape
(2, 3, 2, 1, 5)
>>> comb_array = orig_array.combine_legs([['a', 'd'], ['c', 'e']] )
>>> comb_array.get_leg_labels()
['(a.d)', 'b', '(c.e)']
>>> comb_array.shape
(2, 3, 10)
>>> split_array = comb_array.split_legs()
>>> split_array.get_leg_labels()
['a', 'd', 'b', 'c', 'e']
>>> npc.norm(split_array.transpose(orig_array.get_leg_labels()) - orig_array)
0.0
```

as_completely_blocked()

Gives a version of self which is completely blocked by charges.

Functions like svd() or eigh() require a complete blocking by charges. This can be achieved by encapsulating each leg which is not completely blocked into a LegPipe (containing only that single leg). The LegPipe will then contain all necessary information to revert the blocking.

Returns

- encapsulated_axes (list of int) The leg indices which have been encapsulated into Pipes.
- blocked_self (Array) Self (if len (encapsulated_axes) = 0) or a copy of self, which is completely blocked.

squeeze(axes=None)

Remove single-dimenisional legs, like np.squeeze().

If a squeezed leg has non-zero charge, this charge is added to qtotal.

Parameters axes (None | (iterable of) {int|str}) - Labels or indices of the legs which should be 'squeezed', i.e. the legs removed. The corresponding legs must be trivial, i.e., have ind_len 1.

Returns squeezed – A scalar of self.dtype, if all axes were squeezed. Else a copy of self with reduced rank as specified by *axes*.

Return type :class:Array | scalar

```
astype (dtype, copy=True)
```

Return copy with new dtype, upcasting all blocks in _data.

Parameters

- **dtype** (convertible to a np.dtype) The new data type. If None, deduce the new dtype as common type of self. data.
- **copy** (bool) Whether to make a copy of the blocks even if the type didn't change.

Returns copy – Deep copy of self with new dtype.

Return type Array

ipurge_zeros (*cutoff=2.220446049250313e-15*, *norm_order=None*)

Removes self._data blocks with *norm* less than cutoff; in place.

Parameters

- **cutoff** (float) Blocks with norm <= cutoff are removed. defaults to QCUTOFF.
- norm_order A valid *ord* argument for *np.linalg.norm*. Default None gives the Frobenius norm/2-norm for matrices/everything else. Note that this differs from other methods, e.g. *from_ndarray()*, which use the maximum norm.

iproject (mask, axes)

Applying masks to one or multiple axes; in place.

This function is similar as *np.compress* with boolean arrays For each specified axis, a boolean 1D array *mask* can be given, which chooses the indices to keep.

Warning: Although it is possible to use an 1D int array as a mask, the order is ignored! If you need to permute an axis, use permute () or sort_legcharge().

Parameters

- mask ((list of) 1D array (bool | int)) For each axis specified by axes a mask, which indices of the axes should be kept. If mask is a bool array, keep the indices where mask is True. If mask is an int array, keep the indices listed in the mask, ignoring the order or multiplicity.
- axes ((list of) int | string) The i'th entry in this list specifies the axis for the 'i'th entry of 'mask, either as an int, or with a leg label. If axes is just a single int/string, specify just a single mask.

Returns

- map_qind (list of 1D arrays) The mapping of qindices for each of the specified axes.
- block_masks (list of lists of 1D bool arrays) block_masks[a] [qind] is a boolen mask which indices to keep in block qindex of axes[a].

permute (perm, axis)

Apply a permutation in the indices of an axis.

Similar as np.take with a 1D array. Roughly equivalent to res[:, ...] = self[perm, ...] for the corresponding *axis*. Note: This function is quite slow, and usually not needed!

Parameters

• **perm** (array_like 1D int) - The permutation which should be applied to the leg given by axis.

• axis (str / int) - A leg label or index specifying on which leg to take the permutation.

Returns res – A copy of self with leg *axis* permuted, such that res[i, ...] = self[perm[i], ...] for i along *axis*.

Return type *Array*

See also:

sort_legcharge can also be used to perform a general permutation. Preferable, since it is faster for permutations which don't mix charge blocks.

itranspose (axes=None)

Transpose axes like *np.transpose*; in place.

Parameters axes (iterable (intlstring), len rank | None) – The new order of the axes. By default (None), reverse axes.

transpose (axes=None)

Like *itranspose* (), but on a deep copy.

iswapaxes (axis1, axis2)

Similar as np. swapaxes; in place.

iscale_axis(s, axis=-1)

Scale with varying values along an axis; in place.

```
Rescale to new_self[i1, ..., i_axis, ...] = s[i_axis] * self[i1, ..., i_axis, ...]
```

Parameters

- **s** (1D array, len=self.shape[axis]) The vector with which the axis should be scaled.
- axis (str/int) The leg label or index for the axis which should be scaled.

See also:

iproject can be used to discard indices for which s is zero.

scale_axis(s, axis=-1)

Same as <code>iscale_axis()</code>, but return a (deep) copy.

iunary_blockwise(func, *args, **kwargs)

Roughly self = f(self), block-wise; in place.

Applies an unary function *func* to the non-zero blocks in self._data.

Note: Assumes implicitly that func (np.zeros(...), *args, **kwargs) gives 0, since we don't let func act on zero blocks!

Parameters

- func (function) A function acting on flat arrays, returning flat arrays. It is called like new_block = func(block, *args, **kwargs).
- *args Additional arguments given to function *after* the block.
- ****kwargs** Keyword arguments given to the function.

Examples

```
>>> a = npc.Array.from_ndarray_trivial([1., 2.j])
>>> a.iunary_blockwise(np.conj).to_ndarray() # same data as a.iconj(), but_

doesn't charge conjugate.
array([1.-0.j, 0.-2.j])
>>> a.iunary_blockwise(np.real).to_ndarray() # get real part
array([1., 0.])
```

```
unary_blockwise (func, *args, **kwargs)
```

Roughly return func(self), block-wise. Copies.

Same as *iunary_blockwise()*, but makes a **shallow** copy first.

```
iconj(complex_conj=True)
```

Wraper around self.conj() with inplace=True.

```
conj (complex_conj=True, inplace=False)
```

Conjugate: complex conjugate data, conjugate charge data.

Conjugate all legs, set negative qtotal.

```
Labeling: takes 'a' -> 'a*', 'a*'-> 'a' and '(a,(b*,c))' -> '(a*,(b,c*))'
```

Parameters

- **complex_conj** (bool) Whether the data should be complex conjugated.
- **inplace** (bool) Whether to apply changes to *self*, or to return a *deep* copy.

```
complex_conj()
```

Return copy which is complex conjugated without conjugating the charge data.

```
norm (ord=None, convert_to_float=True)
```

Norm of flattened data.

See norm() for details.

```
ibinary_blockwise (func, other, *args, **kwargs)
```

```
Roughly self = func(self, other), block-wise; in place.
```

Applies a binary function 'block-wise' to the non-zero blocks of self._data and other._data, storing result in place. Assumes that *other* is an <code>Array</code> as well, with the same shape and compatible legs. If leg labels of *other* and *self* are same up to permutations, *other* gets transposed accordingly before the action.

Note: Assumes implicitly that func(np.zeros(...), np.zeros(...), *args, **kwargs) gives 0, since we don't let *func* act on zero blocks!

Parameters

- **func** (function) Binary function, called as new_block = func(block_self, block_other, *args, **kwargs) for blocks (=Numpy arrays) of equal shape.
- other (*Array*) Other Array from which to take blocks. Should have the same leg structure as self.
- *args Extra arguments given to func.

• ****kwargs** – Extra keyword arguments given to *func*.

Examples

```
>>> a = npc.Array.from_ndarray_trivial([1., 3.])
>>> b = npc.Array.from_ndarray_trivial([4., 2.])
>>> a.ibinary_blockwise(np.maximum, b).to_ndarray() # a = max(a, b)
array([4., 3.])
>>> a.ibinary_blockwise(np.add, b).to_ndarray() # roughly ``a += b``
array([8., 5.])
```

binary_blockwise (func, other, *args, **kwargs)

Roughly return func (self, other), block-wise. Copies.

Same as ibinary_blockwise(), but makes a shallow copy first.

matvec(other)

This function is used by the Lanczos algorithm needed for DMRG.

It is supposed to calculate the matrix - vector - product for a rank-2 matrix self and a rank-1 vector other.

iadd_prefactor_other (prefactor, other)

```
self += prefactor * other for scalar prefactor and Array other.
```

Note that we allow the type of *self* to change if necessary. Moreover, if *self* and *other* have the same labels in different order, other gets **transposed** before the action.

iscale_prefactor (prefactor)

```
self *= prefactor for scalar prefactor.
```

Note that we allow the type of *self* to change if necessary.

Functions

concatenate(arrays[, axis, copy])	Stack arrays along a given axis, similar as
	np.concatenate.
detect_grid_outer_legcharge(grid,	Derive a LegCharge for a grid used for
grid_legs)	grid_outer().
detect_legcharge(flat_array, chargeinfo,)	Calculate a missing <i>LegCharge</i> by looking for nonzero
	entries of a flat array.
detect_qtotal(flat_array, legcharges[, cutoff])	Returns the total charge (w.r.t legs) of first non-zero sec-
	tor found in <i>flat_array</i> .
diag(s, leg[, dtype, labels])	Returns a square, diagonal matrix of entries s.
eig(a[, sort])	Calculate eigenvalues and eigenvectors for a non-
	hermitian matrix.
eigh(a[, UPLO, sort])	Calculate eigenvalues and eigenvectors for a hermitian
	matrix.
eigvals(a[, sort])	Calculate eigenvalues for a hermitian matrix.
eigvalsh(a[, UPLO, sort])	Calculate eigenvalues for a hermitian matrix.
expm(a)	Use scipy.linalg.expm to calculate the matrix exponen-
	tial of a square matrix.
eye_like(a[, axis, labels])	Return an identity matrix contractible with the leg axis
	of the Array a.
	continues on next page

18.1. np_conserved 287

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<pre>grid_concat(grid, axes[, copy])</pre>	Given an np.array of npc.Arrays, performs a multi-
	dimensional concatentation along 'axes'.
<pre>grid_outer(grid, grid_legs[, qtotal,])</pre>	Given an np.array of npc.Arrays, return the correspond-
	ing higher-dimensional Array.
inner(a, b[, axes, do_conj])	Contract all legs in a and b, return scalar.
norm(a[, ord, convert_to_float])	Norm of flattened data.
ones(legcharges[, dtype, qtotal, labels])	Short-hand for Array.from_func() with function
	numpy.ones().
outer(a, b)	Forms the outer tensor product, equivalent to
	tensordot(a, b, axes=0).
pinv(a[, cutoff])	Compute the (Moore-Penrose) pseudo-inverse of a ma-
	trix.
qr(a[, mode, inner_labels, cutoff])	Q-R decomposition of a matrix.
speigs(a, charge_sector, k, *args, **kwargs)	Sparse eigenvalue decomposition w , v of square a in a
	given charge sector.
svd(a[, full_matrices, compute_uv, cutoff,])	Singualar value decomposition of an Array a.
tensordot(a, b[, axes])	Similar as np.tensordot but for Array.
to_iterable_arrays(array_list)	Similar as to_iterable(), but also enclose npc Ar-
	rays in a list.
trace(a[, leg1, leg2])	Trace of a, summing over leg1 and leg2.
zeros(legcharges[, dtype, qtotal, labels])	Create a npc array full of zeros (with no _data).

18.1.2 concatenate

- full name: tenpy.linalg.np_conserved.concatenate
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.concatenate(arrays, axis=0, copy=True)

Stack arrays along a given axis, similar as np.concatenate.

Stacks the qind of the array, without sorting/blocking. Labels are inherited from the first array only.

Parameters

- **arrays** (iterable of Array) The arrays to be stacked. They must have the same shape and charge data except on the specified axis.
- **axis** (*int* / *str*) Leg index or label of the first array. Defines the axis along which the arrays are stacked.
- **copy** (bool) Whether to copy the data blocks.

Returns stacked – Concatenation of the given *arrays* along the specified axis.

Return type Array

See also:

Array.sort_legcharge can be used to block by charges along the axis.

18.1.3 detect grid outer legcharge

- full name: tenpy.linalg.np_conserved.detect_grid_outer_legcharge
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.detect_grid_outer_legcharge(grid, grid_legs, qtotal=None, qconj=1, bunch=False)

Derive a LegCharge for a grid used for grid_outer().

Note: The resulting LegCharge is not bunched.

Parameters

- grid (array_like of {Array | None}) The grid as it will be given to grid_outer().
- grid_legs (list of {LegCharge | None}) One LegCharge for each dimension of the grid, except for one entry which is None. This missing entry is to be calculated.
- **qtotal** (*charge*) The desired total charge of the array. Defaults to 0.

Returns new_grid_legs – A copy of the given *grid_legs* with the None replaced by a compatible LegCharge. The new LegCharge is neither bunched nor sorted!

Return type list of LegCharge

See also:

detect_legcharge similar functionality for a flat numpy array instead of a grid.

18.1.4 detect_legcharge

- full name: tenpy.linalg.np_conserved.detect_legcharge
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.detect_legcharge (flat_array, chargeinfo, legcharges, qto tal=None, qconj=1, cutoff=None)

Calculate a missing LegCharge by looking for nonzero entries of a flat array.

Parameters

- flat_array (ndarray) A flat array, in which we look for non-zero entries.
- chargeinfo (ChargeInfo) The nature of the charge.
- **legcharges** (list of LegCharge) One LegCharge for each dimension of flat_array, except for one entry which is None. This missing entry is to be calculated.
- $qconj(\{+1, -1\}) qconj$ for the new calculated LegCharge.
- **qtotal** (*charges*) Desired total charge of the array. Defaults to zeros.
- **cutoff** (*float*) **Blocks** with np.max (np.abs(block)) > cutoff are considered as zero. Defaults to *QCUTOFF*.

Returns new_legcharges – A copy of the given *legcharges* with the None replaced by a compatible LegCharge. The new legcharge is 'bunched', but not sorted!

Return type list of LegCharge

See also:

```
detect_grid_outer_legcharge similar functionality if the flat array is given by a 'grid'.
detect_qtotal detects the total charge, if all legs are known.
```

18.1.5 detect_qtotal

- full name: tenpy.linalg.np_conserved.detect_qtotal
- parent module: tenpy.linalg.np_conserved
- · type: function

```
tenpy.linalg.np_conserved.detect_qtotal (flat_array, legcharges, cutoff=None)

Returns the total charge (w.r.t legs) of first non-zero sector found in flat_array.
```

Parameters

- **flat_array** (*array*) The flat numpy array from which you want to detect the charges.
- legcharges (list of LegCharge) For each leg the LegCharge.
- **cutoff** (*float*) **Blocks** with np.max (np.abs(block)) > cutoff are considered as zero. Defaults to *QCUTOFF*.

Returns qtotal – The total charge fo the first non-zero (i.e. > cutoff) charge block.

Return type charge

See also:

```
detect_legcharge detects the charges of one missing LegCharge if qtotal is known.
detect_grid_outer_legcharge similar functionality if the flat array is given by a 'grid'.
```

18.1.6 diag

- full name: tenpy.linalg.np_conserved.diag
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.diag(s, leg, dtype=None, labels=None)
Returns a square, diagonal matrix of entries s.
```

The resulting matrix has legs (leg, leg.conj()) and charge 0.

Parameters

- **s** (*scalar* / 1D *array*) The entries to put on the diagonal. If scalar, all diagonal entries are the same.
- leg (LegCharge) The first leg of the resulting matrix.
- **dtype** (*None* / *type*) The data type to be used for the result. By default, use dtype of *s*.
- labels (list of {str | None}) Labels associated to each leg, None for non-named labels.

Returns diagonal – A square matrix with diagonal entries s.

Return type Array

See also:

Array.scale_axis similar as tensordot (diag(s), ...), but faster.

18.1.7 eig

- full name: tenpy.linalg.np_conserved.eig
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.eig(a, sort=None)
```

Calculate eigenvalues and eigenvectors for a non-hermitian matrix.

```
W, V = eig(a) yields aV = V diag(w).
```

Parameters

- a (Array) The hermitian square matrix to be diagonalized.
- **sort** ({'m>', 'm<', '>', '<', None}) How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

Returns

- W (1D ndarray) The eigenvalues, sorted within the same charge blocks according to sort.
- **V** (*Array*) Unitary matrix; V[:, i] is normalized eigenvector with eigenvalue W[i]. The first label is inherited from *A*, the second label is 'eig'.

Notes

Requires the legs to be contractible. If a is not blocked by charge, a blocked copy is made via a permutation P, $a' = PaP^{-1} = V'W'(V')^{\dagger}$. The eigenvectors V are then obtained by the reverse permutation, $V = P^{-1}V'$ such that $a = VWV^{\dagger}$.

18.1.8 eigh

- full name: tenpy.linalg.np_conserved.eigh
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.eigh(a, UPLO='L', sort=None)
```

Calculate eigenvalues and eigenvectors for a hermitian matrix.

W, V = eigh(a) yields $a = V diag(w)V^{\dagger}$. Assumes that a is hermitian, a.conj().transpose() == a.

Parameters

- **a** (Array) The hermitian square matrix to be diagonalized.
- **UPLO** ({ 'L', 'U'}) Whether to take the lower ('L', default) or upper ('U') triangular part of *a*.

• **sort** ({'m>', 'm<', '>', '<', None}) – How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

Returns

- W (1D ndarray) The eigenvalues, sorted within the same charge blocks according to sort.
- **V** (*Array*) Unitary matrix; V[:, i] is normalized eigenvector with eigenvalue W[i]. The first label is inherited from *A*, the second label is 'eig'.

Notes

Requires the legs to be contractible. If a is not blocked by charge, a blocked copy is made via a permutation P, $a' = PaP^{-1} = V'W'(V')^{\dagger}$. The eigenvectors V are then obtained by the reverse permutation, $V = P^{-1}V'$ such that $a = VWV^{\dagger}$.

18.1.9 eigvals

- full name: tenpy.linalg.np_conserved.eigvals
- parent module: tenpy.linalg.np_conserved
- · type: function

```
tenpy.linalg.np_conserved.eigvals(a, sort=None)
```

Calculate eigenvalues for a hermitian matrix.

Parameters

- **a** (Array) The hermitian square matrix to be diagonalized.
- **sort** ({'m>', 'm<', '>', '<', None}) How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

Returns W – The eigenvalues, sorted within the same charge blocks according to *sort*.

Return type 1D ndarray

Notes

The eigenvalues are sorted within blocks of the completely blocked legs.

18.1.10 eigvalsh

- full name: tenpy.linalg.np_conserved.eigvalsh
- $\bullet \ \ parent \ module: \ \textit{tenpy.linalg.np_conserved}$
- type: function

```
tenpy.linalg.np_conserved.eigvalsh (a, UPLO='L', sort=None)
Calculate eigenvalues for a hermitian matrix.
```

Assumes that a is hermitian, a.conj().transpose() == a.

Parameters

• **a** (Array) – The hermitian square matrix to be diagonalized.

- **UPLO** ({ 'L', 'U'}) Whether to take the lower ('L', default) or upper ('U') triangular part of *a*.
- **sort** ({'m>', 'm<', '>', '<', None}) How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

Returns W – The eigenvalues, sorted within the same charge blocks according to *sort*.

Return type 1D ndarray

Notes

The eigenvalues are sorted within blocks of the completely blocked legs.

18.1.11 expm

- full name: tenpy.linalg.np_conserved.expm
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.expm(a)
```

Use scipy.linalg.expm to calculate the matrix exponential of a square matrix.

Parameters a (Array) – A square matrix to be exponentiated.

Returns \exp_a – The matrix exponential \exp_a (a), calculated using scipy.linalg.expm. Same legs/labels as a.

Return type Array

18.1.12 eye like

- full name: tenpy.linalg.np_conserved.eye_like
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.eye_like(a, axis=0, labels=None)
```

Return an identity matrix contractible with the leg axis of the Array a.

18.1.13 grid_concat

- full name: tenpy.linalg.np_conserved.grid_concat
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.grid_concat(grid, axes, copy=True)
```

Given an np.array of npc.Arrays, performs a multi-dimensional concatentation along 'axes'.

Similar to numpy.block(), but only for uniform blocking.

Stacks the qind of the array, without sorting/blocking.

Parameters

- grid (array_like of Array) The grid of arrays.
- **axes** (list of int) The axes along which to concatenate the arrays, same len as the dimension of the grid. Concatenate arrays of the *i*'th axis of the grid along the axis ``axes[i]`
- copy (bool) Whether the _data blocks are copied.

Examples

Assume we have prepared rank 2 Arrays A, B, C, D sharing the legs of equal sizes and looking like this:

```
>>> print(A.to_ndarray())
[[0 1]]
>>> print(B.to_ndarray())
[[10 11 12 13]]
>>> print(C.to_ndarray())
[[20 21]
[22 23]
[24 25]]
>>> print(D.to_ndarray())
[[30 31 32 33]
[34 35 36 37]
[38 39 40 41]]
```

Then the following grid will result in a (1+3, 2+4) shaped array:

If A, B, C, D were rank 4 arrays, with the first and last leg as before, and sharing *common* legs 1 and 2 of dimensions 1, 2, then you would get a rank-4 array:

```
>>> g = grid_concat([[A, B], [C, D]], axes=[0, 3])
>>> g.shape
(4, 1, 2, 6)
```

See also:

Array.sort_legcharge can be used to block by charges.

18.1.14 grid_outer

- full name: tenpy.linalg.np_conserved.grid_outer
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.grid_outer (grid, grid_legs, qtotal=None, grid_labels=None) Given an np.array of npc.Arrays, return the corresponding higher-dimensional Array.

Parameters

- **grid** (array_like of {Array | None}) The grid gives the first part of the axes of the resulting array. Entries have to have all the same shape and charge-data, giving the remaining axes. None entries in the grid are interpreted as zeros.
- grid_legs (list of LegCharge) One LegCharge for each dimension of the grid along the grid.
- **qtotal** (*charge*) The total charge of the Array. By default (None), derive it out from a non-trivial entry of the grid.
- grid_labels (list of {str | None}) One label associated to each of the grid axes. None for non-named labels.

Returns res — An Array with shape grid.shape + nontrivial_grid_entry.shape. Constructed such that res[idx] == grid[idx] for any index idx of the *grid* the *grid* entry is not trivial (None).

Return type Array

See also:

detect_grid_outer_legcharge can calculate one missing LegCharge of the grid.

Examples

A typical use-case for this function is the generation of an MPO. Say you have npc.Arrays Splus, Sminus, Sz, Id, each with legs [phys.conj(), phys]. Further, you have to define appropriate LegCharges *l_left* and *l_right*. Then one 'matrix' of the MPO for a nearest neighbour Heisenberg Hamiltonian could look like:

```
>>> s = tenpy.networks.site.SpinHalfSite(conserve='Sz')
>>> Id, Splus, Sminus, Sz = s.Id, s.Sp, s.Sm, s.Sz
>>> J = 1.
>>> leg_wR = npc.LegCharge.from_qflat(s.leg.chinfo,
                                       [op.qtotal for op in [Id, Splus, Sminus, Sz,
. . .

→ Id]],
. . .
                                      qconj=-1)
>>> W_mpo = npc.grid_outer([[Id, Splus, Sminus, Sz, None],
                            [None, None, None, J*0.5*Sminus],
                            [None, None, None, J*0.5*Splus],
. . .
                            [None, None, None, J*Sz],
. . .
                            [None, None, None, Id]],
. . .
                           grid_legs=[leg_wR.conj(), leg_wR],
. . .
                           grid_labels=['wL', 'wR'])
. . .
>>> W_mpo.shape
(5, 5, 2, 2)
>>> W_mpo.get_leg_labels()
['wL', 'wR', 'p', 'p*']
```

18.1.15 inner

• full name: tenpy.linalg.np conserved.inner

• parent module: tenpy.linalg.np conserved

• type: function

tenpy.linalg.np_conserved.inner(a, b, axes=None, do_conj=False)
Contract all legs in a and b, return scalar.

Parameters

- **a** (class:*Array*) The arrays for which to calculate the product. Must have same rank, and compatible LegCharges.
- **b** (class:*Array*) The arrays for which to calculate the product. Must have same rank, and compatible LegCharges.
- axes ((axes_a, axes_b) | 'range', 'labels') axes_a and axes_b specifiy the legs of a and b, respectively, which should be contracted. Legs can be specified with leg labels or indices. We contract leg axes_a[i] of a with leg axes_b[i] of b. The default axes='range' is equivalent to (range(rank), range(rank)).axes='labels' is equivalent to either (a.get_leg_labels(), a.get_leg_labels()) for do_conj=True, or to (a.get_leg_labels(), conj_labels(a.get_leg_labels())) for do_conj=False. In other words, axes='labels' requires a and b to have the same/conjugated labels up to a possible transposition, which is then reverted.
- do_conj (bool) If False (Default), ignore it. If True, conjugate a before, i.e., return inner(a.conj(), b, axes).

Returns inner_product – A scalar (of common dtype of a and b) giving the full contraction of a and b.

Return type dtype

18.1.16 norm

• full name: tenpy.linalg.np conserved.norm

 $\bullet \ \ parent \ module: \ \textit{tenpy.linalg.np_conserved}$

• type: function

tenpy.linalg.np_conserved.norm(a, ord=None, convert_to_float=True)
Norm of flattened data.

Equivalent to np.linalg.norm(a.to_ndarray().flatten(), ord).

In contrast to numpy, we don't distinguish between matrices and vectors, but simply calculate the norm for the **flat** (block) data. The usual *ord*-norm is defined as $(\sum_i |a_i|^{ord})^{1/ord}$.

ord	norm
None/'fro'	Frobenius norm (same as 2-norm)
np.inf	max(abs(x))
-np.inf	min(abs(x))
0	<pre>sum(a != 0) == np.count_nonzero(x)</pre>
other	ususal <i>ord</i> -norm

Parameters

- a (Array | np.ndarray) The array of which the norm should be calculated.
- ord The order of the norm. See table above.
- **convert_to_float** Convert integer to float before calculating the norm, avoiding int overflow.

Returns norm – The norm over the *flat* data of the array.

Return type float

18.1.17 ones

- full name: tenpy.linalg.np_conserved.ones
- parent module: tenpy.linalg.np_conserved
- · type: function

tenpy.linalg.np_conserved.ones (legcharges, dtype=<class 'numpy.float64'>, qtotal=None, labels=None) Short-hand for $Array.from_func()$ with function numpy.ones().

Warning: For non-trivial charges, only blocks with compatible charges are filled with ones!

18.1.18 outer

- full name: tenpy.linalg.np_conserved.outer
- parent module: tenpy.linalg.np_conserved
- · type: function

```
tenpy.linalg.np_conserved.outer (a, b)
```

Forms the outer tensor product, equivalent to tensordot(a, b, axes=0).

Labels are inherited from a and b. In case of a collision (same label in both a and b), they are both dropped.

Parameters

- **a** (Array) The arrays for which to form the product.
- **b** (Array) The arrays for which to form the product.

Returns

c –

Array of rank a.rank + b.rank such that (for Ra = a.rank; Rb = b.rank):

```
c[i_1, ..., i_Ra, j_1, ... j_R] = a[i_1, ..., i_Ra] * b[j_1, ..., j_ \rightarrowrank_b]
```

Return type Array

18.1.19 pinv

- full name: tenpy.linalg.np_conserved.pinv
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.pinv(a, cutoff=1e-15)

Compute the (Moore-Penrose) pseudo-inverse of a matrix.

Equivalent to the following procedure: Perform a SVD, U, S, VH = svd(a, cutoff=cutoff) with a *cutoff* > 0, calculate P = U * diag(1/S) * VH (with * denoting tensordot) and return P.conj. transpose().

Parameters

- **a** ((M, N) Array) Matrix to be pseudo-inverted.
- **cuttof** (*float*) Cutoff for small singular values, as given to *svd*(). (Note: different convetion than numpy.)

Returns B – The pseudo-inverse of a.

Return type (N, M) Array

18.1.20 qr

- full name: tenpy.linalg.np_conserved.qr
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.**qr**(*a*, *mode='reduced'*, *inner_labels=[None*, *None]*, *cutoff=None*) Q-R decomposition of a matrix.

Decomposition such that A == npc.tensordot(q, r, axes=1) up to numerical rounding errors.

Parameters

- a (Array) A square matrix to be exponentiated, shape (M, N).
- mode ('reduced', 'complete') 'reduced': return q and r with shapes (M,K) and (K,N), where K=min(M,N) 'complete': return q with shape (M,M).
- inner_labels ([{str/None}, {str/None}]) The first label is used for Q. legs[1], the second for R.legs[0].
- **cutoff** (None or float) If not None, discard linearly dependent vectors to given precision, which might reduce *K* of the 'reduced' mode even further.

Returns

• \mathbf{q} (Array) – If mode is 'complete', a unitary matrix. For mode 'reduced' such that Otherwise such that $q_{i,i}^*q_{j,k}=\delta_{i,k}$

• **r** (*Array*) – Upper triangular matrix if both legs of A are sorted by charges; Otherwise a simple transposition (performed when sorting by charges) brings it to upper triangular form.

18.1.21 speigs

- full name: tenpy.linalg.np_conserved.speigs
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.speigs (a, charge_sector, k, *args, **kwargs)

Sparse eigenvalue decomposition w, v of square a in a given charge sector.
```

Finds k right eigenvectors (chosen by kwargs['which']) in a given charge sector, tensordot(A, V[i], axes=1) = W[i] * V[i].

Parameters

- a (Array) A square array with contractible legs and vanishing total charge.
- **charge_sector** (*charges*) *ndim* charges to select the block.
- **k** (*int*) How many eigenvalues/vectors should be calculated. If the block of *charge_sector* is smaller than *k*, *k* may be reduced accordingly.
- *args Additional arguments given to scipy.sparse.linalg.eigs.
- ****kwargs** Additional keyword arguments given to *scipy.sparse.linalg.eigs*.

Returns

- W (ndarray) k (or less) eigenvalues
- V (list of Array) k (or less) right eigenvectors of A with total charge *charge_sector*. Note that when interpreted as a matrix, this is the transpose of what np.eigs normally gives.

18.1.22 svd

- · full name: tenpy.linalg.np_conserved.svd
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.\mathbf{svd}(a, full\_matrices=False, compute\_uv=True, cutoff=None, qtotal\_LR=[None, None], inner\_labels=[None, None], inner\_qconj=1)
```

Singualar value decomposition of an Array \bar{a} .

Factorizes U, S, VH = svd(a), such that a = U*diag(S)*VH (where * stands for a tensordot() and diag creates an correctly shaped Array with S on the diagonal). For a non-zero cutoff this holds only approximately.

There is a gauge freedom regarding the charges, see also $Array.gauge_total_charge()$. We ensure contractibility by setting U.legs[1] = VH.legs[0].conj(). Further, we gauge the LegCharge such that U and V have the desired $gtotal_LR$.

Parameters

• a (Array, shape (M, N)) – The matrix to be decomposed.

- full_matrices (bool) If False (default), U and V have shapes (M, K) and (K, N), where K=len(S). If True, U and V are full square unitary matrices with shapes (M, M) and (N, N). Note that the arrays are not directly contractible in that case; diag(S) would need to be a rectangluar (M, N) matrix.
- **compute_uv** (bool) Whether to compute and return U and V.
- **cutoff** (None | float) Keep only singular values which are (strictly) greater than *cutoff*. (Then the factorization holds only approximately). If None (default), ignored.
- qtotal_LR ([{charges|None}, {charges|None}]) The desired qtotal for U and VH, respectively. [None, None] (Default) is equivalent to [None, a.qtotal].
 A single None entry is replaced the unique charge satisfying the requirement U.qtotal + VH.qtotal = a.qtotal (modulo qmod).
- inner_labels_LR ([{str/None}, {str/None}]) The first label corresponds to U.leqs[1], the second to VH.leqs[0].
- inner_qconj ({+1, -1}) Direction of the charges for the new leg. Default +1. The new LegCharge is constructed such that VH.legs[0].qconj = qconj.

Returns

- U (Array) Matrix with left singular vectors as columns. Shape (M, M) or (M, K) depending on *full_matrices*.
- **S** (*1D ndarray*) The singluar values of the array. If no *cutoff* is given, it has length min (M, N).
- VH (Array) Matrix with right singular vectors as rows. Shape (N, N) or (K, N) depending on *full_matrices*.

18.1.23 tensordot

- full name: tenpy.linalg.np_conserved.tensordot
- parent module: tenpy.linalg.np_conserved
- type: function

```
tenpy.linalg.np_conserved.tensordot (a, b, axes=2)
Similar as np.tensordot but for Array.
```

Builds the tensor product of a and b and sums over the specified axes. Does not require complete blocking of the charges.

Labels are inherited from a and b. In case of a collision (= the same label would be inherited from a and b after the contraction), both labels are dropped.

Detailed implementation notes are available in the doc-string of _tensordot_worker().

Parameters

- a (Array) The first and second npc Array for which axes are to be contracted.
- **b** (Array) The first and second npc Array for which axes are to be contracted.
- axes ((axes_a, axes_b) | int) A single integer is equivalent to (range(-axes, 0), range(axes)). Alternatively, axes_a and axes_b specifiy the legs of a and b, respectively, which should be contracted. Legs can be specified with leg labels or indices. Contract leg axes_a[i] of a with leg axes_b[i] of b.

Returns a_dot_b – The tensorproduct of a and b, summed over the specified axes. Returns a scalar in case of a full contraction.

Return type Array

18.1.24 to iterable arrays

- full name: tenpy.linalg.np_conserved.to_iterable_arrays
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.to_iterable_arrays (array_list)
Similar as to_iterable(), but also enclose npc Arrays in a list.

18.1.25 trace

- full name: tenpy.linalg.np_conserved.trace
- parent module: tenpy.linalg.np_conserved
- type: function

```
\texttt{tenpy.linalg.np\_conserved.trace} \ (a, leg1 = 0, leg2 = 1)
```

Trace of a, summing over leg1 and leg2.

Requires that the contracted legs are contractible (i.e. have opposite charges). Labels are inherited from a.

Parameters

- **leg1** (*str/int*) The leg label or index for the two legs which should be contracted (i.e. summed over).
- **leg2** (str/int) The leg label or index for the two legs which should be contracted (i.e. summed over).

Returns traced - A scalar if a.rank == 2, else an Array of rank a.rank - 2. Equivalent to sum([a.take_slice([i, i], [leg1, leg2]) for i in range(a.shape[leg1])]).

Return type Array | a.dtype

18.1.26 zeros

- full name: tenpy.linalg.np_conserved.zeros
- parent module: tenpy.linalg.np_conserved
- type: function

tenpy.linalg.np_conserved.zeros(legcharges, dtype=<class 'numpy.float64'>, qtotal=None, labels=None)

Create a npc array full of zeros (with no _data).

This is just a wrapper around Array (...), detailed documentation can be found in the class doc-string of Array.

Module description

A module to handle charge conservation in tensor networks.

A detailed introduction to this module (including notations) can be found in *Charge conservation with np_conserved*.

This module $np_conserved$ implements a class Array designed to make use of charge conservation in tensor networks. The idea is that the Array class is used in a fashion very similar to the numpy.ndarray, e.g you can call the functions tensordot() or svd() (of this module) on them. The structure of the algorithms (as DMRG) is thus the same as with basic numpy ndarrays.

Internally, an *Array* saves charge meta data to keep track of blocks which are nonzero. All possible operations (e.g. tensordot, svd, ...) on such arrays preserve the total charge structure. In addition, these operations make use of the charges to figure out which of the blocks it has to use/combine - this is the basis for the speed-up.

```
tenpy.linalg.np_conserved.QCUTOFF = 2.220446049250313e-15
   A cutoff to ignore machine precision rounding errors when determining charges
tenpy.linalg.np_conserved.QTYPE = <class 'numpy.int64'>
   the type used for charges
```

18.1.27 Overview

Classes

Array(legcharges[, dtype, qtotal, labels])	A multidimensional array (=tensor) for using charge
	conservation.
ChargeInfo([mod, names])	Meta-data about the charge of a tensor.
LegCharge(chargeinfo, slices, charges[, qconj])	Save the charge data associated to a leg of a tensor.
LegPipe(legs[, qconj, sort, bunch])	A <i>LegPipe</i> combines multiple legs of a tensor to one.

Array creation

Array.from_ndarray_trivial(data_flat[,])	convert a flat numpy ndarray to an Array with trivial
	charge conservation.
Array.from_ndarray(data_flat, legcharges[,])	convert a flat (numpy) ndarray to an Array.
Array.from_func(func, legcharges[, dtype,])	Create an Array from a numpy func.
Array.from_func_square(func, leg[, dtype,	Create an Array from a (numpy) function.
])	
zeros(legcharges[, dtype, qtotal, labels])	Create a npc array full of zeros (with no _data).
eye_like(a[, axis, labels])	Return an identity matrix contractible with the leg axis
	of the Array a.
diag(s, leg[, dtype, labels])	Returns a square, diagonal matrix of entries s.

Concatenation

concatenate(arrays[, axis, copy])	Stack arrays along a given axis, similar as
	np.concatenate.
<pre>grid_concat(grid, axes[, copy])</pre>	Given an np.array of npc.Arrays, performs a multi-
	dimensional concatentation along 'axes'.
<pre>grid_outer(grid, grid_legs[, qtotal,])</pre>	Given an np.array of npc.Arrays, return the correspond-
	ing higher-dimensional Array.

Detecting charges of flat arrays

detect_qtotal(flat_array, legcharges[, cutoff])	Returns the total charge (w.r.t <i>legs</i>) of first non-zero sector found in <i>flat_array</i> .
<pre>detect_legcharge(flat_array, chargeinfo,)</pre>	Calculate a missing <i>LegCharge</i> by looking for nonzero
	entries of a flat array.
detect_grid_outer_legcharge(grid,	Derive a LegCharge for a grid used for
grid_legs)	<pre>grid_outer().</pre>

Contraction of some legs

tensordot(a, b[, axes])	Similar as np.tensordot but for Array.
outer(a, b)	Forms the outer tensor product, equivalent to
	tensordot(a, b, axes=0).
inner(a, b[, axes, do_conj])	Contract all legs in a and b, return scalar.
trace(a[, leg1, leg2])	Trace of a, summing over leg1 and leg2.

Linear algebra

svd(a[, full_matrices, compute_uv, cutoff,])	Singualar value decomposition of an Array a.
pinv(a[, cutoff])	Compute the (Moore-Penrose) pseudo-inverse of a ma-
	trix.
norm(a[, ord, convert_to_float])	Norm of flattened data.
qr(a[, mode, inner_labels, cutoff])	Q-R decomposition of a matrix.
expm(a)	Use scipy.linalg.expm to calculate the matrix exponen-
	tial of a square matrix.

Eigen systems

eigh(a[, UPLO, sort])	Calculate eigenvalues and eigenvectors for a hermitian matrix.
eig(a[, sort])	Calculate eigenvalues and eigenvectors for a non-
J (1)	hermitian matrix.
eigvalsh(a[, UPLO, sort])	Calculate eigenvalues for a hermitian matrix.
eigvals(a[, sort])	Calculate eigenvalues for a hermitian matrix.
	continues on next page

18.1. np_conserved 303

Table 12 – continued from previous page

speigs(a, charge_sector, k, *args, **kwargs)	Sparse eigenvalue decomposition w , v of square a in a
	given charge sector.

18.2 charges

• full name: tenpy.linalg.charges

• parent module: tenpy.linalg

• type: module

Classes



ChargeInfo([mod, names])	Meta-data about the charge of a tensor.
LegCharge(chargeinfo, slices, charges[, qconj])	Save the charge data associated to a leg of a tensor.
LegPipe(legs[, qconj, sort, bunch])	A <i>LegPipe</i> combines multiple legs of a tensor to one.

18.2.1 ChargeInfo

• full name: tenpy.linalg.charges.ChargeInfo

• parent module: tenpy.linalg.charges

• type: class

Inheritance Diagram

ChargeInfo

Methods

ChargeInfoinit([mod, names])	Initialize self.	
ChargeInfo.add(chinfos)	Create a ChargeInfo combining multiple charges.	
ChargeInfo.change(chinfo, charge, new_qmod)	Change the <i>qmod</i> of a given charge.	
ChargeInfo.check_valid(charges)	Check, if charges has all entries as expected from	
	self.mod.	
ChargeInfo.drop(chinfo[, charge])	Remove a charge from a ChargeInfo.	
ChargeInfo.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.	
path)		
ChargeInfo.make_valid([charges])	Take charges modulo self.mod.	
ChargeInfo.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.	
path)		
ChargeInfo.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.	

Class Attributes and Properties

ChargeInfo.mod	Modulo how much each of the charges is taken.
ChargeInfo.qnumber	The number of charges.

class tenpy.linalg.charges.ChargeInfo(mod=[], names=None)

Bases: object

Meta-data about the charge of a tensor.

Saves info about the nature of the charge of a tensor. Provides $make_valid()$ for taking modulo m.

(This class is implemented in tenpy.linalg.charges but also imported in tenpy.linalg. np_conserved for convenience.)

Parameters

- mod (iterable of QTYPE) The len gives the number of charges, qnumber. Each entry is a positive integer, where 1 implies a U(1) charge and N>1 implies a Z_N symmetry. Defaults to "trivial", i.e., no charge.
- names (list of str) Descriptive names for the charges. Defaults to ['']*qnumber.

names

A descriptive name for each of the charges. May have "entries.

Type list of strings

_mask

mask (mod == 1), to speed up *make_valid* in pure python.

Type 1D array bool

_mod_masked

Equivalent to self.mod[self._maks_mod1]

Type 1D array QTYPE

_qnumber, _mod

Storage of qnumber and mod.

18.2. charges 305

Notes

Instances of this class can (should) be shared between different LegCharge and Array's.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

It stores the names under the path "names", and mod as dataset "U1_ZN".

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

The "U1_ZN" dataset is mandatory, 'names' are optional.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

classmethod add(chinfos)

Create a ChargeInfo combining multiple charges.

Parameters chinfos (iterable of ChargeInfo) – ChargeInfo instances to be combined into a single one (in the given order).

Returns chinfo – ChargeInfo combining all the given charges.

Return type ChargeInfo

classmethod drop(chinfo, charge=None)

Remove a charge from a ChargeInfo.

Parameters

- **chinfo** (*ChargeInfo*) The ChargeInfo from where to drop/remove a charge.
- **charge** (*int* / *str*) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.

Returns chinfo – ChargeInfo where the specified charge is dropped.

Return type ChargeInfo

classmethod change (chinfo, charge, new_qmod, new_name=")

Change the *qmod* of a given charge.

Parameters

• **chinfo** (ChargeInfo) – The ChargeInfo for which qmod of charge should be changed.

```
• new_qmod (int) – The new qmod to be set.
```

• **new_name** (str) – The new name of the charge.

Returns chinfo – ChargeInfo where *qmod* of the specified charge was changed.

```
Return type ChargeInfo
```

test sanity()

Sanity check, raises ValueErrors, if something is wrong.

property qnumber

The number of charges.

property mod

Modulo how much each of the charges is taken.

Entries are 1 for a U(1) charge, and N for a Z_N symmetry.

make_valid(charges=None)

Take charges modulo self.mod.

Parameters charges (array_like or None) - 1D or 2D array of charges, last dimension self.qnumber None defaults to trivial charges np.zeros(qnumber, dtype=QTYPE).

Returns A copy of *charges* taken modulo *mod*, but with x % 1 := x

Return type charges

check_valid(charges)

Check, if *charges* has all entries as expected from self.mod.

Parameters charges (2D ndarray QTYPE_t) – Charge values to be checked.

Returns res – True, if all 0 <= charges <= self.mod (wherever self.mod != 1)

Return type bool

18.2.2 LegCharge

- full name: tenpy.linalg.charges.LegCharge
- parent module: tenpy.linalg.charges
- · type: class

Inheritance Diagram

LegCharge

18.2. charges 307

Methods

LegChargeinit(chargeinfo, slices, charges)	Initialize self.
LegCharge.bunch()	Return a copy with bunched self.charges: form blocks
- 9 - 1 9 - 1 - 1 0	for contiguous equal charges.
LegCharge.charge_sectors()	Return unique rows of self.charges.
LegCharge.conj()	Return a (shallow) copy with opposite self.qconj.
LegCharge.copy()	Return a (shallow) copy of self.
LegCharge.extend(extra)	Return a new LegCharge, which extends self with
	futher charges.
LegCharge.flip_charges_qconj()	Return a copy with both negative <i>qconj</i> and <i>charges</i> .
LegCharge.from_add_charge(legs[, charge-	Add the (independent) charges of two or more legs to
info])	get larger qnumber.
LegCharge.from_change_charge(leg, charge,	Remove a charge from a LegCharge.
)	
LegCharge.from_drop_charge(leg[, charge,	Remove a charge from a LegCharge.
])	
LegCharge.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
LegCharge.from_qdict(chargeinfo, qdict[,	Create a LegCharge from qdict form.
qconj])	
LegCharge.from_qflat(chargeinfo, qflat[,	Create a LegCharge from qflat form.
qconj])	
LegCharge.from_qind(chargeinfo, slices,	Just a wrapper around selfinit(), see class doc-
charges)	string for parameters.
LegCharge.from_trivial(ind_len[,])	Create trivial (qnumber=0) LegCharge for given len of
	indices ind_len.
LegCharge.get_block_sizes()	Return the sizes of the individual blocks.
LegCharge.get_charge(qindex)	Return charge self.charges[qindex] *
	self.qconj for a given qindex.
LegCharge.get_qindex(flat_index)	Find qindex containing a flat index.
<pre>LegCharge.get_qindex_of_charges(charges)</pre>	Return the slice selecting the block for given charge val-
	ues.
<pre>LegCharge.get_slice(qindex)</pre>	Return slice selecting the block for a given <i>qindex</i> .
LegCharge.is_blocked()	Returns whether self is blocked, i.e. qindex map 1:1 to
	charge values.
LegCharge.is_bunched()	Checks whether bunch () would change something.
LegCharge.is_sorted()	Returns whether <i>self.charges</i> is sorted lexiographically.
LegCharge.perm_flat_from_perm_qind(perm	
	permutation.
LegCharge.perm_qind_from_perm_flat(perm	
LegCharge.project(mask)	Return copy keeping only the indices specified by <i>mask</i> .
LegCharge.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
LegCharge.sort([bunch])	Return a copy of self sorted by charges (but maybe not
	bunched).
LegCharge.test_contractible(other)	Raises a ValueError if charges are incompatible for con-
	traction with other.
LegCharge.test_equal(other)	Test if charges are <i>equal</i> including <i>qconj</i> .
LegCharge.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
LegCharge.to_qdict()	Return charges in <i>qdict</i> form.
	continues on next page

Table 16 – continued from previous page

LegCharge.to_gflat()

Return charges in *qflat* form.

class tenpy.linalg.charges.LegCharge (chargeinfo, slices, charges, qconj=1)

Bases: object

Save the charge data associated to a leg of a tensor.

This class is more or less a wrapper around a 2D numpy array *charges* and a 1D array *slices*. See *Charge conservation with np_conserved* for more details.

(This class is implemented in tenpy.linalg.charges but also imported in tenpy.linalg. np_conserved for convenience.)

Parameters

- chargeinfo (ChargeInfo) The nature of the charge.
- **slices** (1D array_like, len(block_number+1)) A block with 'qindex' qi correspondes to the leg indices in slice(slices[qi], slices[qi+1]).
- **charges** (2D array_like, shape(block_number, chargeinfo. qnumber)) charges [qi] gives the charges for a block with 'qindex' qi.
- qconj ({+1, -1}) A flag telling whether the charge points inwards (+1, default) or outwards (-1).

ind len

The number of indices for this leg.

Type int

block_number

The number of blocks, i.e., a 'qindex' for this leg is in range (block_number).

chinfo

The nature of the charge. Can be shared between LegCharges.

Type ChargeInfo instance

slices

A block with 'qindex' qi correspondes to the leg indices in slice(self.slices[qi], self. slices[qi+1]). See $get_slice()$.

Type ndarray[np.intp_t,ndim=1] (block_number+1)

charges

charges [qi] gives the charges for a block with 'qindex' qi. Note: the sign might be changed by *qconj*. See also *get_charge()*.

Type ndarray[QTYPE_t,ndim=1] (block_number, chinfo.qnumber)

qconj

A flag telling whether the charge points inwards (+1) or outwards (-1). Whenever charges are added, they should be multiplied with their *qconj* value.

Type $\{-1, 1\}$

sorted

Whether the charges are guaranteed to be sorted.

Type bool

bunched

Whether the charges are guaranteed to be bunched.

18.2. charges 309

Type bool

Notes

Instances of this class can be shared between different *npc.Array*. Thus, functions changing self.slices or self.charges *must* always make copies. Further they *must* set *sorted* and *bunched* to False (if they might not preserve them).

copy()

Return a (shallow) copy of self.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

Checks format for an ouput format key "LegCharge". Possible choices are:

- "blocks" (default) Store slices and charges directly as datasets, and block_number, sorted, bunched as further attributes.
- "compact" A single array np.hstack([self.slices[:-1], self.slices[1:], self. charges]) as dataset "blockcharges", and block_number, sorted, bunched as further attributes.
- "flat" Insufficient (!) to recover the exact blocks; saves only the array returned by to_flat() as
 dataset 'charges'.

The *ind_len*, *qconj*, and the *format* parameter are saved as group attributes under the same names. *chinfo* is always saved as subgroup.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

classmethod from_trivial(ind_len, chargeinfo=None, qconj=1)

Create trivial (qnumber=0) LegCharge for given len of indices ind_len.

```
classmethod from_qflat (chargeinfo, qflat, qconj=1)
```

Create a LegCharge from qflat form.

Does *neither* bunch *nor* sort. We recommend to sort (and bunch) afterwards, if you expect that tensors using the LegCharge have entries at all positions compatible with the charges.

Parameters

- chargeinfo (ChargeInfo) The nature of the charge.
- **qflat** (array_like (ind_len, *qnumber*)) *qnumber* charges for each index of the leg on entry.
- **qconj** ({-1, 1}) A flag telling whether the charge points inwards (+1) or outwards (-1).

See also:

```
sort sorts by charges
```

bunch bunches contiguous blocks of the same charge.

classmethod from_qind(chargeinfo, slices, charges, qconj=1)

Just a wrapper around self.__init__(), see class doc-string for parameters.

See also:

```
sort sorts by charges
```

bunch bunches contiguous blocks of the same charge.

classmethod from_qdict (chargeinfo, qdict, qconj=1)

Create a LegCharge from qdict form.

Parameters

- ${\tt chargeinfo}\,({\it ChargeInfo})$ The nature of the charge.
- **qdict** (dict) A dictionary mapping a tuple of charges to slices.

classmethod from_add_charge(legs, chargeinfo=None)

Add the (independent) charges of two or more legs to get larger qnumber.

Parameters

- **legs** (iterable of *LegCharge*) The legs for which the charges are to be combined/added.
- chargeinfo (ChargeInfo) The ChargeInfo for all charges; create new if None.

Returns combined – A LegCharge with the charges of both legs. Is neither sorted nor bunched!

Return type LegCharge

classmethod from drop charge (leg, charge=None, chargeinfo=None)

Remove a charge from a LegCharge.

Parameters

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (*int* / *str*) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.
- chargeinfo (ChargeInfo) The ChargeInfo with charge dropped; create new if None.

Returns dropped – A LegCharge with the specified charge dropped. Is neither sorted nor bunched!

Return type LegCharge

18.2. charges 311

classmethod from_change_charge (leg, charge, new_qmod, new_name=", chargeinfo=None) Remove a charge from a LegCharge.

Parameters

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (int / str) Number or *name* of the charge (within *chinfo*) for which *mod* is to be changed.
- new_qmod (int) The new mod to be set for charge in the ChargeInfo.
- new_name (str) The new name for *charge*.
- chargeinfo (ChargeInfo) The ChargeInfo with charge changed; create new if None.

Returns leg – A LegCharge with the specified charge changed. Is neither sorted nor bunched!

Return type LegCharge

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

conj()

Return a (shallow) copy with opposite self.qconj.

Returns conjugated – Shallow copy of *self* with flipped *qconj*. *test_contractible()* of *self* with *conjugated* will not raise an error.

Return type LegCharge

flip_charges_qconj()

Return a copy with both negative qconj and charges.

Returns conj_charges – (Shallow) copy of self with negative *qconj* and *charges*, thus representing the very same charges. test_equal() of self with conj_charges will not raise an error.

Return type LegCharge

to_qflat()

Return charges in *qflat* form.

to qdict()

Return charges in qdict form.

Raises ValueError, if not blocked.

is blocked()

Returns whether self is blocked, i.e. qindex map 1:1 to charge values.

is_sorted()

Returns whether self.charges is sorted lexiographically.

is_bunched()

Checks whether bunch () would change something.

test_contractible(other)

Raises a ValueError if charges are incompatible for contraction with other.

Parameters other (*LegCharge*) – The LegCharge of the other leg condsidered for contraction.

Raises ValueError – If the charges are incompatible for direct contraction.

Notes

This function checks that two legs are *ready* for contraction. This is the case, if all of the following conditions are met:

- the ChargeInfo is equal
- the *slices* are equal
- the *charges* are the same up to *opposite* signs qconj:

```
self.charges * self.qconj = - other.charges * other.qconj
```

In general, there could also be a change of the total charge, see *Charge conservation with np_conserved* This special case is not considered here - instead use <code>gauge_total_charge()</code>, if a change of the charge is desired.

If you are sure that the legs should be contractable, check whether the charges are actually valid or whether self and other are blocked or should be sorted.

See also:

```
test_equal self.test_contractible(other) just performs self.
test_equal(other.conj()).
```

test_equal(other)

Test if charges are equal including qconj.

Check that all of the following conditions are met:

- the ChargeInfo is equal
- · the slices are equal
- the *charges* are the same up to the signs qconj:

```
self.charges * self.qconj = other.charges * other.qconj
```

See also:

```
test_contractible self.test_equal(other) is equivalent to self.
test_contractible(other.conj()).
```

get_block_sizes()

Return the sizes of the individual blocks.

Returns sizes - The sizes of the individual blocks; sizes[i] = slices[i+1] - slices[i].

Return type ndarray, shape (block_number,)

get_slice (qindex)

Return slice selecting the block for a given qindex.

get_qindex (flat_index)

Find qindex containing a flat index.

Given a flat index, to find the corresponding entry in an Array, we need to determine the block it is saved in. For example, if slices = [[0, 3], [3, 7], [7, 12]], the flat index 5 corresponds to the second entry, qindex = 1 (since 5 is in [3:7]), and the index within the block would be 2 = 5 - 3.

Parameters flat index (int) – A flat index of the leg. Negative index counts from behind.

18.2. charges 313

Returns

- qindex (int) The qindex, i.e. the index of the block containing flat_index.
- index_within_block (int) The index of flat_index within the block given by qindex.

get_qindex_of_charges (charges)

Return the slice selecting the block for given charge values.

Inverse function of get_charge().

Parameters charges (1D array_like) - Charge values for which the slice of the block is to be determined.

Returns slice(i, j) – Slice of the charge values for

Return type slice

:raises ValueError : if the answer is not unique (because self is not blocked).:

get_charge (qindex)

Return charge self.charges[qindex] * self.qconj for a given qindex.

sort (bunch=True)

Return a copy of *self* sorted by charges (but maybe not bunched).

If bunch=True, the returned copy is completely blocked by charge.

Parameters bunch (bool) – Whether *self.bunch* is called after sorting. If True, the leg is guaranteed to be fully blocked by charge.

Returns

- perm_qind (array (self.block_len,)) The permutation of the qindices (before bunching) used for the sorting. To obtain the flat permutation such that sorted_array[..., :] = unsorted_array[..., perm_flat], use perm_flat = unsorted_leg.perm_flat_from_perm_qind(perm_qind)
- **sorted_copy** (*LegCharge*) A shallow copy of self, with new qind sorted (and thus blocked if bunch) by charges.

See also:

bunch enlarge blocks for contiguous qind of the same charges.

numpy.take can apply perm_flat to a given axis

tenpy.tools.misc.inverse_permutation returns inverse of a permutation

bunch()

Return a copy with bunched self.charges: form blocks for contiguous equal charges.

Returns

- idx (1D array) idx[:-1] are the indices of the old qind which are kept, idx[-1] = old_block_number.
- **cp** (*LegCharge*) A new LegCharge with the same charges at given indices of the leg, but (possibly) shorter self.charges and self.slices.

See also:

sort sorts by charges, thus enforcing complete blocking in combination with bunch.

project (mask)

Return copy keeping only the indices specified by mask.

Parameters mask (1D array (bool)) - Whether to keep of the indices.

Returns

- map_qind (*1D array*) Map of qindices, such that qind_new = map_qind[qind_old], and map_qind[qind_old] = -1 for qindices projected out.
- **block_masks** (1D array) The bool mask for each of the remaining blocks.
- **projected_copy** (*LegCharge*) Copy of self with the qind projected by *mask*.

extend(extra)

Return a new LegCharge, which extends self with futher charges.

This is needed to formally increase the dimension of an Array.

Parameters extra (LegCharge | int) – By what to extend, i.e. the charges to be appended to *self*. An int stands for extending the length of the array by a single new block of that size and zero charges.

Returns extended_leg – Copy of *self* extended by the charge blocks of the *extra* leg.

Return type LegCharge

charge sectors()

Return unique rows of self.charges.

Returns charges – Rows are the rows of self.charges lexsorted and without duplicates.

Return type array[QTYPE, ndim=2]

perm_flat_from_perm_qind(perm_qind)

Convert a permutation of qind (acting on self) into a flat permutation.

perm_qind_from_perm_flat (perm_flat)

Convert flat permutation into qind permutation.

Parameters perm_flat (1D array) - A permutation acting on self, which doesn't mix the blocks of qind.

Returns perm_qind – The permutation of self.qind described by perm_flat.

Return type 1D array

Raises ValueError – If perm_flat mixes blocks of different qindex.

18.2.3 LegPipe

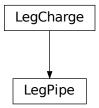
• full name: tenpy.linalg.charges.LegPipe

• parent module: tenpy.linalg.charges

· type: class

18.2. charges 315

Inheritance Diagram



Methods

LegPipeinit(legs[, qconj, sort, bunch])	Initialize self.		
LegPipe.bunch(*args, **kwargs)	Convert to LegCharge and call LegCharge.		
	bunch().		
LegPipe.charge_sectors()	Return unique rows of self.charges.		
LegPipe.conj()	Return a shallow copy with opposite self.qconj.		
LegPipe.copy()	Return a (shallow) copy of self.		
LegPipe.extend(extra)	Return a new LegCharge, which extends self with		
	futher charges.		
LegPipe.flip_charges_qconj()	Return a copy with both negative qconj and charges.		
<pre>LegPipe.from_add_charge(legs[, chargeinfo])</pre>	Add the (independent) charges of two or more legs to		
	get larger qnumber.		
LegPipe.from_change_charge(leg, charge,	Remove a charge from a LegCharge.		
new_qmod)			
LegPipe.from_drop_charge(leg[, charge,])	Remove a charge from a LegCharge.		
LegPipe.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.		
<pre>LegPipe.from_qdict(chargeinfo, qdict[, qconj])</pre>	Create a LegCharge from qdict form.		
LegPipe.from_qflat(chargeinfo, qflat[, qconj])	Create a LegCharge from qflat form.		
LegPipe.from_qind(chargeinfo, slices, charges)	Just a wrapper around selfinit(), see class doc-		
	string for parameters.		
LegPipe.from_trivial(ind_len[, chargeinfo,	Create trivial (qnumber=0) LegCharge for given len of		
])	indices ind_len.		
LegPipe.get_block_sizes()	Return the sizes of the individual blocks.		
LegPipe.get_charge(qindex)	Return charge self.charges[qindex] *		
	self.qconj for a given qindex.		
LegPipe.get_qindex(flat_index)	Find qindex containing a flat index.		
<pre>LegPipe.get_qindex_of_charges(charges)</pre>	Return the slice selecting the block for given charge val-		
	ues.		
LegPipe.get_slice(qindex)	Return slice selecting the block for a given <i>qindex</i> .		
LegPipe.is_blocked()	Returns whether self is blocked, i.e. qindex map 1:1 to		
	charge values.		
LegPipe.is_bunched()	Checks whether bunch () would change something.		
LegPipe.is_sorted()	Returns whether <i>self.charges</i> is sorted lexiographically.		
	continues on next page		

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	1 0
<pre>LegPipe.map_incoming_flat(incoming_indices)</pre>	Map (flat) incoming indices to an index in the outgoing
	pipe.
LegPipe.outer_conj()	Like conj(), but don't change qconj for incoming
	legs.
LegPipe.perm_flat_from_perm_qind(perm_qi	n d)onvert a permutation of qind (acting on self) into a flat
	permutation.
LegPipe.perm_qind_from_perm_flat(perm_flat)	at Convert flat permutation into qind permutation.
LegPipe.project(*args, **kwargs)	Convert self to LegCharge and call LegCharge.
	project().
LegPipe.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
LegPipe.sort(*args, **kwargs)	Convert to LegCharge and call LegCharge.sort().
LegPipe.test_contractible(other)	Raises a ValueError if charges are incompatible for con-
	traction with other.
LegPipe.test_equal(other)	Test if charges are <i>equal</i> including <i>qconj</i> .
LegPipe.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
LegPipe.to_LegCharge()	Convert self to a LegCharge, discarding the information
	how to split the legs.
LegPipe.to_qdict()	Return charges in <i>qdict</i> form.
LegPipe.to_qflat()	Return charges in <i>qflat</i> form.

class tenpy.linalg.charges.LegPipe (legs, qconj=1, sort=True, bunch=True)

Bases: tenpy.linalg.charges.LegCharge

A *LegPipe* combines multiple legs of a tensor to one.

Often, it is necessary to "combine" multiple legs into one: for example to perfom a SVD, the tensor needs to be viewed as a matrix.

This class does exactly this job: it combines multiple LegCharges ('incoming legs') into one 'pipe' (the 'outgoing leg'). The pipe itself is a LegCharge, with indices running from 0 to the product of the individual legs' ind_len, corresponding to all possible combinations of input leg indices.

(This class is implemented in tenpy.linalg.charges but also imported in tenpy.linalg. $np_conserved$ for convenience.)

Parameters

- legs (list of LegCharge) The legs which are to be combined.
- $qconj(\{+1, -1\})$ A flag telling whether the charge of the *resulting* pipe points inwards (+1, default) or outwards (-1).
- **sort** (bool) Whether the outgoing pipe should be sorted. Default True; recommended. Note: calling *sort* () after initialization converts to a LegCharge.
- **bunch** (bool) Whether the outgoing pipe should be bunched. Default True; recommended. Note: calling bunch() after initialization converts to a LegCharge.

nlegs

The number of legs.

Type int

legs

The original legs, which were combined in the pipe.

Type tuple of *LegCharge*

18.2. charges 317

subshape

ind_len for each of the incoming legs.

```
Type tuple of int
```

subqshape

block_number for each of the incoming legs.

```
Type tuple of int
```

q_map

Shape ($block_number$, 3 + nlegs). Rows: [b_j , b_{j+1} , I_s , i_1 , ..., i_{nlegs}], See Notes below for details.

```
Type array[np.intp, ndim=2]
```

q_map_slices

```
Defined such that the row indices of in range (q_map_slices[I_s], q_map_slices[I_s+1]) have q_map[:, 2] == I_s.
```

```
Type array[np.intp, ndim=1]
```

_perm

A permutation such that g map[perm, 3:] is sorted by i l.

```
Type 1D array
```

strides

Strides for mapping incoming qindices i l to the index of q map [perm, :].

```
Type 1D array
```

Notes

For np.reshape, taking, for example, $i, j, ... \to k$ amounted to $k = s_1 * i + s_2 * j + ...$ for appropriate strides s_1, s_2 .

In the charged case, however, we want to block k by charge, so we must implicitly permute as well. This reordering is encoded in q_map .

Each qindex combination of the *nlegs* input legs $(i_1, ..., i_{nlegs})$, will end up getting placed in some slice $a_j: a_{j+1}$ of the outgoing pipe. Within this slice, the data is simply reshaped in usual row-major fashion ('C'-order), i.e., with strides $s_1 > s_2 > ...$.

It will be a subslice of a new total block labeled by qindex I_s . Because many charge combinations fuse to the same total charge, in general there will be many tuples $(i_1,...,i_{nlegs})$ belonging to the same I_s . The rows of q_map are precisely the collections of $[b_j, b_\{j+1\}, I_s, i_1, ..., i_\{nlegs\}]$. Here, $b_j:b_{j+1}$ denotes the slice of this qindex combination within the total block I_s , i.e., $b_j=a_j-self$. slices $[I_s]$.

The rows of q_map are lex-sorted first by \mathbb{I}_s , then the \mathbb{i} . Each \mathbb{I}_s will have multiple rows, and the order in which they are stored in q map is the order the data is stored in the actual tensor, i.e., it might look like

```
[ ...,
 [ b_j, b_{j+1}, I_s, i_1, ..., i_{nlegs} ],
 [ b_{j+1}, b_{j+2}, I_s, i'_1, ..., i'_{nlegs} ],
 [ 0, b_{j+3}, I_s + 1, i''_1, ..., i''_{nlegs} ],
 [ b_{j+3}, b_{j+4}, I_s + 1, i'''_1, ..., i'''_{nlegs}],
 ...]
```

The charge fusion rule is:

```
self.charges[Qi]*self.gcon; == sum([1.charges[qi_1]*1.gcon; for 1 in self.legs])
→mod qmod
Here the qindex Qi of the pipe corresponds to qindices qi_l on the individual legs.
copy()
     Return a (shallow) copy of self.
save_hdf5 (hdf5_saver, h5gr, subpath)
     Export self into a HDF5 file.
     This method saves all the data it needs to reconstruct self with from_hdf5().
     In addition to the data saved for the LegCharge, it just saves the legs as subgroup.
         Parameters
             • hdf5_saver (Hdf5Saver) - Instance of the saving engine.
             • h5gr (:class`Group`) – HDF5 group which is supposed to represent self.
             • subpath (str) – The name of h5gr with a '/' in the end.
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
     Load instance from a HDF5 file.
     This method reconstructs a class instance from the data saved with save_hdf5().
         Parameters
             • hdf5_loader (Hdf5Loader) - Instance of the loading engine.
             • h5gr (Group) – HDF5 group which is represent the object to be constructed.
             • subpath (str) – The name of h5gr with a '/' in the end.
         Returns obj – Newly generated class instance containing the required data.
         Return type cls
test_sanity()
     Sanity check, raises ValueErrors, if something is wrong.
to_LegCharge()
     Convert self to a LegCharge, discarding the information how to split the legs.
     Usually not needed, but called by functions, which are not implemented for a LegPipe.
conj()
     Return a shallow copy with opposite self.qconj.
         Returns conjugated – Shallow copy of self with flipped qconj. Whenever we contract two
             legs, they need to be conjugated to each other. The incoming legs of the pipe are also conju-
             gated.
         Return type LegCharge
     Like conj(), but don't change qconj for incoming legs.
sort (*args, **kwargs)
     Convert to LegCharge and call LegCharge.sort().
bunch (*args, **kwargs)
     Convert to LegCharge and call LegCharge.bunch().
```

18.2. charges 319

```
project (*args, **kwargs)
```

Convert self to LegCharge and call LegCharge.project().

In general, this could be implemented for a LegPipe, but would make <code>split_legs()</code> more complicated, thus we keep it simple. If you really want to project and split afterwards, use the following work-around, which is for example used in <code>exact_diagonalization</code>:

- 1) Create the full pipe and save it separetely.
- 2) Convert the Pipe to a Leg & project the array with it.
- 3) [... do calculations ...]
- 4) To split the 'projected pipe' of A, create and empty array B with the legs of A, but replace the projected leg by the full pipe. Set A as a slice of B. Finally split the pipe.

map_incoming_flat (incoming_indices)

Map (flat) incoming indices to an index in the outgoing pipe.

Parameters incoming_indices (iterable of int) - One (flat) index on each of the incoming legs.

Returns outgoing_index – The index in the outgoing leg.

Return type int

charge sectors()

Return unique rows of self.charges.

Returns charges – Rows are the rows of self.charges lexsorted and without duplicates.

Return type array[QTYPE, ndim=2]

extend(extra)

Return a new LegCharge, which extends self with futher charges.

This is needed to formally increase the dimension of an Array.

Parameters extra (LegCharge | int) – By what to extend, i.e. the charges to be appended to *self*. An int stands for extending the length of the array by a single new block of that size and zero charges.

Returns extended_leg – Copy of *self* extended by the charge blocks of the *extra* leg.

Return type LegCharge

flip_charges_qconj()

Return a copy with both negative qconj and charges.

Returns conj_charges – (Shallow) copy of self with negative *qconj* and *charges*, thus representing the very same charges. test_equal() of self with conj_charges will not raise an error.

Return type LegCharge

$\verb|classmethod from_add_charge| (legs, charge info=None)|$

Add the (independent) charges of two or more legs to get larger qnumber.

Parameters

- **legs** (iterable of *LegCharge*) The legs for which the charges are to be combined/added.
- chargeinfo (ChargeInfo) The ChargeInfo for all charges; create new if None.

Returns combined – A LegCharge with the charges of both legs. Is neither sorted nor bunched!

Return type LegCharge

classmethod from_change_charge (*leg*, *charge*, *new_qmod*, *new_name=''*, *chargeinfo=None*) Remove a charge from a LegCharge.

Parameters

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (int / str) Number or *name* of the charge (within *chinfo*) for which *mod* is to be changed.
- **new_qmod** (int) The new *mod* to be set for *charge* in the *ChargeInfo*.
- **new_name** (str) The new name for *charge*.
- chargeinfo (ChargeInfo) The ChargeInfo with charge changed; create new if None.

Returns leg – A LegCharge with the specified charge changed. Is neither sorted nor bunched!

Return type LegCharge

classmethod from_drop_charge (leg, charge=None, chargeinfo=None)

Remove a charge from a LegCharge.

Parameters

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (int / str) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.
- **chargeinfo** (*ChargeInfo*) The ChargeInfo with *charge* dropped; create new if None.

Returns dropped – A LegCharge with the specified charge dropped. Is neither sorted nor bunched!

Return type LegCharge

classmethod from_qdict(chargeinfo, qdict, qconj=1)

Create a LegCharge from qdict form.

Parameters

- **chargeinfo** (*ChargeInfo*) The nature of the charge.
- **qdict** (dict) A dictionary mapping a tuple of charges to slices.

classmethod from qflat (chargeinfo, qflat, qconj=1)

Create a LegCharge from qflat form.

Does *neither* bunch *nor* sort. We recommend to sort (and bunch) afterwards, if you expect that tensors using the LegCharge have entries at all positions compatible with the charges.

Parameters

- ${\tt chargeinfo}$ (${\tt ChargeInfo}$) The nature of the charge.
- qflat (array_like (ind_len, qnumber)) qnumber charges for each index of the leg on entry.
- qconj ({-1, 1}) A flag telling whether the charge points inwards (+1) or outwards (-1).

See also:

18.2. charges 321

```
sort sorts by charges
     bunch bunches contiguous blocks of the same charge.
classmethod from_qind(chargeinfo, slices, charges, qconj=1)
     Just a wrapper around self.__init__(), see class doc-string for parameters.
     See also:
     sort sorts by charges
     bunch bunches contiguous blocks of the same charge.
classmethod from_trivial(ind_len, chargeinfo=None, qconj=1)
     Create trivial (qnumber=0) LegCharge for given len of indices ind_len.
get_block_sizes()
     Return the sizes of the individual blocks.
         Returns sizes - The sizes of the individual blocks; sizes[i] = slices[i+1] -
             slices[i].
         Return type ndarray, shape (block number,)
get_charge (qindex)
     Return charge self.charges[qindex] * self.qconj for a given qindex.
get_qindex (flat_index)
     Find qindex containing a flat index.
     Given a flat index, to find the corresponding entry in an Array, we need to determine the block it is saved
     in. For example, if slices = [[0, 3], [3, 7], [7, 12]], the flat index 5 corresponds to the
     second entry, qindex = 1 (since 5 is in [3:7]), and the index within the block would be 2 = 5 - 3.
         Parameters flat_index (int) – A flat index of the leg. Negative index counts from behind.
         Returns
             • qindex (int) – The qindex, i.e. the index of the block containing flat_index.
             • index_within_block (int) – The index of flat_index within the block given by qindex.
get_qindex_of_charges (charges)
     Return the slice selecting the block for given charge values.
     Inverse function of get_charge().
         Parameters charges (1D array_like) - Charge values for which the slice of the block is
             to be determined.
         Returns slice(i, j) – Slice of the charge values for
         Return type slice
     :raises ValueError : if the answer is not unique (because self is not blocked).:
get_slice (qindex)
     Return slice selecting the block for a given qindex.
is_blocked()
     Returns whether self is blocked, i.e. qindex map 1:1 to charge values.
is bunched()
     Checks whether bunch () would change something.
```

is sorted()

Returns whether *self.charges* is sorted lexiographically.

perm_flat_from_perm_qind(perm_qind)

Convert a permutation of qind (acting on self) into a flat permutation.

perm_qind_from_perm_flat (perm_flat)

Convert flat permutation into gind permutation.

Parameters perm_flat (1D array) - A permutation acting on self, which doesn't mix the blocks of qind.

Returns perm_qind – The permutation of self.qind described by perm_flat.

Return type 1D array

Raises ValueError - If perm_flat mixes blocks of different qindex.

test_contractible(other)

Raises a ValueError if charges are incompatible for contraction with other.

Parameters other (LegCharge) – The LegCharge of the other leg condsidered for contraction.

Raises ValueError – If the charges are incompatible for direct contraction.

Notes

This function checks that two legs are *ready* for contraction. This is the case, if all of the following conditions are met:

- the ChargeInfo is equal
- the *slices* are equal
- the *charges* are the same up to *opposite* signs qconj:

```
self.charges * self.qconj = - other.charges * other.qconj
```

In general, there could also be a change of the total charge, see *Charge conservation with np_conserved* This special case is not considered here - instead use <code>gauge_total_charge()</code>, if a change of the charge is desired.

If you are sure that the legs should be contractable, check whether the charges are actually valid or whether self and other are blocked or should be sorted.

See also:

```
test_equal self.test_contractible(other) just performs self.
test_equal(other.conj()).
```

test_equal (other)

Test if charges are *equal* including *qconj*.

Check that all of the following conditions are met:

- the ChargeInfo is equal
- the slices are equal
- the *charges* are the same up to the signs qconj:

18.2. charges 323

```
self.charges * self.qconj = other.charges * other.qconj
```

See also:

```
test_contractible self.test_equal(other) is equivalent to self.
test_contractible(other.conj()).
```

to_qdict()

Return charges in qdict form.

Raises ValueError, if not blocked.

to_qflat()

Return charges in qflat form.

Module description

Basic definitions of a charge.

This module contains implementations for handling the quantum numbers ("charges") of the Array.

In particular, the classes Charge Info, LegCharge and LegPipe are implemented here.

Note: The contents of this module are imported in *np_conserved*, so you usually don't need to import this module in your application.

A detailed introduction to np_conserved can be found in Charge conservation with np_conserved.

In this module, some functions have the python decorator <code>@use_cython</code>. Functions with this decorator are replaced by the ones written in Cython, implemented in the file <code>tenpy/linalg/_npc_helper.pyx</code>. For further details, see the definition of <code>use_cython()</code>.

```
tenpy.linalg.charges.QTYPE = <class 'numpy.int64'> Numpy data type for the charges.
```

18.3 svd_robust

• full name: tenpy.linalg.svd_robust

• parent module: tenpy.linalg

• type: module

Functions

svd(a[, full_matrices, compute_uv,])	Wrapper around scipy.linalg.svd() with gesvd	
	backup plan.	
svd_gesvd(a[, full_matrices, compute_uv,])	svd with LAPACK's '#gesvd' (with # = d/z for	
	float/complex).	

18.3.1 svd

- full name: tenpy.linalg.svd_robust.svd
- parent module: tenpy.linalg.svd_robust
- · type: function

```
tenpy.linalg.svd_robust.svd(a, full_matrices=True, compute_uv=True, overwrite_a=False, check_finite=True, lapack_driver='gesdd', warn=True)
Wrapper around scipy.linalg.svd() with gesvd backup plan.
```

Tries to avoid raising an LinAlgError by using using the lapack_driver gesvd, if gesdd failed.

Parameters not described below are as in scipy.linalg.svd()

Parameters

- overwrite_a (bool) Ignored (i.e. set to False) if lapack_driver='gesdd'. Otherwise described in scipy.linalg.svd().
- lapack_driver ({'gesdd', 'gesvd'}, optional) Whether to use the more efficient divide-and-conquer approach ('gesdd') or general rectangular approach ('gesvd') to compute the SVD. MATLAB and Octave use the 'gesvd' approach. Default is 'gesdd'. If 'gesdd' fails, 'gesvd' is used as backup.
- warn (bool) Whether to create a warning when the SVD failed.

Returns U, S, Vh – As described in doc-string of scipy.linalg.svd().

Return type ndarray

18.3.2 svd gesvd

- full name: tenpy.linalg.svd_robust.svd_gesvd
- parent module: tenpy.linalg.svd_robust
- type: function

Similar as numpy.linalg.svd(), but use LAPACK 'gesvd' driver. Works only with 2D arrays. Outer part is based on the code of *numpy.linalg.svd*.

Parameters

- a See numpy.linalg.svd() for details.
- full_matrices See numpy.linalg.svd() for details.
- compute_uv See numpy.linalg.svd() for details.
- **check_finite** check whether input arrays contain 'NaN' or 'inf'.

Returns U, S, Vh – See numpy.linalg.svd() for details.

Return type ndarray

18.3. svd robust 325

Module description

(More) robust version of singular value decomposition.

We often need to perform an SVD. In general, an SVD is a matrix factorization that is always well defined and should also work for ill-conditioned matrices. But sadly, both numpy.linalg.svd() and scipy.linalg.svd() fail from time to time, raising LinalgError("SVD did not converge"). The reason is that both of them call the LAPACK function #gesdd (where # depends on the data type), which takes an iterative approach that can fail. However, it is usually much faster than the alternative (and robust) #gesvd.

Our workaround is as follows: we provide a function svd() with call signature as scipy's svd. This function is basically just a wrapper around scipy's svd, i.e., we keep calling the faster dgesdd. But if that fails, we can still use dgesvd as a backup.

Sadly, *dgesvd* and *zgesvd* were not included into scipy until version '0.18.0' (nor in numpy), which is as the time of this writing the latest stable scipy version. For scipy version newer than '0.18.0', we make use of the new keyword 'lapack_driver' for svd, otherwise we (try to) load *dgesvd* and *zgesvd* from shared LAPACK libraries.

The tribute for the dgesvd wrapper code goes to 'jgarcke', originally posted at http://projects.scipy.org/numpy/ticket/990, which is now hosted at https://github.com/numpy/numpy/issues/1588 He explains a bit more in detail what fails.

The include of *dgesvd* to scipy was done in https://github.com/scipy/scipy/pull/5994.

Examples

The idea is that you just import the *svd* from this module and use it as replacement for np.linalg.svd or scipy. linalg.svd:

```
>>> from tenpy.linalg.svd_robust import svd
>>> U, S, VT = svd([[1., 1.], [0., 1.]])
```

18.4 random_matrix

• full name: tenpy.linalg.random_matrix

• parent module: tenpy.linalq

• type: module

Functions

COE(size)	Circular orthogonal ensemble (COE).
CRE(size)	Circular real ensemble (CRE).
CUE(size)	Circular unitary ensemble (CUE).
GOE(size)	Gaussian orthogonal ensemble (GOE).
GUE(size)	Gaussian unitary ensemble (GUE).
O_close_1(size[, a])	return an random orthogonal matrix 'close' to the Iden-
	tity.
$U_close_1(size[, a])$	return an random orthogonal matrix 'close' to the iden-
	tity.
box(size[, W])	return random number uniform in (-W, W].
	continues on next page

continues on next page

Table 19 – continued from previous page

standard_normal_complex(size)

return (R + 1.j*I) for independent R and I from np.random.standard_normal.

18.4.1 COE

• full name: tenpy.linalg.random_matrix.COE

• parent module: tenpy.linalg.random_matrix

· type: function

tenpy.linalg.random_matrix.COE (size)

Circular orthogonal ensemble (COE).

Parameters size (tuple) – (n, n), where n is the dimension of the output matrix.

Returns U – Unitary, symmetric (complex) matrix drawn from the COE (=Haar measure on this space).

Return type ndarray

18.4.2 CRE

• full name: tenpy.linalg.random_matrix.CRE

• parent module: tenpy.linalg.random_matrix

· type: function

tenpy.linalg.random_matrix.CRE(size)

Circular real ensemble (CRE).

Parameters size (tuple) – (n, n), where n is the dimension of the output matrix.

Returns U – Orthogonal matrix drawn from the CRE (=Haar measure on O(n)).

Return type ndarray

18.4.3 CUE

• full name: tenpy.linalg.random_matrix.CUE

• parent module: tenpy.linalg.random_matrix

• type: function

tenpy.linalg.random_matrix.CUE(size)

Circular unitary ensemble (CUE).

Parameters size (tuple) – (n, n), where n is the dimension of the output matrix.

Returns U – Unitary matrix drawn from the CUE (=Haar measure on U(n)).

Return type ndarray

18.4.4 GOE

```
• full name: tenpy.linalg.random_matrix.GOE
```

• parent module: tenpy.linalg.random_matrix

• type: function

```
tenpy.linalg.random_matrix.GOE(size)
```

Gaussian orthogonal ensemble (GOE).

Parameters size (tuple) – (n, n), where n is the dimension of the output matrix.

Returns H – Real, symmetric numpy matrix drawn from the GOE, i.e. $p(H) = 1/Zexp(-n/4tr(H^2))$

Return type ndarray

18.4.5 GUE

- full name: tenpy.linalg.random_matrix.GUE
- parent module: tenpy.linalg.random_matrix
- type: function

```
tenpy.linalg.random_matrix.GUE(size)
```

Gaussian unitary ensemble (GUE).

Parameters size (tuple) - (n, n), where n is the dimension of the output matrix.

Returns H – Hermitian (complex) numpy matrix drawn from the GUE, i.e. $p(H) = 1/Zexp(-n/4tr(H^2))$.

Return type ndarray

18.4.6 O close 1

- full name: tenpy.linalg.random_matrix.O_close_1
- parent module: tenpy.linalg.random_matrix
- type: function

tenpy.linalg.random_matrix. $\mathbf{0}$ _close_1 (size, a=0.01)

return an random orthogonal matrix 'close' to the Identity.

Parameters

- **size** (tuple) (n, n), where n is the dimension of the output matrix.
- a (float) Parameter determining how close the result is on O; $\lim_{a\to 0} < |O-E| >_a = 0$ (where E is the identity).

Returns \mathbf{O} – Orthogonal matrix close to the identity (for small a).

Return type ndarray

18.4.7 U_close_1

- full name: tenpy.linalg.random_matrix.U_close_1
- parent module: tenpy.linalg.random_matrix
- type: function

tenpy.linalg.random_matrix.**U_close_1** (*size*, *a*=0.01) return an random orthogonal matrix 'close' to the identity.

Parameters

- **size** (tuple) (n, n), where n is the dimension of the output matrix.
- **a** (float) Parameter determining how close the result is to the identity. $\lim_{a\to 0} < |O-E|>_a=0$ (where E is the identity).

Returns U – Unitary matrix close to the identity (for small a). Eigenvalues are chosen i.i.d. as $\exp(1.j*a*x)$ with x uniform in [-1, 1].

Return type ndarray

18.4.8 box

- full name: tenpy.linalg.random_matrix.box
- parent module: tenpy.linalg.random_matrix
- type: function

tenpy.linalg.random_matrix.**box**(*size*, *W*=1.0) return random number uniform in (-W, W].

18.4.9 standard_normal_complex

- full name: tenpy.linalg.random_matrix.standard_normal_complex
- parent module: tenpy.linalg.random_matrix
- · type: function

tenpy.linalg.random_matrix.standard_normal_complex(size) return (R + 1.j*I) for independent R and I from np.random.standard_normal.

Module description

Provide some random matrix ensembles for numpy.

The implemented ensembles are:

ensemble	matrix class drawn from	measure	invariant under	beta
GOE	real, symmetric	$\sim \exp(-n/4 \operatorname{tr}(H^2))$	orthogonal O	1
GUE	hermitian	$\sim \exp(-n/2 \operatorname{tr}(H^2))$	unitary U	2
CRE	O(n)	Haar	orthogonal O	/
COE	U in U(n) with $U = U^T$	Haar	orthogonal O	1
CUE	U(n)	Haar	unitary U	2
O_close_1	O(n)	?	/	/
U_close_1	U(n)	?	/	/

All functions in this module take a tuple (n, n) as first argument, such that we can use the function $from_func()$ to generate a block diagonal Array with the block from the corresponding ensemble, for example:

```
npc.Array.from_func_square(GOE, [leg, leg.conj()])
```

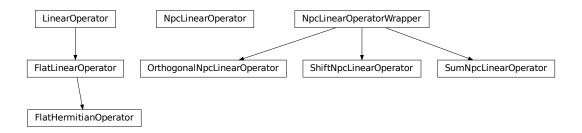
18.5 sparse

• full name: tenpy.linalg.sparse

• parent module: tenpy.linalg

• type: module

Classes



FlatHermitianOperator(*args, **kwargs)	Hermitian variant of FlatLinearOperator.	
FlatLinearOperator(*args, **kwargs)	Square Linear operator acting on numpy arrays based	
	on a matvec acting on npc Arrays.	
NpcLinearOperator()	Prototype for a Linear Operator acting on Array.	
NpcLinearOperatorWrapper(orig_operator)	Base class for wrapping around another	
	NpcLinearOperator.	
OrthogonalNpcLinearOperator(orig_operator,	Replace H -> P H P with the projector P = 1 -	
)	sum_o o> <o .< th=""></o .<>	
ShiftNpcLinearOperator(orig_operator, shift)	Representes original_operator + shift *	
	identity.	
SumNpcLinearOperator(orig_operator,)	Sum of two linear operators.	

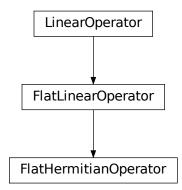
18.5.1 FlatHermitianOperator

 $\bullet \ \ full \ name: tenpy.linalg.sparse.FlatHermitianOperator$

• parent module: tenpy.linalg.sparse

• type: class

Inheritance Diagram



Methods

FlatHermitianOperator.	Initialize this LinearOperator.
init(npc_matvec,)	initialize and zinem operator
FlatHermitianOperator.adjoint()	Hermitian adjoint.
FlatHermitianOperator.dot(x)	Matrix-matrix or matrix-vector multiplication.
FlatHermitianOperator.	Same as FlatLinearOperator(, hermitian=True).
eigenvectors(*args,)	
FlatHermitianOperator.flat_to_npc(vec)	Convert flat numpy vector of selected charge sector into
	npc Array.
FlatHermitianOperator.	Convert flat vector of undetermined charge sectors into
flat_to_npc_None_sector(vec)	npc Array.
FlatHermitianOperator.	Convert flat vector of all charge sectors into npc Array
flat_to_npc_all_sectors(vec)	with extra "charge" leg.
FlatHermitianOperator.	Create a FlatLinearOperator from a square Array.
$from_NpcArray(mat[,])$	
FlatHermitianOperator.	Create a FlatLinearOperator` from a matvec function
from_guess_with_pipe()	acting on multiple legs.
FlatHermitianOperator.matmat(X)	Matrix-matrix multiplication.
FlatHermitianOperator.matvec(x)	Matrix-vector multiplication.
FlatHermitianOperator.	Convert npc Array into a 1D ndarray, inverse of
npc_to_flat(npc_vec)	flat_to_npc().
	continues on next page

Table	21	 continued 	from	previous	page
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FlatHermitianOperator.	Convert npc Array with qtotal = self.charge_sector into
<pre>npc_to_flat_all_sectors(npc_vec)</pre>	ndarray.
FlatHermitianOperator.rmatmat(X)	Adjoint matrix-matrix multiplication.
FlatHermitianOperator.rmatve $c(x)$	Adjoint matrix-vector multiplication.
FlatHermitianOperator.transpose()	Transpose this linear operator.

Class Attributes and Properties

FlatHermitianOperator.H	Hermitian adjoint.
FlatHermitianOperator.T	Transpose this linear operator.
FlatHermitianOperator.charge_sector	Charge sector of the vector which is acted on.

FlatHermitianOperator.ndim

class tenpy.linalg.sparse.FlatHermitianOperator(*args, **kwargs)

Bases: tenpy.linalg.sparse.FlatLinearOperator

Hermitian variant of FlatLinearOperator.

Note that we don't check matvec() to return a hermitian result, we only define an adjoint to be self.

property H

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self.H instead of self.adjoint().

Returns A_H – Hermitian adjoint of self.

Return type LinearOperator

property T

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

adjoint()

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self.H instead of self.adjoint().

Returns A H – Hermitian adjoint of self.

Return type LinearOperator

property charge_sector

Charge sector of the vector which is acted on.

dot(x)

Matrix-matrix or matrix-vector multiplication.

Parameters x (array_like) – 1-d or 2-d array, representing a vector or matrix.

Returns Ax - 1-d or 2-d array (depending on the shape of x) that represents the result of applying this linear operator on x.

Return type array

flat_to_npc(vec)

Convert flat numpy vector of selected charge sector into npc Array.

If charge_sector is not None, convert to a 1D npc vector with leg self.leg. Otherwise convert vec, which can be non-zero in all charge sectors, to a npc matrix with an additional 'charge' leg to allow representing the full vector at once.

Parameters vec (1D ndarray) – Numpy vector to be converted. Should have the entries according to self.charge_sector.

Returns npc_vec – Same as *vec*, but converted into a npc array.

Return type Array

flat_to_npc_None_sector (vec, cutoff=1e-10)

Convert flat vector of undetermined charge sectors into npc Array.

The charge sector to be used is chosen as the block with the maximal norm, not by *self.charge_sector* (which might be *None*).

Parameters vec (1D ndarray) - Numpy vector to be converted.

Returns npc_vec – Same as *vec*, but converted into a npc array.

Return type Array

flat_to_npc_all_sectors(vec)

Convert flat vector of all charge sectors into npc Array with extra "charge" leg.

Deprecated since version 0.7.3: This is merged into $flat_to_npc()$ with self.charge_sector = None.

Parameters vec (1D ndarray) – Numpy vector to be converted.

Returns npc_vec – Same as *vec*, but converted into a npc array.

Return type Array

classmethod from_NpcArray (mat, charge_sector=0)

Create a FlatLinearOperator from a square Array.

Parameters

- mat (Array) A square matrix, with contractable legs.
- **charge_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., *assumes* the dominant eigenvector is in charge sector 0.

Create a FlatLinearOperator` from a matvec function acting on multiple legs.

This function creates a wrapper *matvec* function to allow acting on a "vector" with multiple legs. The wrapper combines the legs into a *LegPipe* before calling the actual *matvec* function, and splits them again in the end.

Parameters

- npc_matvec (function) Function to calculate the action of the linear operator on an npc vector with the given split labels *labels_split*. Has to return an npc vector with the same legs.
- **v0_guess** (Array) Initial guess/starting vector which can be applied to *npc_matvec*.
- labels_split (None / list of str) Labels of v0_guess in the order in which they are to be combined into a LegPipe. None defaults to v0_guess.get_leg_labels().
- **dtype** (np.dtype / None) The data type of the arrays. None defaults to dtype of v0_guess (!).

Returns

- lin_op (cls) Instance of the class to be used as linear operator
- **guess_flat** (*np.ndarray*) Numpy vector representing the guess *v0_guess*.

matmat(X)

Matrix-matrix multiplication.

Performs the operation y=A*X where A is an MxN linear operator and X dense N*K matrix or ndarray.

```
Parameters X ({matrix, ndarray}) - An array with shape (N,K).
```

Returns Y - A matrix or ndarray with shape (M,K) depending on the type of the X argument.

Return type {matrix, ndarray}

Notes

This matmat wraps any user-specified matmat routine or overridden _matmat method to ensure that y has the correct type.

matvec(x)

Matrix-vector multiplication.

Performs the operation y=A*x where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters x (\{matrix, ndarray\}) – An array with shape (N,) or (N,1).
```

Returns y - A matrix or ndarray with shape (M,) or (M,1) depending on the type and shape of the x argument.

Return type {matrix, ndarray}

Notes

This matvec wraps the user-specified matvec routine or overridden _matvec method to ensure that y has the correct shape and type.

```
npc_to_flat (npc_vec)
```

Convert npc Array into a 1D ndarray, inverse of flat_to_npc().

Parameters npc_vec(Array) - Npc Array to be converted. If self.charge_sector is not None, this should be a 1D array with that qtotal. If self.charge_sector is not None, it should have an additional "charge" leg, (as returned by flat_to_npc()).

Returns vec – Same entries as *npc_vec*, but converted into a flat Numpy array.

Return type 1D ndarray

npc to flat all sectors (npc vec)

Convert npc Array with qtotal = self.charge_sector into ndarray.

Deprecated since version 0.7.3: This is merged into $npc_to_flat()$ with self.charge_sector = None.

Parameters npc_vec (Array) – Npc Array to be converted. Should only have entries in self.charge_sector.

Returns vec – Same as *npc_vec*, but converted into a flat Numpy array.

Return type 1D ndarray

${\tt rmatmat}\,(X)$

Adjoint matrix-matrix multiplication.

Performs the operation $y = A^H * x$ where A is an MxN linear operator and x is a column vector or 1-d array, or 2-d array. The default implementation defers to the adjoint.

```
Parameters X({matrix, ndarray}) – A matrix or 2D array.
```

Returns Y – A matrix or 2D array depending on the type of the input.

Return type {matrix, ndarray}

Notes

This rmatmat wraps the user-specified rmatmat routine.

rmatvec(x)

Adjoint matrix-vector multiplication.

Performs the operation $y = A^H * x$ where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters x (\{matrix, ndarray\}) – An array with shape (M,) or (M,1).
```

Returns y - A matrix or ndarray with shape (N,) or (N,1) depending on the type and shape of the x argument.

Return type {matrix, ndarray}

Notes

This rmatvec wraps the user-specified rmatvec routine or overridden _rmatvec method to ensure that y has the correct shape and type.

transpose()

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

eigenvectors (*args, **kwargs)

Same as FlatLinearOperator(..., hermitian=True).

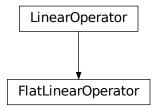
18.5.2 FlatLinearOperator

• full name: tenpy.linalg.sparse.FlatLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

Inheritance Diagram



Methods

FlatLinearOperatorinit(npc_matvec,	Initialize this LinearOperator.
$leg, \dots)$	
FlatLinearOperator.adjoint()	Hermitian adjoint.
FlatLinearOperator.dot(x)	Matrix-matrix or matrix-vector multiplication.
FlatLinearOperator.	Find (dominant) eigenvector(s) of self using scipy.
eigenvectors([num_ev,])	sparse.linalg.eigs().
FlatLinearOperator.flat_to_npc(vec)	Convert flat numpy vector of selected charge sector into
	npc Array.
FlatLinearOperator.	Convert flat vector of undetermined charge sectors into
flat_to_npc_None_sector(vec)	npc Array.
FlatLinearOperator.	Convert flat vector of all charge sectors into npc Array
flat_to_npc_all_sectors(vec)	with extra "charge" leg.
FlatLinearOperator.from_NpcArray(mat[,	Create a FlatLinearOperator from a square Array.
])	
FlatLinearOperator.	Create a FlatLinearOperator` from a matvec function
from_guess_with_pipe()	acting on multiple legs.
FlatLinearOperator. $\mathtt{matmat}(X)$	Matrix-matrix multiplication.
FlatLinearOperator.matvec(x)	Matrix-vector multiplication.
FlatLinearOperator.npc_to_flat(npc_vec)	Convert npc Array into a 1D ndarray, inverse of
	<pre>flat_to_npc().</pre>
FlatLinearOperator.	Convert npc Array with qtotal = self.charge_sector into
<pre>npc_to_flat_all_sectors(npc_vec)</pre>	ndarray.
FlatLinearOperator.rmatmat (X)	Adjoint matrix-matrix multiplication.
FlatLinearOperator.rmatvec(x)	Adjoint matrix-vector multiplication.
FlatLinearOperator.transpose()	Transpose this linear operator.

Class Attributes and Properties

FlatLinearOperator.H	Hermitian adjoint.
FlatLinearOperator.T	Transpose this linear operator.
FlatLinearOperator.charge_sector	Charge sector of the vector which is acted on.

FlatLinearOperator.ndim

class tenpy.linalg.sparse.FlatLinearOperator(*args, **kwargs)

Bases: scipy.sparse.linalg.interface.LinearOperator

Square Linear operator acting on numpy arrays based on a *matvec* acting on npc Arrays.

Note that this class represents a square linear operator. In terms of charges, this means it has legs [self.leg.conj(), self.leg] and trivial (zero) qtotal.

Parameters

- npc_matvec (function) Function to calculate the action of the linear operator on an npc vector (with the specified *leg*). Has to return an npc vector with the same leg.
- leg (LegCharge) Leg of the vector on which npc_matvec can act on.
- **dtype** (np.dtype) The data type of the arrays.
- **charge_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., *assumes* the dominant eigenvector is in charge sector 0.
- **vec_label** (None | str) Label to be set to the npc vector before acting on it with npc_matvec. Ignored if None.

possible_charge_sectors

Each row corresponds to one possible choice for *charge_sector*.

Type ndarray[QTYPE, ndim=2]

shape

The dimensions for the selected charge sector.

Type (int, int)

dtype

The data type of the arrays.

Type np.dtype

leg

Leg of the vector on which *npc_matvec* can act on.

Type LegCharge

vec label

Label to be set to the npc vector before acting on it with *npc_matvec*. Ignored if *None*.

Type None | str

npc_matvec

Function to calculate the action of the linear operator on an npc vector (with one *leg*).

Type function

matvec count

The number of times *npc* matvec was called.

```
Type int
```

mask

The indices of *leg* corresponding to the *charge_sector* to be diagonalized.

Type ndarray[ndim=1, bool]

_npc_matvec_multileg

Only set if initalized with <code>from_guess_with_pipe()</code>. The <code>npc_matvec</code> function to be wrapped around. Takes the npc Array in multidimensional form and returns it that way.

Type function | None

_labels_split

Only set if initalized with from_guess_with_pipe(). Labels of the guess before combining them into a pipe (stored as *leg*).

Type list of str

classmethod from_NpcArray (mat, charge_sector=0)

Create a *FlatLinearOperator* from a square *Array*.

Parameters

- mat (Array) A square matrix, with contractable legs.
- **charge_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., *assumes* the dominant eigenvector is in charge sector 0.

```
classmethod from_guess_with_pipe (npc_matvec, v0_guess, labels_split=None, dtype=None)
```

Create a FlatLinearOperator` from a matvec function acting on multiple legs.

This function creates a wrapper *matvec* function to allow acting on a "vector" with multiple legs. The wrapper combines the legs into a LegPipe before calling the actual *matvec* function, and splits them again in the end.

Parameters

- npc_matvec (function) Function to calculate the action of the linear operator on an npc vector with the given split labels *labels_split*. Has to return an npc vector with the same legs.
- **v0_guess** (*Array*) Initial guess/starting vector which can be applied to *npc_matvec*.
- labels_split (None / list of str) Labels of v0_guess in the order in which they are to be combined into a LegPipe. None defaults to v0_guess. get leg labels().
- dtype (np.dtype / None) The data type of the arrays. None defaults to dtype of vO_guess (!).

Returns

- lin_op (cls) Instance of the class to be used as linear operator
- **guess_flat** (*np.ndarray*) Numpy vector representing the guess *v0_guess*.

property charge_sector

Charge sector of the vector which is acted on.

flat_to_npc(vec)

Convert flat numpy vector of selected charge sector into npc Array.

If charge_sector is not None, convert to a 1D npc vector with leg self.leg. Otherwise convert vec, which can be non-zero in all charge sectors, to a npc matrix with an additional 'charge' leg to allow representing the full vector at once.

Parameters vec (1D ndarray) – Numpy vector to be converted. Should have the entries according to self.charge_sector.

Returns npc_vec – Same as *vec*, but converted into a npc array.

Return type Array

npc_to_flat (npc_vec)

Convert npc Array into a 1D ndarray, inverse of flat_to_npc().

Parameters npc_vec (Array) - Npc Array to be converted. If self.charge_sector is not None, this should be a 1D array with that qtotal. If self.charge_sector is not None, it should have an additional "charge" leg, (as returned by flat_to_npc()).

Returns vec – Same entries as *npc_vec*, but converted into a flat Numpy array.

Return type 1D ndarray

flat_to_npc_all_sectors(vec)

Convert flat vector of all charge sectors into npc Array with extra "charge" leg.

Deprecated since version 0.7.3: This is merged into $flat_to_npc()$ with self.charge_sector = None.

Parameters vec (1D ndarray) – Numpy vector to be converted.

Returns npc_vec – Same as *vec*, but converted into a npc array.

Return type Array

flat_to_npc_None_sector (vec, cutoff=1e-10)

Convert flat vector of undetermined charge sectors into npc Array.

The charge sector to be used is chosen as the block with the maximal norm, not by *self.charge_sector* (which might be *None*).

Parameters vec (1D ndarray) – Numpy vector to be converted.

Returns npc_vec – Same as *vec*, but converted into a npc array.

Return type Array

npc to flat all sectors (npc vec)

Convert npc Array with qtotal = self.charge sector into ndarray.

Deprecated since version 0.7.3: This is merged into $npc_to_flat()$ with self.charge_sector = None.

Parameters npc_vec (Array) - Npc Array to be converted. Should only have entries in self.charge_sector.

Returns vec – Same as *npc_vec*, but converted into a flat Numpy array.

Return type 1D ndarray

If no charge sector was selected, we look in all charge sectors.

Parameters

- num_ev (int) Number of eigenvalues/vectors to look for.
- max_num_ev (int) scipy.sparse.linalg.speigs() somtimes raises a No-ConvergenceError for small num_ev, which might be avoided by increasing num_ev. As a work-around, we try it again in the case of an error, just with larger num_ev up to max_num_ev. None defaults to num_ev + 2.
- max_tol (float) After the first NoConvergenceError we increase the tol argument to that value.
- which (str) Which eigenvalues to look for, see scipy.sparse.linalg. eigs(). More details also in argsort().
- **v0** (Array) Initial guess as a "flat" numpy array.
- v0_npc (Array) Initial guess, to be converted by npc_to_flat().
- **cutoff** (*float*) Only used if self.charge_sector is None; in that case it determines when entries in a given charge-block are considered nonzero, and what counts as degenerate.
- hermitian (bool) If False (default), use scipy.sparse.linalg.eigs() If True, assume that self is hermitian and use scipy.sparse.linalg.eigsh().
- **kwargs Further keyword arguments given to scipy.sparse.linalg.eigsh() or scipy.sparse.linalg.eigs(), respectively.

Returns

- eta (1D ndarray) The eigenvalues, sorted according to which.
- w (list of Array) The eigenvectors corresponding to eta, as npc.Array with LegPipe.

property H

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self. H instead of self.adjoint().

Returns A_H – Hermitian adjoint of self.

Return type LinearOperator

property T

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

adjoint()

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self. H instead of self.adjoint().

Returns A_H – Hermitian adjoint of self.

Return type LinearOperator

dot(x)

Matrix-matrix or matrix-vector multiplication.

Parameters x (array_like) – 1-d or 2-d array, representing a vector or matrix.

Returns Ax - 1-d or 2-d array (depending on the shape of x) that represents the result of applying this linear operator on x.

Return type array

matmat(X)

Matrix-matrix multiplication.

Performs the operation y=A*X where A is an MxN linear operator and X dense N*K matrix or ndarray.

```
Parameters X (\{matrix, ndarray\}) – An array with shape (N,K).
```

Returns Y – A matrix or ndarray with shape (M,K) depending on the type of the X argument.

Return type {matrix, ndarray}

Notes

This matmat wraps any user-specified matmat routine or overridden _matmat method to ensure that y has the correct type.

matvec(x)

Matrix-vector multiplication.

Performs the operation y=A*x where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters \mathbf{x} ({matrix, ndarray}) - An array with shape (N,) or (N,1).
```

Returns y - A matrix or ndarray with shape (M,) or (M,1) depending on the type and shape of the x argument.

Return type {matrix, ndarray}

Notes

This matvec wraps the user-specified matvec routine or overridden _matvec method to ensure that y has the correct shape and type.

rmatmat(X)

Adjoint matrix-matrix multiplication.

Performs the operation $y = A^H * x$ where A is an MxN linear operator and x is a column vector or 1-d array, or 2-d array. The default implementation defers to the adjoint.

Parameters X({matrix, ndarray}) – A matrix or 2D array.

Returns Y – A matrix or 2D array depending on the type of the input.

Return type {matrix, ndarray}

Notes

This rmatmat wraps the user-specified rmatmat routine.

rmatvec(x)

Adjoint matrix-vector multiplication.

Performs the operation $y = A^H * x$ where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters x (\{matrix, ndarray\}) – An array with shape (M,) or (M,1).
```

Returns y - A matrix or ndarray with shape (N,) or (N,1) depending on the type and shape of the x argument.

Return type {matrix, ndarray}

Notes

This rmatvec wraps the user-specified rmatvec routine or overridden _rmatvec method to ensure that y has the correct shape and type.

transpose()

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

18.5.3 NpcLinearOperator

• full name: tenpy.linalg.sparse.NpcLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

Inheritance Diagram

NpcLinearOperator

Methods

NpcLinearOperatorinit()	Initialize self.
NpcLinearOperator.adjoint()	Return the hermitian conjugate of self
NpcLinearOperator.matvec(vec)	Calculate the action of the operator on a vector <i>vec</i> .
NpcLinearOperator.to_matrix()	Contract <i>self</i> to a matrix.

Class Attributes and Properties

NpcLinearOperator.acts_on

class tenpy.linalg.sparse.NpcLinearOperator

Bases: object

Prototype for a Linear Operator acting on Array.

Note that an *Array* implements a matvec function. Thus you can use any (square) npc Array as an NpcLinear-Operator.

dtype

The data type of its action.

Type np.type

acts_on

Labels of the state on which the operator can act. NB: Class attribute.

Type list of str

matvec(vec)

Calculate the action of the operator on a vector vec.

Note that we don't require *vec* to be one-dimensional. However, for square operators we require that the result of *matvec* has the same legs (in the same order) as *vec* such that they can be added. Note that this excludes a non-trivial *qtotal* for square operators.

to_matrix()

Contract self to a matrix.

If *self* represents an operator with very small shape, e.g. because the MPS bond dimension is very small, an algorithm might choose to contract *self* to a single tensor.

Returns matrix – Contraction of the represented operator.

Return type Array

adjoint()

Return the hermitian conjugate of self

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

18.5.4 NpcLinearOperatorWrapper

• full name: tenpy.linalg.sparse.NpcLinearOperatorWrapper

• parent module: tenpy.linalg.sparse

• type: class

Inheritance Diagram

NpcLinearOperatorWrapper

Methods

NpcLinearOperatorWrapper.	Initialize self.
init(orig_operator)	
NpcLinearOperatorWrapper.adjoint()	Return the hermitian conjugate of <i>self</i> .
NpcLinearOperatorWrapper.to_matrix()	Contract <i>self</i> to a matrix.
NpcLinearOperatorWrapper.unwrapped()	Return to the original NpcLinearOperator.

class tenpy.linalg.sparse.NpcLinearOperatorWrapper(orig_operator)

Bases: object

Base class for wrapping around another NpcLinearOperator.

Attributes not explicitly set with self.attribute = value (or by defining methods) default to the attributes of the wrapped $orig_operator$.

Warning: If there are multiple levels of wrapping operators, the order might be critical to get correct results; e.g. <code>OrthogonalNpcLinearOperator</code> needs to be the outer-most wrapper to produce correct results and/or be efficient.

Parameters orig_operator (NpcLinearOperator) - The original operator implementing the *matvec*.

orig_operator

The original operator implementing the *matvec*.

Type NpcLinearOperator

unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

to matrix()

Contract self to a matrix.

adjoint()

Return the hermitian conjugate of self.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

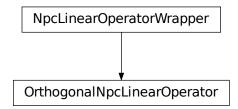
18.5.5 OrthogonalNpcLinearOperator

• full name: tenpy.linalg.sparse.OrthogonalNpcLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

Inheritance Diagram



Methods

OrthogonalNpcLinearOperatorinit()	Initialize self.
OrthogonalNpcLinearOperator.adjoint()	Return the hermitian conjugate of <i>self</i> .
OrthogonalNpcLinearOperator.	
matvec(vec)	
OrthogonalNpcLinearOperator.	Contract self to a matrix.
to_matrix()	
OrthogonalNpcLinearOperator.	Return to the original NpcLinearOperator.
unwrapped()	

```
\verb|class| tenpy.linalg.sparse.OrthogonalNpcLinearOperator| (|orig\_operator, ortho\_vecs)|
```

Bases: tenpy.linalg.sparse.NpcLinearOperatorWrapper

Replace $H \rightarrow P H P$ with the projector $P = 1 - sum_o |o> <o|$.

Here, | o> are the vectors from ortho_vecs.

Parameters

• orig_operator (EffectiveH) – The original *EffectiveH* instance to wrap around.

• ortho_vecs (list of Array) – The vectors to orthogonalize against.

to_matrix()

Contract *self* to a matrix.

adjoint()

Return the hermitian conjugate of *self*.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

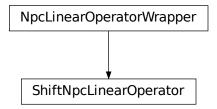
18.5.6 ShiftNpcLinearOperator

• full name: tenpy.linalg.sparse.ShiftNpcLinearOperator

• parent module: tenpy.linalg.sparse

· type: class

Inheritance Diagram



Methods

ShiftNpcLinearOperatorinit()	Initialize self.
ShiftNpcLinearOperator.adjoint()	Return the hermitian conjugate of <i>self</i> .
ShiftNpcLinearOperator.matvec(vec)	
billienpelinearoperacor: macvec(vec)	
omitempelimentoperator.matvee(100)	
ShiftNpcLinearOperator.to_matrix()	Contract <i>self</i> to a matrix.

```
class tenpy.linalg.sparse.ShiftNpcLinearOperator(orig_operator, shift)
```

Bases: tenpy.linalg.sparse.NpcLinearOperatorWrapper

Representes original_operator + shift * identity.

This can be useful e.g. for better Lanczos convergence.

to matrix()

Contract self to a matrix.

adjoint()

Return the hermitian conjugate of *self*.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

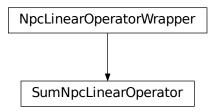
18.5.7 SumNpcLinearOperator

• full name: tenpy.linalg.sparse.SumNpcLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

Inheritance Diagram



Methods

SumNpcLinearOperator.	Initialize self.
init(orig_operator,)	
SumNpcLinearOperator.adjoint()	Return the hermitian conjugate of <i>self</i> .
SumNpcLinearOperator.matvec(vec)	
SumNpcLinearOperator.to_matrix()	Contract self to a matrix.
SumNpcLinearOperator.unwrapped()	Return to the original NpcLinearOperator.

class tenpy.linalg.sparse.SumNpcLinearOperator(orig_operator, other_operator)

Bases: tenpy.linalg.sparse.NpcLinearOperatorWrapper

Sum of two linear operators.

to_matrix()

Contract self to a matrix.

adjoint()

Return the hermitian conjugate of self.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

Module description

Providing support for sparse algorithms (using matrix-vector products only).

Some linear algebra algorithms, e.g. Lanczos, do not require the full representations of a linear operator, but only the action on a vector, i.e., a matrix-vector product *matvec*. Here we define the structure of such a general operator, NpcLinearOperator, as it is used in our own implementations of these algorithms (e.g., lanczos). Moreover, the FlatLinearOperator allows to use all the scipy sparse methods by providing functionality to convert flat numpy arrays to and from np_conserved arrays.

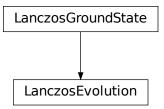
18.6 lanczos

• full name: tenpy.linalg.lanczos

• parent module: tenpy.linalg

• type: module

Classes



LanczosEvolution(H, psi0, options)	Calculate $exp(deltaH) psi0>$ using Lanczos.
$\texttt{LanczosGroundState}(H, psi0, options[, \dots])$	Lanczos algorithm working on npc arrays.

Functions

<pre>gram_schmidt(vecs[, rcond, verbose])</pre>	In place Gram-Schmidt Orthogonalization and normal-
	ization for npc Arrays.
lanczos(H, psi[, options, orthogonal_to])	Simple wrapper calling LanczosGroundState (H,
	<pre>psi, options, orthogonal_to).run()</pre>
<pre>lanczos_arpack(H, psi[, options, orthogonal_to])</pre>	Use scipy.sparse.linalg.eigsh() to find the
	ground state of H .
plot_stats(ax, Es)	Plot the convergence of the energies.

18.6.1 gram_schmidt

• full name: tenpy.linalg.lanczos.gram_schmidt

• parent module: tenpy.linalg.lanczos

• type: function

tenpy.linalg.lanczos.gram_schmidt (vecs, rcond=1e-14, verbose=None) In place Gram-Schmidt Orthogonalization and normalization for npc Arrays.

Parameters

- **vecs** (list of *Array*) The vectors which should be orthogonalized. All with the same *order* of the legs. Entries are modified *in place*. if a norm < rcond, the entry is set to *None*.
- rcond (float) Vectors of norm < rcond (after projecting out previous vectors) are discarded.

Returns

- vecs (*list of Array*) The ortho-normalized vectors (without any None).
- ov (2D Array) For j >= i, ov[j, i] = npc.inner(vecs[j], vecs[i], 'range', do_conj=True) (where vecs[j] was orthogonalized to all vecs[k], k < i).

18.6.2 lanczos

• full name: tenpy.linalg.lanczos.lanczos

• parent module: tenpy.linalg.lanczos

· type: function

tenpy.linalg.lanczos.lanczos(*H*, *psi*, *options={}*, *orthogonal_to={}*])
Simple wrapper calling LanczosGroundState(H, psi, options, orthogonal_to).run()

Deprecated since version 0.6.0: Going to remove the *orthogonal_to* argument. Instead, replace H with *OrthogonalNpcLinearOperator(H, orthogonal_to)* using the *OrthogonalNpcLinearOperator*.

Parameters

- **H See** LanczosGroundState.
- **psi** See LanczosGroundState.
- options See LanczosGroundState.
- orthogonal_to See LanczosGroundState.

18.6. lanczos 349

```
Returns See LanczosGroundState.run().
Return type E0, psi0, N
```

18.6.3 lanczos arpack

• full name: tenpy.linalg.lanczos.lanczos arpack

• parent module: tenpy.linalg.lanczos

• type: function

```
tenpy.linalg.lanczos.lanczos_arpack (H, psi, options={}, orthogonal_to=[]) Use scipy.sparse.linalg.eigsh() to find the ground state of H.
```

This function has the same call/return structure as <code>lanczos()</code>, but uses the ARPACK package through the functions <code>speigsh()</code> instead of the custom lanczos implementation in <code>LanczosGroundState</code>. This function is mostly intended for debugging, since it requires to convert the vector from np_conserved <code>Array</code> into a flat numpy array and back during <code>each matvec-operation!</code>

Deprecated since version 0.6.0: Going to remove the *orthogonal_to* argument. Instead, replace H with *OrthogonalNpcLinearOperator(H, orthogonal_to)* using the *OrthogonalNpcLinearOperator*.

Parameters

- **H** See LanczosGroundState. *H* and *psi* should have/use labels.
- **psi** See LanczosGroundState. *H* and *psi* should have/use labels.
- options See LanczosGroundState. H and psi should have/use labels.
- orthogonal_to See LanczosGroundState. H and psi should have/use labels.

Returns

- **E0** (*float*) Ground state energy.
- **psi0** (*Array*) Ground state vector.

18.6.4 plot stats

• full name: tenpy.linalg.lanczos.plot_stats

• parent module: tenpy.linalg.lanczos

• type: function

tenpy.linalg.lanczos.plot_stats(ax, Es)

Plot the convergence of the energies.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \mathbf{Es} (list of ndarray.) \mathbf{The} energies Lanczos. \mathbf{Es} .

Module description

Lanczos algorithm for np_conserved arrays.

18.6. lanczos 351

CHAPTER

NINETEEN

MODELS

full name: tenpy.modelsparent module: tenpy

• type: module

Module description

Definition of the various models.

For an introduction to models see *Models*.

The module <code>tenpy.models.model</code> contains base classes for models. The module <code>tenpy.models.lattice</code> contains base classes and implementations of lattices. All other modules in this folder contain model classes derived from these base classes.

Submodules

lattice	Classes to define the lattice structure of a model.
model	This module contains some base classes for models.

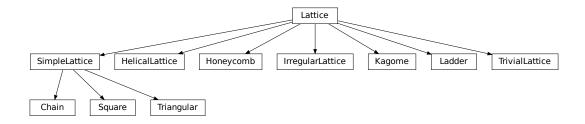
19.1 lattice

• full name: tenpy.models.lattice

• parent module: tenpy.models

• type: module

Classes



Chain(L, site, **kwargs)	A chain of L equal sites.
HelicalLattice(regular_lattice, N_unit_cells)	Translation invariant version of a tilted, regular 2D lat-
	tice.
Honeycomb(Lx, Ly, sites, **kwargs)	A honeycomb lattice.
<pre>IrregularLattice(regular_lattice[, remove,])</pre>	A variant of a regular lattice, where we might have extra
	sites or sites missing.
Kagome(Lx, Ly, sites, **kwargs)	A Kagome lattice.
Ladder(L, sites, **kwargs)	A ladder coupling two chains.
Lattice(Ls, unit_cell[, order, bc, bc_MPS,])	A general, regular lattice.
SimpleLattice(Ls, site, **kwargs)	A lattice with a unit cell consiting of just a single site.
Square(Lx, Ly, site, **kwargs)	A square lattice.
Triangular(Lx, Ly, site, **kwargs)	A triangular lattice.
TrivialLattice(mps_sites, **kwargs)	Trivial lattice consisting of a single (possibly large) unit
	cell in 1D.

19.1.1 Chain

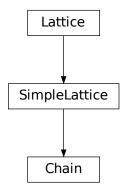
• full name: tenpy.models.lattice.Chain

• parent module: tenpy.models.lattice

• type: class

354 Chapter 19. models

Inheritance Diagram



Methods

Chaininit(L, site, **kwargs)	Initialize self.		
Chain.count_neighbors([u, key])	Count e.g.		
Chain.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.		
Chain.distance(u1, u2, dx)	Get the distance for a given coupling between two sites		
	in the lattice.		
Chain.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-		
	tions; in place.		
Chain.find_coupling_pairs([max_dx, cutoff,	Automatically find coupling pairs grouped by distances.		
eps])			
Chain.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.		
Chain.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,		
	u) to MPS index i.		
Chain.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,		
	x_{dim-1} , u).		
Chain.mps2lat_values(A[, axes, u])	<pre>same as Lattice.mps2lat_values(), but ignore</pre>		
	u, setting it to 0.		
Chain.mps2lat_values_masked($A[, axes,]$)	Reshape/reorder an array A to replace an MPS index by		
	lattice indices.		
Chain.mps_idx_fix_u([u])	return an index array of MPS indices for which the site		
	within the unit cell is u .		
Chain.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the		
	corresponding lattice indices.		
Chain.mps_sites()	Return a list of sites for all MPS indices.		
Chain.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a		
	multi_coupling.		
Chain.number_nearest_neighbors([u])	Deprecated.		
Chain.number_next_nearest_neighbors($[u]$)			
Chain.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.		
	continues on next page		

Table 3 – continued from previous page

Chain.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.	
Chain.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-	
])	tions.	
Chain.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.	
Chain.plot_order(ax[, order, textkwargs])	Plot a line connecting sites in the specified "order" and	
	text labels enumerating them.	
Chain.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.	
Chain.position(lat_idx)	return 'space' position of one or multiple sites.	
Chain.possible_couplings(u1, u2, dx[,	Find possible MPS indices for two-site couplings.	
strength])		
Chain.possible_multi_couplings(ops[,	Generalization of possible_couplings() to cou-	
strength])	plings with more than 2 sites.	
Chain.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.	
Chain.site(i)	return Site instance corresponding to an MPS index i	
Chain.test_sanity()	Sanity check.	

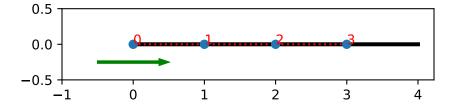
Class Attributes and Properties

Chain.Lu	the (expected) number of sites in the unit cell,
	len(unit_cell).
Chain.boundary_conditions	Human-readable list of boundary conditions from bc
	<pre>and bc_shift.</pre>
Chain.dim	the dimension of the lattice
Chain.nearest_neighbors	
Chain.next_nearest_neighbors	
Chain.next_next_nearest_neighbors	
Chain.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.Chain (L, site, **kwargs)

Bases: tenpy.models.lattice.SimpleLattice

A chain of L equal sites.



Parameters

- $\mathbf{L}(int)$ The length of the chain.
- ${\tt site}\,({\it Site})\,{\tt -}\,{\tt The}\,\,{\tt local}\,\,{\tt lattice}\,\,{\tt site}.$ The ${\it unit_cell}\,\,{\tt of}\,\,{\tt the}\,\,\,{\it Lattice}\,\,{\tt is}\,\,{\tt just}\,\,\,[\,{\tt site}]\,.$

**kwargs - Additional keyword arguments given to the Lattice. pairs are initialize
with [next_]next_]nearest_neighbors. positions can be specified as a single
vector.

dim = 1

the dimension of the lattice

ordering(order)

Provide possible orderings of the *N* lattice sites.

The following orders are defined in this method compared to Lattice.ordering():

or-	Resulting order			
der				
'def	aull,t'1, 2, 3, 4,, L-1			
'fol	olded,' L-1, 1, L-2,, L//2. This order might be usefull if you want to consider			
	ring with periodic boundary conditions with a finite MPS: It avoids the ultra-long range of the			
	coupling from site 0 to L present in the default order.			

property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

```
count_neighbors (u=0, key='nearest_neighbors')
```

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- u (int) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever key specifies).
- **key** (str) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The *u1*, *u2*, *dx* parameters are defined in analogy with *add_coupling()*, i.e., this function calculates the distance between a pair of operators added with *add_coupling* (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- **u1** (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

```
enlarge_mps_unit_cell(factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

```
find_coupling_pairs (max_dx=3, cutoff=None, eps=1e-10)
```

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

• hdf5_loader (Hdf5Loader) - Instance of the loading engine.

- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps idx(lat idx)

Translate lattice indices (x_0, \dots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx (array_like [.., dim+1]) - The last dimension corresponds to lattice indices (x_0, ..., x_{D-1}, u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS, an x_0 outside indicates shifts across the boundary.

Returns i – MPS index/indices corresponding to lat_idx . Has the same shape as lat_idx without the last dimension.

Return type array_like

$mps2lat_idx(i)$

Translate MPS index i to lattice indices $(x_0, \ldots, x_{\min-1})$, u).

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``(x_0 , ..., x_d) corresponding to i. For i accross the MPS unit cell and "infinite" bc_M PS, we shift x_0 accordingly.

Return type array

$mps2lat_values(A, axes=0, u=None)$

same as Lattice.mps2lat_values(), but ignore u, setting it to 0.

mps21at_values_masked(A, axes=-1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps2lat_values$ () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

mps_idx_fix_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u (u=None)

Similar as $mps_idx_fix_u$ (), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- **lat_idx** (2D array) The row j contains the lattice index (without u) corresponding to mps_idx[j].

mps sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape $(N_{ops}, dim)) - dx[i, :]$ is the translation vector in the lattice for the *i*-th operator. Corresponds to the *dx* of each operator given in the argument *ops* of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

number nearest neighbors (u=0)

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot order ().

```
plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)
```

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

```
plot_bc_identified (ax, direction=- 1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.
```

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

```
plot_coupling (ax, coupling=None, wrap=False, **kwargs)
```

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- coupling (list of (u1, u2, dx)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (i0, i1, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

```
plot_order(ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- **textkwargs** (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

position (lat_idx)

return 'space' position of one or multiple sites.

Parameters lat_idx (ndarray, (..., dim+1)) – Lattice indices.

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

Return type ndarray, (..., dim)

possible_couplings (u1, u2, dx, strength=None)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (1D array) (Only returend if strength is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible_multi_couplings (ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

• **mps_ijkl** (2D int array) – Each row contains MPS indices *i,j,k,l,...* for each of the operators positions. The positions are defined by dx (j,k,l,... relative to *i*) and boundary coundary conditions of *self* (how much the *box* for given dx can be shifted around without hitting a boundary - these are the different rows).

- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

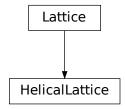
19.1.2 HelicalLattice

• full name: tenpy.models.lattice.HelicalLattice

• parent module: tenpy.models.lattice

• type: class

Inheritance Diagram



Methods

<pre>HelicalLatticeinit(regular_lattice,)</pre>	Initialize self.			
<pre>HelicalLattice.count_neighbors([u, key])</pre>	Count e.g.			
HelicalLattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.			
HelicalLattice.distance(u1, u2, dx)	Get the distance for a given coupling between two sites			
	in the lattice.			
HelicalLattice.enlarge_mps_unit_cell([fa	ncRepeat the unit cell for infinite MPS boundary condi-			
	tions; in place.			
HelicalLattice.find_coupling_pairs([max	_Axutomatically find coupling pairs grouped by distances.			
])				
HelicalLattice.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.			
)				
<pre>HelicalLattice.lat2mps_idx(lat_idx)</pre>	Translate lattice indices (x_0, \ldots, x_{D-1}) ,			
	u) to MPS index i.			
$ extit{HelicalLattice.mps2lat_idx(i)}$	Translate MPS index i to lattice indices (x_0,,			
	x_{dim-1} , u).			
HelicalLattice.mps2lat_values(*args,	Not implemented, use			
**kwargs)	mps2lat_values_masked() instead.			
HelicalLattice.mps2lat_values_masked(*a	• • •			
)	lattice indices.			
HelicalLattice.mps_idx_fix_u([u])	return an index array of MPS indices for which the site			
	within the unit cell is <i>u</i> .			
<pre>HelicalLattice.mps_lat_idx_fix_u([u])</pre> <pre>Similar as mps_idx_fix_u(), but return all</pre>				
	corresponding lattice indices.			
<pre>HelicalLattice.mps_sites()</pre>	Return a list of sites for all MPS indices.			
HelicalLattice.multi_coupling_shape(dx) Calculate correct shape of the street				
	multi_coupling.			
HelicalLattice.number_nearest_neighbor				
<pre>HelicalLattice.number_next_nearest_nei</pre>				
HelicalLattice.ordering(order) Provide possible orderings of the lattice sites.				
HelicalLattice.plot_basis(ax[, origin,	Plot arrows indicating the basis vectors of the lattice.			
shade])				
<pre>HelicalLattice.plot_bc_identified(ax[,</pre>	Mark two sites indified by periodic boundary condi-			
])	tions.			
	continues on next page			

Table 5 – continued from previous page				
HelicalLattice.plot_coupling(ax[, cou-	Plot lines connecting nearest neighbors of the lattice.			
pling,])				
<pre>HelicalLattice.plot_order(ax[, order,])</pre>	Plot a line connecting sites in the specified "order" and			
	text labels enumerating them.			
<pre>HelicalLattice.plot_sites(ax[, markers])</pre>	Plot the sites of the lattice with markers.			
HelicalLattice.position(lat_idx)	return 'space' position of one or multiple sites.			
HelicalLattice.possible_couplings(u1,	Find possible MPS indices for two-site couplings.			
u2, dx)				
HelicalLattice.possible_multi_coupling	s (ralization of possible_couplings () to cou-			
	plings with more than 2 sites.			
HelicalLattice.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.			
)				
HelicalLattice.site(i)	return $Site$ instance corresponding to an MPS index i			
<pre>HelicalLattice.test_sanity()</pre>	Sanity check.			

Class Attributes and Properties

HelicalLattice.Lu	the (expected) number of sites in the unit cell,	
	len(unit_cell).	
HelicalLattice.boundary_conditions	Human-readable list of boundary conditions from bc	
	<pre>and bc_shift.</pre>	
HelicalLattice.dim	The dimension of the lattice.	
HelicalLattice.nearest_neighbors		
HelicalLattice.next_nearest_neighbors		
HelicalLattice.next_next_nearest_neighbors		
HelicalLattice.order	Defines an ordering of the lattice sites, thus mapping the	
	lattice to a 1D chain.	

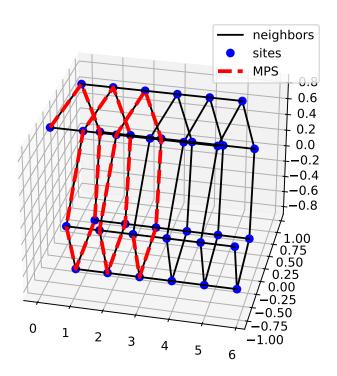
class tenpy.models.lattice.HelicalLattice(regular_lattice, N_unit_cells)
 Bases: tenpy.models.lattice.Lattice

Translation invariant version of a tilted, regular 2D lattice.

A 2D lattice on an infinite cylinder becomes translation invariant by a single *lattice* unit cell if we tilt/shift the boundary conditions around the cylinder such that the unit cell at (x, y=Ly-1) is neighbored by (x+1, y=0), and the MPS winds as a helix around the cylinder. Let's illustrate this for the Square lattice with a single-site unit cell - for a multi-site unit cell, imagine it being inserted at each of the sites of a Square lattice.

Warning: Some assumptions of the regular lattice like "the number of the sites in the MPS unit cell is product (lat.shape)" no longer hold for this model! Be very careful e.g. for getting the units of the correlation_length() right.

Parameters N_unit_cells (int) - Number of lattice unit cells to include into the MPS unit cell. The total number of sites will be N_unit_cells * len(regular_lattice.unit_cell).



```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

ordering(order)

Provide possible orderings of the lattice sites.

Parameters order — Argument for the *Lattice.ordering()* of the regular_lattice, or 2D ndarray providing the order of the *regular lattice*. Note that really the only freedom is the order of the sites in the unit_cell.

Returns order – The order to be used for *order*, i.e. *order* with added/removed sites as specified by remove and add.

Return type array, shape (N, D+1)

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot_order().

mps2lat_idx(i)

Translate MPS index i to lattice indices (x_0, ..., x_{dim-1}, u).

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

$lat2mps_idx(lat_idx)$

Translate lattice indices (x_0, \ldots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx (array_like [.., dim+1]) - The last dimension corresponds to lattice indices (x_0, ..., x_{D-1}, u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS, an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

mps2lat_values (*args, **kwargs)

Not implemented, use mps2lat_values_masked() instead.

mps2lat_values_masked(*args, **kwargs)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps21at_va1ues$ () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool)—Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.
- **Returns** res_A Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

possible_couplings (u1, u2, dx, strength=None)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

• **strength** (array_like / None) - If given, instead of returning lat_indices and coupling shape directly return the correct strength 12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible_multi_couplings (ops, strength=None)

Generalization of possible_couplings() to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,... for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with *s* being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

plot_coupling(ax, coupling=None, wrap=True, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.

- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

property boundary_conditions

Human-readable list of boundary conditions from bc and bc shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count_neighbors (u=0, key='nearest_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

coupling shape (dx)

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to *dx* argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (array) Translation vector from origin to the lower left corner of box spanned by dx.

property dim

The dimension of the lattice.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()

• dx (array) – Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

find coupling pairs (max dx=3, cutoff=None, eps=1e-10)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (float) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

mps_idx_fix_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u (u=None)

Similar as mps idx fix u(), but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- **lat_idx** (2D array) The row j contains the lattice index (without u) corresponding to mps_idx[j].

mps_sites()

Return a list of sites for all MPS indices.

Equivalent to [self.site(i) for i in range(self.N_sites)].

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the i-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

number_nearest_neighbors (u=0)

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

```
plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)
```

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=- 1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit cell positions.
- **kwargs Keyword arguments for the used ax.plot.

```
plot_order (ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', 'n', 's', 'p', 'h', 'D'], **kwargs)
Plot the sites of the lattice with markers.
```

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

position (lat_idx)

return 'space' position of one or multiple sites.

Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

Return type ndarray, (..., dim)

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

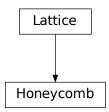
Sanity check.

Raises ValueErrors, if something is wrong.

19.1.3 Honeycomb

- full name: tenpy.models.lattice.Honeycomb
- parent module: tenpy.models.lattice
- type: class

Inheritance Diagram



Methods

Honeycombinit(Lx, Ly, sites, **kwargs)	Initialize self.		
Honeycomb.count_neighbors([u, key])	Count e.g.		
Honeycomb.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.		
Honeycomb.distance(u1, u2, dx)	Get the distance for a given coupling between two sites		
-	in the lattice.		
Honeycomb.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary conditions; in place.		
<pre>Honeycomb.find_coupling_pairs([max_dx,])</pre>	Automatically find coupling pairs grouped by distances.		
Honeycomb.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.		
Honeycomb.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \dots, x_{D-1}) , u) to MPS index <i>i</i> .		
Honeycomb.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,, x_{dim-1}, u).		
Honeycomb.mps2lat_values(A[, axes, u])	Reshape/reorder <i>A</i> to replace an MPS index by lattice indices.		
Honeycomb.mps2lat_values_masked(A[,	Reshape/reorder an array A to replace an MPS index by		
axes,])	lattice indices.		
Honeycomb.mps_idx_fix_u([u])	return an index array of MPS indices for which the site within the unit cell is u .		
Honeycomb.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the corresponding lattice indices.		
Honeycomb.mps_sites()	Return a list of sites for all MPS indices.		
Honeycomb.multi_coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a multi_coupling.		
Honeycomb.number_nearest_neighbors([u])	Deprecated.		
Honeycomb.number_next_nearest_neighbor.			
Honeycomb.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.		
Honeycomb.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.		
Honeycomb.plot_bc_identified(ax[,])			
noneycomb.proc_bc_rdencrrred(ax[,])	Mark two sites indified by periodic boundary condi-		
noneycomb.proc_bc_rdencirred(ax[,])	Mark two sites indified by periodic boundary conditions.		
Honeycomb.plot_coupling(ax[, coupling, wrap])	· ·		
Honeycomb.plot_coupling(ax[, coupling,	tions.		
Honeycomb.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.		
Honeycomb.plot_coupling(ax[, coupling, wrap])	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and		
Honeycomb.plot_coupling(ax[, coupling, wrap]) Honeycomb.plot_order(ax[, order, textkwargs])	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them.		
Honeycomb.plot_coupling(ax[, coupling, wrap]) Honeycomb.plot_order(ax[, order, textkwargs]) Honeycomb.plot_sites(ax[, markers]) Honeycomb.position(lat_idx) Honeycomb.possible_couplings(u1, u2, dx[,	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers.		
Honeycomb.plot_coupling(ax[, coupling, wrap]) Honeycomb.plot_order(ax[, order, textkwargs]) Honeycomb.plot_sites(ax[, markers]) Honeycomb.position(lat_idx) Honeycomb.possible_couplings(u1, u2, dx[,]) Honeycomb.possible_multi_couplings(ops[,	Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to cou-		
Honeycomb.plot_coupling(ax[, coupling, wrap]) Honeycomb.plot_order(ax[, order, textkwargs]) Honeycomb.plot_sites(ax[, markers]) Honeycomb.position(lat_idx) Honeycomb.possible_couplings(u1, u2, dx[,]) Honeycomb.possible_multi_couplings(ops[,])	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to couplings with more than 2 sites.		
Honeycomb.plot_coupling(ax[, coupling, wrap]) Honeycomb.plot_order(ax[, order, textkwargs]) Honeycomb.plot_sites(ax[, markers]) Honeycomb.position(lat_idx) Honeycomb.possible_couplings(u1, u2, dx[,]) Honeycomb.possible_multi_couplings(ops[,]) Honeycomb.save_hdf5(hdf5_saver, h5gr, sub-	Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to cou-		
Honeycomb.plot_coupling(ax[, coupling, wrap]) Honeycomb.plot_order(ax[, order, textkwargs]) Honeycomb.plot_sites(ax[, markers]) Honeycomb.position(lat_idx) Honeycomb.possible_couplings(u1, u2, dx[,]) Honeycomb.possible_multi_couplings(ops[,]) Honeycomb.save_hdf5(hdf5_saver, h5gr, subpath)	Plot a line connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to couplings with more than 2 sites. Export self into a HDF5 file.		
Honeycomb.plot_coupling(ax[, coupling, wrap]) Honeycomb.plot_order(ax[, order, textkwargs]) Honeycomb.plot_sites(ax[, markers]) Honeycomb.position(lat_idx) Honeycomb.possible_couplings(u1, u2, dx[,]) Honeycomb.possible_multi_couplings(ops[,]) Honeycomb.save_hdf5(hdf5_saver, h5gr, sub-	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to couplings with more than 2 sites.		

Class Attributes and Properties

Honeycomb.Lu	the (expected) number of sites in the unit cell,
	len(unit_cell).
Honeycomb.boundary_conditions	Human-readable list of boundary conditions from bc
	and bc_shift.
Honeycomb.dim	the dimension of the lattice
Honeycomb.fifth_nearest_neighbors	
Honeycomb.fourth_nearest_neighbors	
Honeycomb.nearest_neighbors	
Honeycomb.next_nearest_neighbors	
Honeycomb.next_next_nearest_neighbors	
Honeycomb.order	Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

class tenpy.models.lattice.**Honeycomb** (*Lx*, *Ly*, *sites*, **kwargs)

Bases: tenpy.models.lattice.Lattice

A honeycomb lattice.

Parameters

- **Lx** (*int*) The length in each direction.
- Ly (int) The length in each direction.
- **sites** ((list of) *Site*) The two local lattice sites making the *unit_cell* of the *Lattice*. If only a single *Site* is given, it is used for both sites.
- **kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs are set accordingly. For the Honeycomb lattice 'fourth_nearest_neighbors', 'fifth_nearest_neighbors' are set in pairs.

dim = 2

the dimension of the lattice

Lu = 2

the (expected) number of sites in the unit cell, len (unit_cell).

${\tt ordering}\,(order)$

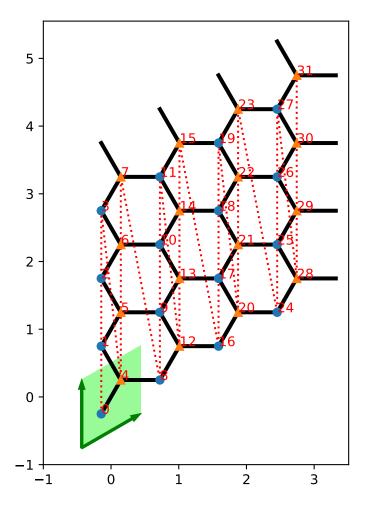
Provide possible orderings of the *N* lattice sites.

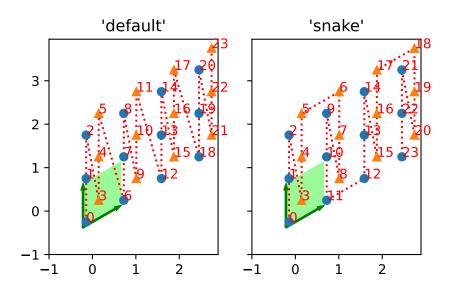
The following orders are defined in this method compared to Lattice.ordering():

order	equivalent <i>priority</i>	equivalent snake_winding	
'default'	(0, 2, 1)	(False, False, False)	
'snake'	(0, 2, 1)	(False, True, False)	

property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.





Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

```
count_neighbors (u=0, key='nearest_neighbors')
```

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (array) Translation vector from origin to the lower left corner of box spanned by dx.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

find_coupling_pairs (*max_dx=3*, *cutoff=None*, *eps=1e-10*)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps_idx (lat_idx)

Translate lattice indices (x 0, ..., x $\{D-1\}$, u) to MPS index i.

Parameters lat_idx ($array_like[.., dim+1]$) - The last dimension corresponds to lattice indices (x_0, ..., x_{D-1}, u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

$mps2lat_idx(i)$

Translate MPS index i to lattice indices (x_0, ..., x_{dim-1}, u).

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``($x_0, \ldots, x_{dim-1}, u$)` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder A to replace an MPS index by lattice indices.

Parameters

- A (ndarray) Some values. Must have A.shape[axes] = self.N_sites if u is None, or A.shape[axes] = self.N_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit_cell[u], as returned by $mps_idx_fix_u()$. The resulting array will not have the additional dimension(s) of u.

Returns res_A – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[tuple(lat.mps2lat_idx(5))] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use $mps_idx_fix_u()$ to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

mps2lat_values_masked (A, axes=- 1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps2lat_values$ () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

mps_idx_fix_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u (u=None)

Similar as $mps_idx_fix_u$ (), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- $lat_idx (2D \ array)$ The row j contains the lattice index (without u) corresponding to $mps_idx[j]$.

mps_sites()

Return a list of sites for all MPS indices.

Equivalent to [self.site(i) for i in range(self.N_sites)].

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the i-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot_order().

```
plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)
```

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=- 1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

plot_coupling(ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

• ax (matplotlib.axes.Axes) - The axes on which we should plot.

- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

```
plot_order (ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- **textkwargs** (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', 'n', 's', 'p', 'h', 'D'], **kwargs)
Plot the sites of the lattice with markers.
```

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

position (lat_idx)

return 'space' position of one or multiple sites.

Parameters lat_idx (ndarray, (..., dim+1)) – Lattice indices.

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

Return type ndarray, (..., dim)

```
possible couplings (u1, u2, dx, strength=None)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to 0 <= x_a < Ls[a] and 0 <= x_a+dx[a] < lat.Ls[a].

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling shape directly return the correct strength 12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible_multi_couplings(ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument ops of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from hdf5().

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

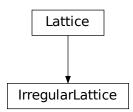
19.1.4 IrregularLattice

• full name: tenpy.models.lattice.IrregularLattice

• parent module: tenpy.models.lattice

• type: class

Inheritance Diagram



Methods

True and and attical	Initialize self.	
<pre>IrregularLatticeinit(regular_lattice[,</pre>	initianze sen.	
])		
IrregularLattice.count_neighbors([u,	Count e.g.	
key])		
IrregularLattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.	
IrregularLattice.distance(u1, u2, dx)	Get the distance for a given coupling between two sites	
	in the lattice.	
IrregularLattice.	Repeat the unit cell for infinite MPS boundary condi-	
enlarge_mps_unit_cell([factor])	tions; in place.	
IrregularLattice.	Automatically find coupling pairs grouped by distances.	
find_coupling_pairs([])		
IrregularLattice.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.	
)		
<pre>IrregularLattice.lat2mps_idx(lat_idx)</pre>	Translate lattice indices (x_0, \ldots, x_{D-1}) ,	
	u) to MPS index <i>i</i> .	
IrregularLattice.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,	
	x_{dim-1} , u).	
IrregularLattice.mps2lat_values(A[,	Reshape/reorder A to replace an MPS index by lattice	
axes, u])	indices.	
	continues on next page	

Table	9 –	continued	from	previous	page
IGOIO	•	Continuou		piovious	pago

IrregularLattice.	Reshape/reorder an array A to replace an MPS index by
${\it mps2lat_values_masked}(A[,\dots])$	lattice indices.
<pre>IrregularLattice.mps_idx_fix_u([u])</pre>	return an index array of MPS indices for which the site
	within the unit cell is u .
IrregularLattice.	Similar as mps_idx_fix_u(), but return also the
$mps_lat_idx_fix_u([u])$	corresponding lattice indices.
<pre>IrregularLattice.mps_sites()</pre>	Return a list of sites for all MPS indices.
IrregularLattice.	Calculate correct shape of the strengths for a
multi_coupling_shape(dx)	multi_coupling.
IrregularLattice.	Deprecated.
<pre>number_nearest_neighbors([u])</pre>	
IrregularLattice.	Deprecated.
$number_next_nearest_neighbors ([u])$	
IrregularLattice.ordering(order)	Provide possible orderings of the lattice sites.
IrregularLattice.plot_basis(ax[, origin,	Plot arrows indicating the basis vectors of the lattice.
shade])	
-/	
IrregularLattice.	Mark two sites indified by periodic boundary condi-
	tions.
IrregularLattice.	tions. Plot lines connecting nearest neighbors of the lattice.
<pre>IrregularLattice. plot_bc_identified(ax[,])</pre>	tions.
<pre>IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,])</pre>	tions. Plot lines connecting nearest neighbors of the lattice.
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,])	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them.
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,]) IrregularLattice.plot_sites(ax[, markers])	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers.
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,]) IrregularLattice.plot_sites(ax[, markers]) IrregularLattice.position(lat_idx)	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites.
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,]) IrregularLattice.plot_sites(ax[, markers]) IrregularLattice.position(lat_idx) IrregularLattice.	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites.
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,]) IrregularLattice.plot_sites(ax[, markers]) IrregularLattice.position(lat_idx) IrregularLattice. possible_couplings(u1, u2, dx)	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings.
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,]) IrregularLattice.plot_sites(ax[, markers]) IrregularLattice.position(lat_idx) IrregularLattice. possible_couplings(u1, u2, dx) IrregularLattice.	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to cou-
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,]) IrregularLattice.plot_sites(ax[, markers]) IrregularLattice.position(lat_idx) IrregularLattice. possible_couplings(u1, u2, dx) IrregularLattice. possible_multi_couplings(ops) IrregularLattice.save_hdf5(hdf5_saver, h5gr,)	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to couplings with more than 2 sites.
IrregularLattice. plot_bc_identified(ax[,]) IrregularLattice.plot_coupling(ax[,]) IrregularLattice.plot_order(ax[, order,]) IrregularLattice.plot_sites(ax[, markers]) IrregularLattice.position(lat_idx) IrregularLattice. possible_couplings(u1, u2, dx) IrregularLattice. possible_multi_couplings(ops) IrregularLattice.save_hdf5(hdf5_saver,	tions. Plot lines connecting nearest neighbors of the lattice. Plot a line connecting sites in the specified "order" and text labels enumerating them. Plot the sites of the lattice with markers. return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to couplings with more than 2 sites.

Class Attributes and Properties

IrregularLattice.Lu	the (expected) number of sites in the unit cell,
	len(unit_cell).
IrregularLattice.boundary_conditions	Human-readable list of boundary conditions from bc
	<pre>and bc_shift.</pre>
IrregularLattice.dim	The dimension of the lattice.
IrregularLattice.nearest_neighbors	
IrregularLattice.	
next_nearest_neighbors	
IrregularLattice.	
next_next_nearest_neighbors	
IrregularLattice.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

Bases: tenpy.models.lattice.Lattice

A variant of a regular lattice, where we might have extra sites or sites missing.

Note: The lattice defines only the geometry of the sites, not the couplings; you can have position-dependent couplings/onsite terms despite having a regular lattice.

By adjusting the order and a few private attributes and methods, we can make functions like possible_couplings() work with a more "irregular" lattice structure, where some of the sites are missing and other sites added instead.

Parameters

- regular_lattice (Lattice) The lattice this is based on.
- **remove** (2D array | None) Each row is a lattice index (x_0, ..., x_{dim-1}, u) of a site to be removed. If None, don't remove something.
- add (Tuple[2D array, list] / None) Each row of the 2D array is a lattice index (x_0, ..., x_{dim-1}, u) specifying where a site is to be added; u is the index of the site within the final unit_cell of the irregular lattice. For each row of the 2D array, there is one entry in the list specifying where the site is inserted in the MPS; the values are compared to the MPS indices of the regular lattice and sorted into it, so "2.5" goes between what was site 2 and 3 in the regular lattice. An entry None indicates that the site should be inserted after the lattice site (x_0, ..., x_{dim-1}, -1) of the regular_lattice.
- add_unit_cell (list of Site) Extra sites to be added to the unit cell.
- add_positions (iterable of 1D arrays) For each extra site in add_unit_cell the position within the unit cell. Defaults to np.zeros((len(add_unit_cell), dim)).

regular_lattice

The lattice this is based on.

```
Type Lattice
```

remove, add

See above. Used in ordering () only.

Type 2D array | None

Examples

Let's imagine that we have two different sites; for concreteness we can thing of a fermion site, which we represent with 'F', and a spin site 'S'. If you want to preserve charges, take a look at set_common_charges()
for the proper way to initialize the sites.

You could now imagine that to have fermion chain with spins on the "bonds". In the periodic/infinite case, you would simply define

```
>>> lat = Lattice([2], unit_cell=['F', 'S'], bc='periodic', bc_MPS='infinite')
>>> lat.mps_sites()
['F', 'S', 'F', 'S']
```

For a finite system, you don't want to terminate with a spin on the right, so you need to remove the very last site by specifying the lattice index [L-1, 1] of that site:

```
>>> L = 4
>>> reg_lat = Lattice([L], unit_cell=['F', 'S'], bc='open', bc_MPS='finite')
>>> irr_lat = IrregularLattice(reg_lat, remove=[[L - 1, 1]])
>>> irr_lat.mps_sites()
['F', 'S', 'F', 'S', 'F', 'S', 'F']
```

Another simple example would be to add a spin in the center of a fermion chain. In that case, we add another site to the unit cell and specify the lattice index as [(L-1)//2, 1] (where the 1 is the index of 'S' in the unit cell ['F', 'S'] of the irregular lattice). The *None* for the MPS index is equivalent to (L-1)/2 in this case.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

ordering(order)

Provide possible orderings of the lattice sites.

Parameters order – Argument for the *Lattice.ordering()* of the *regular_lattice*, or 2D ndarray providing the order of the *regular lattice*.

Returns order – The order to be used for *order*, i.e. *order* with added/removed sites as specified by remove and add.

Return type array, shape (N, D+1)

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot_order().

mps_idx_fix_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

property boundary_conditions

Human-readable list of boundary conditions from bc and bc shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count neighbors (u=0, key='nearest neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

property dim

The dimension of the lattice.

distance (u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

find_coupling_pairs (*max_dx=3*, *cutoff=None*, *eps=1e-10*)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of $nearest_neighbors$, $next_nearest_neighbors$ etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given cutoff distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

lat2mps_idx (lat_idx)

Translate lattice indices (x_0, \dots, x_{D-1}) , u) to MPS index i.

Parameters $lat_idx(array_like [.., dim+1])$ - The last dimension corresponds to lattice indices (x_0, ..., x_{D-1}, u). All lattice indices should be positive and

smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

```
mps2lat\_idx(i)
```

Translate MPS index i to lattice indices $(x \ 0, \ldots, x \ \{dim-1\}, u)$.

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices ``(x_0, \ldots, x_d) corresponding to i. For i across the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder A to replace an MPS index by lattice indices.

Parameters

- A (ndarray) Some values. Must have A.shape[axes] = self.N_sites if u is None, or A.shape[axes] = self.N_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit_cell[u], as returned by $mps_idx_fix_u()$. The resulting array will not have the additional dimension(s) of u.

Returns res_A – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[tuple(lat.mps2lat_idx(5))] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use $mps_idx_fix_u()$ to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

mps2lat_values_masked (A, axes=- 1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps2lat_values()$ allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u ((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

mps_lat_idx_fix_u (u=None)

Similar as mps_idx_fix_u(), but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- lat_idx (2D array) The row j contains the lattice index (without u) corresponding to mps_idx[j].

mps_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the i-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

```
plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)
```

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

 $\verb|plot_bc_identified| (ax, direction =- 1, origin = None, cylinder_axis = False, **kwargs)|$

Mark two sites indified by periodic boundary conditions. Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit cell positions.
- **kwargs Keyword arguments for the used ax.plot.

plot_coupling(ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- coupling (list of (u1, u2, dx)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (i0, i1, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.

- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

```
plot order(ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers [u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

```
position (lat_idx)
```

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible_couplings (u1, u2, dx, strength=None)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- **u1** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

• **mps1**, **mps2** (1D array) – For each possible two-site coupling the MPS indices for the u1 and u2.

- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible_multi_couplings (ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

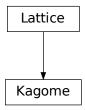
19.1.5 Kagome

• full name: tenpy.models.lattice.Kagome

• parent module: tenpy.models.lattice

· type: class

Inheritance Diagram



Methods

Kagomeinit(Lx, Ly, sites, **kwargs)	Initialize self.	
Kagome.count_neighbors([u, key])	Count e.g.	
Kagome.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.	
Kagome.distance(u1, u2, dx)	Get the distance for a given coupling between two sites	
, , , , , , , , , , , , , , , , , , ,	in the lattice.	
Kagome.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-	
	tions; in place.	
Kagome.find_coupling_pairs([max_dx, cut-	Automatically find coupling pairs grouped by distances.	
off, eps])		
Kagome.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.	
Kagome.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,	
	u) to MPS index i.	
Kagome.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,	
	x_{dim-1} , u).	
Kagome.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice	
	indices.	
Kagome.mps2lat_values_masked(A[, axes,	Reshape/reorder an array A to replace an MPS index by	
])	lattice indices.	
Kagome.mps_idx_fix_u([u])	return an index array of MPS indices for which the site	
	within the unit cell is u .	
$Kagome.mps_lat_idx_fix_u([u])$	Similar as mps_idx_fix_u(), but return also the	
	corresponding lattice indices.	
Kagome.mps_sites()	Return a list of sites for all MPS indices.	
Kagome.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a	
	multi_coupling.	
Kagome.number_nearest_neighbors([u])	Deprecated.	
Kagome.number_next_nearest_neighbors([u])Deprecated.	
Kagome.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.	
<pre>Kagome.plot_basis(ax[, origin, shade])</pre>	Plot arrows indicating the basis vectors of the lattice.	
Kagome.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-	
])	tions.	
<pre>Kagome.plot_coupling(ax[, coupling, wrap])</pre>	Plot lines connecting nearest neighbors of the lattice.	
	continues on next page	

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	, , ,
<pre>Kagome.plot_order(ax[, order, textkwargs])</pre>	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
<pre>Kagome.plot_sites(ax[, markers])</pre>	Plot the sites of the lattice with markers.
Kagome.position(lat_idx)	return 'space' position of one or multiple sites.
Kagome.possible_couplings(u1, u2, dx[,	Find possible MPS indices for two-site couplings.
strength])	
Kagome.possible_multi_couplings(ops[,	Generalization of possible_couplings() to cou-
strength])	plings with more than 2 sites.
Kagome.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
Kagome.site(i)	return $Site$ instance corresponding to an MPS index i
Kagome.test_sanity()	Sanity check.

Class Attributes and Properties

Kagome.Lu	the (expected) number of sites in the unit cell,
	<pre>len(unit_cell).</pre>
Kagome.boundary_conditions	Human-readable list of boundary conditions from bc
	and bc_shift.
Kagome.dim	the dimension of the lattice
Kagome.nearest_neighbors	
Kagome.next_nearest_neighbors	
Kagome.next_next_nearest_neighbors	
Kagome.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.Kagome (Lx, Ly, sites, **kwargs)

Bases: tenpy.models.lattice.Lattice

A Kagome lattice.

Parameters

- **Lx** (*int*) The length in each direction.
- Ly (int) The length in each direction.
- **sites** ((list of) *Site*) The two local lattice sites making the *unit_cell* of the *Lattice*. If only a single *Site* is given, it is used for both sites.
- **kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs are set accordingly.

property boundary_conditions

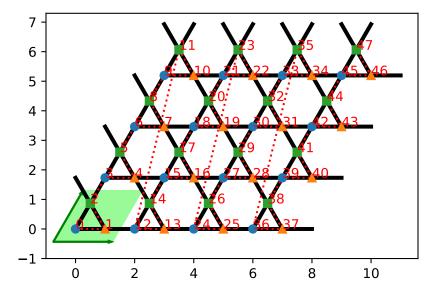
Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count_neighbors (u=0, key='nearest_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.



Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

find_coupling_pairs (max_dx=3, cutoff=None, eps=1e-10)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps_idx (lat_idx)

Translate lattice indices (x_0, \dots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx ($array_1ike[..,dim+1]$) - The last dimension corresponds to lattice indices ($x_0, ..., x_{D-1}, u$). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

$mps2lat_idx(i)$

Translate MPS index i to lattice indices $(x_0, \dots, x_{\min}, u)$.

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices $(x_0, \ldots, x_{dim-1}, u)$ corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

$mps2lat_values(A, axes=0, u=None)$

Reshape/reorder A to replace an MPS index by lattice indices.

Parameters

- A (ndarray) Some values. Must have A.shape[axes] = self.N_sites if u is None, or A.shape[axes] = self.N_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit_cell[u], as returned by mps_idx_fix_u(). The resulting array will not have the additional dimension(s) of u.

Returns res_A – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[tuple(lat.mps2lat_idx(5))] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use $mps_idx_fix_u$ () to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

mps2lat_values_masked (A, axes=-1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps21at_values()$ allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

$mps_idx_fix_u (u=None)$

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps lat idx fix u(u=None)

Similar as $mps_idx_fix_u$ (), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- $lat_idx (2D \ array)$ The row j contains the lattice index (without u) corresponding to $mps_idx[j]$.

mps_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use <code>count_neighbors()</code> instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

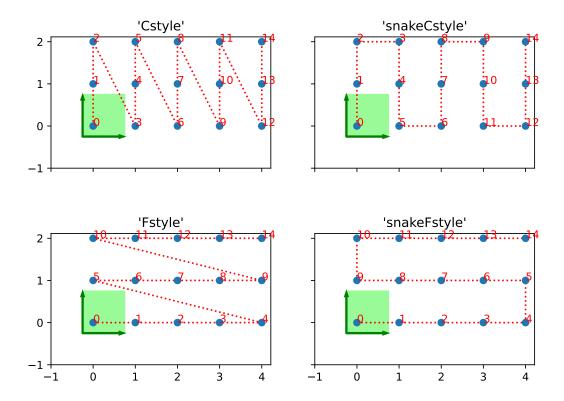
You can visualize the order with plot_order().

ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	$(\dim -1, \ldots, 1, 0, \dim)$	(False,, False, False)
'snakeFstyle'	$(\dim -1, \ldots, 1, 0, \dim)$	(False,, False, False)



Note: For lattices with a non-trivial unit cell (e.g. Honeycomb, Kagome), the grouped order might be more appropriate, see <code>get_order_grouped()</code>

Parameters order (str | ('standard', snake_winding, priority) | ('grouped', groups, ...)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get_order() and 'grouped' for get_order_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

Returns order – the order to be used for order.

Return type array, shape (N, D+1), dtype np.intp

See also:

```
get_order generates the order from equivalent priority and snake_winding.
get_order_grouped variant of get_order.
plot_order visualizes the resulting order.
```

plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

plot_coupling (ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

plot_order (ax, order=None, textkwargs={'color': 'r'}, **kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.

- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers [u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

position (lat_idx)

return 'space' position of one or multiple sites.

```
Parameters lat idx (ndarray, (..., dim+1)) – Lattice indices.
```

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

```
Return type ndarray, (..., dim)
```

possible_couplings (u1, u2, dx, strength=None)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible multi couplings(ops, strength=None)

Generalization of possible couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,... for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

dim = 2

the dimension of the lattice

Lu = 3

the (expected) number of sites in the unit cell, len (unit_cell).

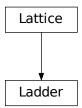
19.1.6 Ladder

• full name: tenpy.models.lattice.Ladder

• parent module: tenpy.models.lattice

• type: class

Inheritance Diagram



Methods

- 11 (T to shold	T 1:1 11 10
Ladderinit(L, sites, **kwargs)	Initialize self.
Ladder.count_neighbors([u, key])	Count e.g.
Ladder.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Ladder.distance(u1, u2, dx)	Get the distance for a given coupling between two sites
	in the lattice.
Ladder.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Ladder.find_coupling_pairs([max_dx, cut-	Automatically find coupling pairs grouped by distances.
off, eps])	
Ladder.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Ladder.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,
	u) to MPS index i.
Ladder.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,
	x_{dim-1} , u).
Ladder.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice
	indices.
Ladder.mps2lat_values_masked(A[, axes,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
Ladder.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is u .
Ladder.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Ladder.mps_sites()	Return a list of sites for all MPS indices.
Ladder.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
	continues on next page

Table 13 – continued from previous page

Ladder.number_nearest_neighbors([u])	Deprecated.
Ladder.number_next_nearest_neighbors([u])Deprecated.
Ladder.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
Ladder.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.
Ladder.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-
])	tions.
Ladder.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.
Ladder.plot_order(ax[, order, textkwargs])	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
<pre>Ladder.plot_sites(ax[, markers])</pre>	Plot the sites of the lattice with markers.
Ladder.position(lat_idx)	return 'space' position of one or multiple sites.
Ladder.possible_couplings(u1, u2, dx[,	Find possible MPS indices for two-site couplings.
strength])	
Ladder.possible_multi_couplings(ops[,	Generalization of possible_couplings() to cou-
strength])	plings with more than 2 sites.
Ladder.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
Ladder.site(i)	return $Site$ instance corresponding to an MPS index i
Ladder.test_sanity()	Sanity check.

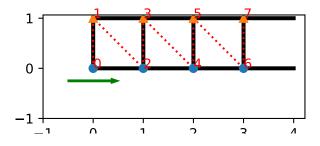
Class Attributes and Properties

Ladder.Lu	the (expected) number of sites in the unit cell,
	len(unit_cell).
Ladder.boundary_conditions	Human-readable list of boundary conditions from bc
	<pre>and bc_shift.</pre>
Ladder.dim	the dimension of the lattice
Ladder.nearest_neighbors	
Ladder.next_nearest_neighbors	
Ladder.next_next_nearest_neighbors	
Ladder.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

 ${\tt class}$ tenpy.models.lattice.Ladder(L, sites, **kwargs)

Bases: tenpy.models.lattice.Lattice

A ladder coupling two chains.



Parameters

- **L** (*int*) The length of each chain, we have 2*L sites in total.
- **sites** ((list of) Site) The two local lattice sites making the *unit_cell* of the Lattice. If only a single Site is given, it is used for both chains.
- **kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs
 are set accordingly.

Lu = 2

the (expected) number of sites in the unit cell, len (unit cell).

dim = 1

the dimension of the lattice

property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count_neighbors (u=0, key='nearest_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to *dx* argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The *u1*, *u2*, *dx* parameters are defined in analogy with *add_coupling()*, i.e., this function calculates the distance between a pair of operators added with *add_coupling* (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

find_coupling_pairs (*max_dx=3*, *cutoff=None*, *eps=1e-10*)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps_idx (lat_idx)

Translate lattice indices (x_0, \dots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx ($array_like[..,dim+1]$) – The last dimension corresponds to lattice indices ($x_0, ..., x_{D-1}, u$). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to lat_idx . Has the same shape as lat_idx without the last dimension.

Return type array like

$mps2lat_idx(i)$

Translate MPS index i to lattice indices $(x_0, \dots, x_{\min-1}, u)$.

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices ``($x_0, \ldots, x_{dim-1}, u$)` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder A to replace an MPS index by lattice indices.

Parameters

- A (ndarray) Some values. Must have A.shape[axes] = self.N_sites if u is None, or A.shape[axes] = self.N_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit_cell[u], as returned by $mps_idx_fix_u()$. The resulting array will not have the additional dimension(s) of u.

Returns res_A – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by $self.mps2lat_idx(i)$. Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[tuple(lat.mps2lat_idx(5))] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
```

(continues on next page)

(continued from previous page)

```
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use $mps_idx_fix_u()$ to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

mps2lat_values_masked (A, axes=-1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of mps2lat_values() allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u ((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

mps_idx_fix_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u (u=None)

Similar as $mps_idx_fix_u$ (), but return also the corresponding lattice indices.

Parameters u (*None* / *int*) – Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- lat_idx (2D array) The row j contains the lattice index (without u) corresponding to mps_idx[j].

mps_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the strengths for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

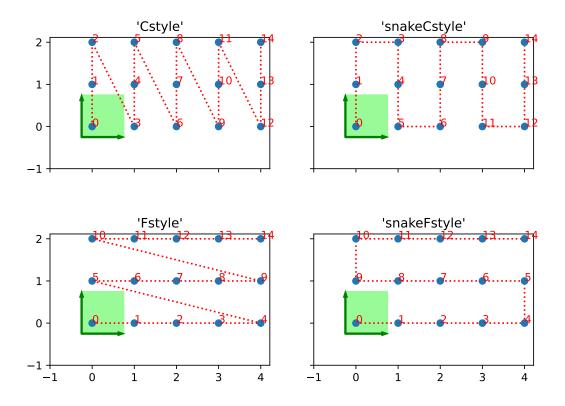
You can visualize the order with plot_order().

ordering (order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)



Note: For lattices with a non-trivial unit cell (e.g. Honeycomb, Kagome), the grouped order might be more appropriate, see get_order_grouped()

Parameters order (str | ('standard', snake_winding, priority) | ('grouped', groups, ...)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get_order() and 'grouped' for get_order_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

Returns order – the order to be used for order.

Return type array, shape (N, D+1), dtype np.intp

See also:

```
get_order generates the order from equivalent priority and snake_winding.
get_order_grouped variant of get_order.
plot_order visualizes the resulting order.
```

plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

 $\verb"plot_bc_identified" (ax, direction =- 1, origin = None, cylinder_axis = False, **kwargs)$

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- cylinder_axis (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

plot_coupling (ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ...,

- u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

```
plot order (ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site *u* in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

```
position (lat_idx)
```

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible_couplings (u1, u2, dx, strength=None)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le x_$

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible_multi_couplings(ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument ops of add multi coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- **strength_vals** (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

$\mathtt{site}\left(i\right)$

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

19.1.7 Lattice

• full name: tenpy.models.lattice.Lattice

• parent module: tenpy.models.lattice

• type: class

Inheritance Diagram

Lattice

Methods

Latticeinit(Ls, unit_cell[, order, bc,])	Initialize self.
Lattice.count_neighbors([u, key])	Count e.g.
Lattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Lattice.distance(u1, u2, dx)	Get the distance for a given coupling between two sites
	in the lattice.
Lattice.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Lattice.find_coupling_pairs([max_dx,	Automatically find coupling pairs grouped by distances.
])	
Lattice.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Lattice.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,
	u) to MPS index i.
Lattice.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,
	x_{dim-1} , u).
Lattice.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice
	indices.
Lattice.mps2lat_values_masked(A[, axes,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
Lattice.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is <i>u</i> .
Lattice.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Lattice.mps_sites()	Return a list of sites for all MPS indices.
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T	4 -		•		
Table	15 -	 continued 	trom	previous	nage

Lattice.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a	
	multi_coupling.	
Lattice.number_nearest_neighbors([u])	Deprecated.	
Lattice.number_next_nearest_neighbors(uDeprecated.	
Lattice.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.	
Lattice.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.	
Lattice.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-	
])	tions.	
Lattice.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.	
Lattice.plot_order(ax[, order, textkwargs]) Plot a line connecting sites in the specified "order"		
	text labels enumerating them.	
Lattice.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.	
Lattice.position(lat_idx)	return 'space' position of one or multiple sites.	
Lattice.possible_couplings(u1, u2, dx[,	Find possible MPS indices for two-site couplings.	
])		
Lattice.possible_multi_couplings(ops[,	Generalization of possible_couplings() to cou-	
strength])	plings with more than 2 sites.	
Lattice.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.	
Lattice.site(i)	return Site instance corresponding to an MPS index i	
Lattice.test_sanity()	Sanity check.	

Class Attributes and Properties

Lattice.Lu	the (expected) number of sites in the unit cell,	
	<pre>len(unit_cell).</pre>	
Lattice.boundary_conditions	Human-readable list of boundary conditions from bc	
	<pre>and bc_shift.</pre>	
Lattice.dim	the dimension of the lattice	
Lattice.nearest_neighbors		
Lattice.next_nearest_neighbors		
Lattice.next_next_nearest_neighbors		
Lattice.order	Defines an ordering of the lattice sites, thus mapping the	
	lattice to a 1D chain.	

Bases: object

A general, regular lattice.

The lattice consists of a **unit cell** which is repeated in *dim* different directions. A site of the lattice is thus identified by **lattice indices** (x_0 , ..., $x_{\dim -1}$, u), where $0 \le x_1 \le Ls[1]$ pick the position of the unit cell in the lattice and $0 \le u \le len(unit_cell)$ picks the site within the unit cell. The site is located in 'space' at $sum_1 x_1 *basis[1] + unit_cell_positions[u]$ (see position()). (Note that the position in space is only used for plotting, not for defining the couplings.)

In addition to the pure geometry, this class also defines an order of all sites. This order maps the lattice to

a finite 1D chain and defines the geometry of MPSs and MPOs. The MPS index i corresponds thus to the lattice sites given by $(x_0, \ldots, x_{\dim-1}, u) = \text{tuple(self.order[i])}$. Infinite boundary conditions of the MPS repeat in the first spatial direction of the lattice, i.e., if the site at $(x_0, x_1, \ldots, x_{\dim-1}, u)$ has MPS index i, the site at $(x_0 + Ls[0], x_1, \ldots, x_{\dim-1}, u)$ corresponds to MPS index i + N_sites. Use $mps2lat_idx(i)$ and $lat2mps_idx(i)$ for conversion of indices. The function $mps2lat_values(i)$ performs the necessary reshaping and re-ordering from arrays indexed in MPS form to arrays indexed in lattice form.

Deprecated since version 0.5.0: The parameters and attributes *nearest_neighbors*, *next_nearest_neighbors* and *next_next_nearest_neighbors* are deprecated. Instead, we use a dictionary *pairs* with those names as keys and the corresponding values as specified before.

Parameters

- Ls (list of int) the length in each direction
- unit_cell (list of Site) The sites making up a unit cell of the lattice. If you want to specify it only after initialization, use None entries in the list.
- order (str | ('standard', snake_winding, priority) | ('grouped', groups, ...)) A string or tuple specifying the order, given to ordering().
- **bc** ((iterable of) {'open' | 'periodic' | int}) Boundary conditions in each direction of the lattice. A single string holds for all directions. An integer *shift* means that we have periodic boundary conditions along this direction, but shift/tilt by -shift*lattice.basis[0] (~cylinder axis for bc_MPS='infinite') when going around the boundary along this direction.
- bc_MPS ('finite' | 'segment' | 'infinite') Boundary conditions for an MPS/MPO living on the ordered lattice. If the system is 'infinite', the infinite direction is always along the first basis vector (justifying the definition of N_rings and N_sites_per_ring).
- basis (iterable of 1D arrays) For each direction one translation vectors shifting the unit cell. Defaults to the standard ONB np.eye (dim).
- **positions** (*iterable of 1D arrays*) For each site of the unit cell the position within the unit cell. Defaults to np.zeros ((len(unit_cell), dim)).
- nearest_neighbors (None | list of (u1, u2, dx)) Deprecated. Specify as pairs['nearest_neighbors'] instead.
- next_nearest_neighbors (None | list of (u1, u2, dx)) Deprecated. Specify as pairs ['next_nearest_neighbors'] instead.
- next_next_nearest_neighbors (None | list of (u1, u2, dx)) Deprecated. Specify as pairs ['next_next_nearest_neighbors'] instead.
- pairs (dict) Of the form {'nearest_neighbors': [(u1, u2, dx), ...], ...}. Typical keys are 'nearest_neighbors', 'next_nearest_neighbors'. For each of them, it specifies a list of tuples (u1, u2, dx) which can be used as parameters for add_coupling() to generate couplings over each pair of ,e.g., 'nearest_neighbors'. Note that this adds couplings for each pair only in one direction!

Ls

the length in each direction.

Type tuple of int

shape

the 'shape' of the lattice, same as Ls + (len(unit_cell),)

```
Type tuple of int
```

N cells

the number of unit cells in the lattice, np.prod(self.Ls).

Type int

N sites

the number of sites in the lattice, np.prod(self.shape).

Type int

N_sites_per_ring

Defined as N_sites / Ls[0], for an infinite system the number of cites per "ring".

Type int

N_rings

Alias for Ls [0], for an infinite system the number of "rings" in the unit cell.

Type int

unit cell

the sites making up a unit cell of the lattice.

Type list of Site

bc

Boundary conditions of the couplings in each direction of the lattice, translated into a bool array with the global $bc_choices$.

Type bool ndarray

bc_shift

The shift in x-direction when going around periodic boundaries in other directions.

Type None | ndarray(int)

bc MPS

Boundary conditions for an MPS/MPO living on the ordered lattice. If the system is 'infinite', the infinite direction is always along the first basis vector (justifying the definition of N_rings and $N_sites_per_ring$).

Type 'finite' | 'segment' | 'infinite'

basis

translation vectors shifting the unit cell. The row i gives the vector shifting in direction i.

Type ndarray (dim, Dim)

unit_cell_positions

for each site in the unit cell a vector giving its position within the unit cell.

Type ndarray, shape (len(unit_cell), Dim)

pairs

See above.

Type dict

_order

The place where order is stored.

Type ndarray (N_sites, dim+1)

```
strides
     necessary for lat2mps_idx().
         Type ndarray (dim, )
perm
     permutation needed to make order lexsorted, _perm = np.lexsort(_order.T).
         Type ndarray (N, )
_mps2lat_vals_idx
     index array for reshape/reordering in mps2lat_vals()
         Type ndarray shape
_mps_fix_u
    for each site of the unit cell an index array selecting the mps indices of that site.
         Type tuple of ndarray (N_cells, ) np.intp
_mps2lat_vals_idx_fix_u
     similar as _mps2lat_vals_idx, but for a fixed u picking a site from the unit cell.
         Type tuple of ndarray of shape Ls
Lu = None
    the (expected) number of sites in the unit cell, len (unit_cell).
test sanity()
     Sanity check.
     Raises ValueErrors, if something is wrong.
save_hdf5 (hdf5_saver, h5gr, subpath)
     Export self into a HDF5 file.
     This method saves all the data it needs to reconstruct self with from_hdf5().
     Specifically,
                          saves
                                   unit_cell,
                                                     Ls.
                                                             unit_cell_positions,
                                                                                            basis.
     boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS
     ", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.
         Parameters
             • hdf5_saver (Hdf5Saver) – Instance of the saving engine.
             • h5gr (:class `Group `) - HDF5 group which is supposed to represent self.
             • subpath (str) – The name of h5gr with a '/' in the end.
classmethod from hdf5 (hdf5 loader, h5gr, subpath)
     Load instance from a HDF5 file.
     This method reconstructs a class instance from the data saved with save_hdf5().
         Parameters
             • hdf5_loader (Hdf5Loader) - Instance of the loading engine.
             • h5gr (Group) – HDF5 group which is represent the object to be constructed.
             • subpath (str) – The name of h5gr with a '/' in the end.
         Returns obj – Newly generated class instance containing the required data.
         Return type cls
```

property dim

the dimension of the lattice

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

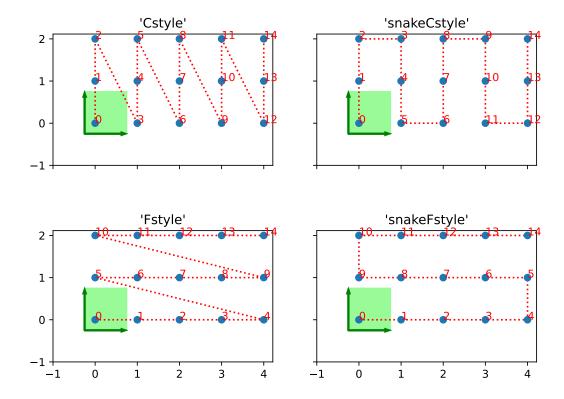
You can visualize the order with plot_order().

ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)



site(i)

Note: For lattices with a non-trivial unit cell (e.g. Honeycomb, Kagome), the grouped order might be more appropriate, see get order grouped()

```
Parameters order
                                   ('standard', snake_winding, priority)
                              (str
             ('grouped', groups, ...)) - Specifies the desired ordering using one of the
            strings of the above tables. Alternatively, an ordering is specified by a tuple with
            first entry specifying a function, 'standard' for get_order() and 'grouped'
            for get_order_grouped(), and other arguments in the tuple as specified in the
            documentation of these functions.
        Returns order – the order to be used for order.
        Return type array, shape (N, D+1), dtype np.intp
    See also:
    get_order generates the order from equivalent priority and snake_winding.
    get_order_grouped variant of get_order.
    plot_order visualizes the resulting order.
property boundary_conditions
    Human-readable list of boundary conditions from bc and bc shift.
        Returns boundary conditions – List of "open" or "periodic", one entry for each direc-
            tion of the lattice.
        Return type list of str
enlarge mps unit cell(factor=2)
    Repeat the unit cell for infinite MPS boundary conditions; in place.
        Parameters factor (int) – The new number of sites in the MPS unit cell will be increased
            from N_sites to factor*N_sites_per_ring. Since MPS unit cells are repeated in
            the x-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to
             (Lx*factor, Ly, ..., Lu).
position (lat idx)
    return 'space' position of one or multiple sites.
        Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
        Returns pos – The position of the lattice sites specified by lat idx in real-space.
        Return type ndarray, (..., dim)
    return Site instance corresponding to an MPS index i
mps_sites()
    Return a list of sites for all MPS indices.
    Equivalent to [self.site(i) for i in range(self.N_sites)].
    This should be used for sites of 1D tensor networks (MPS, MPO,...).
mps2lat_idx(i)
```

424 Chapter 19. models

Translate MPS index i to lattice indices $(x \ 0, \ldots, x \ dim-1)$, u).

Parameters i (int | array_like of int) – MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim +1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

lat2mps idx(lat idx)

Translate lattice indices (x 0, ..., x $\{D-1\}$, u) to MPS index i.

Parameters lat_idx ($array_1ike[..,dim+1]$) - The last dimension corresponds to lattice indices ($x_0, ..., x_{D-1}, u$). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

mps_idx_fix_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit_cell[u].

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u (u=None)

Similar as $mps_idx_fix_u$ (), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- $lat_idx (2D \ array)$ The row j contains the lattice index (without u) corresponding to mps idx[j].

$mps2lat_values(A, axes=0, u=None)$

Reshape/reorder *A* to replace an MPS index by lattice indices.

Parameters

- A (ndarray) Some values. Must have A.shape[axes] = self.N_sites if u is None, or A.shape[axes] = self.N_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit_cell[u], as returned by $mps_idx_fix_u()$. The resulting array will not have the additional dimension(s) of u.

Returns res_A – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by $self.mps2lat_idx(i)$. Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[tuple(lat.mps2lat_idx(5))] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use $mps_idx_fix_u$ () to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

mps2lat_values_masked(A, axes=-1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps21at_values()$ allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

```
count_neighbors (u=0, key='nearest_neighbors')
```

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of *pairs* to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count neighbors () instead.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and $unit_cell_positions$ of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

find_coupling_pairs (*max_dx=3*, *cutoff=None*, *eps=1e-10*)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of nearest_neighbors, next_nearest_neighbors etc at a given distance are basically fixed (although not uniquely, since we take out

half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible *pairs* at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (float) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

coupling_shape (dx)

Calculate correct shape of the *strengths* for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

possible_couplings (u1, u2, dx, strength=None)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (int) Indices within the unit cell; the u1 and u2 of add coupling ()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (1D array) (Only returend if strength is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.

- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi-coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the i-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

possible_multi_couplings (ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

plot_sites (ax, markers=['o', 'n', 's', 'p', 'h', 'D'], **kwargs)

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

plot order(ax, order=None, textkwargs={'color': 'r'}, **kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

plot_coupling(ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

plot_basis (*ax*, *origin*=(0.0, 0.0), *shade*=*None*, ***kwargs*)

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- ****kwargs** Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- cylinder_axis (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of *unit_cell_positions*.
- **kwargs Keyword arguments for the used ax.plot.

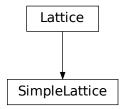
19.1.8 SimpleLattice

• full name: tenpy.models.lattice.SimpleLattice

• parent module: tenpy.models.lattice

• type: class

Inheritance Diagram



Methods

SimpleLatticeinit(Ls, site, **kwargs)	Initialize self.
SimpleLattice.count_neighbors([u, key])	Count e.g.
SimpleLattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
SimpleLattice.distance(u1, u2, dx)	Get the distance for a given coupling between two sites
	in the lattice.
SimpleLattice.enlarge_mps_unit_cell([fac	to peat the unit cell for infinite MPS boundary condi-
	tions; in place.
SimpleLattice.find_coupling_pairs([max_c	dxAutomatically find coupling pairs grouped by distances.
])	
SimpleLattice.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
SimpleLattice.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,
	u) to MPS index i.
$SimpleLattice.mps2lat_idx(i)$	Translate MPS index i to lattice indices (x_0 ,,
	x_{dim-1}, u).
SimpleLattice.mps2lat_values(A[, axes, u])	same as Lattice.mps2lat_values(), but ignore
	u, setting it to 0.
$SimpleLattice.mps2lat_values_masked (A[,$	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
$SimpleLattice.mps_idx_fix_u([u])$	return an index array of MPS indices for which the site
	within the unit cell is u .
$SimpleLattice.mps_lat_idx_fix_u([u])$	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
SimpleLattice.mps_sites()	Return a list of sites for all MPS indices.
	continues on next page

Table 17 – continued from previous page		
SimpleLattice.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a	
	multi_coupling.	
SimpleLattice.number_nearest_neighbors	([D])eprecated.	
SimpleLattice.number_next_nearest_neig	hDepre([at])d.	
SimpleLattice.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.	
SimpleLattice.plot_basis(ax[, origin,	Plot arrows indicating the basis vectors of the lattice.	
shade])		
SimpleLattice.plot_bc_identified(ax[,	Mark two sites indified by periodic boundary condi-	
])	tions.	
SimpleLattice.plot_coupling(ax[, coupling,	Plot lines connecting nearest neighbors of the lattice.	
wrap])		
SimpleLattice.plot_order(ax[, order, textk-	Plot a line connecting sites in the specified "order" and	
wargs])	text labels enumerating them.	
SimpleLattice.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.	
SimpleLattice.position(lat_idx)	return 'space' position of one or multiple sites.	
SimpleLattice.possible_couplings(u1,	Find possible MPS indices for two-site couplings.	
u2, dx)		
SimpleLattice.possible_multi_couplings	(openeralization of possible_couplings() to cou-	
	plings with more than 2 sites.	
SimpleLattice.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.	
)		
SimpleLattice.site(i)	return $Site$ instance corresponding to an MPS index i	
SimpleLattice.test_sanity()	Sanity check.	

Class Attributes and Properties

SimpleLattice.Lu	the (expected) number of sites in the unit cell,
	<pre>len(unit_cell).</pre>
SimpleLattice.boundary_conditions	Human-readable list of boundary conditions from bc
	and bc_shift.
SimpleLattice.dim	The dimension of the lattice.
SimpleLattice.nearest_neighbors	
SimpleLattice.next_nearest_neighbors	
SimpleLattice.next_next_nearest_neighb	oors
SimpleLattice.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.SimpleLattice (Ls, site, **kwargs)

 $Bases: \ \texttt{tenpy.models.lattice.Lattice}$

A lattice with a unit cell consiting of just a single site.

In many cases, the unit cell consists just of a single site, such that the the last entry of u of an 'lattice index' can only be 0. From the point of internal algorithms, we handle this class like a Lattice – in that way we don't need to distinguish special cases in the algorithms.

Yet, from the point of a tenpy user, for example if you measure an expectation value on each site in a *SimpleLattice*, you expect to get an ndarray of dimensions self.Ls, not self.shape. To avoid that problem, *SimpleLattice* overwrites just the meaning of u=None in mps2lat_values() to be the same as u=0.

Parameters

- Ls (list of int) the length in each direction
- **site** (Site) the lattice site. The *unit_cell* of the *Lattice* is just [site].
- **kwargs Additional keyword arguments given to the Lattice. If order is specified in the form ('standard', snake_windingi, priority), the snake_winding and priority should only be specified for the spatial directions. Similarly, positions can be specified as a single vector.

Lu = 1

the (expected) number of sites in the unit cell, len (unit_cell).

$mps2lat_values(A, axes=0, u=None)$

same as Lattice.mps2lat_values(), but ignore u, setting it to 0.

property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count_neighbors (u=0, key='nearest_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

property dim

The dimension of the lattice.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The *u1*, *u2*, *dx* parameters are defined in analogy with *add_coupling()*, i.e., this function calculates the distance between a pair of operators added with *add_coupling* (using the basis and unit cell positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

find_coupling_pairs (*max_dx=3*, *cutoff=None*, *eps=1e-10*)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps idx(lat idx)

Translate lattice indices (x_0, \ldots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx ($array_1ike[..,dim+1]$) - The last dimension corresponds to lattice indices ($x_0, ..., x_{D-1}, u$). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

$mps2lat_idx(i)$

Translate MPS index i to lattice indices $(x_0, \dots, x_{\min}, u)$.

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

mps2lat_values_masked(A, axes=-1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps2lat_values()$ allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

$mps_idx_fix_u(u=None)$

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u (u=None)

Similar as $mps_idx_fix_u()$, but return also the corresponding lattice indices.

Parameters u (*None* / *int*) – Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- lat_idx (2D array) The row j contains the lattice index (without u) corresponding to mps_idx[j].

mps_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape $(N_{ops}, dim)) - dx[i, :]$ is the translation vector in the lattice for the *i*-th operator. Corresponds to the *dx* of each operator given in the argument *ops* of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

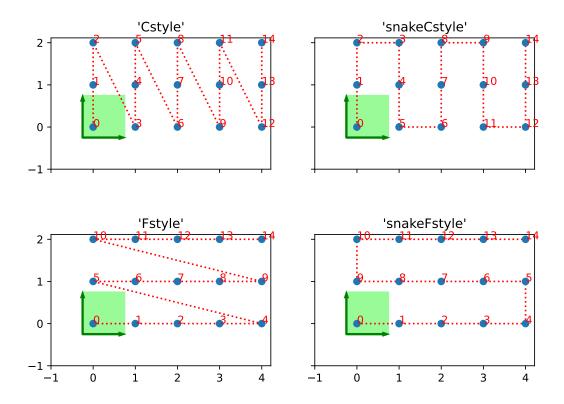
You can visualize the order with plot_order().

ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)



Note: For lattices with a non-trivial unit cell (e.g. Honeycomb, Kagome), the grouped order might be more appropriate, see <code>get_order_grouped()</code>

Parameters order (str | ('standard', snake_winding, priority) | ('grouped', groups, ...)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get_order() and 'grouped' for get_order_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

Returns order – the order to be used for order.

Return type array, shape (N, D+1), dtype np.intp

See also:

```
get_order generates the order from equivalent priority and snake_winding.
get_order_grouped variant of get_order.
plot_order visualizes the resulting order.
```

plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

plot_coupling (ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

plot_order (ax, order=None, textkwargs={'color': 'r'}, **kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.

- **textkwargs** (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers [u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

position (lat_idx)

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

```
Return type ndarray, (..., dim)
```

possible_couplings (u1, u2, dx, strength=None)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible multi couplings(ops, strength=None)

Generalization of possible couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,... for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

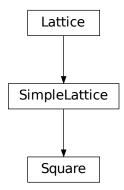
19.1.9 Square

• full name: tenpy.models.lattice.Square

• parent module: tenpy.models.lattice

• type: class

Inheritance Diagram



Methods

Squareinit(Lx, Ly, site, **kwargs)	Initialize self.
Square.count_neighbors([u, key])	Count e.g.
Square.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Square.distance(u1, u2, dx)	Get the distance for a given coupling between two sites
	in the lattice.
Square.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Square.find_coupling_pairs([max_dx, cut-	Automatically find coupling pairs grouped by distances.
off, eps])	
Square.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Square.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,
	u) to MPS index i.
$Square.mps2lat_idx(i)$	Translate MPS index i to lattice indices (x_0,,
	x_{dim-1} , u).
Square.mps2lat_values(A[, axes, u])	<pre>same as Lattice.mps2lat_values(), but ignore</pre>
	u, setting it to 0.
Square.mps2lat_values_masked(A[, axes,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
Square.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is <i>u</i> .
Square.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Square.mps_sites()	Return a list of sites for all MPS indices.
$Square.multi_coupling_shape(dx)$	Calculate correct shape of the strengths for a
	multi_coupling.
$Square.number_nearest_neighbors([u])$	Deprecated.
Square.number_next_nearest_neighbors([u	
Square.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
	continues on next page

Table 19 – continued from previous page

Square.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.	
Square.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-	
])	tions.	
Square.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.	
Square.plot_order(ax[, order, textkwargs])	Plot a line connecting sites in the specified "order" and	
	text labels enumerating them.	
Square.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.	
Square.position(lat_idx)	return 'space' position of one or multiple sites.	
Square.possible_couplings(u1, u2, dx[,	Find possible MPS indices for two-site couplings.	
strength])		
Square.possible_multi_couplings(ops[,	Generalization of possible_couplings() to cou-	
strength])	plings with more than 2 sites.	
Square.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.	
Square.site(i)	return $Site$ instance corresponding to an MPS index i	
Square.test_sanity()	Sanity check.	

Class Attributes and Properties

Square.Lu	the (expected) number of sites in the unit cell,
	len(unit_cell).
Square.boundary_conditions	Human-readable list of boundary conditions from bc
	<pre>and bc_shift.</pre>
Square.dim	the dimension of the lattice
Square.nearest_neighbors	
Square.next_nearest_neighbors	
Square.next_next_nearest_neighbors	
Square.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.Square (Lx, Ly, site, **kwargs)

Bases: tenpy.models.lattice.SimpleLattice

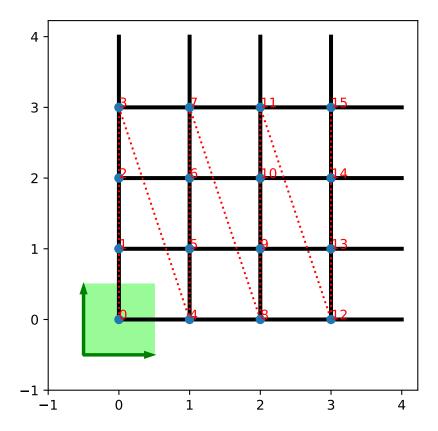
A square lattice.

Parameters

- **Lx** (*int*) The length in each direction.
- Ly (int) The length in each direction.
- **site** (Site) The local lattice site. The *unit_cell* of the Lattice is just [site].
- **kwargs Additional keyword arguments given to the Lattice. pairs are set accordingly. If order is specified in the form ('standard', snake_winding, priority), the snake_winding and priority should only be specified for the spatial directions. Similarly, positions can be specified as a single vector.

dim = 2

the dimension of the lattice



property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

```
count_neighbors (u=0, key='nearest_neighbors')
```

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- **u1** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- **u2** (int) Indices within the unit cell; the u1 and u2 of add coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

find_coupling_pairs (max_dx=3, cutoff=None, eps=1e-10)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps_idx (lat_idx)

Translate lattice indices (x_0, \ldots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx ($array_1ike[.., dim+1]$) - The last dimension corresponds to lattice indices (x_0, ..., x_{D-1}, u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

mps2lat idx(i)

Translate MPS index i to lattice indices $(x \ 0, \ldots, x \ \{dim-1\}, u)$.

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

$mps2lat_values(A, axes=0, u=None)$

same as Lattice.mps2lat values(), but ignore u, setting it to 0.

```
mps2lat_values_masked (A, axes=- 1, mps_inds=None, include_u=None)
```

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps2lat_values()$ allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc MPS, we shift x 0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

mps_idx_fix_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u(u=None)

Similar as mps_idx_fix_u(), but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- $lat_idx (2D \ array)$ The row j contains the lattice index (without u) corresponding to $mps_idx[j]$.

mps sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim.* The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot_order().

ordering(order)

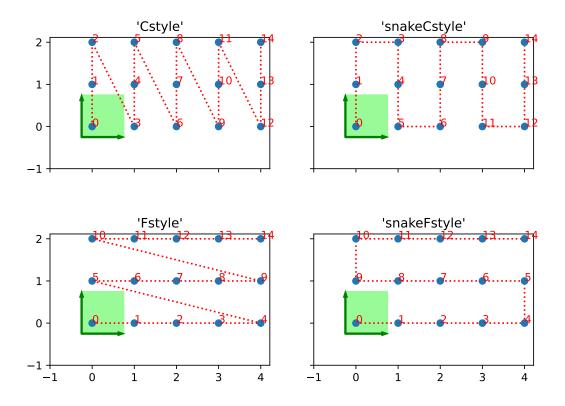
Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Note: For lattices with a non-trivial unit cell (e.g. Honeycomb, Kagome), the grouped order might be more appropriate, see get_order_grouped()

Parameters order (str | ('standard', snake_winding, priority) | ('grouped', groups, ...)) - Specifies the desired ordering using one of the



strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for $get_order()$ and 'grouped' for $get_order_grouped()$, and other arguments in the tuple as specified in the documentation of these functions.

Returns order – the order to be used for order.

Return type array, shape (N, D+1), dtype np.intp

See also:

```
get_order generates the order from equivalent priority and snake_winding.
get_order_grouped variant of get_order.
plot_order visualizes the resulting order.
```

plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

plot_coupling (ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (*i0*, *i1*, ..., *u1*) to the site (*i0*+dx[0], *i1*+dx[1], ..., *u2*), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

```
plot order(ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
\verb"plot_sites" (ax, markers = ['o', '^', 's', 'p', 'h', 'D'], **kwargs")
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

```
position (lat_idx)
```

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible_couplings (u1, u2, dx, strength=None)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (1D array) (Only returend if strength is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.

- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible_multi_couplings(ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,... for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- **strength_vals** (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

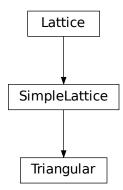
19.1.10 Triangular

• full name: tenpy.models.lattice.Triangular

• parent module: tenpy.models.lattice

• type: class

Inheritance Diagram



Methods

Triangularinit(Lx, Ly, site, **kwargs)	Initialize self.
Triangular.count_neighbors([u, key])	Count e.g.
Triangular.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Triangular.distance(u1, u2, dx)	Get the distance for a given coupling between two sites
	in the lattice.
<pre>Triangular.enlarge_mps_unit_cell([factor])</pre>	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Triangular.find_coupling_pairs([max_dx,	Automatically find coupling pairs grouped by distances.
])	
Triangular.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
Triangular.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,
	u) to MPS index i.
Triangular.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,
	$x_{dim-1}, u).$
Triangular.mps2lat_values(A[, axes, u])	<pre>same as Lattice.mps2lat_values(), but ignore</pre>
	u, setting it to 0.
Triangular.mps2lat_values_masked(A[,	Reshape/reorder an array A to replace an MPS index by
axes,])	lattice indices.
Triangular.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is u .
	continues on next page

Table	21	 continued 	from	previous page	
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Triangular.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
- 5:	corresponding lattice indices.
Triangular.mps_sites()	Return a list of sites for all MPS indices.
Triangular.multi_coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a
iriangular · marci_coapring_bnape(ax)	multi_coupling.
Triangular.number_next_nearest_neighbors	-
Triangular.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
	<u> </u>
Triangular.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.
Triangular.plot_bc_identified($\operatorname{ax}[,\ldots]$)	Mark two sites indified by periodic boundary condi-
	tions.
Triangular.plot_coupling(ax[, coupling,	Plot lines connecting nearest neighbors of the lattice.
wrap])	
Triangular.plot_order(ax[, order, textk-	Plot a line connecting sites in the specified "order" and
wargs])	text labels enumerating them.
Triangular.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
Triangular.position(lat_idx)	return 'space' position of one or multiple sites.
Triangular.possible_couplings(u1, u2, dx[,	Find possible MPS indices for two-site couplings.
])	
Triangular.possible_multi_couplings(ops	[,Generalization of possible_couplings() to cou-
])	plings with more than 2 sites.
Triangular.save_hdf5(hdf5_saver, h5gr, sub-	Export <i>self</i> into a HDF5 file.
path)	•
Triangular.site(i)	return Site instance corresponding to an MPS index i
Triangular.test_sanity()	Sanity check.

Class Attributes and Properties

Triangular.Lu	the (expected) number of sites in the unit cell,	
	<pre>len(unit_cell).</pre>	
Triangular.boundary_conditions	Human-readable list of boundary conditions from bc	
	and bc_shift.	
Triangular.dim	the dimension of the lattice	
Triangular.nearest_neighbors		
Triangular.next_nearest_neighbors		
Triangular.next_next_nearest_neighbors		
Triangular.order	Defines an ordering of the lattice sites, thus mapping the	
	lattice to a 1D chain.	

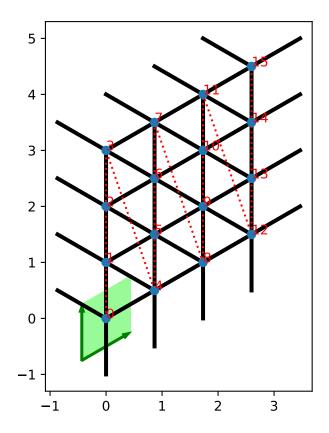
class tenpy.models.lattice.Triangular(Lx, Ly, site, **kwargs)

 $Bases: \verb|tenpy.models.lattice.SimpleLattice|\\$

A triangular lattice.

Parameters

- Lx (int) The length in each direction.
- Ly (int) The length in each direction.



- **site** (Site) The local lattice site. The *unit_cell* of the Lattice is just [site].
- **kwargs Additional keyword arguments given to the Lattice. pairs are set accordingly. If order is specified in the form ('standard', snake_windingi, priority), the snake_winding and priority should only be specified for the spatial directions. Similarly, positions can be specified as a single vector.

dim = 2

the dimension of the lattice

property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count_neighbors (u=0, key='nearest_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

Parameters dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

• **u1** (*int*) – Indices within the unit cell; the *u1* and *u2* of add_coupling()

- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

```
enlarge mps unit cell(factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

```
find_coupling_pairs (max_dx=3, cutoff=None, eps=1e-10)
```

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps_idx (lat_idx)

Translate lattice indices (x_0, \ldots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx $(array_like [.., dim+1])$ - The last dimension corresponds to lattice indices $(x_0, ..., x_\{D-1\}, u)$. All lattice indices should be positive and

smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an $x \ \theta$ outside indicates shifts across the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

$mps2lat_idx(i)$

Translate MPS index i to lattice indices $(x \ 0, \ldots, x \ \{dim-1\}, u)$.

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``($x_0, \ldots, x_{dim-1}, u$)` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

$mps2lat_values(A, axes=0, u=None)$

same as Lattice.mps2lat_values(), but ignore u, setting it to 0.

mps2lat_values_masked(A, axes=-1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps2lat_values()$ allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- **A** (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

mps_idx_fix_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None | int) – Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps lat idx fix u(u=None)

Similar as mps_idx_fix_u(), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- **lat_idx** (2D array) The row j contains the lattice index (without u) corresponding to mps_idx[j].

mps_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use <code>count_neighbors()</code> instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

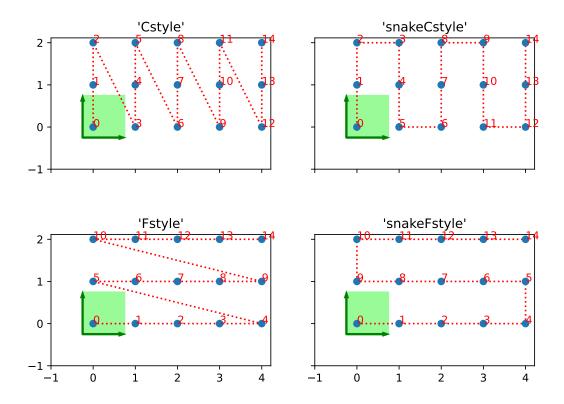
You can visualize the order with plot_order().

ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	$(\dim -1, \ldots, 1, 0, \dim)$	(False,, False, False)
'snakeFstyle'	$(\dim -1,, 1, 0, \dim)$	(False,, False, False)



Note: For lattices with a non-trivial unit cell (e.g. Honeycomb, Kagome), the grouped order might be more appropriate, see <code>get_order_grouped()</code>

Parameters order (str | ('standard', snake_winding, priority) | ('grouped', groups, ...)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get_order() and 'grouped' for get_order_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

Returns order – the order to be used for order.

Return type array, shape (N, D+1), dtype np.intp

See also:

```
get_order generates the order from equivalent priority and snake_winding.
get_order_grouped variant of get_order.
plot_order visualizes the resulting order.
```

plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

plot_coupling (ax, coupling=None, wrap=False, **kwargs)

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

plot_order (ax, order=None, textkwargs={'color': 'r'}, **kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.

- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

position (lat_idx)

return 'space' position of one or multiple sites.

```
Parameters lat idx (ndarray, (..., dim+1)) – Lattice indices.
```

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

```
Return type ndarray, (..., dim)
```

possible_couplings (u1, u2, dx, strength=None)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le x_$

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible multi couplings(ops, strength=None)

Generalization of possible couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

19.1.11 TrivialLattice

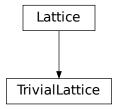
• full name: tenpy.models.lattice.TrivialLattice

• parent module: tenpy.models.lattice

• type: class

463

Inheritance Diagram



Methods

19.1. lattice

TrivialLatticeinit(mps_sites,	Initialize self.
**kwargs)	
TrivialLattice.count_neighbors([u, key])	Count e.g.
$TrivialLattice.coupling_shape(dx)$	Calculate correct shape of the <i>strengths</i> for a coupling.
TrivialLattice.distance(u1, u2, dx)	Get the distance for a given coupling between two sites
	in the lattice.
TrivialLattice.enlarge_mps_unit_cell([fa	cRepeat the unit cell for infinite MPS boundary condi-
	tions; in place.
TrivialLattice.find_coupling_pairs([max	_dautomatically find coupling pairs grouped by distances.
])	
TrivialLattice.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
TrivialLattice.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,
	u) to MPS index i.
TrivialLattice.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,
	x_{dim-1} , u).
TrivialLattice.mps2lat_values(A[, axes,	Reshape/reorder A to replace an MPS index by lattice
u])	indices.
TrivialLattice.mps2lat_values_masked(A	, Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
TrivialLattice.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is <i>u</i> .
TrivialLattice.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
TrivialLattice.mps_sites()	Return a list of sites for all MPS indices.
TrivialLattice.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
TrivialLattice.number_nearest_neighbor	s (Pep)recated.
TrivialLattice.number_next_nearest_nei	gDiepreca([nd])
TrivialLattice.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
TrivialLattice.plot_basis(ax[, origin,	Plot arrows indicating the basis vectors of the lattice.
shade])	
	continues on next page

Table 23 – continued from previous page		
TrivialLattice.plot_bc_identified(ax[,	Mark two sites indified by periodic boundary condi-	
])	tions.	
TrivialLattice.plot_coupling(ax[, cou-	Plot lines connecting nearest neighbors of the lattice.	
pling,])		
TrivialLattice.plot_order(ax[, order,])	Plot a line connecting sites in the specified "order" and	
	text labels enumerating them.	
TrivialLattice.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.	
TrivialLattice.position(lat_idx)	return 'space' position of one or multiple sites.	
TrivialLattice.possible_couplings(u1,	Find possible MPS indices for two-site couplings.	
u2, dx)		
TrivialLattice.possible_multi_coupling	s@pseralization of possible_couplings() to cou-	
	plings with more than 2 sites.	
TrivialLattice.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.	
)		
TrivialLattice.site(i)	return Site instance corresponding to an MPS index i	
TrivialLattice.test_sanity()	Sanity check.	

Class Attributes and Properties

TrivialLattice.Lu	the (expected) number of sites in the unit cell,	
	len(unit_cell).	
TrivialLattice.boundary_conditions	Human-readable list of boundary conditions from bc	
	and bc_shift.	
TrivialLattice.dim	The dimension of the lattice.	
TrivialLattice.nearest_neighbors		
TrivialLattice.next_nearest_neighbors		
TrivialLattice.next_next_nearest_neighbors		
TrivialLattice.order	Defines an ordering of the lattice sites, thus mapping the	
	lattice to a 1D chain.	

class tenpy.models.lattice.TrivialLattice(mps_sites, **kwargs)

Bases: tenpy.models.lattice.Lattice

Trivial lattice consisting of a single (possibly large) unit cell in 1D.

This is usefull if you need a valid *Lattice* with given *mps_sites*() and don't care about the actual geometry, e.g, because you don't intend to use the *CouplingModel*.

Parameters

- mps_sites (list of Site) The sites making up a unit cell of the lattice.
- **kwargs Further keyword arguments given to Lattice.

property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count_neighbors (u=0, key='nearest_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

property dim

The dimension of the lattice.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The u1, u2, dx parameters are defined in analogy with $add_coupling()$, i.e., this function calculates the distance between a pair of operators added with $add_coupling$ (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- **u1** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (int) – The new number of sites in the MPS unit cell will be increased from N_sites to factor*N $_sites_per_ring$. Since MPS unit cells are repeated in

19.1. lattice 465

```
the x-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).
```

find_coupling_pairs (*max_dx=3*, *cutoff=None*, *eps=1e-10*)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (*float*) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

lat2mps_idx (lat_idx)

Translate lattice indices (x_0, \ldots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx ($array_like[..,dim+1]$) – The last dimension corresponds to lattice indices ($x_0, ..., x_{D-1}, u$). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

mps2lat idx(i)

Translate MPS index i to lattice indices (x_0, \dots, x_{dim-1}) , u).

Parameters i (int / array_like of int)-MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder A to replace an MPS index by lattice indices.

Parameters

- A (ndarray) Some values. Must have A.shape[axes] = self.N_sites if u is None, or A.shape[axes] = self.N_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit_cell[u], as returned by mps_idx_fix_u(). The resulting array will not have the additional dimension(s) of u.

Returns res_A – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[tuple(lat.mps2lat_idx(5))] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use $mps_idx_fix_u()$ to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

19.1. lattice 467

mps2lat_values_masked (A, axes=- 1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps2lat_values$ () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices accross the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

mps_idx_fix_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

mps_lat_idx_fix_u (u=None)

Similar as $mps_idx_fix_u$ (), but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices *i* for which self.site(i) is self. unit_cell[u].
- $lat_idx (2D \ array)$ The row j contains the lattice index (without u) corresponding to $mps_idx[j]$.

mps_sites()

Return a list of sites for all MPS indices.

Equivalent to [self.site(i) for i in range(self.N_sites)].

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the i-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

$number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot_order().

ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

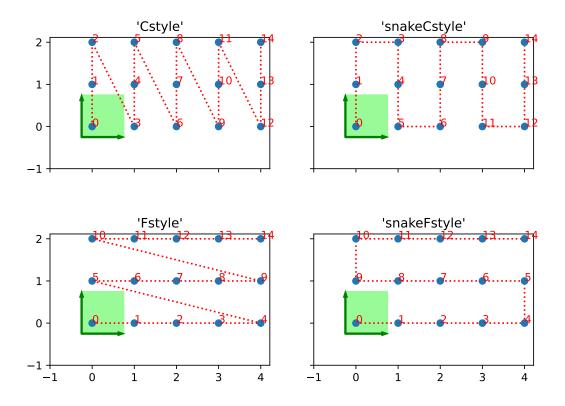
order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Note: For lattices with a non-trivial unit cell (e.g. Honeycomb, Kagome), the grouped order might be more appropriate, see get_order_grouped()

Parameters order (str | ('standard', snake_winding, priority) | ('grouped', groups, ...)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get_order() and 'grouped' for get_order_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

Returns order – the order to be used for order.

19.1. lattice 469



Return type array, shape (N, D+1), dtype np.intp

See also:

```
get_order generates the order from equivalent priority and snake_winding.
get_order_grouped variant of get_order.
plot_order visualizes the resulting order.
```

plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **kwargs Keyword arguments for ax.arrow.

```
plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.
```

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit_cell_positions.
- **kwargs Keyword arguments for the used ax.plot.

```
plot_coupling(ax, coupling=None, wrap=False, **kwargs)
```

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (*i0*, *i1*, ..., *u1*) to the site (*i0*+dx[0], *i1*+dx[1], ..., *u2*), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

```
plot_order(ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

• ax (matplotlib.axes.Axes) - The axes on which we should plot.

19.1. lattice 471

- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering (); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

```
position (lat idx)
```

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

Returns pos – The position of the lattice sites specified by *lat idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible_couplings (u1, u2, dx, strength=None)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le Ls[a]$ and $0 \le x_a + dx[a] \le lat.Ls[a]$.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible multi couplings(ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add_multi_coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- **strength_vals** (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

Functions

<pre>get_lattice(lattice_name)</pre>	Given the name of a Lattice class, get the lattice class
	itself.
<pre>get_order(shape, snake_winding[, priority])</pre>	Built the Lattice.order in (Snake-) C-Style for a
	given lattice shape.
<pre>get_order_grouped(shape, groups[, priority])</pre>	Variant of get_order(), grouping some sites of the
	unit cell.

19.1. lattice 473

19.1.12 get lattice

```
• full name: tenpy.models.lattice.get_lattice
```

• parent module: tenpy.models.lattice

• type: function

```
tenpy.models.lattice.get_lattice(lattice_name)
```

Given the name of a Lattice class, get the lattice class itself.

```
Parameters lattice_name (str) - Name of a Lattice class defined in the module lattice, for example "Chain", "Square", "Honeycomb", ....
```

Returns LatticeClass – The lattice class (type, not instance) specified by *lattice_name*.

Return type Lattice

19.1.13 get_order

• full name: tenpy.models.lattice.get_order

• parent module: tenpy.models.lattice

• type: function

tenpy.models.lattice.get_order(shape, snake_winding, priority=None)

Built the Lattice.order in (Snake-) C-Style for a given lattice shape.

Note: In this doc-string, the word 'direction' referst to a physical direction of the lattice or the index u of the unit cell as an "artificial direction".

Parameters

- **shape** (tuple of int) The shape of the lattice, i.e., the length in each direction.
- **snake_winding** (tuple of bool) For each direction one bool, whether we should wind as a "snake" (True) in that direction (i.e., going forth and back) or simply repeat ascending (False)
- **priority** (None | tuple of float) If None (default), use C-Style ordering. Otherwise, this defines the priority along which direction to wind first; the direction with the highest priority increases fastest. For example, "C-Style" order is enforced by priority=(0, 1, 2, ...), and Fortrans F-style order is enforced by priority=(dim, dim-1, ..., 1, 0)
- **group** (None | tuple of tuple) If None (default), ignore it. Otherwise, it specifies that we group the fastests changing dimension

Returns order – An order of the sites for Lattice.order in the specified ordering.

Return type ndarray (np.prod(shape), len(shape))

See also:

```
Lattice.ordering method in Lattice to obtain the order from parameters.
```

Lattice.plot_order visualizes the resulting order in a Lattice.

get_order_grouped a variant grouping sites of the unit cell.

19.1.14 get order grouped

• full name: tenpy.models.lattice.get_order_grouped

• parent module: tenpy.models.lattice

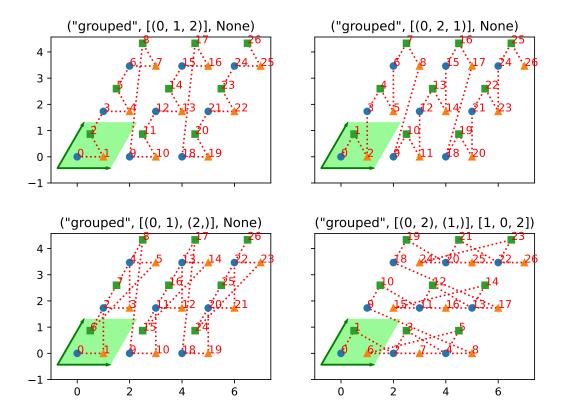
• type: function

tenpy.models.lattice.get_order_grouped(shape, groups, priority=None)

Variant of get_order(), grouping some sites of the unit cell.

This function is usefull for lattices with a unit cell of more than 2 sites (e.g. Kagome). For 2D lattices with a unit cell, the ordering goes first within a group, then along y, then the next group (for the same x-value), again along y, and finally along x when all groups are done.

As an example, consider the Kagome lattice.



Note: In this doc-string, the word 'direction' referst to a physical direction of the lattice or the index u of the unit cell as an "artificial direction".

Parameters

• **shape** (tuple of int) - The shape of the lattice, i.e., the length in each direction.

19.1. lattice 475

- groups (tuple of tuple of int) A partition and reordering of range (shape [-1]) into smaller groups. The ordering goes first within a group, then along the last spatial dimensions, then changing between different groups and finally in Cstyle order along the remaining spatial dimensions.
- **priority** (None | tuple of ints) By default (None), use C-style order for everything except the unit cell, as shown above. If a tuple, it should have length len(shape) and specifies which order to go first, similarly as in get_order(). To group sites in the unit cell, you should make the last entry of priority the largest. However, you can also choose to group along another direction in that case groups should be a partitioning of range(shape(argmax(priority))). Try and plot it, if you need it!

Returns order – An order of the sites for *Lattice.order* in the specified *ordering*.

Return type ndarray (np.prod(shape), len(shape))

See also:

```
Lattice.ordering() method in Lattice to obtain the order from parameters.

Lattice.plot_order() visualizes the resulting order in a Lattice.
```

Module description

Classes to define the lattice structure of a model.

The base class Lattice defines the general structure of a lattice, you can subclass this to define you own lattice. The SimpleLattice is a slight simplification for lattices with a single-site unit cell. Further, we have some predefined lattices, namely Chain, Ladder in 1D and Square, Triangular, Honeycomb, and Kagome in 2D.

The IrregularLattice provides a way to remove or add sites to an existing, regular lattice.

See also the Models and Details on the lattice geometry.

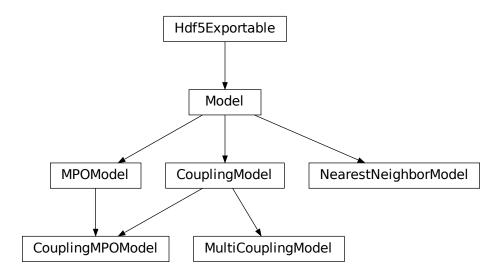
19.2 model

• full name: tenpy.models.model

• parent module: tenpy.models

• type: module

Classes



CouplingMPOModel(model_params)	Combination of the CouplingModel and
	MPOModel.
CouplingModel(lattice[, bc_coupling,])	Base class for a general model of a Hamiltonian consist-
	ing of two-site couplings.
MPOMode1(lattice, H_MPO)	Base class for a model with an MPO representation of
	the Hamiltonian.
Model(lattice)	Base class for all models.
MultiCouplingModel(lattice[, bc_coupling,])	Deprecated class which was a generalization of the Cou-
	plingModel.
NearestNeighborModel(lattice, H_bond)	Base class for a model of nearest neighor interactions
	w.r.t.

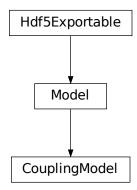
19.2.1 CouplingModel

• full name: tenpy.models.model.CouplingModel

• parent module: tenpy.models.model

• type: class

Inheritance Diagram



Methods

CouplingModelinit(lattice[,])	Initialize self.
CouplingModel.add_coupling(strength, u1,	Add twosite coupling terms to the Hamiltonian, sum-
)	ming over lattice sites.
CouplingModel.add_coupling_term(strength,	Add a two-site coupling term on given MPS sites.
i,)	
CouplingModel.add_exponentially_decayi	nAdd an exponentially decaying long-range coupling.
CouplingModel.add_local_term(strength,	Add a single term to <i>self</i> .
term)	
CouplingModel.add_multi_coupling(strength	, Add multi-site coupling terms to the Hamiltonian, sum-
ops)	ming over lattice sites.
	. Add a general M-site coupling term on given MPS sites.
])	
CouplingModel.add_onsite(strength, u, op-	Add onsite terms to onsite_terms.
name)	
CouplingModel.add_onsite_term(strength, i,	Add an onsite term on a given MPS site.
op)	
CouplingModel.all_coupling_terms()	Sum of all coupling_terms.
CouplingModel.all_onsite_terms()	Sum of all onsite_terms.
CouplingModel.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
CouplingModel.calc_H_bond([tol_zero])	calculate <i>H_bond</i> from coupling_terms and
	onsite_terms.
CouplingModel.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
CouplingModel.coupling_strength_add_ex	t Add an external flux to the coupling strength.
CouplingModel.enlarge_mps_unit_cell([fac	etoRepeat the unit cell for infinite MPS boundary condi-
	tions; in place.
CouplingModel.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
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Table	27	- continued	from	previous	page

		, , ,
CouplingModel.group_sites([n,		Modify <i>self</i> in place to group sites.
grouped_sites])		
CouplingModel.save_hdf5(hdf5_saver,	h5gr,	Export self into a HDF5 file.
)		
CouplingModel.test_sanity()		Sanity check, raises ValueErrors, if something is wrong.

Class Attributes and Properties

CouplingModel.logger	class attribute.	

_

lattice, bc_coupling=None, plicit_plus_hc=False)

ex-

Bases: tenpy.models.model.Model

class tenpy.models.model.CouplingModel(lattice,

Base class for a general model of a Hamiltonian consisting of two-site couplings.

In this class, the terms of the Hamiltonian are specified explicitly as OnsiteTerms or CouplingTerms.

Deprecated since version 0.4.0: $bc_coupling$ will be removed in 1.0.0. To specify the full geometry in the lattice, use the bc parameter of the Lattice.

Parameters

- lattice (Lattice) The lattice defining the geometry and the local Hilbert space(s).
- bc_coupling ((iterable of) {'open' | 'periodic' | int}) Boundary conditions of the couplings in each direction of the lattice. Defines how the couplings are added in add_coupling(). A single string holds for all directions. An integer shift means that we have periodic boundary conditions along this direction, but shift/tilt by -shift*lattice.basis[0] (~cylinder axis for bc_MPS='infinite') when going around the boundary along this direction.
- **explicit_plus_hc** (bool) If True, the Hermitian conjugate of the MPO is computed at runtime, rather than saved in the MPO.

onsite_terms

The OnsiteTerms ordered by category.

Type {'category': OnsiteTerms}

coupling_terms

The CouplingTerms ordered by category. In case we've added terms with more than 2 operators, e.g. with add_multi_coupling(), the values of the dictionary may also be MultiCouplingTerms.

Type {'category': CouplingTerms}

exp_decaying_terms

Collection of coupling terms with exponentially decaying long-range interactions. Filled by add_exponentially_decaying_coupling().

Type ExponentiallyDecayingTerms

explicit_plus_hc

If *True*, *self* represents the terms in *onsite_terms*, *coupling_terms* and *exp_decaying_terms plus* their hermitian conjugate added. The flag will be carried on to the MPO, which will have a reduced bond dimension if self.add_coupling(..., plus_hc=True) was used. Note that *add_onsite()*, *add_coupling()*, *add_multi_coupling()* and

add_exponentially_decaying_coupling() respect this flag, ensuring that the represented Hamiltonian is indepentent of the explicit_plus_hc flag.

```
Type bool
```

test sanity()

Sanity check, raises ValueErrors, if something is wrong.

```
add_local_term (strength, term, category=None, plus_hc=False)
```

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to *onsite_terms* or *coupling_terms*, respectively.

Parameters

- **strength** (*float/complex*) The prefactor of the term.
- **term** (list of (str, array_like)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index lat_idx. Here, lat_idx is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms or coupling_terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)

Add onsite terms to onsite_terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat.unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to <code>onsite_terms</code>; doesn't rebuild the MPO.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

See also:

```
add_coupling Add a terms acting on two sites.
```

add_onsite_term Add a single term without summing over vecx.

```
add_onsite_term (strength, i, op, category=None, plus_hc=False)
```

Add an onsite term on a given MPS site.

Wrapper for self.onsite_terms[category].add_onsite_term(...).

Parameters

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < I.
- op (str) Name of the involved operator.
- category (str) Descriptive name used as key for onsite_terms. Defaults to op.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

all_onsite_terms()

Sum of all onsite terms.

add_coupling (strength, u1, op1, u2, op2, dx, op_string=None, str_on_first=True, raise_op2_left=False, category=None, plus_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,...,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str_on_first* and *raise_op2_left* will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit_cell[u1] for OP1.
- **op1** (str) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op_needs_JW().
- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a

well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.

- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

When initializing a model, you can add a term $J\sum_{\langle i,j\rangle}S_i^zS_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus_hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

$\to Cdagger_down C_up$
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

add_onsite Add terms acting on one site only.

add_multi_coupling_term for terms on more than two sites.

add coupling term Add a single term without summing over \vec{x} .

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- j (*int*) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j . If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- op1 (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

all_coupling_terms()

Sum of all coupling_terms.

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by $self.lat.unit_cell[u].get_op(opname)$.

The coupling strength may vary spatially if the given strength is a numpy array. The correct shape of this array is the $coupling_shape$ returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of strength is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.
- op_string (str | None) If a string is given, we use this as the name of an operator to be used inbetween the operators, excluding the sites on which any operators act. This operator should be defined on all sites in the unit cell.

If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.

- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D 
>>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), \rightarrowdx_0, u1)])
```

See also:

```
add_onsite Add terms acting on one site only.
```

add coupling Add terms acting on two sites.

add_multi_coupling_term Add a single term, not summing over the possible \vec{x} .

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

• **strength** (*float*) – The strength of the coupling term.

- ijkl (list of int) The MPS indices of the sites on which the operators acts. With $i, j, k, \ldots = ijkl$, we require that they are ordered ascending, $i < j < k < \ldots$ and that $0 <= i < N_sites$. Inidces $>= N_sites$ indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k,
- **op_string** (list of str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- op i (string) Names for the operators.
- **op_j** (*string*) Names for the operators.
- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- **op_string** (*None | str*) The operator to be inserted between *A* and *B*; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right op_j acts first.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use $fit_with_sum_of_exp()$ to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...    return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
>>> x = np.arange(1, fit_range + 1)
>>> print('error in fit: {0:.3e}'.format(np.sum(np.abs(decay(x) - sum_of_exp(lam, pref, x)))))
error in fit: 1.073e-04
>>> for pr, la in zip(pref, lam):
... self.add_exponentially_decaying_coupling(pr, la, 'N', 'N')
```

calc_H_onsite(tol_zero=1e-15)

Calculate *H_onsite* from *self.onsite_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take $explicit_plus_hc$ into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

- **H_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If <code>explicit_plus_hc</code> is True, Hermitian conjugates of the onsite terms will be included.

calc_H_bond(tol_zero=1e-15)

calculate H_bond from coupling_terms and onsite_terms.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are
 considered to be zero.</pre>

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_MPO(tol_zero=1e-15)
```

Calculate MPO representation of the Hamiltonian.

Uses onsite_terms and coupling_terms to build an MPOGraph (and then an MPO).

Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in add_coupling().

Parameters

• **strength** (*scalar* / *array*) – The strength to be used in *add_coupling()*, when no external flux would be present.

- dx (iterable of int) Translation vector (of the unit cell) between op1 and op2 in add coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

Returns strength – The strength array to be used as *strength* in add_coupling() with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... strength_with_flux = self.coupling_strength_add_ext_flux(strength, dx,

| [0, phi])
... self.add_coupling(strength_with_flux, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(strength_with_flux), u2, 'Cd', u1, 'C', -dx)
```

enlarge_mps_unit_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each n sites together using the GroupedSite. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

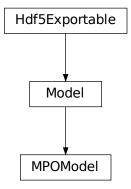
Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

19.2.2 MPOModel

- full name: tenpy.models.model.MPOModel
- parent module: tenpy.models.model
- type: class

Inheritance Diagram



Methods

MPOModelinit(lattice, H_MPO)	Initialize self.
MPOModel.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
MPOModel.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
<pre>MPOModel.from_hdf5(hdf5_loader, h5gr, subpath)</pre>	Load instance from a HDF5 file.
<pre>MPOModel.group_sites([n, grouped_sites])</pre>	Modify <i>self</i> in place to group sites.
MPOModel.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
MPOModel.test_sanity()	

Class Attributes and Properties

```
class tenpy.models.model.MPOModel(lattice, H_MPO)
```

Bases: tenpy.models.model.Model

Base class for a model with an MPO representation of the Hamiltonian.

In this class, the Hamiltonian gets represented by an MPO. Thus, instances of this class are suitable for MPO-based algorithms like DMRG dmrg and MPO time evolution.

Parameters H_MPO (MPO) – The Hamiltonian rewritten as an MPO.

н мро

MPO representation of the Hamiltonian. If the *explicit_plus_hc* flag of the MPO is *True*, the represented Hamiltonian is H_MPO + hermitian_cojugate(H_MPO).

```
Type tenpy.networks.mpo.MPO
```

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (int) – The new number of sites in the MPS unit cell will be increased from N_sites to factor*N $_sites_per_ring$. Since MPS unit cells are repeated in the x-direction in our convetion, the lattice shape goes from (Lx, Ly, \ldots, Lu) to $(Lx*factor, Ly, \ldots, Lu)$.

group_sites (n=2, grouped_sites=None)

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of *GroupedSite*) The sites grouped together.

Returns grouped_sites – The sites grouped together.

```
Return type list of GroupedSite
```

calc_H_bond_from_MPO (tol_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

```
Returns H_bond - Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']
```

```
Return type list of Array
```

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

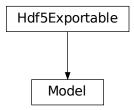
Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

19.2.3 Model

- full name: tenpy.models.model.Model
- parent module: tenpy.models.model
- · type: class

Inheritance Diagram



Methods

Modelinit(lattice)	Initialize self.
Model.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Model.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
<pre>Model.group_sites([n, grouped_sites])</pre>	Modify <i>self</i> in place to group sites.
Model.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.

Class Attributes and Properties

Model.logger class attribute.

class tenpy.models.model.Model(lattice)

Bases: tenpy.tools.hdf5_io.Hdf5Exportable

Base class for all models.

The common base to all models is the underlying Hilbert space and geometry, specified by a Lattice.

Parameters lattice (Lattice) – The lattice defining the geometry and the local Hilbert space(s).

lat

The lattice defining the geometry and the local Hilbert space(s).

Type Lattice

dtype

The data type of the Hamiltonian

Type dtype

logger = <Logger tenpy.models.model.Model (WARNING) >
 class attribute.

Type logging.Logger

Type An instance of a logger; see *Logging and terminal output*. NB

```
enlarge mps unit cell(factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of *GroupedSite*) The sites grouped together.

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

```
classmethod from hdf5 (hdf5 loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

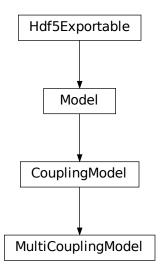
19.2.4 MultiCouplingModel

 $\bullet \ \ full \ name: tenpy.models.model.MultiCouplingModel$

• parent module: tenpy.models.model

• type: class

Inheritance Diagram



Methods

MultiCouplingModelinit(lattice[,])	Initialize self.
MultiCouplingModel.	Add twosite coupling terms to the Hamiltonian, sum-
add_coupling(strength,)	ming over lattice sites.
MultiCouplingModel.	Add a two-site coupling term on given MPS sites.
$ ext{add_coupling_term}(\dots[,\dots])$	
MultiCouplingModel.	Add an exponentially decaying long-range coupling.
add_exponentially_decaying_coupling()	
MultiCouplingModel.	Add a single term to <i>self</i> .
<pre>add_local_term(strength, term)</pre>	
MultiCouplingModel.	Add multi-site coupling terms to the Hamiltonian, sum-
$add_multi_coupling([,])$	ming over lattice sites.
MultiCouplingModel.	Add a general M-site coupling term on given MPS sites.
add_multi_coupling_term (\dots)	
MultiCouplingModel.add_onsite(strength, u,	Add onsite terms to onsite_terms.
)	
	continues on next page

rable 35 – continued from previous page		
MultiCouplingModel.	Add an onsite term on a given MPS site.	
<pre>add_onsite_term(strength,)</pre>		
MultiCouplingModel.	Sum of all coupling_terms.	
all_coupling_terms()		
MultiCouplingModel.all_onsite_terms()	Sum of all onsite_terms.	
MultiCouplingModel.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.	
MultiCouplingModel.	calculate H_bond from coupling_terms and	
calc_H_bond([tol_zero])	onsite_terms.	
MultiCouplingModel.	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .	
calc_H_onsite([tol_zero])		
MultiCouplingModel.	Add an external flux to the coupling strength.	
${\it coupling_strength_add_ext_flux}(\ldots)$		
MultiCouplingModel.	Repeat the unit cell for infinite MPS boundary condi-	
$enlarge_mps_unit_cell([])$	tions; in place.	
MultiCouplingModel.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.	
)		
MultiCouplingModel.group_sites([n,])	Modify <i>self</i> in place to group sites.	
MultiCouplingModel.save_hdf5(hdf5_saver,	Export self into a HDF5 file.	
)		
MultiCouplingModel.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.	

Table 33 – continued from previous page

Class Attributes and Properties

MultiCouplingModel.logger	class attribute.	

Bases: tenpy.models.model.CouplingModel

Deprecated class which was a generalization of the CouplingModel.

Deprecated since version 0.7.2: In earlier versions of TeNPy, this class contained the methods <code>add_multi_coupling()</code> and <code>add_multi_coupling_term()</code>. However, since we introduced the <code>MultiCouplingTerms</code>, this separation within the Model class is no longer necessary. We hence merged the <code>MultiCouplingModel</code> with the <code>CouplingModel</code>.

add_coupling (strength, u1, op1, u2, op2, dx, op_string=None, str_on_first=True, raise_op2_left=False, category=None, plus_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible_couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str_on_first* and *raise_op2_left* will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit_cell[u1] for OP1.
- **op1** (*str*) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- u2 (int) Picks the site lat.unit_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.
- dx (*iterable of int*) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op_needs_JW().
- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

When initializing a model, you can add a term $J\sum_{\langle i,j\rangle} S_i^z S_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus_hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

```
add_onsite Add terms acting on one site only. 
add_multi_coupling_term for terms on more than two sites. 
add_coupling_term Add a single term without summing over \vec{x}.
```

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **j** (*int*) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., *op_i* acts "left" of *op_j*. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **op1** (str) Names of the involved operators.
- **op2** (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.

- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- **op_i** (*string*) Names for the operators.
- **op_j** (*string*) Names for the operators.
- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- op_string (None / str) The operator to be inserted between A and B; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right op_j acts first.
- plus hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use $fit_with_sum_of_exp()$ to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...     return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
>>> x = np.arange(1, fit_range + 1)
>>> print('error in fit: {0:.3e}'.format(np.sum(np.abs(decay(x) - sum_of_exp(lam, pref, x)))))
error in fit: 1.073e-04
>>> for pr, la in zip(pref, lam):
...     self.add_exponentially_decaying_coupling(pr, la, 'N', 'N')
```

add_local_term (strength, term, category=None, plus_hc=False)

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite_terms or coupling_terms, respectively.

Parameters

- **strength** (*float/complex*) The prefactor of the term.
- **term** (*list of* (*str*, *array_like*)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat_idx*. Here, *lat_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms or coupling_terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

```
add_multi_coupling (strength, ops, __deprecate_l='DEPRECATED', __depre-cate_2='DEPRECATED', op_string=None, category=None, plus_hc=False)
Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.
```

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by $self.lat.unit_cell[u].get_op(opname)$.

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.
- op_string (str | None) If a string is given, we use this as the name of an operator to be used inbetween the operators, excluding the sites on which any operators act. This operator should be defined on all sites in the unit cell.
 - If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D 
>>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), dx_0, u1)])
```

See also:

add_onsite Add terms acting on one site only.

add_coupling Add terms acting on two sites.

add_multi_coupling_term Add a single term, not summing over the possible \vec{x} .

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With i, j, k, ... = ijkl, we require that they are ordered ascending, i < j < k < ... and that 0 <= i < N_sites. Inidees >= N_sites indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots
- op_string (list of str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)
Add onsite terms to onsite terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat.unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to onsite terms; doesn't rebuild the MPO.

Parameters

19.2. model 499

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

See also:

```
add_coupling Add a terms acting on two sites.
```

add_onsite_term Add a single term without summing over vecx.

add_onsite_term (strength, i, op, category=None, plus_hc=False)

Add an onsite term on a given MPS site.

Wrapper for self.onsite_terms[category].add_onsite_term(...).

Parameters

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- op (str) Name of the involved operator.
- **category** (str) Descriptive name used as key for onsite_terms. Defaults to op.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

all_coupling_terms()

Sum of all coupling_terms.

all_onsite_terms()

Sum of all onsite_terms.

calc_H_MPO(tol_zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite_terms and coupling_terms to build an MPOGraph (and then an MPO).

Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

$calc_H_bond(tol_zero=1e-15)$

calculate H_bond from coupling_terms and onsite_terms.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_bond - Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc H onsite (tol zero=1e-15)
```

Calculate *H_onsite* from *self.onsite_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take explicit_plus_hc into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

- **H_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit_plus_hc is True, Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in add_coupling().

Parameters

- **strength** (*scalar* / *array*) The strength to be used in *add_coupling()*, when no external flux would be present.
- **dx** (iterable of int) Translation vector (of the unit cell) between op1 and op2 in add_coupling().
- phase (iterable of float) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase phi when hopping around the cylinder.

Returns strength – The strength array to be used as strength in add_coupling() with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

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19.2. model 501

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```
self.add_coupling(strength_with_flux, u1, 'Cd', u2, 'C', dx)
self.add_coupling(np.conj(strength_with_flux), u2, 'Cd', u1, 'C', -dx)
```

```
enlarge_mps_unit_cell (factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each *n* sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5qr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

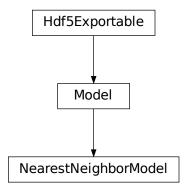
19.2.5 NearestNeighborModel

• full name: tenpy.models.model.NearestNeighborModel

• parent module: tenpy.models.model

• type: class

Inheritance Diagram



Methods

NearestNeighborModelinit(lattice,	Initialize self.
H_bond)	
NearestNeighborModel.	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
bond_energies(psi)	
NearestNeighborModel.	Calculate the MPO Hamiltonian from the bond Hamil-
${\it calc}_{\it H}_{\it MPO}_{\it from}_{\it bond}([\dots])$	tonian.
NearestNeighborModel.	Repeat the unit cell for infinite MPS boundary condi-
enlarge_mps_unit_cell([])	tions; in place.
NearestNeighborModel.	Initialize a NearestNeighborModel from a model class
<pre>from_MPOModel(mpo_model)</pre>	defining an MPO.
NearestNeighborModel.	Load instance from a HDF5 file.
<pre>from_hdf5(hdf5_loader,)</pre>	
NearestNeighborModel.group_sites([n,	Modify <i>self</i> in place to group sites.
])	
NearestNeighborModel.	Export self into a HDF5 file.
save_hdf5(hdf5_saver,)	
	continues on poyt page

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19.2. model 503

Table 35 – continued from previous page

NearestNeighborModel.test sanity()

NearestNeighborModel.	Return a NearestNeighborModel with same lattice, but
$trivial_like_NNModel()$	trivial (H=0) bonds.

Class Attributes and Properties

NearestNeighborModel.logger	class attribute.	

class tenpy.models.model.NearestNeighborModel(lattice, H_bond)

Bases: tenpy.models.model.Model

Base class for a model of nearest neigbor interactions w.r.t. the MPS index.

In this class, the Hamiltonian $H = \sum_i H_{i,i+1}$ is represented by "bond terms" $H_{i,i+1}$ acting only on two neighboring sites i and i+1, where i is an integer. Instances of this class are suitable for $t \in \mathcal{D}d$.

Note that the "nearest-neighbor" in the name refers to the MPS index, not the lattice. In short, this works only for 1-dimensional (1D) nearest-neighbor models: A 2D lattice is internally mapped to a 1D MPS "snake", and even a nearest-neighbor coupling in 2D becomes long-range in the MPS chain.

Parameters

- **lattice** (tenpy.model.lattice.Lattice) The lattice defining the geometry and the local Hilbert space(s).
- H_bond (list of {Array | None}) The Hamiltonian rewritten as sum_i H_bond[i] for MPS indices i. H_bond[i] acts on sites (i-1, i); we require len (H_bond) == lat.N_sites. Legs of each H_bond[i] are ['p0', 'p0*', 'p1', 'p1*'].

H bond

The Hamiltonian rewritten as $sum_i H_bond[i]$ for MPS indices i. $H_bond[i]$ acts on sites (i-1, i), None represents 0. Legs of each $H_bond[i]$ are ['p0', 'p0*', 'p1', 'p1*']. H_bond is not affected by the *explicit_plus_hc* flag of a *CouplingModel*.

Type list of {Array | None}

classmethod from_MPOModel (mpo_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group_sites().

Parameters mpo_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

Examples

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> grouped_sites = nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define H_bond . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
>>> isinstance(nnn_chain_for_tebd, NearestNeighborModel)
True
```

trivial_like_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

bond_energies (psi)

Calculate bond energies <psilH_bondlpsi>.

Parameters psi (MPS) – The MPS for which the bond energies should be calculated.

Returns $E_bond - List$ of bond energies: for finite bc, $E_Bond[i]$ is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc $E_bond[i]$ is the energy of bond i-1, i.

Return type 1D ndarray

```
enlarge_mps_unit_cell(factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the <code>GroupedSite</code>. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- n (int) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

Returns grouped sites – The sites grouped together.

Return type list of GroupedSite

```
calc_H_MPO_from_bond(tol_zero=1e-15)
```

Calculate the MPO Hamiltonian from the bond Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

19.2. model 505

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Module description

This module contains some base classes for models.

A 'model' is supposed to represent a Hamiltonian in a generalized way. The Lattice specifies the geometry and underlying Hilbert space, and is thus common to all models. It is needed to intialize the common base class Model of all models.

Different algorithms require different representations of the Hamiltonian. For example for DMRG, the Hamiltonian needs to be given as an MPO, while TEBD needs the Hamiltonian to be represented by 'nearest neighbor' bond terms. This module contains the base classes defining these possible representations, namley the MPOModel and NearestNeighborModel.

A particular model like the XXZChain should then yet another class derived from these classes. In it's __init__, it needs to explicitly call the MPOModel.__init__ (self, lattice, H_MPO), providing an MPO representation of H, and also the NearestNeighborModel.__init__ (self, lattice, H_bond), providing a representation of H by bond terms H_bond .

The CouplingModel is the attempt to generalize the representation of H by explicitly specifying the couplings in a general way, and providing functionality for converting them into H_MPO and H_bond. This allows to quickly generate new model classes for a very broad class of Hamiltonians.

The CouplingMPOModel aims at structuring the initialization for most models and is used as base class in (most of) the predefined models in TeNPy.

See also the introduction in *Models*.

Specific models

tf_ising	Prototypical example of a quantum model: the trans-
	verse field Ising model.
xxz_chain	Prototypical example of a 1D quantum model: the spin-
	1/2 XXZ chain.
spins	Nearest-neighbour spin-S models.
spins_nnn	Next-Nearest-neighbour spin-S models.
fermions_spinless	Spinless fermions with hopping and interaction.
hubbard	Bosonic and fermionic Hubbard models.
hofstadter	Cold atomic (Harper-)Hofstadter model on a strip or
	cylinder.
haldane	Bosonic and fermionic Haldane models.
toric_code	Kitaev's exactly solvable toric code model.

19.3 tf_ising

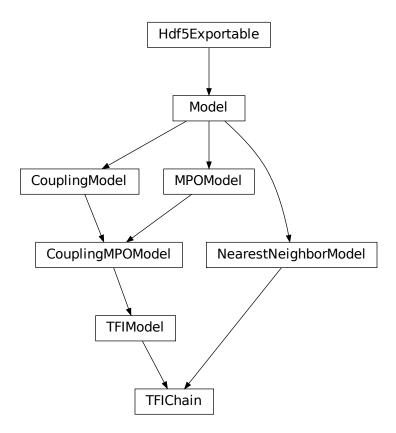
• full name: tenpy.models.tf_ising

• parent module: tenpy.models

• type: module

19.3. tf_ising 507

Classes



TFIChain(model_params)	The TFIModel on a Chain, suitable for TEBD.
TFIModel(model_params)	Transverse field Ising model on a general lattice.

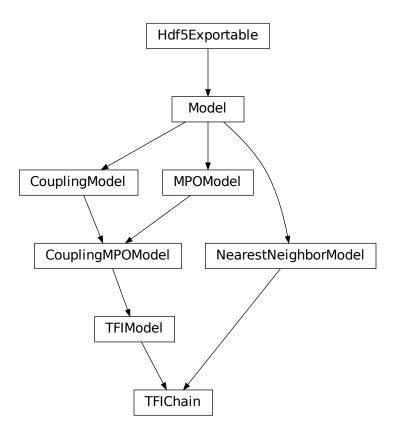
19.3.1 TFIChain

• full name: tenpy.models.tf_ising.TFIChain

• parent module: tenpy.models.tf_ising

• type: class

Inheritance Diagram



Methods

TFIChaininit(model_params)	Initialize self.
TFIChain.add_coupling(strength, u1, op1, u2,	Add twosite coupling terms to the Hamiltonian, sum-
)	ming over lattice sites.
TFIChain.add_coupling_term(strength, i, j ,	Add a two-site coupling term on given MPS sites.
)	
TFIChain.add_exponentially_decaying_co	uAdd an (exp) nentially decaying long-range coupling.
TFIChain.add_local_term(strength, term[,	Add a single term to <i>self</i> .
])	
TFIChain.add_multi_coupling(strength,	A did monthi site a sometime termine to the III and the mission some
ii i chain : ada_mai ci_coapiing (stichgui,	Add multi-site coupling terms to the Hamiltonian, sum-
ops[,])	ming over lattice sites.
	ming over lattice sites.
ops[,])	ming over lattice sites.
<pre>ops[,]) TFIChain.add_multi_coupling_term(strength)</pre>	ming over lattice sites.

19.3. tf_ising 509

Table 39 – continued from previous page

TFIChain.add_onsite_term(strength, i, op[,	Add an onsite term on a given MPS site.
])	Add all olisite term on a given ivii o site.
TFIChain.all_coupling_terms()	Sum of all coupling_terms.
TFIChain.all_onsite_terms()	Sum of all onsite_terms.
TFIChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
TFIChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
TFIChain.calc_H_MPO_from_bond([tol_zero])	Calculate the MPO Hamiltonian from the bond Hamil-
	tonian.
TFIChain.calc_H_bond([tol_zero])	calculate H_bond from coupling_terms and
	onsite_terms.
TFIChain.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
TFIChain.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
TFIChain.coupling_strength_add_ext_flu	x(Add) an external flux to the coupling strength.
TFIChain.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
TFIChain.from_MPOModel(mpo_model)	Initialize a NearestNeighborModel from a model class
	defining an MPO.
TFIChain.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
<pre>TFIChain.group_sites([n, grouped_sites])</pre>	Modify <i>self</i> in place to group sites.
TFIChain.init_H_from_terms()	Initialize H_MPO (and H_bond) from the terms of the
	CouplingModel.
TFIChain.init_lattice(model_params)	Initialize a lattice for the given model parameters.
TFIChain.init_sites(model_params)	Define the local Hilbert space and operators; needs to
•	be implemented in subclasses.
TFIChain.init_terms(model_params)	Add the onsite and coupling terms to the model; sub-
_ , _ _ ,	classes should implement this.
TFIChain.save_hdf5(hdf5_saver, h5gr, subpath)	Export <i>self</i> into a HDF5 file.
TFIChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
TFIChain.trivial_like_NNModel()	Return a NearestNeighborModel with same lattice, but
	trivial (H=0) bonds.

Class Attributes and Properties

TFIChain.force_default_lattice	If True, init_lattice() asserts that the initialized
	lattice is (a subclass of) default_lattice
TFIChain.logger	class attribute.
TFIChain.verbose	

class tenpy.models.tf_ising.TFIChain(model_params)

Bases: tenpy.models.tf_ising.TFIModel, tenpy.models.model.

NearestNeighborModel

The TFIModel on a Chain, suitable for TEBD.

See the TFIModel for the documentation of parameters.

default_lattice

alias of tenpy.models.lattice.Chain

add_coupling (strength, u1, op1, u2, op2, dx, op_string=None, str_on_first=True, raise_op2_left=False, category=None, plus_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,...,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str_on_first* and *raise_op2_left* will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- **u1** (*int*) Picks the site lat.unit_cell[u1] for OP1.
- **op1** (*str*) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op_needs_JW().
- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

19.3. tf ising 511

When initializing a model, you can add a term $J\sum_{\langle i,j\rangle}S_i^zS_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus_hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

→Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

```
add_onsite Add terms acting on one site only.
add_multi_coupling_term for terms on more than two sites.
add_coupling_term Add a single term without summing over \( \vec{x} \).
```

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

```
Wrapper for self.coupling_terms[category].add_coupling_term(...).
```

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **j** (*int*) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., *op_i* acts "left" of *op_j*. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- op1 (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

 $\begin{tabular}{llll} {\bf add_exponentially_decaying_coupling} (strength, lambda_, op_i, op_j, subsites=None, op_string=None, plus_hc=False) \\ \end{tabular}$

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- op_i (string) Names for the operators.
- op_j (string) Names for the operators.
- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- op_string (None / str) The operator to be inserted between A and B; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right op_j acts first.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use fit_with_sum_of_exp() to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...    return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
```

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19.3. tf ising 513

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 $\verb"add_local_term" (strength, term, category = None, plus_hc = False)$

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite_terms or coupling_terms, respectively.

Parameters

- **strength** (*float/complex*) The prefactor of the term.
- **term** (*list of* (*str*, *array_like*)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat_idx*. Here, *lat_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms or coupling_terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add_multi_coupling (strength, ops, _deprecate_1='DEPRECATED', _deprecate_2='DEPRECATED', op_string=None, category=None, plus_hc=False)

Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by $self.lat.unit_cell[u].get_op(opname)$.

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length

lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.

• op_string (str | None) - If a string is given, we use this as the name of an operator to be used inbetween the operators, excluding the sites on which any operators act. This operator should be defined on all sites in the unit cell.

If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.

- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D >>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), \rightarrowdx_0, u1)])
```

See also:

add_onsite Add terms acting on one site only.

add_coupling Add terms acting on two sites.

add_multi_coupling_term Add a single term, not summing over the possible \vec{x} .

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With i, j, k, ... = ijkl, we require that they are ordered ascending, i < j < k < ... and that 0 <= i < N_sites. Inidees >= N_sites indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots

19.3. tf_ising 515

- **op_string** (*list of str*) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between *i* and *j*.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If True, the hermitian conjugate of the term is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)

Add onsite terms to onsite terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat.unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to onsite_terms; doesn't rebuild the MPO.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

See also:

```
add_coupling Add a terms acting on two sites.
```

add_onsite_term Add a single term without summing over vecx.

add_onsite_term(strength, i, op, category=None, plus_hc=False)

Add an onsite term on a given MPS site.

Wrapper for self.onsite_terms[category].add_onsite_term(...).

Parameters

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- op (str) Name of the involved operator.
- category (str) Descriptive name used as key for onsite_terms. Defaults to op.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

all_coupling_terms()

Sum of all coupling_terms.

all_onsite_terms()

Sum of all onsite_terms.

bond_energies (psi)

Calculate bond energies <psi|H_bond|psi>.

Parameters psi (MPS) – The MPS for which the bond energies should be calculated.

Returns $E_bond - List$ of bond energies: for finite bc, $E_Bond[i]$ is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc $E_bond[i]$ is the energy of bond i-1, i.

Return type 1D ndarray

calc_H_MPO (tol_zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite terms and coupling terms to build an MPOGraph (and then an MPO).

Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_MPO_from_bond(tol_zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_bond(tol_zero=1e-15)

calculate H_bond from coupling_terms and onsite_terms.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are
considered to be zero.</pre>

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of *Array*

:raises ValueError : if the Hamiltonian contains longer-range terms.:

calc_H_bond_from_MPO(tol_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1*, 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

calc_H_onsite(tol_zero=1e-15)

Calculate *H_onsite* from *self.onsite_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take explicit_plus_hc into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

• **H_onsite** (list of npc.Array)

19.3. tf ising 517

• onsite terms of the Hamiltonian. If explicit_plus_hc is True, - Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in $add_coupling()$.

Parameters

- **strength** (*scalar* / *array*) The strength to be used in *add_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

Returns strength – The strength array to be used as *strength* in $add_coupling()$ with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... strength_with_flux = self.coupling_strength_add_ext_flux(strength, dx,

-> [0, phi])
... self.add_coupling(strength_with_flux, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(strength_with_flux), u2, 'Cd', u1, 'C', -dx)
```

enlarge_mps_unit_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

classmethod from MPOModel(mpo model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group_sites().

Parameters mpo_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

Examples

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> grouped_sites = nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define H_bond . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
>>> isinstance(nnn_chain_for_tebd, NearestNeighborModel)
True
```

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

19.3. tf ising 519

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

```
init_H_from_terms()
```

Initialize H_MPO (and H_bond) from the terms of the CouplingModel.

This function is called automatically during *CouplingMPOModel*.__init__.

If you use one of the *add_** methods of the *CouplingModel after* initialization, you will need to call *init_H_from_terms* in the end by yourself, in order to update the *H_MPO* (and possibly *H_bond*) representations. (You should get a warning about this... The way to avoid it is to initialize all the terms in *init_terms* by defining your own model, as outlined in *Models*.

```
init_lattice (model_params)
```

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

Returns lat – An initialized lattice.

Return type Lattice

Options

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, directly a subclass of Lattice instead of the name. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
```

```
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite" bc_MPS , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on bc_y).

```
option CouplingMPOModel.bc y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc_MPS="finite" and "periodic" for bc_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

init_sites (model_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

Note: Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

If you need to initialize more than one site, the function tenpy.networks.site. set_common_charges() should be helpful.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

Returns sites – The local sites of the lattice, defining the local basis states and operators.

```
Return type (tuple of) Site
```

init_terms (model_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with <code>save_dict_content()</code>, storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

trivial_like_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

Module description

Prototypical example of a quantum model: the transverse field Ising model.

Like the XXZChain, the transverse field ising chain *TFIChain* is contained in the more general *SpinChain*; the idea is more to serve as a pedagogical example for a 'model'.

We choose the field along z to allow to conserve the parity, if desired.

19.3. tf_ising 521

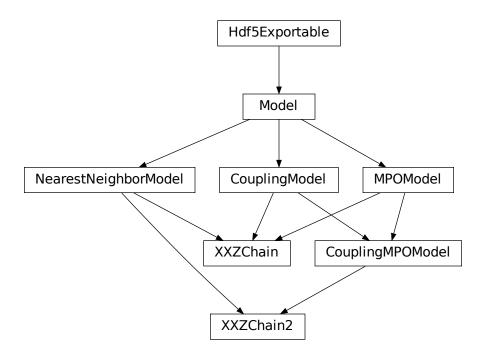
19.4 xxz_chain

• full name: tenpy.models.xxz_chain

• parent module: tenpy.models

• type: module

Classes



XXZChain(model_params)	Spin-1/2 XXZ chain with Sz conservation.
XXZChain2(model_params)	Another implementation of the Spin-1/2 XXZ chain
	with Sz conservation.

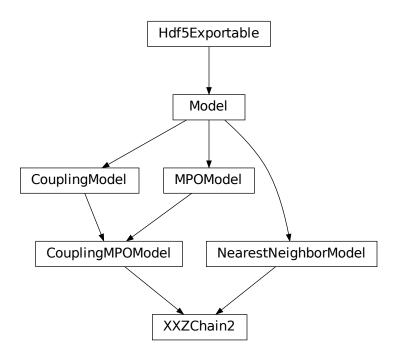
19.4.1 XXZChain2

• full name: tenpy.models.xxz_chain.XXZChain2

• parent module: tenpy.models.xxz_chain

• type: class

Inheritance Diagram



Methods

XXZChain2init(model_params)	Initialize self.
XXZChain2.add_coupling(strength, u1, op1,)	Add twosite coupling terms to the Hamiltonian, sum-
	ming over lattice sites.
XXZChain2.add_coupling_term(strength, i, j,	Add a two-site coupling term on given MPS sites.
)	
XXZChain2.add_exponentially_decaying_c	Add an exponentially decaying long-range coupling.
XXZChain2.add_local_term(strength, term[,	Add a single term to <i>self</i> .
])	
XXZChain2.add_multi_coupling(strength,	Add multi-site coupling terms to the Hamiltonian, sum-
ops)	ming over lattice sites.
XXZChain2.add_multi_coupling_term(strength	hAdd a general M-site coupling term on given MPS sites.
)	
XXZChain2.add_onsite(strength, u, opname[,	Add onsite terms to onsite_terms.
])	
XXZChain2.add_onsite_term(strength, i, op[,	Add an onsite term on a given MPS site.
])	
XXZChain2.all_coupling_terms()	Sum of all coupling_terms.
XXZChain2.all_onsite_terms()	Sum of all onsite_terms.
XXZChain2.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
	continues on next page

19.4. xxz_chain 523

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XXZChain2.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
XXZChain2.calc_H_MPO_from_bond([tol_zero])	Calculate the MPO Hamiltonian from the bond Hamil-
	tonian.
XXZChain2.calc_H_bond([tol_zero])	calculate <i>H_bond</i> from coupling_terms and
	onsite_terms.
XXZChain2.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
XXZChain2.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
XXZChain2.coupling_strength_add_ext_fl	
<pre>XXZChain2.enlarge_mps_unit_cell([factor])</pre>	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
$XXZChain2.from_MPOModel(mpo_model)$	Initialize a NearestNeighborModel from a model class
	defining an MPO.
XXZChain2.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
XXZChain2.group_sites([n, grouped_sites])	Modify self in place to group sites.
XXZChain2.group_sites([n, grouped_sites]) XXZChain2.init_H_from_terms()	Initialize H_MPO (and H_bond) from the terms of the
XXZChain2.init_H_from_terms()	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> .
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params)</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters.
XXZChain2.init_H_from_terms()	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params)</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to be implemented in subclasses.
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params)</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to be implemented in subclasses. Add the onsite and coupling terms to the model; sub-
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params) XXZChain2.init_sites(model_params) XXZChain2.init_terms(model_params)</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to be implemented in subclasses. Add the onsite and coupling terms to the model; subclasses should implement this.
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params) XXZChain2.init_sites(model_params)</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to be implemented in subclasses. Add the onsite and coupling terms to the model; sub-
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params) XXZChain2.init_sites(model_params) XXZChain2.init_terms(model_params) XXZChain2.save_hdf5(hdf5_saver, h5gr, sub-path)</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to be implemented in subclasses. Add the onsite and coupling terms to the model; subclasses should implement this. Export <i>self</i> into a HDF5 file.
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params) XXZChain2.init_sites(model_params) XXZChain2.init_terms(model_params) XXZChain2.save_hdf5(hdf5_saver, h5gr, sub-</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to be implemented in subclasses. Add the onsite and coupling terms to the model; subclasses should implement this. Export <i>self</i> into a HDF5 file. Sanity check, raises ValueErrors, if something is wrong.
<pre>XXZChain2.init_H_from_terms() XXZChain2.init_lattice(model_params) XXZChain2.init_sites(model_params) XXZChain2.init_terms(model_params) XXZChain2.save_hdf5(hdf5_saver, h5gr, sub-path)</pre>	Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the <i>CouplingModel</i> . Initialize a lattice for the given model parameters. Define the local Hilbert space and operators; needs to be implemented in subclasses. Add the onsite and coupling terms to the model; subclasses should implement this. Export <i>self</i> into a HDF5 file.

Class Attributes and Properties

XXZChain2.default_lattice	The default lattice class or class name to be used in init_lattice().
XXZChain2.force_default_lattice	If True, init_lattice() asserts that the initialized lattice is (a subclass of) default_lattice
XXZChain2.logger	class attribute.
XXZChain2.verbose	

class tenpy.models.xxz_chain.XXZChain2 (model_params)

Bases: tenpy.models.model.CouplingMPOModel, tenpy.models.model.

NearestNeighborModel

Another implementation of the Spin-1/2 XXZ chain with Sz conservation.

This implementation takes the same parameters as the XXZChain, but is implemented based on the CouplingMPOModel.

Parameters model_params (dict | Config) - See XXZChain

init_sites(model_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by init_lattice() to get the Site for the lattice unit cell.

Note: Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

If you need to initialize more than one site, the function tenpy.networks.site. set_common_charges() should be helpful.

Parameters model_params (dict) – The model parameters given to __init__.

Returns sites – The local sites of the lattice, defining the local basis states and operators.

Return type (tuple of) Site

init_terms (model_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible_couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str_on_first* and *raise_op2_left* will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit_cell[u1] for OP1.
- op1 (str) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- **u2** (int) Picks the site lat.unit_cell[u2] for OP2.
- **op2** (*str*) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.
- dx (*iterable of int*) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op_needs_JW().

19.4. xxz chain 525

- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

When initializing a model, you can add a term $J\sum_{\langle i,j\rangle}S_i^zS_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus_hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c.

→ Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

```
add_onsite Add terms acting on one site only.
add_multi_coupling_term for terms on more than two sites.
add_coupling_term Add a single term without summing over \( \vec{x} \).
```

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- j (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- op1 (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- op_i (string) Names for the operators.
- op_j (string) Names for the operators.
- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.

19.4. xxz chain 527

- op_string (None / str) The operator to be inserted between A and B; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right op j acts first.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use $fit_with_sum_of_exp()$ to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...    return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
>>> x = np.arange(1, fit_range + 1)
>>> print('error in fit: {0:.3e}'.format(np.sum(np.abs(decay(x) - sum_of_exp(lam, pref, x))))
error in fit: 1.073e-04
>>> for pr, la in zip(pref, lam):
... self.add_exponentially_decaying_coupling(pr, la, 'N', 'N')
```

 $\verb"add_local_term" (strength, term, category = None, plus_hc = False)$

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite_terms or coupling_terms, respectively.

Parameters

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array_like*)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat_idx*. Here, *lat_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms of coupling terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

```
add_multi_coupling (strength, ops, __deprecate_1='DEPRECATED', __deprecate_2='DEPRECATED', op_string=None, category=None, plus_hc=False)

Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.
```

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by self.lat.unit_cell[u].get_op(opname).

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the *dx* entries of *ops* and is chosen such that the first entry strength[0, 0, ...] of *strength*

is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.
- op_string (str | None) If a string is given, we use this as the name of an operator to be used inbetween the operators, excluding the sites on which any operators act. This operator should be defined on all sites in the unit cell.
 - If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D >>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), \rightarrowdx_0, u1)])
```

See also:

```
add_onsite Add terms acting on one site only.

add_coupling Add terms acting on two sites.

add_multi_coupling_term Add a single term, not summing over the possible \vec{x}.
```

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

19.4. xxz chain 529

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With $i, j, k, \ldots = ijkl$, we require that they are ordered ascending, $i < j < k < \ldots$ and that $0 <= i < N_sites$. Inidces $>= N_sites$ indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots
- **op_string** (list of str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)

Add onsite terms to onsite_terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat.unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to onsite_terms; doesn't rebuild the MPO.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc $(b \circ \circ 1)$ If *True*, the hermitian conjugate of the terms is added automatically.

See also:

add coupling Add a terms acting on two sites.

add onsite term Add a single term without summing over vecx.

 $\verb"add_onsite_term" (strength, i, op, category = None, plus_hc = False)$

Add an onsite term on a given MPS site.

Wrapper for self.onsite_terms[category].add_onsite_term(...).

Parameters

- strength (float) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.

```
• op (str) – Name of the involved operator.
```

- category (str) Descriptive name used as key for onsite_terms. Defaults to op.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

all_coupling_terms()

Sum of all coupling terms.

all_onsite_terms()

Sum of all onsite terms.

bond_energies(psi)

Calculate bond energies <psilH_bondlpsi>.

Parameters psi (MPS) – The MPS for which the bond energies should be calculated.

Returns E_bond - List of bond energies: for finite bc, E_Bond[i] is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the energy of bond i-1, i.

Return type 1D ndarray

calc_H_MPO (tol_zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite_terms and coupling_terms to build an MPOGraph (and then an MPO).

Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_MPO_from_bond(tol_zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

Parameters tol_zero (float) – Arrays with norm < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_bond(tol_zero=1e-15)

calculate H_bond from coupling_terms and onsite_terms.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

calc_H_bond_from_MPO (tol_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

19.4. xxz chain 531

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_onsite(tol_zero=1e-15)
```

Calculate *H* onsite from self.onsite terms.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take explicit_plus_hc into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

- **H_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit_plus_hc is True, Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in add_coupling().

Parameters

- **strength** (*scalar* / *array*) The strength to be used in *add_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add_coupling().
- **phase** (*iterable* of float) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

Returns strength – The strength array to be used as *strength* in $add_coupling()$ with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
```

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enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

classmethod from_MPOModel(mpo_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group_sites().

Parameters mpo_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

Examples

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> grouped_sites = nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define H_bond . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
>>> isinstance(nnn_chain_for_tebd, NearestNeighborModel)
True
```

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.

19.4. xxz chain 533

• **subpath** (str) – The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

```
init_H_from_terms()
```

Initialize *H_MPO* (and *H_bond*) from the terms of the *CouplingModel*.

This function is called automatically during *CouplingMPOModel*.__init__.

If you use one of the *add_** methods of the *CouplingModel after* initialization, you will need to call *init_H_from_terms* in the end by yourself, in order to update the *H_MPO* (and possibly *H_bond*) representations. (You should get a warning about this... The way to avoid it is to initialize all the terms in *init_terms* by defining your own model, as outlined in *Models*.

init_lattice (model_params)

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) - The model parameters given to __init__.
```

Returns lat – An initialized lattice.

Return type Lattice

Options

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, directly a subclass of Lattice instead of the name. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
Boundary conditions for the MPS.
```

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite" bc_MPS , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on bc_y).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc_MPS="finite" and "periodic" for bc_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

```
trivial_like_NNModel()
```

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

Module description

Prototypical example of a 1D quantum model: the spin-1/2 XXZ chain.

The XXZ chain is contained in the more general *SpinChain*; the idea of this module is more to serve as a pedagogical example for a model.

19.5 spins

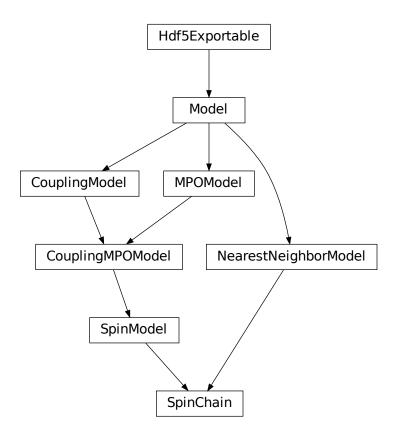
• full name: tenpy.models.spins

• parent module: tenpy.models

• type: module

19.5. spins 535

Classes



SpinChain(model_params)	The SpinModel on a Chain, suitable for TEBD.
SpinModel(model_params)	Spin-S sites coupled by nearest neighbour interactions.

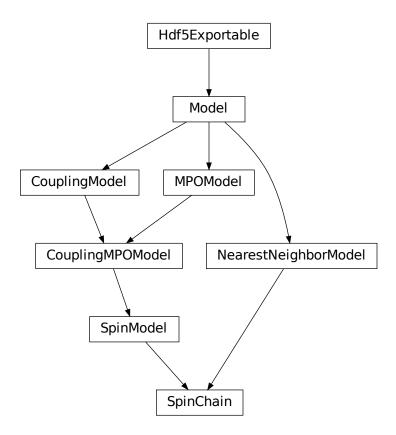
19.5.1 SpinChain

• full name: tenpy.models.spins.SpinChain

• parent module: tenpy.models.spins

• type: class

Inheritance Diagram



Methods

SpinChaininit(model_params)	Initialize self.
SpinChain.add_coupling(strength, u1, op1,)	Add twosite coupling terms to the Hamiltonian, sum-
	ming over lattice sites.
SpinChain.add_coupling_term(strength, i, j,	Add a two-site coupling term on given MPS sites.
)	
SpinChain.add_exponentially_decaying_c	Add an exponentially decaying long-range coupling.
SpinChain.add_local_term(strength, term[,	Add a single term to <i>self</i> .
])	
SpinChain.add_multi_coupling(strength,	Add multi-site coupling terms to the Hamiltonian, sum-
ops)	ming over lattice sites.
SpinChain.add_multi_coupling_term(strengt	hAdd a general M-site coupling term on given MPS sites.
)	
SpinChain.add_onsite(strength, u, opname[,	Add onsite terms to onsite_terms.
70	
])	

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19.5. spins 537

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SpinChain.add_onsite_term(strength, i, op[,	Add an onsite term on a given MPS site.
])	•
SpinChain.all_coupling_terms()	Sum of all coupling_terms.
SpinChain.all_onsite_terms()	Sum of all onsite_terms.
SpinChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
SpinChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
SpinChain.calc_H_MPO_from_bond([tol_zero])	Calculate the MPO Hamiltonian from the bond Hamil-
	tonian.
SpinChain.calc_H_bond([tol_zero])	calculate H_bond from coupling_terms and
	onsite_terms.
SpinChain.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
SpinChain.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
SpinChain.coupling_strength_add_ext_fl	
SpinChain.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
SpinChain.from_MPOModel(mpo_model)	Initialize a NearestNeighborModel from a model class
2 / 21 / 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	defining an MPO.
SpinChain.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	Modify self in place to anouncities
SpinChain.group_sites([n, grouped_sites])	Modify <i>self</i> in place to group sites. Initialize <i>H_MPO</i> (and <i>H_bond</i>) from the terms of the
SpinChain.init_H_from_terms()	CouplingModel.
SpinChain.init_lattice(model_params)	Initialize a lattice for the given model parameters.
SpinChain.init_sites(model_params)	Define the local Hilbert space and operators; needs to
Spinenain.init_sites(model_params)	be implemented in subclasses.
SpinChain.init_terms(model_params)	Add the onsite and coupling terms to the model; sub-
Spinenain. inite_ceims(model_params)	classes should implement this.
SpinChain.save_hdf5(hdf5_saver, h5gr, sub-	Export <i>self</i> into a HDF5 file.
path)	zapore son and a ribito mo.
SpinChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
SpinChain.trivial_like_NNModel()	Return a NearestNeighborModel with same lattice, but
	trivial (H=0) bonds.
	· · ·

Class Attributes and Properties

SpinChain.force_default_lattice	If True, init_lattice() asserts that the initialized
	lattice is (a subclass of) default_lattice
SpinChain.logger	class attribute.
SpinChain.verbose	

class tenpy.models.spins.SpinChain(model_params)

 $Bases: \verb|tenpy.models.spins.SpinModel||, tenpy.models.model.NearestNeighborModel||$

The SpinModel on a Chain, suitable for TEBD.

See the SpinModel for the documentation of parameters.

default_lattice

 $alias \ of \ tenpy. {\it models.lattice.Chain}$

add_coupling (strength, u1, op1, u2, op2, dx, op_string=None, str_on_first=True, raise_op2_left=False, category=None, plus_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,...,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str_on_first* and *raise_op2_left* will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit cell[u1] for OP1.
- op1 (str) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op_needs_JW().
- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

19.5. spins 539

When initializing a model, you can add a term $J\sum_{\langle i,j\rangle}S_i^zS_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus_hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

-> Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

```
add_onsite Add terms acting on one site only.

add_multi_coupling_term for terms on more than two sites.

add_coupling_term Add a single term without summing over \vec{x}.
```

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

```
Wrapper for self.coupling_terms[category].add_coupling_term(...).
```

```
Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.
```

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **j** (*int*) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., *op_i* acts "left" of *op_j*. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- op1 (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- op_i (string) Names for the operators.
- $op_j(string)$ Names for the operators.
- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- op_string (None / str) The operator to be inserted between A and B; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right op_j acts first.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use fit_with_sum_of_exp() to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...    return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
```

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19.5. spins 541

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add_local_term (strength, term, category=None, plus_hc=False)
Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite_terms or coupling_terms, respectively.

Parameters

- **strength** (*float/complex*) The prefactor of the term.
- **term** (*list of* (*str*, *array_like*)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat_idx*. Here, *lat_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms or coupling_terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add_multi_coupling (strength, ops, _deprecate_1='DEPRECATED', _deprecate_2='DEPRECATED', op_string=None, category=None, plus_hc=False)

Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by $self.lat.unit_cell[u].get_op(opname)$.

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length

lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.

• op_string (str | None) - If a string is given, we use this as the name of an operator to be used inbetween the operators, excluding the sites on which any operators act. This operator should be defined on all sites in the unit cell.

If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.

- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D >>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), \rightarrowdx_0, u1)])
```

See also:

add_onsite Add terms acting on one site only.

add_coupling Add terms acting on two sites.

add_multi_coupling_term Add a single term, not summing over the possible \vec{x} .

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With i, j, k, ... = ijkl, we require that they are ordered ascending, i < j < k < ... and that 0 <= i < N_sites. Inidees >= N_sites indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots

19.5. spins 543

- **op_string** (list of str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If True, the hermitian conjugate of the term is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)

Add onsite terms to onsite terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat.unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to onsite_terms; doesn't rebuild the MPO.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

See also:

```
add_coupling Add a terms acting on two sites.
```

add_onsite_term Add a single term without summing over vecx.

add_onsite_term(strength, i, op, category=None, plus_hc=False)

Add an onsite term on a given MPS site.

Wrapper for self.onsite_terms[category].add_onsite_term(...).

Parameters

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- op (str) Name of the involved operator.
- **category** (str) Descriptive name used as key for onsite_terms. Defaults to op.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

all_coupling_terms()

Sum of all coupling_terms.

all_onsite_terms()

Sum of all onsite_terms.

bond_energies (psi)

Calculate bond energies <psi|H_bond|psi>.

 $\textbf{Parameters} \hspace{0.2cm} \textbf{psi} \hspace{0.1cm} (\texttt{MPS}) - \textbf{The} \hspace{0.1cm} \textbf{MPS} \hspace{0.1cm} \text{for which the bond energies should be calculated.}$

Returns E_bond – List of bond energies: for finite bc, $E_Bond[i]$ is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc $E_bond[i]$ is the energy of bond i-1, i.

Return type 1D ndarray

calc H MPO(tol zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite terms and coupling terms to build an MPOGraph (and then an MPO).

Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_MPO_from_bond(tol_zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_bond(tol_zero=1e-15)

calculate H_bond from coupling_terms and onsite_terms.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

calc_H_bond_from_MPO (tol_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1*, 'p1*']

Return type list of Array

:raises ValueError: if the Hamiltonian contains longer-range terms.:

calc_H_onsite(tol_zero=1e-15)

Calculate *H_onsite* from *self.onsite_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take explicit_plus_hc into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

• **H_onsite** (list of npc.Array)

19.5. spins 545

onsite terms of the Hamiltonian. If explicit_plus_hc is True, – Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in $add_coupling()$.

Parameters

- **strength** (*scalar* / *array*) The strength to be used in *add_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

Returns strength – The strength array to be used as *strength* in $add_coupling()$ with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... strength_with_flux = self.coupling_strength_add_ext_flux(strength, dx,

-> [0, phi])
... self.add_coupling(strength_with_flux, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(strength_with_flux), u2, 'Cd', u1, 'C', -dx)
```

enlarge_mps_unit_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

classmethod from MPOModel(mpo model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group_sites().

Parameters mpo_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

Examples

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> grouped_sites = nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define H_bond . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
>>> isinstance(nnn_chain_for_tebd, NearestNeighborModel)
True
```

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites(n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

19.5. spins 547

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

```
init_H_from_terms()
```

Initialize H_MPO (and H_bond) from the terms of the CouplingModel.

This function is called automatically during *CouplingMPOModel*.__init__.

If you use one of the *add_** methods of the *CouplingModel after* initialization, you will need to call *init_H_from_terms* in the end by yourself, in order to update the *H_MPO* (and possibly *H_bond*) representations. (You should get a warning about this... The way to avoid it is to initialize all the terms in *init_terms* by defining your own model, as outlined in *Models*.

```
init_lattice (model_params)
```

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

Returns lat – An initialized lattice.

Return type Lattice

Options

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, directly a subclass of Lattice instead of the name. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
```

```
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite" bc_MPS , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on bc_y).

```
option CouplingMPOModel.bc y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc_MPS="finite" and "periodic" for bc_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

init sites(model params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

Note: Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

If you need to initialize more than one site, the function tenpy.networks.site. set_common_charges() should be helpful.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

Returns sites – The local sites of the lattice, defining the local basis states and operators.

Return type (tuple of) Site

init_terms (model_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

trivial_like_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

Module description

Nearest-neighbour spin-S models.

Uniform lattice of spin-S sites, coupled by nearest-neighbour interactions.

19.6 spins_nnn

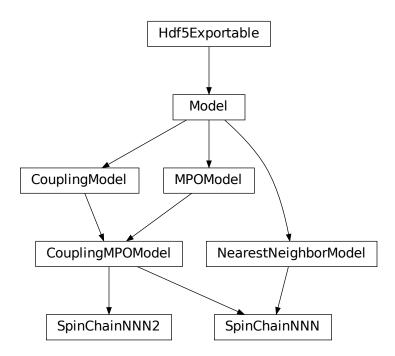
• full name: tenpy.models.spins_nnn

• parent module: tenpy.models

• type: module

19.6. spins nnn 549

Classes



SpinChainNNN(model_params)	Spin-S sites coupled by (next-)nearest neighbour interactions on a <i>GroupedSite</i> .
SpinChainNNN2(model_params)	Spin-S sites coupled by next-nearest neighbour interactions.

Module description

Next-Nearest-neighbour spin-S models.

Uniform lattice of spin-S sites, coupled by next-nearest-neighbour interactions. We have two variants implementing the same hamiltonian. The SpinChainNNN uses the <code>GroupedSite</code> to keep it a <code>NearestNeighborModel</code> suitable for TEBD, while the SpinChainNNN2 just involves longer-range couplings in the MPO. The latter is preferable for pure DMRG calculations and avoids having to add each of the short range couplings twice for the grouped sites.

Note that you can also get a NearestNeighborModel for TEBD from the latter by using $group_sites()$ and $from_MPOModel()$. An example for such a case is given in the file examples/c_tebd.py.

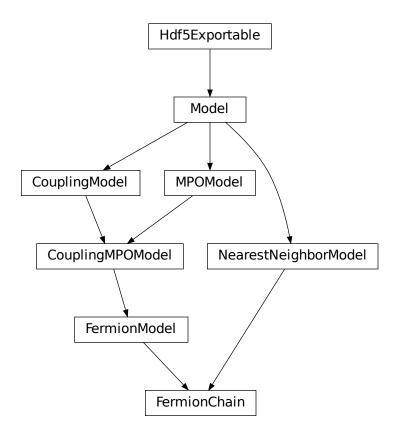
19.7 fermions_spinless

• full name: tenpy.models.fermions_spinless

• parent module: tenpy.models

• type: module

Classes

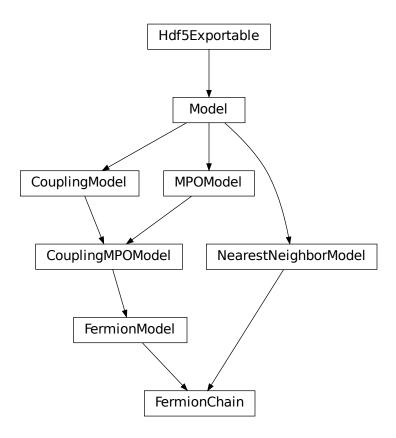


FermionChain(model_params)	The FermionModel on a Chain, suitable for TEBD.
FermionModel(model_params)	Spinless fermions with particle number conservation.

19.7.1 FermionChain

- full name: tenpy.models.fermions_spinless.FermionChain
- parent module: tenpy.models.fermions_spinless
- type: class

Inheritance Diagram



Methods

FermionChaininit(model_params)	Initialize self.
FermionChain.add_coupling(strength, u1,	Add twosite coupling terms to the Hamiltonian, sum-
op1,)	ming over lattice sites.
FermionChain.add_coupling_term(strength,	Add a two-site coupling term on given MPS sites.
i,)	
FermionChain.add exponentially decayi	ngAdd ap exporentially decaying long-range coupling.

ermionenain, add_exponentially_decayingholantany decaying long-lange coupling

continues on next page

Table 49 – continued from previous page

trom previous page
Add a single term to <i>self</i> .
Add multi-site coupling terms to the Hamiltonian, sum-
ming over lattice sites.
[Add a general M-site coupling term on given MPS sites.
Add onsite terms to onsite_terms.
Add an onsite term on a given MPS site.
Sum of all coupling_terms.
Sum of all onsite_terms.
Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
Calculate MPO representation of the Hamiltonian.
rolalculate the MPO Hamiltonian from the bond Hamil-
tonian.
calculate <i>H_bond</i> from coupling_terms and
onsite_terms.
r@alculate the bond Hamiltonian from the MPO Hamil-
tonian.
Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
_Add a(n ex)ernal flux to the coupling strength.
prepare the unit cell for infinite MPS boundary condi-
tions; in place.
Initialize a NearestNeighborModel from a model class
defining an MPO.
Load instance from a HDF5 file.
Load instance from a 11D1 5 me.
Modify <i>self</i> in place to group sites.
riodity self in place to group sites.
Initialize H_MPO (and H_bond) from the terms of the
CouplingModel.
Initialize a lattice for the given model parameters.
Define the local Hilbert space and operators; needs to
be implemented in subclasses.
Add the onsite and coupling terms to the model; sub-
ž ž
classes should implement this. Export <i>self</i> into a HDF5 file.
EXPORT SELL THE A LIDES THE.
1
Sanity check, raises ValueErrors, if something is wrong.

Class Attributes and Properties

If True, init_lattice() asserts that the initialized
lattice is (a subclass of) default_lattice
class attribute.

class tenpy.models.fermions_spinless.FermionChain (model_params)

Bases: tenpy.models.fermions_spinless.FermionModel, tenpy.models.model. NearestNeighborModel

The FermionModel on a Chain, suitable for TEBD.

See the FermionModel for the documentation of parameters.

default lattice

alias of tenpy.models.lattice.Chain

add_coupling (strength, u1, op1, u2, op2, dx, op_string=None, str_on_first=True, raise_op2_left=False, category=None, plus_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible_couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments str_on_first and $raise_op2_left$ will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit_cell[u1] for OP1.
- **op1** (*str*) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str / None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator

should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using <code>op_needs_JW()</code>.

- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

When initializing a model, you can add a term $J \sum_{\langle i,j \rangle} S_i^z S_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

Gdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

```
add_onsite Add terms acting on one site only.

add_multi_coupling_term for terms on more than two sites.

add_coupling_term Add a single term without summing over \vec{x}.
```

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- j (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j . If $j >= N_s$ it indicates couplings between unit cells of an infinite MPS.
- op1 (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

 $\begin{tabular}{llll} {\bf add_exponentially_decaying_coupling} (strength, lambda_, op_i, op_j, subsites=None, op_string=None, plus_hc=False) \\ \end{tabular}$

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- **op_i** (*string*) Names for the operators.
- **op_j** (*string*) Names for the operators.

- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- op_string (None / str) The operator to be inserted between A and B; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right op_j acts first.
- plus hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use fit_with_sum_of_exp() to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...    return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
>>> x = np.arange(1, fit_range + 1)
>>> print('error in fit: {0:.3e}'.format(np.sum(np.abs(decay(x) - sum_of_exp(lam, pref, x)))))
error in fit: 1.073e-04
>>> for pr, la in zip(pref, lam):
... self.add_exponentially_decaying_coupling(pr, la, 'N', 'N')
```

add_local_term (strength, term, category=None, plus_hc=False)
Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite_terms or coupling_terms, respectively.

Parameters

- **strength** (float/complex) The prefactor of the term.
- **term** (*list* of (*str*, *array_like*)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat_idx*. Here, *lat_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms or coupling_terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

```
add_multi_coupling (strength, ops, __deprecate_1='DEPRECATED', __deprecate_2='DEPRECATED', op_string=None, category=None, plus_hc=False)

Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.
```

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by $self.lat.unit_cell[u].get_op(opname)$.

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.
- op_string (str | None) If a string is given, we use this as the name of an operator to be used inbetween the operators, excluding the sites on which any operators act. This operator should be defined on all sites in the unit cell.
 - If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D 
>>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the strength, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), \rightarrowdx_0, u1)])
```

See also:

```
add_onsite Add terms acting on one site only.
add_coupling Add terms acting on two sites.
add_multi_coupling_term Add a single term, not summing over the possible \(\vec{x}\).
```

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With $i, j, k, \ldots = ijkl$, we require that they are ordered ascending, $i < j < k < \ldots$ and that $0 <= i < N_sites$. Inidces >= N_sites indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots
- op_string (list of str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)

Add onsite terms to onsite_terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat. unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to onsite_terms; doesn't rebuild the MPO.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- **u** (*int*) Picks a Site lat.unit_cell[u] out of the unit cell.
- **opname** (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

See also:

```
add_coupling Add a terms acting on two sites.
```

add_onsite_term Add a single term without summing over vecx.

add_onsite_term (strength, i, op, category=None, plus_hc=False)

Add an onsite term on a given MPS site.

Wrapper for self.onsite_terms[category].add_onsite_term(...).

Parameters

```
• strength (float) – The strength of the term.
```

- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- op (str) Name of the involved operator.
- category (str) Descriptive name used as key for onsite_terms. Defaults to op.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

all_coupling_terms()

Sum of all coupling_terms.

all_onsite_terms()

Sum of all onsite_terms.

bond_energies (psi)

Calculate bond energies <psilH_bondlpsi>.

Parameters psi (MPS) – The MPS for which the bond energies should be calculated.

Returns E_bond - List of bond energies: for finite bc, E_Bond[i] is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the energy of bond i-1, i.

Return type 1D ndarray

calc_H_MPO (tol_zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite_terms and coupling_terms to build an MPOGraph (and then an MPO).

Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are
 considered to be zero.</pre>

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_MPO_from_bond(tol_zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_bond(tol_zero=1e-15)

calculate H_bond from coupling_terms and onsite_terms.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_bond - Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

calc_H_bond_from_MPO (tol_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

```
Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']
```

Return type list of *Array*

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_onsite(tol_zero=1e-15)
```

Calculate *H_onsite* from *self.onsite_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take explicit_plus_hc into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

- **H_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit_plus_hc is True, Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in add_coupling().

Parameters

- **strength** (*scalar* / *array*) The strength to be used in *add_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

Returns strength – The strength array to be used as *strength* in $add_coupling()$ with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... strength_with_flux = self.coupling_strength_add_ext_flux(strength, dx,
->> [0, phi])
... self.add_coupling(strength_with_flux, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(strength_with_flux), u2, 'Cd', u1, 'C', -dx)
```

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

classmethod from MPOModel(mpo model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group_sites().

Parameters mpo_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

Examples

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> grouped_sites = nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define H_bond . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
>>> isinstance(nnn_chain_for_tebd, NearestNeighborModel)
True
```

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

• hdf5_loader (Hdf5Loader) - Instance of the loading engine.

- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each *n* sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

init_H_from_terms()

Initialize *H_MPO* (and *H_bond*) from the terms of the *CouplingModel*.

This function is called automatically during *CouplingMPOModel*.__init__.

If you use one of the *add_** methods of the *CouplingModel after* initialization, you will need to call *init_H_from_terms* in the end by yourself, in order to update the *H_MPO* (and possibly *H_bond*) representations. (You should get a warning about this... The way to avoid it is to initialize all the terms in *init_terms* by defining your own model, as outlined in *Models*.

init lattice(model params)

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

Returns lat – An initialized lattice.

Return type Lattice

Options

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, directly a subclass of Lattice instead of the name. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite" bc_MPS , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on bc y).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc_MPS="finite" and "periodic" for bc_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

init_sites (model_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

Note: Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

If you need to initialize more than one site, the function tenpy.networks.site. set_common_charges() should be helpful.

Parameters model_params (dict) – The model parameters given to __init__.

Returns sites – The local sites of the lattice, defining the local basis states and operators.

Return type (tuple of) Site

init_terms (model_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

trivial_like_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

Module description

Spinless fermions with hopping and interaction.

Todo: add further terms (e.g. c^dagger c^dagger + h.c.) to the Hamiltonian.

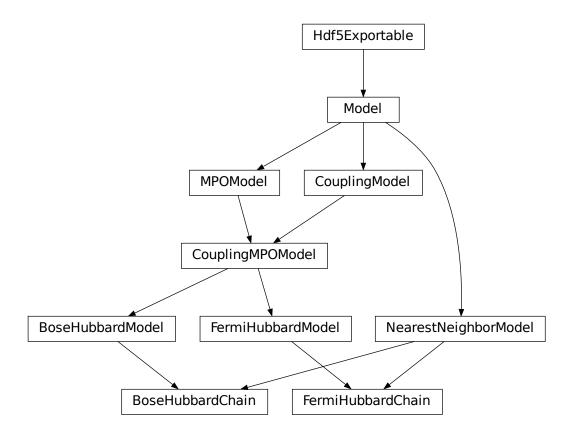
19.8 hubbard

• full name: tenpy.models.hubbard

• parent module: tenpy.models

• type: module

Classes



19.8. hubbard 565

BoseHubbardChain(model_params)	The BoseHubbardModel on a Chain, suitable for
	TEBD.
BoseHubbardModel(model_params)	Spinless Bose-Hubbard model.
FermiHubbardChain(model_params)	The FermiHubbardModel on a Chain, suitable for
	TEBD.
FermiHubbardModel(model_params)	Spin-1/2 Fermi-Hubbard model.

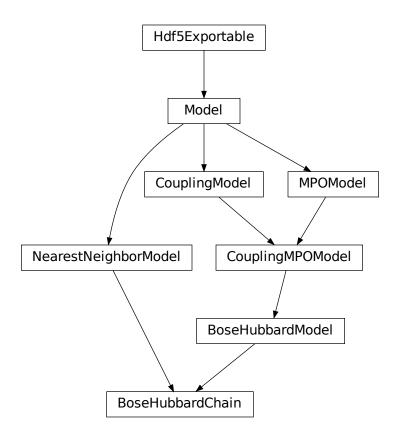
19.8.1 BoseHubbardChain

• full name: tenpy.models.hubbard.BoseHubbardChain

 $\bullet \ \ parent \ module: \ \textit{tenpy.models.hubbard}$

• type: class

Inheritance Diagram



Methods

BoseHubbardChaininit(model_params)	Initialize self.
BoseHubbardChain.add_coupling(strength,	Add twosite coupling terms to the Hamiltonian, sum-
u1,)	ming over lattice sites.
BoseHubbardChain.	Add a two-site coupling term on given MPS sites.
add_coupling_term(strength,)	red a two site coupling term on given wit o sites.
BoseHubbardChain.	Add an exponentially decaying long-range coupling.
add_exponentially_decaying_coupling(
BoseHubbardChain.	Add a single term to <i>self</i> .
add_local_term(strength, term)	rad a single term to sey.
BoseHubbardChain.	Add multi-site coupling terms to the Hamiltonian, sum-
add_multi_coupling([,])	ming over lattice sites.
BoseHubbardChain.	Add a general M-site coupling term on given MPS sites.
add_multi_coupling_term()	rad a general wishe coupling term on given wit o sites.
BoseHubbardChain.add_onsite(strength, u,	Add onsite terms to onsite_terms.
opname)	rad onsite terms to onsited_terms.
BoseHubbardChain.	Add an onsite term on a given MPS site.
add_onsite_term(strength, i, op)	rad an onotic term on a given in o site.
BoseHubbardChain.all_coupling_terms()	Sum of all coupling_terms.
BoseHubbardChain.all_onsite_terms()	Sum of all onsite_terms.
BoseHubbardChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
BoseHubbardChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
BoseHubbardChain.	Calculate the MPO Hamiltonian from the bond Hamil-
calc_H_MPO_from_bond([tol_zero])	tonian.
BoseHubbardChain.calc_H_bond([tol_zero])	calculate <i>H_bond</i> from coupling_terms and
bosenassaraenam, care_n_sona([tol_bolo])	onsite_terms.
BoseHubbardChain.	Calculate the bond Hamiltonian from the MPO Hamil-
calc_H_bond_from_MPO([tol_zero])	tonian.
BoseHubbardChain.	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
calc_H_onsite([tol_zero])	_ , _
BoseHubbardChain.	Add an external flux to the coupling strength.
<pre>coupling_strength_add_ext_flux()</pre>	1 0 0
BoseHubbardChain.	Repeat the unit cell for infinite MPS boundary condi-
<pre>enlarge_mps_unit_cell([factor])</pre>	tions; in place.
BoseHubbardChain.	Initialize a NearestNeighborModel from a model class
<pre>from_MPOModel(mpo_model)</pre>	defining an MPO.
BoseHubbardChain.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
BoseHubbardChain.group_sites([n,	Modify <i>self</i> in place to group sites.
grouped_sites])	
BoseHubbardChain.init_H_from_terms()	Initialize H_MPO (and H_bond) from the terms of the
	CouplingModel.
BoseHubbardChain.	Initialize a lattice for the given model parameters.
<pre>init_lattice(model_params)</pre>	
BoseHubbardChain.	Define the local Hilbert space and operators; needs to
<pre>init_sites(model_params)</pre>	be implemented in subclasses.
BoseHubbardChain.	Add the onsite and coupling terms to the model; sub-
<pre>init_terms(model_params)</pre>	classes should implement this.
BoseHubbardChain.save_hdf5(hdf5_saver,	Export self into a HDF5 file.
h5gr,)	
	continues on next page

19.8. hubbard 567

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BoseHubbardChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
BoseHubbardChain.	Return a NearestNeighborModel with same lattice, but
trivial_like_NNModel()	trivial (H=0) bonds.

Class Attributes and Properties

BoseHubbardChain.default_lattice	The default lattice class or class name to be used in		
	<pre>init_lattice().</pre>		
BoseHubbardChain.	If True, init_lattice() asserts that the initialized		
force_default_lattice	lattice is (a subclass of) default_lattice		
BoseHubbardChain.logger	class attribute.		
BoseHubbardChain.verbose			

class tenpy.models.hubbard.BoseHubbardChain (model_params)

The BoseHubbardModel on a Chain, suitable for TEBD.

See the BoseHubbardModel for the documentation of parameters.

add_coupling (strength, u1, op1, u2, op2, dx, op_string=None, str_on_first=True, raise_op2_left=False, category=None, plus_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible_couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str_on_first* and *raise_op2_left* will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit cell[u1] for OP1.
- op1 (str) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.

- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op_needs_JW().
- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

When initializing a model, you can add a term $J \sum_{\langle i,j \rangle} S_i^z S_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus_hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

19.8. hubbard 569

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

\( \to Cdagger_down C_up \)
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

```
add_onsite Add terms acting on one site only.

add_multi_coupling_term for terms on more than two sites.

add_coupling_term Add a single term without summing over \vec{x}.
```

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j . If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **j** (*int*) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., *op_i* acts "left" of *op_j*. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- op1 (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add_exponentially_decaying_coupling(strength, lambda_, op_i, op_j, subsites=None, op_string=None, plus_hc=False)

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- **op_i** (*string*) Names for the operators.
- op_j (string) Names for the operators.
- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- **op_string** (*None* / *str*) The operator to be inserted between *A* and *B*; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right *op_j* acts first.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use $fit_with_sum_of_exp()$ to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...     return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
>>> x = np.arange(1, fit_range + 1)
>>> print('error in fit: {0:.3e}'.format(np.sum(np.abs(decay(x) - sum_of_exp(lam, pref, x)))))
error in fit: 1.073e-04
>>> for pr, la in zip(pref, lam):
...     self.add_exponentially_decaying_coupling(pr, la, 'N', 'N')
```

add_local_term (strength, term, category=None, plus_hc=False)

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite_terms or coupling_terms, respectively.

Parameters

- **strength** (*float/complex*) The prefactor of the term.
- **term** (list of (str, array_like)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index lat_idx. Here, lat_idx is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms or coupling_terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add_multi_coupling (strength, ops, _deprecate_1='DEPRECATED', _deprecate_2='DEPRECATED', op_string=None, category=None, plus_hc=False)
Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by self.lat.unit_cell[u].get_op(opname).

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.
- op_string(str / None) If a string is given, we use this as the name of an operator to be used inbetween the operators, *excluding* the sites on which any operators act. This operator should be defined on all sites in the unit cell.
 - If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D 
>>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), dx_0, u1)])
```

See also:

```
add_onsite Add terms acting on one site only.
add_coupling Add terms acting on two sites.
add_multi_coupling_term Add a single term, not summing over the possible \(\vec{x}\).
```

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With $i, j, k, \ldots = ijkl$, we require that they are ordered ascending, $i < j < k < \ldots$ and that $0 <= i < N_sites$. Inidces $>= N_sites$ indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots
- op_string (list of str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)
Add onsite terms to onsite_terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat.unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to onsite_terms; doesn't rebuild the MPO.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

See also:

```
add_coupling Add a terms acting on two sites.
add_onsite_term Add a single term without summing over vecx.
```

```
add_onsite_term (strength, i, op, category=None, plus_hc=False)
    Add an onsite term on a given MPS site.
    Wrapper for self.onsite_terms[category].add_onsite_term(...).
        Parameters
            • strength (float) – The strength of the term.
            • i (int) - The MPS index of the site on which the operator acts. We require 0 <= i <
              L.
            • op (str) – Name of the involved operator.
            • category (str) - Descriptive name used as key for onsite_terms. Defaults to op.
            • plus_hc (bool) – If True, the hermitian conjugate of the term is added automatically.
all_coupling_terms()
    Sum of all coupling_terms.
all_onsite_terms()
    Sum of all onsite terms.
bond energies (psi)
    Calculate bond energies <psilH_bondlpsi>.
        Parameters psi (MPS) – The MPS for which the bond energies should be calculated.
         Returns E bond – List of bond energies: for finite bc, E Bond[i] is the energy of bond i,
            i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the
            energy of bond i-1, i.
         Return type 1D ndarray
calc_H_MPO(tol_zero=1e-15)
    Calculate MPO representation of the Hamiltonian.
    Uses onsite_terms and coupling_terms to build an MPOGraph (and then an MPO).
        Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are</pre>
            considered to be zero.
        Returns H MPO – MPO representation of the Hamiltonian.
        Return type MPO
calc_H_MPO_from_bond(tol_zero=1e-15)
    Calculate the MPO Hamiltonian from the bond Hamiltonian.
        Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.
         Returns H MPO – MPO representation of the Hamiltonian.
        Return type MPO
calc_H_bond(tol_zero=1e-15)
    calculate H_bond from coupling_terms and onsite_terms.
        Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are
            considered to be zero.
        Returns H_bond - Bond terms as required by the constructor of NearestNeighborModel.
            Legs are ['p0', 'p0*', 'p1', 'p1*']
        Return type list of Array
```

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc H bond from MPO (tol zero=1e-15)
```

Calculate the bond Hamiltonian from the MPO Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_bond - Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_onsite(tol_zero=1e-15)
```

Calculate *H_onsite* from *self.onsite_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take explicit_plus_hc into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

- **H_onsite** (*list of npc.Array*)
- onsite terms of the Hamiltonian. If explicit_plus_hc is True, Hermitian conjugates of the onsite terms will be included.

coupling_strength_add_ext_flux (strength, dx, phase)

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in add coupling().

Parameters

- **strength** (*scalar* / *array*) The strength to be used in *add_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

Returns strength – The strength array to be used as *strength* in $add_coupling()$ with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... strength_with_flux = self.coupling_strength_add_ext_flux(strength, dx,

-> [0, phi])
... self.add_coupling(strength_with_flux, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(strength_with_flux), u2, 'Cd', u1, 'C', -dx)
```

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

classmethod from_MPOModel(mpo_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group_sites().

Parameters mpo_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

Examples

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> grouped_sites = nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define H_bond . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
>>> isinstance(nnn_chain_for_tebd, NearestNeighborModel)
True
```

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each *n* sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

Returns grouped_sites – The sites grouped together.

Return type list of GroupedSite

init_H_from_terms()

Initialize H_MPO (and H_bond) from the terms of the CouplingModel.

This function is called automatically during *CouplingMPOModel*.__init__.

If you use one of the *add_** methods of the *CouplingModel after* initialization, you will need to call *init_H_from_terms* in the end by yourself, in order to update the *H_MPO* (and possibly *H_bond*) representations. (You should get a warning about this... The way to avoid it is to initialize all the terms in *init_terms* by defining your own model, as outlined in *Models*.

init_lattice (model_params)

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

Parameters model_params (dict) – The model parameters given to __init__.

Returns lat – An initialized lattice.

Return type Lattice

Options

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, directly a subclass of Lattice instead of the name. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

option CouplingMPOModel.bc_MPS: str

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite" bc_MPS , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on bc_y).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc_MPS="finite" and "periodic" for bc_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

init_sites (model_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

Note: Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

If you need to initialize more than one site, the function tenpy.networks.site.set_common_charges() should be helpful.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

Returns sites – The local sites of the lattice, defining the local basis states and operators.

Return type (tuple of) Site

```
init terms (model params)
```

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with <code>save_dict_content()</code>, storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

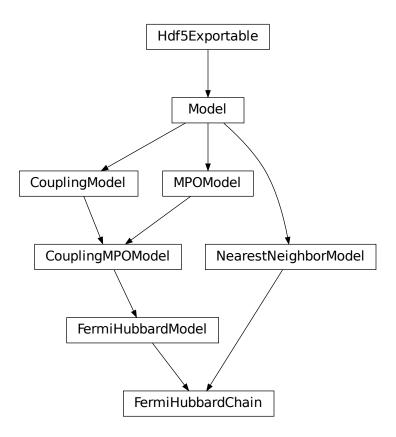
```
trivial_like_NNModel()
```

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

19.8.2 FermiHubbardChain

- full name: tenpy.models.hubbard.FermiHubbardChain
- parent module: tenpy.models.hubbard
- type: class

Inheritance Diagram



Methods

FermiHubbardChaininit(model_params)	Initialize self.
FermiHubbardChain.add_coupling(strength,	Add twosite coupling terms to the Hamiltonian, sum-
u1,)	ming over lattice sites.
FermiHubbardChain.	Add a two-site coupling term on given MPS sites.
add_coupling_term $([,])$	
FermiHubbardChain.	Add an exponentially decaying long-range coupling.
add_exponentially_decaying_coupling (\ldots)	
FermiHubbardChain.	Add a single term to <i>self</i> .
<pre>add_local_term(strength, term)</pre>	
FermiHubbardChain.	Add multi-site coupling terms to the Hamiltonian, sum-
$add_multi_coupling([,])$	ming over lattice sites.
FermiHubbardChain.	Add a general M-site coupling term on given MPS sites.
add_multi_coupling_term (\ldots)	

continues on next page

Table 54 – continued from previous page

	u irom previous page
FermiHubbardChain.add_onsite(strength, u,	Add onsite terms to onsite_terms.
opname)	
FermiHubbardChain.	Add an onsite term on a given MPS site.
_add_onsite_term(strength,)	
FermiHubbardChain.	Sum of all coupling_terms.
all_coupling_terms()	
FermiHubbardChain.all_onsite_terms()	Sum of all onsite_terms.
FermiHubbardChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
FermiHubbardChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
FermiHubbardChain.	Calculate the MPO Hamiltonian from the bond Hamil-
$ extit{calc_H_MPO_from_bond([])}$	tonian.
FermiHubbardChain.calc_H_bond([tol_zero])	<pre>calculate H_bond from coupling_terms and</pre>
	onsite_terms.
FermiHubbardChain.	Calculate the bond Hamiltonian from the MPO Hamil-
$calc_H_bond_from_MPO([])$	tonian.
FermiHubbardChain.	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
calc_H_onsite([tol_zero])	
FermiHubbardChain.	Add an external flux to the coupling strength.
${\it coupling_strength_add_ext_flux}()$	
FermiHubbardChain.	Repeat the unit cell for infinite MPS boundary condi-
enlarge_mps_unit_cell([factor])	tions; in place.
FermiHubbardChain.	Initialize a NearestNeighborModel from a model class
<pre>from_MPOModel(mpo_model)</pre>	defining an MPO.
FermiHubbardChain.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
FermiHubbardChain.group_sites([n,	Modify <i>self</i> in place to group sites.
grouped_sites])	
FermiHubbardChain.init_H_from_terms()	Initialize H_MPO (and H_bond) from the terms of the
	CouplingModel.
FermiHubbardChain.	Initialize a lattice for the given model parameters.
initlattice(model_params)	
FermiHubbardChain.	Define the local Hilbert space and operators; needs to
<pre>init_sites(model_params)</pre>	be implemented in subclasses.
FermiHubbardChain.	Add the onsite and coupling terms to the model; sub-
<pre>init_terms(model_params)</pre>	classes should implement this.
FermiHubbardChain.save_hdf5(hdf5_saver,	Export self into a HDF5 file.
)	
FermiHubbardChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
FermiHubbardChain.	Return a NearestNeighborModel with same lattice, but
trivial_like_NNModel()	trivial (H=0) bonds.

Class Attributes and Properties

FermiHubbardChain.	If True, init_lattice() asserts that the initialized
force_default_lattice	lattice is (a subclass of) default_lattice
FermiHubbardChain.logger	class attribute.
FermiHubbardChain.verbose	

class tenpy.models.hubbard.FermiHubbardChain (model_params)

Bases: tenpy.models.hubbard.FermiHubbardModel, tenpy.models.model. NearestNeighborModel

The Fermi Hubbard Model on a Chain, suitable for TEBD.

See the FermiHubbardModel for the documentation of parameters.

default_lattice

```
alias of tenpy.models.lattice.Chain
```

Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$, where OP0 := lat.unit_cell[u0].get_op(op0) acts on the site (x_0, ..., x_{dim-1}, u1), and OP1 := lat.unit_cell[u1].get_op(op1) acts on the site (x_0+dx[0], ..., x_{dim-1}+dx[dim-1], u1). Possible combinations x_0, ..., x_{dim-1} are determined from the boundary conditions in possible_couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling_shape* returned by tenpy.models.lattice.coupling_shape() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str_on_first* and *raise_op2_left* will be removed in version 1.0.0.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- **u1** (*int*) Picks the site lat.unit_cell[u1] for OP1.
- **op1** (*str*) Valid operator name of an onsite operator in lat.unit_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op_needs_JW().
- **str_on_first** (bool) Whether the provided *op_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.

- raise_op2_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

When initializing a model, you can add a term $J \sum_{\langle i,j \rangle} S_i^z S_j^z$ on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], u1, 'Sz', u2, 'Sz', dx)
```

Make sure to use the *plus_hc* argument if necessary, e.g. for hoppings:

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add_coupling(t, u1, 'A', u2, 'B', dx) is add_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see $get_hc_op_name()$. For spin-less fermions (FermionSite), this would be

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

$\times Cdagger_down C_up$
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

See also:

```
add_onsite Add terms acting on one site only.

add_multi_coupling_term for terms on more than two sites.

add_coupling_term Add a single term without summing over \vec{x}.
```

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id', category=None, plus_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- j (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **op1** (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op1}_i {op2}_j".
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Add an exponentially decaying long-range coupling.

$$strength \sum_{i < j} \lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Where the operator A is given by op_i , and B is given by op_j . Note that the sum over i,j is long-range, for infinite systems going beyond the MPS unit cell. Moreover, note that the distance in the exponent is the distance within *subsites*.

Parameters

- strength (float) Overall prefactor.
- lambda (float) Decay-rate
- **op_i** (*string*) Names for the operators.
- op_j (string) Names for the operators.
- **subsites** (*None* | 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- op_string (None / str) The operator to be inserted between A and B; If None, this function checks whether a fermionic "JW" string is needed for the given operators; in this case the right op_j acts first.
- plus_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

Examples

At least for simple enough 1D chains (or ladders), you can use $fit_with_sum_of_exp()$ to approximate a long-range function with a few sum of exponentials and then add them with this function.

```
>>> def decay(x):
...    return np.exp(-0.1*x) / x**2
>>> from tenpy.tools.fit import fit_with_sum_of_exp, sum_of_exp
>>> n_exp = 5
>>> fit_range = 50
>>> lam, pref = fit_with_sum_of_exp(decay, n_exp, fit_range)
>>> x = np.arange(1, fit_range + 1)
>>> print('error in fit: {0:.3e}'.format(np.sum(np.abs(decay(x) - sum_of_exp(lam, pref, x)))))
error in fit: 1.073e-04
>>> for pr, la in zip(pref, lam):
... self.add_exponentially_decaying_coupling(pr, la, 'N', 'N')
```

add_local_term (strength, term, category=None, plus_hc=False)
Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite_terms or coupling_terms, respectively.

Parameters

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array_like*)) List of tuples (opname, lat_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat_idx*. Here, *lat_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite_terms or coupling_terms.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

```
add_multi_coupling (strength, ops, _deprecate_1='DEPRECATED', _deprecate_2='DEPRECATED', op_string=None, category=None, plus_hc=False)

Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.
```

Represents couplings of the form $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$, involving M operators. Here, OP_m stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position $\vec{x} + d\vec{x}$ and unit-cell index u of the site it acts on; the actual operator is given by $self_ilat_iunit\ cell[u]_set\ op(opname)$.

The coupling strength may vary spatially if the given strength is a numpy array. The correct shape of this array is the $coupling_shape$ returned by tenpy.models.lattice. possible_multi_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of strength is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and $other_op$ with other_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]*self.lat.dim. Note the changed order inside the tuples!

Parameters

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially, and is tiled to the required shape.
- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length lat.dim) is a translation vector, and u (int) is the index of $lat.unit_cell$ on which the operator acts. The first entry of ops corresponds to OP_0 and acts last in the physical sense.
- **op_string** (str | None) If a string is given, we use this as the name of an operator to be used inbetween the operators, *excluding* the sites on which any operators act. This operator should be defined on all sites in the unit cell.

If None, auto-determine whether a Jordan-Wigner string is needed (using $op_needs_JW()$) for each of the segments inbetween the operators and also on the sites of the left operators.

- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {other_ops[0]}_j {other_ops[1]}_k ...".
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

Examples

A call to add_coupling() with arguments add_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D >>> self.add_multi_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate (instead of simply using $plus_hc = True$), you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names (indicated by the hc(...), see $get_hc_op_name()$):

```
>>> self.add_multi_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), dx_0, u1)])
```

See also:

```
add_onsite Add terms acting on one site only.
add_coupling Add terms acting on two sites.
```

add_multi_coupling_term Add a single term, not summing over the possible \vec{x} .

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string, category=None, plus_hc=False) Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling_terms[category].add_multi_coupling_term(...).

Warning: This function does not handle Jordan-Wigner strings! You might want to use add_local_term() instead.

Parameters

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With $i, j, k, \ldots = ijkl$, we require that they are ordered ascending, $i < j < k < \ldots$ and that $0 <= i < N_sites$. Inidces $>= N_sites$ indicate couplings between different unit cells of an infinite MPS.
- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots
- op_string (list of str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling_terms. Defaults to a string of the form "{op0}_i {op1}_j {op2}_k ...".
- plus_hc (bool) If True, the hermitian conjugate of the term is added automatically.

add_onsite (strength, u, opname, category=None, plus_hc=False)

Add onsite terms to onsite_terms.

Adds $\sum_{\vec{x}} strength[\vec{x}] * OP$ to the represented Hamiltonian, where the operator OP=lat.unit_cell[u].get_op(opname) acts on the site given by a lattice index (x_0, ..., x_{dim-1}, u),

The necessary terms are just added to onsite_terms; doesn't rebuild the MPO.

Parameters

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit_cell[u].
- category (str) Descriptive name used as key for onsite_terms. Defaults to opname.
- plus_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

See also:

add coupling Add a terms acting on two sites.

add onsite term Add a single term without summing over vecx.

 $\verb"add_onsite_term" (strength, i, op, category = None, plus_hc = False)$

Add an onsite term on a given MPS site.

Wrapper for self.onsite_terms[category].add_onsite_term(...).

Parameters

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.

```
• op (str) – Name of the involved operator.
```

• category (str) - Descriptive name used as key for onsite_terms. Defaults to op.

• plus_hc (bool) – If *True*, the hermitian conjugate of the term is added automatically.

all_coupling_terms()

Sum of all coupling_terms.

all_onsite_terms()

Sum of all onsite terms.

bond_energies(psi)

Calculate bond energies <psilH_bondlpsi>.

Parameters psi (MPS) – The MPS for which the bond energies should be calculated.

Returns E_bond - List of bond energies: for finite bc, E_Bond[i] is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the energy of bond i-1, i.

Return type 1D ndarray

calc_H_MPO (tol_zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite_terms and coupling_terms to build an MPOGraph (and then an MPO).

Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_MPO_from_bond(tol_zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

Parameters tol_zero (float) – Arrays with norm < tol_zero are considered to be zero.

Returns H_MPO – MPO representation of the Hamiltonian.

Return type MPO

calc_H_bond(tol_zero=1e-15)

calculate H_bond from coupling_terms and onsite_terms.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns H_bond - Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

calc_H_bond_from_MPO (tol_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.

Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_onsite(tol_zero=1e-15)
```

Calculate *H* onsite from self.onsite terms.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all_onsite_terms().remove_zeros(tol_zero).to_Arrays(self.lat.mps_sites()). You might also want to take explicit_plus_hc into account.

Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are considered to be zero.

Returns

- **H_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit_plus_hc is True, Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [[resta1998]]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up $exp(+i\ phase)$.

Warning: For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in $add_coupling()$.

Parameters

- **strength** (*scalar* / *array*) The strength to be used in *add_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add_coupling().
- **phase** (*iterable* of float) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

Returns strength – The strength array to be used as strength in add_coupling() with the given dx.

Return type complex array

Examples

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
```

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enlarge_mps_unit_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

classmethod from_MPOModel(mpo_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group sites().

Parameters mpo_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

Examples

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> grouped_sites = nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define H_bond . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
>>> isinstance(nnn_chain_for_tebd, NearestNeighborModel)
True
```

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.

• **subpath** (str) – The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each *n* sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

Parameters

- **n** (*int*) Number of sites to be grouped together.
- grouped_sites (None | list of GroupedSite) The sites grouped together.

Returns grouped_sites - The sites grouped together.

Return type list of GroupedSite

```
init_H_from_terms()
```

Initialize *H_MPO* (and *H_bond*) from the terms of the *CouplingModel*.

This function is called automatically during CouplingMPOModel.__init__.

If you use one of the *add_** methods of the *CouplingModel after* initialization, you will need to call *init_H_from_terms* in the end by yourself, in order to update the *H_MPO* (and possibly *H_bond*) representations. (You should get a warning about this... The way to avoid it is to initialize all the terms in *init_terms* by defining your own model, as outlined in *Models*.

init_lattice (model_params)

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

Returns lat – An initialized lattice.

Return type Lattice

Options

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, directly a subclass of Lattice instead of the name. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
    Boundary conditions for the MPS.
```

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite" bc_MPS , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on bc y).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc_MPS="finite" and "periodic" for bc_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

init_sites (model_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

Note: Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

If you need to initialize more than one site, the function tenpy.networks.site. set_common_charges() should be helpful.

Parameters model_params (dict) – The model parameters given to __init__.

Returns sites – The local sites of the lattice, defining the local basis states and operators.

Return type (tuple of) Site

init_terms (model_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

trivial_like_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

Module description

Bosonic and fermionic Hubbard models.

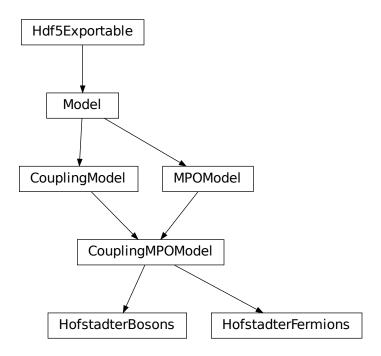
19.9 hofstadter

• full name: tenpy.models.hofstadter

• parent module: tenpy.models

• type: module

Classes



<pre>HofstadterBosons(model_params)</pre>	Bosons on a square lattice with magnetic flux.
<pre>HofstadterFermions(model_params)</pre>	Fermions on a square lattice with magnetic flux.

19.9. hofstadter 593

Functions

gauge_hopping(model_params)	Compute hopping amplitudes for the Hofstadter models
	based on a gauge choice.

19.9.1 gauge hopping

- full name: tenpy.models.hofstadter.gauge_hopping
- parent module: tenpy.models.hofstadter
- · type: function

tenpy.models.hofstadter.gauge hopping (model params)

Compute hopping amplitudes for the Hofstadter models based on a gauge choice.

In the Hofstadter model, the magnetic field enters as an Aharonov-Bohm phase. This phase is dependent on a choice of gauge, which simultaneously defines a 'magnetic unit cell' (MUC).

The magnetic unit cell is the smallest set of lattice plaquettes that encloses an integer number of flux quanta. It can be user-defined by setting mx and my, but for common gauge choices is computed based on the flux density.

The gauge choices are:

- 'landau_x': Landau gauge along the x-axis. The magnetic unit cell will have shape :math`(mathtt{mx}, 1)`. For flux densities p/q, mx will default to q. Example: at a flux density 1/3, the magnetic unit cell will have shape (3, 1), so it encloses exactly 1 flux quantum.
- 'landau_y': Landau gauge along the y-axis. The magnetic unit cell will have shape :math`(1, mathtt{my})`. For flux densities :math`p/q`, my will default to q. Example: at a flux density 3/7, the magnetic unit cell will have shape (1,7), so it encloses axactly 3 flux quanta.
- 'symmetric': symmetric gauge. The magnetic unit cell will have shape (mx, my), with mx = my. For flux densities p/q, mx and my will default to q Example: at a flux density 4/9, the magnetic unit cell will have shape (9.9).

Parameters

- gauge ('landau_x' | 'landau_y' | 'symmetric') Choice of the gauge, see table above.
- mx (int / None) Dimensions of the magnetic unit cell in terms of lattice sites. None defaults to the minimal choice compatible with gauge and phi_pq.
- my (int / None) Dimensions of the magnetic unit cell in terms of lattice sites. None defaults to the minimal choice compatible with gauge and phi_pq.
- **Jx** (*float*) 'Bare' hopping amplitudes (without phase). Without any flux we have hop_x = -Jx and hop_y = -Jy.
- **Jy** (*float*) 'Bare' hopping amplitudes (without phase). Without any flux we have hop_x = -Jx and hop_y = -Jy.
- phi_pq(tuple (int, int)) Magnetic flux as a fraction p/q, defined as (p, q)

Returns hop_x, hop_y – Hopping amplitudes to be used as prefactors for $c_{x,y}^{\dagger}c_{x+1,y}$ (hop_x) and $c_{x,y}^{\dagger}c_{x,y+1}$ (hop_x), respectively, with the necessary phases for the gauge.

Return type float | array

Module description

Cold atomic (Harper-)Hofstadter model on a strip or cylinder.

Todo: WARNING: These models are still under development and not yet tested for correctness. Use at your own risk! Replicate known results to confirm models work correctly. Long term: implement different lattices. Long term: implement variable hopping strengths Jx, Jy.

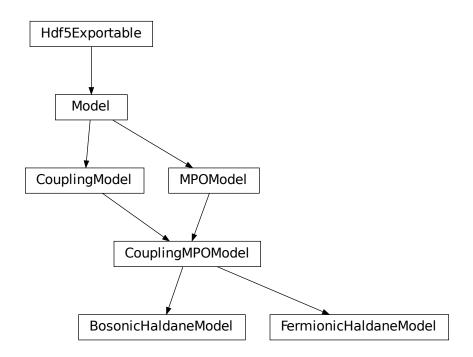
19.10 haldane

• full name: tenpy.models.haldane

• parent module: tenpy.models

• type: module

Classes



BosonicHaldaneModel(model_params)	Hardcore bosonic Haldane model.
FermionicHaldaneModel(model_params)	Spinless fermionic Haldane model.

19.10. haldane 595

Module description

Bosonic and fermionic Haldane models.

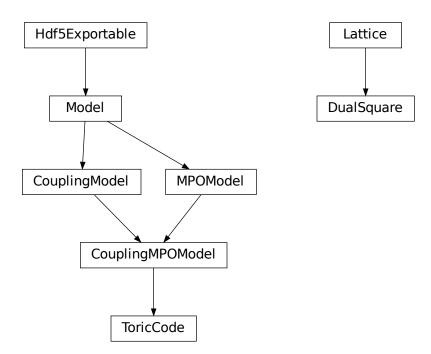
19.11 toric_code

• full name: tenpy.models.toric_code

• parent module: tenpy.models

• type: module

Classes



DualSquare(Lx, Ly, sites, **kwargs)	The dual lattice of the square lattice (again square).
ToricCode(model_params)	Toric code model.

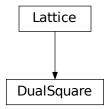
19.11.1 DualSquare

• full name: tenpy.models.toric_code.DualSquare

• parent module: tenpy.models.toric_code

• type: class

Inheritance Diagram



Methods

DualSquareinit(Lx, Ly, sites, **kwargs)	Initialize self.
DualSquare.count_neighbors([u, key])	Count e.g.
	Calculate correct shape of the <i>strengths</i> for a coupling.
DualSquare.coupling_shape(dx)	1 0 1 0
DualSquare.distance(u1, u2, dx)	Get the distance for a given coupling between two sites
	in the lattice.
DualSquare.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
DualSquare.find_coupling_pairs([max_dx,	Automatically find coupling pairs grouped by distances.
])	
DualSquare.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
DualSquare.lat2mps_idx(lat_idx)	Translate lattice indices (x_0, \ldots, x_{D-1}) ,
	u) to MPS index i.
DualSquare.mps2lat_idx(i)	Translate MPS index i to lattice indices (x_0,,
	x_{dim-1} , u).
DualSquare.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice
	indices.
DualSquare.mps2lat_values_masked(A[,	Reshape/reorder an array A to replace an MPS index by
axes,])	lattice indices.
DualSquare.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is u .
DualSquare.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
DualSquare.mps_sites()	Return a list of sites for all MPS indices.
	continues on next page

19.11. toric_code 597

Tahla	6N _	continued	from	previous page

DualSquare.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
DualSquare.number_nearest_neighbors([u])	Deprecated.
DualSquare.number_next_nearest_neighbo	r D(=pn)=cated.
DualSquare.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
DualSquare.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.
DualSquare.plot_bc_identified(ax[,])	Mark two sites indified by periodic boundary condi-
	tions.
DualSquare.plot_coupling(ax[, coupling,	Plot lines connecting nearest neighbors of the lattice.
wrap])	
DualSquare.plot_order(ax[, order, textk-	Plot a line connecting sites in the specified "order" and
wargs])	text labels enumerating them.
DualSquare.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
DualSquare.position(lat_idx)	return 'space' position of one or multiple sites.
DualSquare.possible_couplings(u1, u2, dx[,	Find possible MPS indices for two-site couplings.
])	
DualSquare.possible_multi_couplings(ops	[,Generalization of possible_couplings() to cou-
])	plings with more than 2 sites.
DualSquare.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
DualSquare.site(i)	return Site instance corresponding to an MPS index i

Class Attributes and Properties

DualSquare.Lu	the (expected) number of sites in the unit cell,
	<pre>len(unit_cell).</pre>
DualSquare.boundary_conditions	Human-readable list of boundary conditions from bc
	and bc_shift.
DualSquare.dim	the dimension of the lattice
DualSquare.nearest_neighbors	
DualSquare.next_nearest_neighbors	
DualSquare.next_next_nearest_neighbors	
DualSquare.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.toric_code.DualSquare(Lx, Ly, sites, **kwargs)

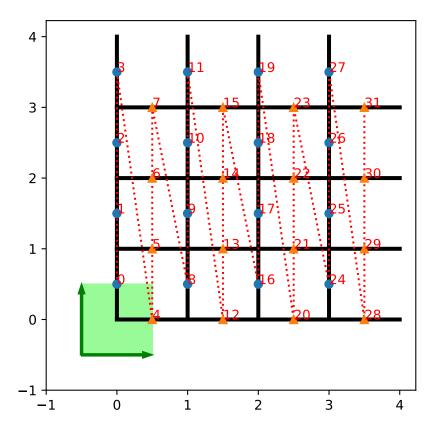
Bases: tenpy.models.lattice.Lattice

The dual lattice of the square lattice (again square).

The sites in this lattice correspond to the vertical and horizontal (nearest neighbor) bonds of a common Square lattice with the same dimensions Lx, Ly.

Parameters

- Lx (int) Dimensions of the original lattice. This lattice has 2*Lx*Ly sites.
- Ly (int) Dimensions of the original lattice. This lattice has 2*Lx*Ly sites.



19.11. toric_code 599

- **sites** (Site) The sites for the horizontal (first entry) and vertical (second entry) bonds.
- **kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs
 are set accordingly.

dim = 2

the dimension of the lattice

ordering(order)

Provide possible orderings of the *N* lattice sites.

The following orders are defined in this method compared to tenpy.models.lattice.Lattice.ordering():

order	equivalent <i>priority</i>	equivalent snake_winding
'default'	(0, 2, 1)	(False, False, False)

property boundary_conditions

Human-readable list of boundary conditions from bc and bc_shift.

Returns boundary_conditions – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

count neighbors (u=0, key='nearest neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

Parameters

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

Returns number – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

Return type int

$coupling_shape(dx)$

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add_multi_coupling().

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

distance(u1, u2, dx)

Get the distance for a given coupling between two sites in the lattice.

The *u1*, *u2*, *dx* parameters are defined in analogy with *add_coupling()*, i.e., this function calculates the distance between a pair of operators added with *add_coupling* (using the basis and unit_cell_positions of the lattice).

Warning: This function ignores "wrapping" arround the cylinder in the case of periodic boundary conditions.

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (*int*) Indices within the unit cell; the *u1* and *u2* of add_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

Returns distance – The distance between site at lattice indices [x, y, u1] and [x + dx[0], y + dx[1], u2], **ignoring** any boundary effects.

Return type float

enlarge_mps_unit_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

Parameters factor (*int*) – The new number of sites in the MPS unit cell will be increased from *N_sites* to factor*N_sites_per_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).

find_coupling_pairs (*max_dx=3*, *cutoff=None*, *eps=1e-10*)

Automatically find coupling pairs grouped by distances.

Given the unit_cell_positions and basis, the coupling pairs of *nearest_neighbors*, *next_nearest_neighbors* etc at a given distance are basically fixed (although not uniquely, since we take out half of them to avoid double-counting couplings in both directions A_i B_j + B_i A_i). This function iterates through all possible couplings up to a given *cutoff* distance and then determines the possible pairs at fixed distances (up to round-off errors).

Parameters

- max_dx (int) Maximal index for each index of dx to iterate over. You need large enough values to include every possible coupling up to the desired distance, but choosing it too large might make this function run for a long time.
- **cutoff** (*float*) Maximal distance (in the units in which basis and unit_cell_positions is given).
- **eps** (float) Tolerance up to which to distances are considered the same.

Returns coupling_pairs – Keys are distances of nearest-neighbors, next-nearest-neighbors etc. Values are [(u1, u2, dx), ...] as in pairs.

Return type dict

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

19.11. toric code 601

Return type cls

lat2mps_idx (lat_idx)

Translate lattice indices (x_0, \dots, x_{D-1}) , u) to MPS index i.

Parameters lat_idx ($array_1ike[..,dim+1]$) - The last dimension corresponds to lattice indices ($x_0, ..., x_{D-1}, u$). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc_MPS , an x_0 outside indicates shifts accross the boundary.

Returns i – MPS index/indices corresponding to *lat_idx*. Has the same shape as *lat_idx* without the last dimension.

Return type array_like

$mps2lat_idx(i)$

Translate MPS index i to lattice indices $(x_0, \dots, x_{\min}, u)$.

Parameters i (int | array_like of int) - MPS index/indices.

Returns lat_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite" bc_MPS , we shift x_0 accordingly.

Return type array

$mps2lat_values(A, axes=0, u=None)$

Reshape/reorder A to replace an MPS index by lattice indices.

Parameters

- A (ndarray) Some values. Must have A.shape[axes] = self.N_sites if u is None, or A.shape[axes] = self.N_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit_cell[u], as returned by mps_idx_fix_u(). The resulting array will not have the additional dimension(s) of u.
- **Returns res_A** Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[tuple(lat.mps2lat_idx(5))] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use $mps_idx_fix_u$ () to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

mps2lat_values_masked (A, axes=-1, mps_inds=None, include_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of $mps21at_values()$ allowing for the case of an arbitrary set of MPS indices present in each axis of A.

Parameters

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps_inds.
- mps_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc_MPS, we shift x_0 accordingly.
- include_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res_A. Defaults to len(self.unit_cell) > 1.

Returns res_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res_A[..., x0, x1, x2, ...] = A[..., mps_inds[j], ...].

Return type np.ma.MaskedArray

$mps_idx_fix_u (u=None)$

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns mps_idx - MPS indices for which self.site(i) is self.unit_cell[u]. Ordered ascending.

Return type array

19.11. toric code 603

mps_lat_idx_fix_u (u=None)

Similar as $mps_idx_fix_u$ (), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

Returns

- mps_idx (array) MPS indices i for which self.site(i) is self. unit_cell[u].
- lat_idx (2D array) The row j contains the lattice index (without u) corresponding to mps_idx[j].

mps_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

multi_coupling_shape (dx)

Calculate correct shape of the *strengths* for a multi_coupling.

Parameters dx (2D array, shape (N_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of $tenpy.models.model.CouplingModel.add_multi_coupling().$

Returns

- **coupling_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat indices* has only rows within this shape.
- **shift_lat_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling_shape()* it can also contain entries > 0)

number nearest neighbors (u=0)

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors() instead.

$number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count_neighbors () instead.

property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot_order().

```
plot_basis (ax, origin=(0.0, 0.0), shade=None, **kwargs)
```

Plot arrows indicating the basis vectors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- ****kwargs** Keyword arguments for ax.arrow.

plot_bc_identified (ax, direction=- 1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit cell positions.
- **kwargs Keyword arguments for the used ax.plot.

```
plot_coupling(ax, coupling=None, wrap=False, **kwargs)
```

Plot lines connecting nearest neighbors of the lattice.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- coupling (list of (u1, u2, dx)) By default (None), use self. pairs['nearest_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (i0, i1, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, plot couplings going around the boundary by directly connecting the sites it connects. This might be hard to see, as this puts lines from one end of the lattice to the other. If False, plot the couplings as dangling lines.
- **kwargs Further keyword arguments given to ax.plot().

```
plot_order(ax, order=None, textkwargs={'color': 'r'}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- **kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site *u* in the unit cell is plotted with a marker markers[u % len(markers)].
- **kwargs Further keyword arguments given to ax.plot().

position (lat_idx)

return 'space' position of one or multiple sites.

Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.

19.11. toric code 605

Returns pos – The position of the lattice sites specified by *lat_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible_couplings (u1, u2, dx, strength=None)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x_a is limited to $0 \le x_a \le x_$

Parameters

- u1 (int) Indices within the unit cell; the u1 and u2 of add_coupling()
- **u2** (int) Indices within the unit cell; the u1 and u2 of add coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.
- **strength** (array_like / None) If given, instead of returning lat_indices and coupling_shape directly return the correct strength_12.

Returns

- mps1, mps2 (1D array) For each possible two-site coupling the MPS indices for the u1 and u2.
- strength_vals (*ID array*) (Only returend if *strength* is not None.) Such that for (i, j, s) in zip(mps1, mps2, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returend if strength is None.) Rows of lat_indices correspond to entries of mps1 and mps2 and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling_shape** (*tuple of int*) (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

possible_multi_couplings (ops, strength=None)

Generalization of possible_couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument *ops* of add multi coupling().

Returns

- mps_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- strength_vals (*1D array*) (Only returend if *strength* is not None.) Such that for (ijkl, s) in zip(mps_ijkl, strength_vals): iterates over all possible couplings with s being the strength of that coupling. Couplings where strength_vals == 0. are filtered out.
- lat_indices (2D int array) (Only returned if strength is None.) Rows of lat_indices correspond to rows of mps_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.

• **coupling_shape** (*tuple of int*) – (Only returend if *strength* is None.) Len <code>dim</code>. The correct shape for an array specifying the coupling strength. *lat_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

Specifically, it saves unit_cell, Ls, unit_cell_positions, basis, boundary_conditions, pairs under their name, bc_MPS as "boundary_conditions_MPS", and order as "order_for_MPS". Moreover, it saves dim and N_sites as HDF5 attributes.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

site(i)

return Site instance corresponding to an MPS index i

test_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

Module description

Kitaev's exactly solvable toric code model.

As we put the model on a cylinder, the name "toric code" is a bit misleading, but it is the established name for this model...

19.11. toric_code 607

608 Chapter 19. models

CHAPTER

TWENTY

NETWORKS

• full name: tenpy.networks

• parent module: tenpy

• type: module

Module description

Definitions of tensor networks like MPS and MPO.

Here, 'tensor network' refers just to the (parital) contraction of tensors. For example an MPS represents the contraction along the 'virtual' legs/bonds of its *B*.

Submodules

site	Defines a class describing the local physical Hilbert
	space.
mps	This module contains a base class for a Matrix Product
	State (MPS).
mpo	Matrix product operator (MPO).
terms	Classes to store a collection of operator names and sites
	they act on, together with prefactors.
purification_mps	This module contains an MPS class representing an den-
	sity matrix by purification.

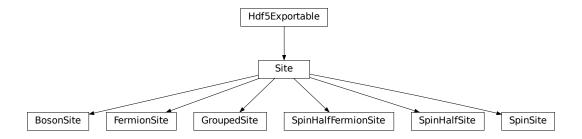
20.1 site

• full name: tenpy.networks.site

ullet parent module: tenpy.networks

• type: module

Classes



BosonSite([Nmax, conserve, filling])	Create a Site for up to Nmax bosons.
FermionSite([conserve, filling])	Create a Site for spin-less fermions.
GroupedSite(sites[, labels, charges])	Group two or more Site into a larger one.
Site(leg[, state_labels])	Collects necessary information about a single local site
	of a lattice.
SpinHalfFermionSite([cons_N, cons_Sz, fill-	Create a Site for spinful (spin-1/2) fermions.
ing])	
SpinHalfSite([conserve])	Spin-1/2 site.
SpinSite([S, conserve])	General Spin S site.

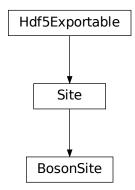
20.1.1 BosonSite

• full name: tenpy.networks.site.BosonSite

• parent module: tenpy.networks.site

• type: class

Inheritance Diagram



Methods

BosonSiteinit([Nmax, conserve, filling])	Initialize self.
BosonSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
BosonSite.change_charge([new_leg_charge,	Change the charges of the site (in place).
])	
BosonSite.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
BosonSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
BosonSite.get_op(name)	Return operator of given name.
BosonSite.multiply_op_names(names)	Multiply operator names together.
BosonSite.multiply_operators(operators)	Multiply local operators (possibly given by their names)
	together.
BosonSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
BosonSite.remove_op(name)	Remove an added operator.
BosonSite.rename_op(old_name, new_name)	Rename an added operator.
BosonSite.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
BosonSite.state_index(label)	Return index of a basis state from its label.
BosonSite.state_indices(labels)	Same as state_index(), but for multiple labels.
BosonSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
BosonSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

Class Attributes and Properties

BosonSite.dim	Dimension of the local Hilbert space.
BosonSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.BosonSite(Nmax=1, conserve='N', filling=0.0)

Bases: tenpy.networks.site.Site

Create a *Site* for up to *Nmax* bosons.

Local states are vac, 1, 2, ..., No. (Exception: for parity conservation, we sort as vac, 2, 4, ..., 1, 3, 5, ...)

operator	description	
Id, JW	Identity ⊮	
В	Annihilation operator b	
Bd	Creation operator b^{\dagger}	
N	Number operator $n = b^{\dagger}b$	
NN	n^2	
dN	$\delta n := n - filling$	
dNdN	$(\delta n)^2$	
Р	Parity $Id - 2(n \mod 2)$.	

conserve	qmod	excluded onsite operators
'N'	[1]	_
'parity'	[2]	_
None	[]	_

Parameters

- Nmax (int) Cutoff defining the maximum number of bosons per site. The default Nmax=1 describes hard-core bosons.
- **conserve** (*str*) Defines what is conserved, see table above.
- **filling** (*float*) Average filling. Used to define dN.

conserve

Defines what is conserved, see table above.

Type str

filling

Average filling. Used to define dN.

Type float

add_op (name, op, need_JW=False, hc=None)

Add one on-site operators.

Parameters

• name (str) – A valid python variable name, used to label the operator. The name under which *op* is added as attribute to self.

- op (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p*'.
- need_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need_JW_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc_ops.

change_charge (new_leg_charge=None, permute=None)

Change the charges of the site (in place).

Parameters

- new_leg_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state_labels and perm. If you sorted the previous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg. perm_flat_from_perm_qind(perm_qind). Ignored if None.

property dim

Dimension of the local Hilbert space.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5qr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

get_hc_op_name (name)

Return the hermitian conjugate of a given operator.

Parameters name (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as get op() does.

Returns hc_op_name – Operator name for the hermi such that $get_op()$ of

Return type str

get_op (name)

Return operator of given name.

Parameters name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

Returns op – The operator given by *name*, with labels 'p', 'p*'. If name already was an npc Array, it's directly returned.

Return type np_conserved

multiply_op_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get_op* returns the product of the corresponding operators.

Parameters names (list of str) – List of valid operator labels.

Returns combined_opname – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

multiply_operators (operators)

Multiply local operators (possibly given by their names) together.

Parameters operators (list of {str | Array}) – List of valid operator names (to be translated with get_op()) or directly on-site operators in the form of npc arrays with 'p', 'p*' label. The operators are multiplied left-to-right.

Returns combined_operator – The product of the given *operators* in a left-to-right multiplication following the usual mathematical convention. For example, if operators=['Sz', 'Sp', 'Sx'], the final operator is equivalent to site.get_op('Sz Sp Sx'), with the 'Sx' operator acting first on any physical state.

Return type Array

property onsite_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

op_needs_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

Parameters name (str) – The name of the operator, as in $get_op()$.

Returns needs_JW — Whether the operator needs a Jordan-Wigner string, judging from $need_JW_string$.

Return type bool

remove_op (name)

Remove an added operator.

Parameters name (str) – The name of the operator to be removed.

rename_op (old_name, new_name)

Rename an added operator.

Parameters

- old_name (str) The old name of the operator.
- **new_name** (*str*) The new name of the operator.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with <code>save_dict_content()</code>, storing the format under the attribute 'format'.

Parameters

• hdf5_saver (Hdf5Saver) - Instance of the saving engine.

- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

$state_index(label)$

Return index of a basis state from its label.

Parameters label (int | string) – eather the index directly or a label (string) set before.

Returns state_index – the index of the basis state associated with the label.

Return type int

state_indices(labels)

Same as state_index(), but for multiple labels.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

valid_opname (name)

Check whether 'name' labels a valid onsite-operator.

Parameters name (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

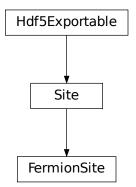
Returns valid – True if *name* is a valid argument to $get_op()$.

Return type bool

20.1.2 FermionSite

- full name: tenpy.networks.site.FermionSite
- parent module: tenpy.networks.site
- type: class

Inheritance Diagram



Methods

FermionSiteinit([conserve, filling])	Initialize self.
FermionSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
FermionSite.change_charge([new_leg_charge,	Change the charges of the site (in place).
])	
FermionSite.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
FermionSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
FermionSite.get_op(name)	Return operator of given name.
FermionSite.multiply_op_names(names)	Multiply operator names together.
FermionSite.multiply_operators(operators)	Multiply local operators (possibly given by their names)
	together.
FermionSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
FermionSite.remove_op(name)	Remove an added operator.
FermionSite.rename_op(old_name, new_name)	Rename an added operator.
FermionSite.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
FermionSite.state_index(label)	Return index of a basis state from its label.
FermionSite.state_indices(labels)	Same as state_index(), but for multiple labels.
FermionSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
FermionSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

Class Attributes and Properties

FermionSite.dim	Dimension of the local Hilbert space.
FermionSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.FermionSite(conserve='N', filling=0.5)

Bases: tenpy.networks.site.Site

Create a Site for spin-less fermions.

Local states are empty and full.

Warning: Using the Jordan-Wigner string (JW) is crucial to get correct results, otherwise you just describe hardcore bosons! Further details in *Fermions and the Jordan-Wigner transformation*.

operator	description
Id	Identity ⊮
JW	Sign for the Jordan-Wigner string.
С	Annihilation operator c (up to 'JW'-string left of it)
Cd	Creation operator c^{\dagger} (up to 'JW'-string left of it)
N	Number operator $n = c^{\dagger}c$
dN	$\delta n := n - filling$
dNdN	$(\delta n)^2$

conserve	qmod	exluded onsite operators
'N'	[1]	_
'parity'	[2]	_
None	[]	_

Parameters

- **conserve** (*str*) Defines what is conserved, see table above.
- **filling** (*float*) Average filling. Used to define dN.

conserve

Defines what is conserved, see table above.

Type str

filling

Average filling. Used to define dN.

Type float

add_op (name, op, need_JW=False, hc=None)

Add one on-site operators.

Parameters

- name (str) A valid python variable name, used to label the operator. The name under which *op* is added as attribute to self.
- op (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p*'.
- need_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need_JW_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc_ops.

change_charge (new_leg_charge=None, permute=None)

Change the charges of the site (in place).

Parameters

- new_leg_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state_labels and perm. If you sorted the previous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg. perm_flat_from_perm_qind(perm_qind). Ignored if None.

property dim

Dimension of the local Hilbert space.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

get hc op name (name)

Return the hermitian conjugate of a given operator.

Parameters name (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as $get_op()$ does.

Returns hc_op_name – Operator name for the hermi such that $get_op()$ of

Return type str

get_op (name)

Return operator of given name.

Parameters name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

Returns op – The operator given by *name*, with labels 'p', 'p*'. If name already was an npc Array, it's directly returned.

Return type np_conserved

multiply_op_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get_op* returns the product of the corresponding operators.

Parameters names (list of str) – List of valid operator labels.

Returns combined_opname – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

multiply_operators (operators)

Multiply local operators (possibly given by their names) together.

Parameters operators (list of {str | Array}) – List of valid operator names (to be translated with get_op()) or directly on-site operators in the form of npc arrays with 'p', 'p*' label. The operators are multiplied left-to-right.

Returns combined_operator – The product of the given *operators* in a left-to-right multiplication following the usual mathematical convention. For example, if operators=['Sz', 'Sp', 'Sx'], the final operator is equivalent to site.get_op('Sz Sp Sx'), with the 'Sx' operator acting first on any physical state.

Return type Array

property onsite_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

op_needs_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

```
Parameters name (str) – The name of the operator, as in get\_op().
```

Returns needs_JW — Whether the operator needs a Jordan-Wigner string, judging from ${\tt need_JW_string}$.

Return type bool

remove op (name)

Remove an added operator.

Parameters name (str) – The name of the operator to be removed.

rename_op (old_name, new_name)

Rename an added operator.

Parameters

- **old_name** (*str*) The old name of the operator.
- **new_name** (str) The new name of the operator.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

state_index(label)

Return index of a basis state from its label.

Parameters label (int | string) – eather the index directly or a label (string) set before.

Returns state_index - the index of the basis state associated with the label.

Return type int

state indices(labels)

Same as $state_index()$, but for multiple labels.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

valid_opname (name)

Check whether 'name' labels a valid onsite-operator.

Parameters name (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

Returns valid – True if *name* is a valid argument to $qet_op()$.

Return type bool

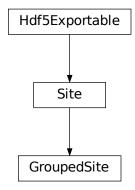
20.1.3 GroupedSite

• full name: tenpy.networks.site.GroupedSite

• parent module: tenpy.networks.site

• type: class

Inheritance Diagram



Methods

GroupedSiteinit(sites[, labels, charges])	Initialize self.
GroupedSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
<pre>GroupedSite.change_charge([new_leg_charge,</pre>	Change the charges of the site (in place).
])	
GroupedSite.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
GroupedSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
GroupedSite.get_op(name)	Return operator of given name.
GroupedSite.kroneckerproduct(ops)	Return the Kronecker product $op0\otimes op1$ of local oper-
	ators.
GroupedSite.multiply_op_names(names)	Multiply operator names together.
<pre>GroupedSite.multiply_operators(operators)</pre>	Multiply local operators (possibly given by their names)
	together.
GroupedSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
GroupedSite.remove_op(name)	Remove an added operator.
<pre>GroupedSite.rename_op(old_name, new_name)</pre>	Rename an added operator.
GroupedSite.save_hdf5(hdf5_saver, h5gr, sub-	Export <i>self</i> into a HDF5 file.
path)	-
GroupedSite.state_index(label)	Return index of a basis state from its label.
	continues on next page

Table 7 – continued from previous page

GroupedSite.state_indices(labels)	Same as state_index(), but for multiple labels.
GroupedSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
GroupedSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

Class Attributes and Properties

GroupedSite.dim	Dimension of the local Hilbert space.
GroupedSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.GroupedSite(sites, labels=None, charges='same')

Bases: tenpy.networks.site.Site

Group two or more Site into a larger one.

A typical use-case is that you want a NearestNeighborModel for TEBD although you have next-nearest neighbor interactions: you just double your local Hilbertspace to consist of two original sites. Note that this is a 'hack' at the cost of other things (e.g., measurements of 'local' operators) getting more complicated/computationally expensive.

If the individual sites indicate fermionic operators (with entries in *need_JW_string*), we construct the new onsite oerators of *site1* to include the JW string of *site0*, i.e., we use the Kronecker product of [JW, op] instead of [Id, op] if necessary (but always [op, Id]). In that way the onsite operators of this DoubleSite automatically fulfill the expected commutation relations. See also *Fermions and the Jordan-Wigner transformation*.

Parameters

- **sites** (list of *Site*) The individual sites being grouped together. Copied before use if charges!='same'.
- labels Include the Kronecker product of the each onsite operator *op* on sites[i] and identities on other sites with the name opname+labels[i]. Similarly, set state labels for ' '.join(state[i]+'_'+labels[i]). Defaults to [str(i) for i in range(n_sites)], which for example grouping two SpinSites gives operators name like "Sz0" and sites labels like 'up_0 down_1'.
- **charges** ('same' | 'drop' | 'independent') How to handle charges, defaults to 'same'. "same' means that all *sites* have the same *ChargeInfo*, and the total charge is the sum of the charges on the individual *sites*. 'independent' means that the *sites* have possibly different *ChargeInfo*, and the charges are conserved separately, i.e., we have *n_sites* conserved charges. For 'drop', we drop any charges, such that the remaining legcharges are trivial. For more complex situations, you can call *multi_sites_combine_charges()* beforehand.

n sites

The number of sites grouped together, i.e. len(sites).

Type int

sites

The sites grouped together into self.

Type list of Site

labels

The labels using which the single-site operators are added during construction.

Type list of str

kroneckerproduct (ops)

Return the Kronecker product $op0 \otimes op1$ of local operators.

Parameters ops (list of Array) – One operator (or operator name) on each of the ungrouped sites. Each operator should have labels ['p', 'p*'].

Returns prod – Kronecker product $ops[0] \otimes ops[1] \otimes \cdots$, with labels ['p', 'p*'].

Return type Array

add_op (name, op, need_JW=False, hc=None)

Add one on-site operators.

Parameters

- name (str) A valid python variable name, used to label the operator. The name under which op is added as attribute to self.
- **op** (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p*'.
- need_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need_JW_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc_ops.

change_charge (new_leg_charge=None, permute=None)

Change the charges of the site (in place).

Parameters

- new_leg_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state_labels and perm. If you sorted the previous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg. perm_flat_from_perm_qind(perm_qind). Ignored if None.

property dim

Dimension of the local Hilbert space.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

get_hc_op_name (name)

Return the hermitian conjugate of a given operator.

Parameters name (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as $get_op()$ does.

Returns hc_op_name – Operator name for the hermi such that $get_op()$ of

Return type str

get_op (name)

Return operator of given name.

Parameters name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

Returns op – The operator given by *name*, with labels 'p', 'p*'. If name already was an npc Array, it's directly returned.

Return type np_conserved

multiply_op_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get_op* returns the product of the corresponding operators.

Parameters names (list of str) – List of valid operator labels.

Returns combined_opname – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

multiply_operators (operators)

Multiply local operators (possibly given by their names) together.

Parameters operators (list of {str | Array}) – List of valid operator names (to be translated with get_op()) or directly on-site operators in the form of npc arrays with 'p', 'p*' label. The operators are multiplied left-to-right.

Returns combined_operator - The product of the given *operators* in a left-to-right multiplication following the usual mathematical convention. For example, if operators=['Sz', 'Sp', 'Sx'], the final operator is equivalent to site.get_op('Sz Sp Sx'), with the 'Sx' operator acting first on any physical state.

Return type Array

property onsite_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

op_needs_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

Parameters name (str) – The name of the operator, as in $get_op()$.

Returns needs_JW — Whether the operator needs a Jordan-Wigner string, judging from ${\tt need_JW_string}.$

Return type bool

remove_op (name)

Remove an added operator.

Parameters name (str) – The name of the operator to be removed.

rename_op (old_name, new_name)

Rename an added operator.

Parameters

- old_name (str) The old name of the operator.
- **new_name** (str) The new name of the operator.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

state index(label)

Return index of a basis state from its label.

Parameters label (int / string) – eather the index directly or a label (string) set before.

Returns state_index – the index of the basis state associated with the label.

Return type int

state_indices(labels)

Same as state_index(), but for multiple labels.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

valid_opname (name)

Check whether 'name' labels a valid onsite-operator.

Parameters name (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

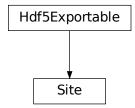
Returns valid – True if *name* is a valid argument to $get_op()$.

Return type bool

20.1.4 Site

- full name: tenpy.networks.site.Site
- parent module: tenpy.networks.site
- type: class

Inheritance Diagram



Methods

	T '4' 1' 16
Siteinit(leg[, state_labels])	Initialize self.
Site.add_op(name, op[, need_JW, hc])	Add one on-site operators.
Site.change_charge([new_leg_charge, per-	Change the charges of the site (in place).
mute])	
Site.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Site.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
Site.get_op(name)	Return operator of given name.
Site.multiply_op_names(names)	Multiply operator names together.
Site.multiply_operators(operators)	Multiply local operators (possibly given by their names)
	together.
Site.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
Site.remove_op(name)	Remove an added operator.
Site.rename_op(old_name, new_name)	Rename an added operator.
Site.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
Site.state_index(label)	Return index of a basis state from its label.
Site.state_indices(labels)	Same as state_index(), but for multiple labels.
Site.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
Site.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

Class Attributes and Properties

Site.dim	Dimension of the local Hilbert space.
Site.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.Site(leg, state_labels=None, **site_ops)
 Bases: tenpy.tools.hdf5_io.Hdf5Exportable

Collects necessary information about a single local site of a lattice.

This class defines what the local basis states are: it provides the *leg* defining the charges of the physical leg for this site. Moreover, it stores (local) on-site operators, which are directly available as attribute, e.g., self.Sz

is the Sz operator for the SpinSite. Alternatively, operators can be obtained with $get_op()$. The operator names Id and JW are reserved for the identy and Jordan-Wigner strings.

Warning: The order of the local basis can change depending on the charge conservation! This is a *necessary* feature since we need to sort the basis by charges for efficiency. We use the <code>state_labels</code> and <code>perm</code> to keep track of these permutations.

Parameters

- leg (LegCharge) Charges of the physical states, to be used for the physical leg of MPS.
- **state_labels** (*None | list of str*) Optionally a label for each local basis states. None entries are ignored / not set.
- **site_ops Additional keyword arguments of the form name=op given to add_op(). The identity operator 'Id' is automatically included. If no 'JW' for the Jordan-Wigner string is given, 'JW' is set as an alias to 'Id'.

leg

Charges of the local basis states.

```
Type LegCharge
```

state labels

(Optional) labels for the local basis states.

```
Type {str: int}
```

opnames

Labels of all onsite operators (i.e. self.op exists if 'op' in self.opnames). Note that $get_op()$ allows arbitrary concatenations of them.

```
Type set
```

need JW string

Labels of all onsite operators that need a Jordan-Wigner string. Used in <code>op_needs_JW()</code> to determine whether an operator anticommutes or commutes with operators on other sites.

```
Type set
```

ops

Onsite operators are added directly as attributes to self. For example after self.add_op('Sz', Sz) you can use self.Sz for the Sz operator. All onsite operators have labels 'p', 'p*'.

```
Type Array
```

perm

Index permutation of the physical leg compared to conserve=None, i.e. OP_conserved =
OP_nonconserved[np.ix_(perm,perm)] and perm[state_labels_conserved[
"some_state"]] == state_labels_nonconserved["some_state"].

```
Type 1D array
```

JW_exponent

Exponents of the 'JW' operator, such that self.JW.to_ndarray() = np.diag(np.exp(1. $j*np.pi*JW_exponent)$)

```
Type 1D array
```

hc_ops

Mapping from operator names to their hermitian conjugates. Use get_hc_op_name () to obtain entries.

Type dict(str->str)

Examples

The following generates a site for spin-1/2 with Sz conservation. Note that Sx = (Sp + Sm)/2 violates Sz conservation and is thus not a valid on-site operator.

change_charge (new_leg_charge=None, permute=None)

Change the charges of the site (in place).

Parameters

- new_leg_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- **permute** (*ndarray* | *None*) The permuation applied to the physical leg, which gets used to adjust *state_labels* and *perm*. If you sorted the previous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg. perm_flat_from_perm_qind(perm_qind). Ignored if None.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

property dim

Dimension of the local Hilbert space.

property onsite_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

add_op (name, op, need_JW=False, hc=None)

Add one on-site operators.

Parameters

- name (str) A valid python variable name, used to label the operator. The name under which op is added as attribute to self.
- **op** (np.ndarray | *Array*) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to *Array*. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p*'.

- need_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need_JW_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc_ops.

rename op (old name, new name)

Rename an added operator.

Parameters

- old_name (str) The old name of the operator.
- **new_name** (str) The new name of the operator.

remove_op (name)

Remove an added operator.

Parameters name (str) – The name of the operator to be removed.

state index(label)

Return index of a basis state from its label.

Parameters label (int / string) – eather the index directly or a label (string) set before.

Returns state index – the index of the basis state associated with the label.

Return type int

state indices(labels)

Same as state_index(), but for multiple labels.

get_op (name)

Return operator of given name.

Parameters name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

Returns op – The operator given by *name*, with labels 'p', 'p*'. If name already was an npc Array, it's directly returned.

Return type np_conserved

get_hc_op_name (name)

Return the hermitian conjugate of a given operator.

Parameters name (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as $get \circ p()$ does.

Returns hc_op_name – Operator name for the hermi such that $get_op()$ of

Return type str

op_needs_JW(name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

Parameters name (str) – The name of the operator, as in $get_op()$.

Returns needs_JW – Whether the operator needs a Jordan-Wigner string, judging from need_JW_string.

Return type bool

valid opname(name)

Check whether 'name' labels a valid onsite-operator.

Parameters name (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

Returns valid – True if *name* is a valid argument to $get_op()$.

Return type bool

multiply_op_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get_op* returns the product of the corresponding operators.

Parameters names (list of str) – List of valid operator labels.

Returns combined_opname – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

multiply_operators (operators)

Multiply local operators (possibly given by their names) together.

Parameters operators (list of {str | Array}) – List of valid operator names (to be translated with get_op()) or directly on-site operators in the form of npc arrays with 'p', 'p*' label. The operators are multiplied left-to-right.

Returns combined_operator – The product of the given *operators* in a left-to-right multiplication following the usual mathematical convention. For example, if operators=['Sz', 'Sp', 'Sx'], the final operator is equivalent to site.get_op('Sz Sp Sx'), with the 'Sx' operator acting first on any physical state.

Return type Array

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.

• **subpath** (str) – The *name* of h5gr with a '/' in the end.

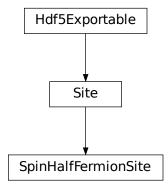
20.1.5 SpinHalfFermionSite

 $\bullet \ \ full \ name: tenpy.networks.site.SpinHalfFermionSite$

• parent module: tenpy.networks.site

• type: class

Inheritance Diagram



Methods

SpinHalfFermionSiteinit([cons_N,	Initialize self.
])	
SpinHalfFermionSite.add_op(name, op[,])	Add one on-site operators.
SpinHalfFermionSite.	Change the charges of the site (in place).
change_charge([])	
SpinHalfFermionSite.	Load instance from a HDF5 file.
<pre>from_hdf5(hdf5_loader,)</pre>	
SpinHalfFermionSite.	Return the hermitian conjugate of a given operator.
<pre>get_hc_op_name(name)</pre>	
SpinHalfFermionSite.get_op(name)	Return operator of given name.
SpinHalfFermionSite.	Multiply operator names together.
multiply_op_names(names)	
SpinHalfFermionSite.	Multiply local operators (possibly given by their names)
multiply_operators(operators)	together.
SpinHalfFermionSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
SpinHalfFermionSite.remove_op(name)	Remove an added operator.
	continues on next page

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Table 11 – continued from previous page

SpinHalfFermionSite.rename_op(old_name,	Rename an added operator.
new_name)	
SpinHalfFermionSite.	Export self into a HDF5 file.
<pre>save_hdf5(hdf5_saver,)</pre>	
SpinHalfFermionSite.state_index(label)	Return index of a basis state from its label.
SpinHalfFermionSite.	Same as state_index(), but for multiple labels.
state_indices(labels)	
SpinHalfFermionSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
SpinHalfFermionSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

Class Attributes and Properties

SpinHalfFermionSite.dim	Dimension of the local Hilbert space.
SpinHalfFermionSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.SpinHalfFermionSite($cons_N='N'$, $cons_Sz='Sz'$, filling=1.0)
Bases: tenpy.networks.site.Site

Bases. compy . meeworks . Siece . Siece

Create a Site for spinful (spin-1/2) fermions.

Local states are: empty (vacuum), up (one spin-up electron), down (one spin-down electron), and full (both electrons)

Local operators can be built from creation operators.

Warning: Using the Jordan-Wigner string (JW) in the correct way is crucial to get correct results, otherwise you just describe hardcore bosons!

operator	description
Id	Identity ⊮
JW	Sign for the Jordan-Wigner string $(-1)^{n_{\uparrow}+n_{\downarrow}}$
JWu	Partial sign for the Jordan-Wigner string $(-1)^{n_{\uparrow}}$
JWd	Partial sign for the Jordan-Wigner string $(-1)^{n_{\downarrow}}$
Cu	Annihilation operator spin-up c_{\uparrow} (up to 'JW'-string on sites left of it).
Cdu	Creation operator spin-up c^{\dagger}_{\uparrow} (up to 'JW'-string on sites left of it).
Cd	Annihilation operator spin-down c_{\downarrow} (up to 'JW'-string on sites left of it). Includes JWu such
	that it anti-commutes onsite with Cu, Cdu.
Cdd	Creation operator spin-down c_{\perp}^{\dagger} (up to 'JW'-string on sites left of it). Includes JWu such that it
	anti-commutes onsite with Cu, Cdu.
Nu	Number operator $n_{\uparrow} = c_{\uparrow}^{\dagger} c_{\uparrow}$
Nd	Number operator $n_{\downarrow} = c_{\downarrow}^{\dagger} c_{\downarrow}$
NuNd	Dotted number operators $n_{\uparrow}n_{\downarrow}$
Ntot	Total number operator $n_t = n_{\uparrow} + n_{\downarrow}$
dN	Total number operator compared to the filling $\Delta n = n_t - filling$
Sx,	Spin operators $S^{x,y,z}$, in particular $S^z=\frac{1}{2}(n_{\uparrow}-n_{\downarrow})$
Sy, Sz	
Sp, Sm	Spin flips $S^{\pm}=S^x\pm iS^y$, e.g. $S^+=c^{\dagger}_{\uparrow}c_{\downarrow}$

The spin operators are defined as $S^{\gamma}=(c_{\uparrow}^{\dagger},c_{\downarrow}^{\dagger})\sigma^{\gamma}(c_{\uparrow},c_{\downarrow})^{T}$, where σ^{γ} are spin-1/2 matrices (i.e. half the pauli matrices).

cons_N	cons_Sz	qmod	excluded onsite operators
'N'	'Sz'	[1, 1]	Sx, Sy
'N'	'parity'	[1, 2]	_
'N'	None	[1]	_
'parity'	'Sz'	[2, 1]	Sx, Sy
'parity'	'parity'	[2, 2]	_
'parity'	None	[2]	_
None	'Sz'	[1]	Sx, Sy
None	'parity'	[2]	_
None	None	[]	_

Parameters

- cons_N('N' | 'parity' | None) Whether particle number is conserved, c.f. table above.
- cons_Sz('Sz' | 'parity' | None) Whether spin is conserved, c.f. table above.
- **filling** (*float*) Average filling. Used to define dN.

cons N

Whether particle number is conserved, c.f. table above.

cons Sz

Whether spin is conserved, c.f. table above.

filling

Average filling. Used to define dN.

add_op (name, op, need_JW=False, hc=None)

Add one on-site operators.

Parameters

- name (str) A valid python variable name, used to label the operator. The name under which op is added as attribute to self.
- **op** (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p*'.
- need_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need_JW_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc_ops.

change_charge (new_leg_charge=None, permute=None)

Change the charges of the site (in place).

Parameters

- new_leg_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state_labels and perm. If you sorted the previous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg. perm_flat_from_perm_qind(perm_qind). Ignored if None.

property dim

Dimension of the local Hilbert space.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

get_hc_op_name (name)

Return the hermitian conjugate of a given operator.

Parameters name (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as $get_op()$ does.

Returns hc_op_name – Operator name for the hermi such that $get_op()$ of

Return type str

get_op (name)

Return operator of given name.

Parameters name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

Returns op – The operator given by *name*, with labels 'p', 'p*'. If name already was an npc Array, it's directly returned.

Return type np_conserved

multiply_op_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get_op* returns the product of the corresponding operators.

Parameters names (list of str) – List of valid operator labels.

Returns combined_opname – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

multiply_operators (operators)

Multiply local operators (possibly given by their names) together.

Parameters operators (list of {str | Array}) – List of valid operator names (to be translated with get_op()) or directly on-site operators in the form of npc arrays with 'p', 'p*' label. The operators are multiplied left-to-right.

Returns combined_operator – The product of the given *operators* in a left-to-right multiplication following the usual mathematical convention. For example, if operators=['Sz', 'Sp', 'Sx'], the final operator is equivalent to site.get_op('Sz Sp Sx'), with the 'Sx' operator acting first on any physical state.

Return type Array

property onsite_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

op needs JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

Parameters name (str) – The name of the operator, as in $get_op()$.

Returns needs_JW — Whether the operator needs a Jordan-Wigner string, judging from need_JW_string.

Return type bool

remove_op (name)

Remove an added operator.

Parameters name (str) – The name of the operator to be removed.

rename_op (old_name, new_name)

Rename an added operator.

Parameters

- old_name (str) The old name of the operator.
- **new_name** (str) The new name of the operator.

```
save\_hdf5 (hdf5\_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5 saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

${\tt state_index}\,(label)$

Return index of a basis state from its label.

Parameters label (int | string) – eather the index directly or a label (string) set before.

Returns state_index - the index of the basis state associated with the label.

Return type int

state_indices(labels)

Same as state_index(), but for multiple labels.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

valid_opname (name)

Check whether 'name' labels a valid onsite-operator.

Parameters name (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

Returns valid – True if *name* is a valid argument to $get_op()$.

Return type bool

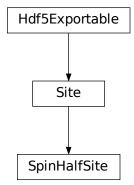
20.1.6 SpinHalfSite

• full name: tenpy.networks.site.SpinHalfSite

• parent module: tenpy.networks.site

• type: class

Inheritance Diagram



Methods

SpinHalfSiteinit([conserve])	Initialize self.
SpinHalfSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
SpinHalfSite.change_charge([new_leg_charge	, Change the charges of the site (in place).
])	
SpinHalfSite.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
SpinHalfSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
SpinHalfSite.get_op(name)	Return operator of given name.
SpinHalfSite.multiply_op_names(names)	Multiply operator names together.
	continues on next nage

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Table 13 – continued from previous page

SpinHalfSite.multiply_operators(operators)	Multiply local operators (possibly given by their names)
	together.
SpinHalfSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
SpinHalfSite.remove_op(name)	Remove an added operator.
SpinHalfSite.rename_op(old_name,	Rename an added operator.
new_name)	
SpinHalfSite.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
subpath)	
SpinHalfSite.state_index(label)	Return index of a basis state from its label.
SpinHalfSite.state_indices(labels)	Same as state_index(), but for multiple labels.
SpinHalfSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
SpinHalfSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

Class Attributes and Properties

SpinHalfSite.dim	Dimension of the local Hilbert space.
SpinHalfSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.SpinHalfSite(conserve='Sz')

Bases: tenpy.networks.site.Site

Spin-1/2 site.

Local states are up (0) and down (1). Local operators are the usual spin-1/2 operators, e.g. Sz = [[0.5, 0.], [0., -0.5]], $Sx = 0.5 \times sigma_x$ for the Pauli matrix $sigma_x$.

operator	description
Id, JW	Identity ⊮
Sx, Sy, Sz	Spin components $S^{x,y,z}$, equal to half the Pauli matrices.
Sigmax, Sigmay, Sigmaz	Pauli matrices $\sigma^{x,y,z}$
Sp, Sm	Spin flips $S^{\pm} = S^x \pm i S^y$

conserve	qmod	excluded onsite operators			
'Sz'	[1]	Sx,	Sy,	Sigmax,	Sigmay
'parity'	[2]	_			
None	[]	_			

Parameters conserve (str) – Defines what is conserved, see table above.

conserve

Defines what is conserved, see table above.

Type str

add_op (name, op, need_JW=False, hc=None)
Add one on-site operators.

Parameters

• name (str) – A valid python variable name, used to label the operator. The name under which *op* is added as attribute to self.

- op (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p*'.
- need_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need_JW_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc_ops.

change_charge (new_leg_charge=None, permute=None)

Change the charges of the site (in place).

Parameters

- new_leg_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state_labels and perm. If you sorted the previous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg. perm_flat_from_perm_qind(perm_qind). Ignored if None.

property dim

Dimension of the local Hilbert space.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

get_hc_op_name (name)

Return the hermitian conjugate of a given operator.

Parameters name (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as get op() does.

Returns hc_op_name – Operator name for the hermi such that $get_op()$ of

Return type str

get_op (name)

Return operator of given name.

Parameters name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

Returns op – The operator given by *name*, with labels 'p', 'p*'. If name already was an npc Array, it's directly returned.

Return type np_conserved

multiply_op_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get_op* returns the product of the corresponding operators.

Parameters names (list of str) – List of valid operator labels.

Returns combined_opname – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

multiply_operators (operators)

Multiply local operators (possibly given by their names) together.

Parameters operators (list of {str | Array}) – List of valid operator names (to be translated with get_op()) or directly on-site operators in the form of npc arrays with 'p', 'p*' label. The operators are multiplied left-to-right.

Returns combined_operator – The product of the given *operators* in a left-to-right multiplication following the usual mathematical convention. For example, if operators=['Sz', 'Sp', 'Sx'], the final operator is equivalent to $site.get_op('Sz Sp Sx')$, with the 'Sx' operator acting first on any physical state.

Return type Array

property onsite_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

op_needs_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

Parameters name (str) – The name of the operator, as in $get_op()$.

Returns needs_JW — Whether the operator needs a Jordan-Wigner string, judging from $\texttt{need_JW_string}$.

Return type bool

remove_op (name)

Remove an added operator.

Parameters name (str) – The name of the operator to be removed.

rename_op (old_name, new_name)

Rename an added operator.

Parameters

- old_name (str) The old name of the operator.
- **new_name** (*str*) The new name of the operator.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with <code>save_dict_content()</code>, storing the format under the attribute 'format'.

Parameters

• hdf5_saver (Hdf5Saver) - Instance of the saving engine.

- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

$state_index(label)$

Return index of a basis state from its label.

Parameters label (int | string) – eather the index directly or a label (string) set before.

Returns state_index – the index of the basis state associated with the label.

Return type int

state_indices(labels)

Same as state_index(), but for multiple labels.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

valid_opname (name)

Check whether 'name' labels a valid onsite-operator.

Parameters name (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

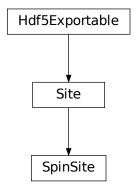
Returns valid – True if *name* is a valid argument to $get_op()$.

Return type bool

20.1.7 SpinSite

- full name: tenpy.networks.site.SpinSite
- parent module: tenpy.networks.site
- type: class

Inheritance Diagram



Methods

SpinSiteinit([S, conserve])	Initialize self.	
SpinSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.	
SpinSite.change_charge([new_leg_charge,	Change the charges of the site (in place).	
permute])		
SpinSite.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.	
SpinSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.	
SpinSite.get_op(name)	Return operator of given name.	
SpinSite.multiply_op_names(names)	Multiply operator names together.	
SpinSite.multiply_operators(operators)	Multiply local operators (possibly given by their names)	
	together.	
SpinSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and	
	needs a Jordan-Wigner string.	
SpinSite.remove_op(name)	Remove an added operator.	
SpinSite.rename_op(old_name, new_name)	Rename an added operator.	
SpinSite.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.	
SpinSite.state_index(label)	Return index of a basis state from its label.	
SpinSite.state_indices(labels)	Same as state_index(), but for multiple labels.	
SpinSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.	
SpinSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.	

Class Attributes and Properties

SpinSite.dim	Dimension of the local Hilbert space.
SpinSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.SpinSite(S=0.5, conserve='Sz')

Bases: tenpy.networks.site.Site

General Spin S site.

There are 2S+1 local states range from down (0) to up (2S+1), corresponding to Sz=-S, -S+1, ..., S-1, S. Local operators are the spin-S operators, e.g. Sz = [[0.5, 0.], [0., -0.5]], $Sx = 0.5*sigma_x$ for the Pauli matrix $sigma_x$.

operator	description
Id, JW	Identity ⊮
Sx, Sy, Sz	1 1 / 1
Sp, Sm	Spin flips $S^{\pm} = S^x \pm iS^y$

conserve	qmod	excluded onsite operators
'Sz'	[1]	Sx, Sy
'parity'	[2]	_
None	[]	_

Parameters conserve (str) – Defines what is conserved, see table above.

s

The 2S+1 states range from m = -S, -S+1, ... +S.

```
Type {0.5, 1, 1.5, 2, ..}
```

conserve

Defines what is conserved, see table above.

Type str

add_op (name, op, need_JW=False, hc=None)

Add one on-site operators.

Parameters

- name (str) A valid python variable name, used to label the operator. The name under which *op* is added as attribute to self.
- op (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p*'.
- need_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need_JW_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc_ops.

change_charge (new_leg_charge=None, permute=None)

Change the charges of the site (in place).

Parameters

- new_leg_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state_labels and perm. If you sorted the previous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg.perm_flat_from_perm_qind(perm_qind). Ignored if None.

property dim

Dimension of the local Hilbert space.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

get_hc_op_name (name)

Return the hermitian conjugate of a given operator.

Parameters name (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as get op() does.

Returns hc_op_name – Operator name for the hermi such that $get_op()$ of

Return type str

```
get_op (name)
```

Return operator of given name.

Parameters name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

Returns op – The operator given by *name*, with labels 'p', 'p*'. If name already was an npc Array, it's directly returned.

Return type np_conserved

multiply_op_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get_op* returns the product of the corresponding operators.

Parameters names (list of str) – List of valid operator labels.

Returns combined_opname – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

multiply_operators (operators)

Multiply local operators (possibly given by their names) together.

Parameters operators (list of {str | Array}) – List of valid operator names (to be translated with get_op()) or directly on-site operators in the form of npc arrays with 'p', 'p*' label. The operators are multiplied left-to-right.

Returns combined_operator — The product of the given *operators* in a left-to-right multiplication following the usual mathematical convention. For example, if operators=['Sz', 'Sp', 'Sx'], the final operator is equivalent to site.get_op('Sz Sp Sx'), with the 'Sx' operator acting first on any physical state.

Return type Array

property onsite_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

op_needs_JW(name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

Parameters name (str) – The name of the operator, as in get op().

Returns needs_JW — Whether the operator needs a Jordan-Wigner string, judging from need_JW_string.

Return type bool

remove_op (name)

Remove an added operator.

Parameters name (str) – The name of the operator to be removed.

rename_op (old_name, new_name)

Rename an added operator.

Parameters

• old_name (str) - The old name of the operator.

• **new_name** (str) – The new name of the operator.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

state_index(label)

Return index of a basis state from its label.

Parameters label (int | string) – eather the index directly or a label (string) set before.

Returns state_index – the index of the basis state associated with the label.

Return type int

state indices(labels)

Same as state_index(), but for multiple labels.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

valid_opname (name)

Check whether 'name' labels a valid onsite-operator.

Parameters name (stx) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

Returns valid – True if *name* is a valid argument to *get_op()*.

Return type bool

Functions

<pre>group_sites(sites[, n, labels, charges])</pre>	Given a list of sites, group each <i>n</i> sites together.
multi_sites_combine_charges(sites[,])	Adjust the charges of the given sites (in place) such that
	they can be used together.
set_common_charges(sites[, new_charges,])	Adjust the charges of the given sites in place such that
	they can be used together.

20.1. site 643

20.1.8 group sites

```
• full name: tenpy.networks.site.group_sites
```

```
• parent module: tenpy.networks.site
```

· type: function

```
tenpy.networks.site.group_sites (sites, n=2, labels=None, charges='same') Given a list of sites, group each n sites together.
```

Parameters

- **sites** (list of *Site*) The sites to be grouped together.
- n (int) We group each n consecutive sites from sites together in a GroupedSite.
- labels See GroupedSites.
- charges See GroupedSites.

Returns grouped_sites - The grouped sites. Has length (len(sites)-1)//n + 1.

Return type list of GroupedSite

20.1.9 multi_sites_combine_charges

- full name: tenpy.networks.site.multi_sites_combine_charges
- parent module: tenpy.networks.site
- · type: function

```
tenpy.networks.site.multi_sites_combine_charges (sites, same_charges=[]) Adjust the charges of the given sites (in place) such that they can be used together.
```

When we want to contract tensors corresponding to different Site instances, these sites need to share a single ChargeInfo. This function adjusts the charges of these sites such that they can be used together.

Deprecated since version 0.7.3: Deprecated in favore of the new, more powerful <code>set_common_charges()</code>. Be aware of the slightly different argument structure though, namely that this function keeps charges not included in <code>same_charges</code>, whereas you need to include them explicitly into the <code>new_charges</code> argument of <code>set_common_charges</code>.

Parameters

- **sites** (list of *Site*) The sites to be combined. Modified **in place**.
- same_charges ([[(int, int|str), (int, int|str), ...], ...]) Defines which charges actually are the same, i.e. their quantum numbers are added up. Each charge is specified by a tuple (s, i) = (int, int|str), where s gives the index of the site within sites and i the index or name of the charge in the ChargeInfo of this site.

Returns perms – For each site the permutation performed on the physical leg to sort by charges.

Return type list of ndarray

Examples

```
>>> from tenpy.networks.site import *
>>> ferm = SpinHalfFermionSite(cons N='N', cons Sz='Sz')
>>> spin = SpinSite(1.0, 'Sz')
>>> ferm.leg.chinfo is spin.leg.chinfo
False
>>> print(spin.leg)
+1
0 [[-2]
1 [ 0]
2 [2]]
3
>>> multi_sites_combine_charges([ferm, spin], same_charges=[[(0, 1), (1, 0)]])
[array([0, 1, 2, 3]), array([0, 1, 2])]
>>> # no permutations where needed
>>> ferm.leg.chinfo is spin.leg.chinfo
>>> ferm.leg.chinfo.names
['N', '2*Sz']
>>> print(spin.leg)
+1
0 [[ 0 -2]
1 [0 0]
2 [ 0 2]]
3
```

20.1.10 set_common_charges

- full name: tenpy.networks.site.set_common_charges
- parent module: tenpy.networks.site
- type: function

tenpy.networks.site.set_common_charges(sites, new_charges='same', new_names=None, new mod=None)

Adjust the charges of the given sites *in place* such that they can be used together.

(This function replaces the now deprecated mutli_sites_combine_charges().)

Before we can contract operators (and tensors) corresponding to different Site instances, we first need to define the overall conserved charges, i.e., we need to merge the ChargeInfo of them to a single, global chinfo and adjust the charges of the physical legs. That's what this function does.

A typical place to do this would be in tenpy.models.model.CouplingMPOModel.init_sites().

Parameters

- **sites** (list of Site) The sites to be combined. The sites are modified **in place**.
- new_charges ('same' | 'drop' | 'independent' | list of list of tuple) Defines the new, common charges in terms of the old ones.

list of lists of tuple If a list is given, each entry <code>new_charge</code> of the list defines one new charge, i.e. the new number of charges is <code>qnumber=len(new_charges)</code>. Each entry <code>new_charge</code> of the outer list is itself a list of 3-tuples, <code>new_charge = [(factor, site_index, old_charge_index), ...]</code>. where the value of the new charge is the sum of <code>factor</code> times the value of the old charge, (specified by the <code>site_index</code> and the <code>old_charge_index</code> within that site), and the sum runs over all entries in that list

20.1. site 645

new_charge. old_charge_index can be an integer (=the index) or a string (=the name) of the charge in the corresponding sites[site_index].leg.chinfo.

- 'same' defaults to charges with the same name to match, and charges with different names to be independently conserved (see example below); None-set names are considered different.
- 'drop' Drop/remove all charges, equivalent to new_charges=[].
- 'independent' For the case that the charges of the different sites are independent and individually conserved, even if they have the same name.
- new_names (list of str) Names for each of the new charges. Defaults to name of the first old charge specified.
- new_mod (list of int) mod for the new charges, one entry for each list in new_charges. Defaults to the mod of the old charges, if not specified otherwise.

Returns perms – For each site the permutation performed on the physical leg to sort by charges.

Return type list of ndarray

Examples

When we just initialize some sites, they will in general have different charges. For example, we could have a *SpinHalfFermionSite* a spin-1 *SpinSite*. For reference, let's also print the names and values of the charges.

```
>>> from tenpy.networks.site import *
>>> ferm = SpinHalfFermionSite(cons_N='N', cons_Sz='Sz')
>>> ferm.leg.chinfo.names
['N', '2*Sz']
>>> print(ferm.leg.to_qflat())
[[1 -1]
[ 0 0]
[20]
[ 1 1]]
>>> spin = SpinSite(1.0, conserve='Sz')
>>> spin.leg.chinfo.names
['2*Sz']
>>> print(spin.leg.to_qflat())
[[-2]
[0]
 [ 2]]
```

With the default new_charges='same', this function will combine charges with the same name, and hence we will have two conserved quantities, namley the fermion particle number 'N' = N_{up_fermions} + N_{down-fermions}, and the total Sz spin '2*Sz' = N_{up-fermions} + N_{up-spins} - N_{down-fermions} - N_{down-spins}. In this case, there will only appear an extra column of zeros for the charges of the spin leg.

```
>>> set_common_charges([ferm, spin], new_charges='same')
[array([0, 1, 2, 3]), array([0, 1, 2])]
>>> ferm.leg.chinfo.names
['N', '2*Sz']
>>> print(ferm.leg.to_qflat()) # didn't change (except making a copy)
[[ 1 -1]
  [ 0      0]
  [ 2      0]
```

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```
[ 1 1]]
>>> spin.leg.chinfo.names # additional 'N' chargename
['N', '2*Sz']
>>> print(spin.leg.to_qflat()) # additional column of zeros for the 'N' charge
[[ 0 -2]
  [ 0 0]
  [ 0 2]]
```

With new_charges='independent', we preserve the charges of the old sites individually. In this example, we get 3 conserved quantities, namely the fermion particle number 'N_ferm' = N_{up_fermions} + N_{down-fermions}, and the fermionic Sz spin '2*Sz_ferm' = N_{up-fermions} - N_{down-fermions} and the Sz spin of the spin sites, '2*Sz_spin' = N_{up-spins} - N_{down-spins}. (We give the charges new names for clearer distinction.) Corresponding zero columns are added to the LegCharges.

With the full specification of the *new_charges* through a list of list of tuples, you can create new charges as linear combinations of the charges of the individual sites. For example, the *SpinHalfFermionSite* is essentially the product of two *FermionSite*, one for the up electrons, and one for the down electrons. The '2*Sz' charge of the *SpinHalfFermionSite* is then equivalent to the difference of individual particle numbers, '2*Sz' = $N_{up} - N_{down}$.

```
>>> f_up = FermionSite(conserve='N')
>>> f_down = FermionSite(conserve='N')
>>> print(f_up.leg.to_qflat())
[[0]]
[1]]
>>> print(f_down.leg.to_qflat())
[[0]]
[1]]
>>> f_down.state_labels
{'empty': 0, 'full': 1}
>>> set_common_charges([f_up, f_down],
                       new_charges=[[(1, 0, 'N'), (1, 1, 'N')],
. . .
                                     [(1, 0, 'N'), (-1, 1, 'N')]],
. . .
                        new_names=['N_tot', '2*Sz=(N_up-N_down)'])
. . .
[array([0, 1]), array([1, 0])]
>>> f_down.state_labels # sorting charges caused permutation of local states
{'empty': 1, 'full': 0}
>>> print(f_up.leg.to_qflat())
[[0 0]]
[1 1]]
```

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20.1. site 647

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```
>>> print(f_down.leg.to_qflat()) # top row = full, bottom row=empty
[[ 1 -1]
  [ 0  0]]
```

Another example could be that you have both fermions and bosons, and that you have terms $c_i c_j b_k^{\dagger} + c_i^{\dagger} c_j^{\dagger} b_k$, where two fermions can merge into a pair forming a boson. In this case, neither the fermion number nor the boson number is preserved individually, but the combination N_{fermions} + 2 * N_{bosons} is preserved.

```
>>> ferm = FermionSite(conserve='N')
>>> bos = BosonSite(Nmax=3, conserve='N')
>>> set_common_charges([ferm, bos], [[(1, 0, 'N'), (2, 1, 'N')]], ['N_f + 2 N_b'])
[array([0, 1]), array([0, 1, 2, 3])]
```

Finally, it can sometimes be convenient to change the charges of the The new_charges='drop' or new_charges=[] option is a quick way to remove any charges.

Module description

Defines a class describing the local physical Hilbert space.

The Site is the prototype, read it's docstring.

20.2 mps

• full name: tenpy.networks.mps

• parent module: tenpy.networks

• type: module

Classes



InitialStateBuilder(lattice, options[,])	Class to simplify providing common sets of intial states.
	A Matrix Product State, finite (MPS) or infinite (iMPS).
MPS(sites, Bs, SVs[, bc, form, norm])	
<pre>MPSEnvironment(bra, ket[, init_LP, init_RP,])</pre>	Stores partial contractions of $< bra Op ket >$ for local
	operators <i>Op</i> .
TransferMatrix(bra, ket[, shift_bra,])	Transfer matrix of two MPS (bra & ket).

20.2.1 MPSEnvironment

• full name: tenpy.networks.mps.MPSEnvironment

• parent module: tenpy.networks.mps

• type: class

Inheritance Diagram

MPSEnvironment

Methods

<pre>MPSEnvironmentinit(bra, ket[, init_LP,</pre>	Initialize self.
])	
MPSEnvironment.del_LP(i)	Delete stored part strictly to the left of site <i>i</i> .
MPSEnvironment.del_RP(i)	Delete storde part scrictly to the right of site <i>i</i> .
MPSEnvironment.expectation_value(ops[,	Expectation value <bra ket="" ops="" =""> of (n-site) opera-</bra>
])	tor(s).

continues on next page

20.2. mps 649

Table	19 - continued	from	previous	page
-------	----------------	------	----------	------

$ extit{MPSEnvironment.full_contraction}(i0)$	Calculate the overlap by a full contraction of the net-		
	work.		
MPSEnvironment.get_LP(i[, store])	Calculate LP at given site from nearest available one.		
MPSEnvironment.get_LP_age(i)	Return number of physical sites in the contractions of		
	get_LP(i).		
MPSEnvironment.get_RP(i[, store])	Calculate RP at given site from nearest available one.		
MPSEnvironment.get_RP_age(i)	Return number of physical sites in the contractions of		
	get_RP(i).		
MPSEnvironment.get_initialization_data	a()Return data for (re-)initialization.		
MPSEnvironment.init_LP(i)	Build initial left part LP.		
MPSEnvironment.init_RP(i)	Build initial right part RP for an		
	MPS/MPOEnvironment.		
MPSEnvironment.set_LP(i, LP, age)	Store part to the left of site <i>i</i> .		
MPSEnvironment.set_RP(i, RP, age)	Store part to the right of site <i>i</i> .		
MPSEnvironment.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.		

Bases: object

Stores partial contractions of < bra|Op|ket > for local operators Op.

The network for a contraction < bra|Op|ket > of a local operator Op, say exemplary at sites i, i+1 looks like:

Of course, we can also calculate the overlap $\langle bralket \rangle$ by using the special case Op = Id.

We use the following label convention (where arrows indicate *qconj*):

To avoid recalculations of the whole network e.g. in the DMRG sweeps, we store the contractions up to some site index in this class. For bc='finite', 'segment', the very left and right part LP[0] and RP[-1] are trivial and don't change, but for bc='infinite' they are might be updated (by inserting another unit cell to the left/right).

The MPS bra and ket have to be in canonical form. All the environments are constructed without the singular values on the open bond. In other words, we contract left-canonical A to the left parts LP and right-canonical B to the right parts RP. Thus, the special case ket=bra should yield identity matrices for LP and RP.

Parameters

• **bra** (MPS) – The MPS to project on. Should be given in usual 'ket' form; we call *conj()* on the matrices directly. Stored in place, without making copies. If necessary to match charges, we call <code>gauge_total_charge()</code>.

- **ket** (MPO | None) The MPS on which the local operator acts. Stored in place, without making copies. If None, use *bra*.
- init_LP (None | Array) Initial very left part LP. If None, build trivial one with init_LP().
- init_RP (None | Array) Initial very right part RP. If None, build trivial one with init RP().
- age_LP (int) The number of physical sites involved into the contraction yielding firstLP.
- age_RP (int) The number of physical sites involved into the contraction yielding lastRP.

L

Number of physical sites involved into the Environment, i.e. the least common multiple of bra.L and ket.L.

```
Type int
```

bra, ket

The two MPS for the contraction.

```
Type MPS
```

dtype

The data type.

Type type

finite

Whether the boundary conditions of the MPS are finite.

```
Type bool
```

_LP

Left parts of the environment, len L. LP [i] contains the contraction strictly left of site i (or None, if we don't have it calculated).

```
Type list of {None | Array}
```

_RP

Right parts of the environment, len L. RP [i] contains the contraction strictly right of site i (or None, if we don't have it calculated).

```
Type list of {None | Array}
```

_LP_age

Used for book-keeping, how large the DMRG system grew: _LP_age[i] stores the number of physical sites invovled into the contraction network which yields self._LP[i].

```
Type list of int | None
```

_RP_age

Used for book-keeping, how large the DMRG system grew: _RP_age[i] stores the number of physical sites invovled into the contraction network which yields self._RP[i].

Type list of int | None

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

$init_{LP}(i)$

Build initial left part LP.

20.2. mps 651

Parameters i(int) – Build LP left of site i.

Returns init_LP – Identity contractible with the νL leg of ket.get_B(i), labels 'vR*', 'vR'.

Return type Array

init RP(i)

Build initial right part RP for an MPS/MPOEnvironment.

Parameters i (int) – Build RP right of site i.

Returns init_RP – Identity contractible with the νR leg of ket.get_B(i), labels 'vL*', 'vL'.

Return type Array

get_LP (i, store=True)

Calculate LP at given site from nearest available one.

The returned LP_i corresponds to the following contraction, where the M's and the N's are in the 'A' form:

Parameters

- i(int) The returned *LP* will contain the contraction *strictly* left of site *i*.
- **store** (bool) Wheter to store the calculated *LP* in *self* (True) or discard them (False).

Returns LP_i – Contraction of everything left of site i, with labels 'vR*', 'vR' for bra, ket.

Return type Array

get_RP (i, store=True)

Calculate RP at given site from nearest available one.

The returned RP_i corresponds to the following contraction, where the M's and the N's are in the 'B' form:

Parameters

- i (int) The returned RP will contain the contraction strictly right of site i.
- **store** (bool) Wheter to store the calculated RP in self (True) or discard them (False).

Returns RP_i – Contraction of everything left of site i, with labels 'vL*', 'vL' for bra, ket.

Return type Array

$get_LP_age(i)$

Return number of physical sites in the contractions of get_LP(i).

Might be None.

$get_RP_age(i)$

Return number of physical sites in the contractions of get_RP(i).

Might be None.

$set_{LP}(i, LP, age)$

Store part to the left of site i.

$set_RP(i, RP, age)$

Store part to the right of site *i*.

$del_LP(i)$

Delete stored part strictly to the left of site i.

$del_RP(i)$

Delete storde part scrictly to the right of site *i*.

get_initialization_data()

Return data for (re-)initialization.

The returned parameters are collected in a dictionary with the following names.

Returns

- init_LP, init_RP (Array) LP on the left of site 0 and RP on the right of site L-1, which can be used as $init_LP$ and $init_RP$ for the initialization of a new environment.
- age_LP, age_RP (int) The number of physical sites involved into the contraction yielding init_LP and init_RP, respectively.

$full_contraction(i\theta)$

Calculate the overlap by a full contraction of the network.

The full contraction of the environments gives the overlap
bra|ket>, taking into account MPS.norm of both *bra* and *ket*. For this purpose, this function contracts get_LP(i0+1, store=False) and get_RP(i0, store=False) with appropriate singular values in between.

Parameters iO (int) – Site index.

expectation_value (ops, sites=None, axes=None)

Expectation value

bra | ops | ket > of (n-site) operator(s).

Calculates n-site expectation values of operators sandwiched between bra and ket. For examples the contraction for a two-site operator on site *i* would look like:

Here, the B are taken from ket, the B^* from bra. The call structure is the same as for MPS. expectation_value().

20.2. mps 653

Warning: In contrast to MPS.expectation_value(), this function does not normalize, thus it also takes into account MPS.norm of both *bra* and *ket*.

Parameters

- **ops** ((list of) { *Array* | str }) The operators, for wich the expectation value should be taken, All operators should all have the same number of legs (namely 2 n). If less than len(sites) operators are given, we repeat them periodically. Strings (like 'Id', 'Sz') are translated into single-site operators defined by sites.
- **sites** (list) List of site indices. Expectation values are evaluated there. If None (default), the entire chain is taken (clipping for finite b.c.)
- **axes** (None | (list of str, list of str)) Two lists of each n leg labels giving the physical legs of the operator used for contraction. The first n legs are contracted with conjugated B, the second n legs with the non-conjugated B. None defaults to (['p'], ['p*']) for single site (n=1), or (['p0', 'p1', ... 'p{n-1}'], ['p0*', 'p1*', ... 'p{n-1}*']) for n>1.

Returns exp_vals - Expectation values, exp_vals[i] =
bra|ops[i]|ket>, where ops[i] acts on site(s) j, j+1, ..., j+{n-1} with j=sites[i].

Return type 1D ndarray

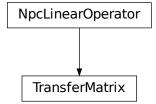
20.2.2 TransferMatrix

• full name: tenpy.networks.mps.TransferMatrix

• parent module: tenpy.networks.mps

· type: class

Inheritance Diagram



Methods

TransferMatrixinit(bra, ket[,])	Initialize self.
TransferMatrix.adjoint()	Return the hermitian conjugate of self
TransferMatrix.eigenvectors(*args,	Find (dominant) eigenvector(s) of self using scipy.
**kwargs)	sparse.
TransferMatrix.initial_guess([diag])	Return a diagonal matrix as initial guess for the eigen-
	vector.
TransferMatrix.matvec(vec)	Given <i>vec</i> as an npc.Array, apply the transfer matrix.
TransferMatrix.to_matrix()	Contract <i>self</i> to a matrix.

Class Attributes and Properties

```
TransferMatrix.acts_on
```

Bases: tenpy.linalg.sparse.NpcLinearOperator

Transfer matrix of two MPS (bra & ket).

For an iMPS in the thermodynamic limit, we often need to find the 'dominant RP' (and LP). This mean nothing else than to take the transfer matrix of the unit cell and find the (right/left) eigenvector with the largest (magnitude) eigenvalue, since it will dominate $(TM)^nRP$ (or $LP(TM)^n$) in the limit $n \to \infty$ - whatever the initial RP is. This class provides exactly that functionality with eigenvectors().

Given two MPS, we define the transfer matrix as:

Here the M denotes the matrices of the bra and N the ones of the ket, respectively. To view it as a matrix, we combine the left and right indices to pipes:

```
 | \quad (vL.vL*) \quad ->-TM->- \quad (vR.vR*) \quad \text{ acting on } \quad (vL.vL*) \quad ->-RP
```

Note that we keep all M and N as copies.

Deprecated since version 0.6.0: The default for *shift_ket* was the value of *shift_bra*, this will be changed to 0.

Parameters

- **bra** (MPS) The MPS which is to be (complex) conjugated.
- **ket** (MPS) The MPS which is not (complex) conjugated.
- **shift_bra** (*int*) We start the *N* of the bra at site *shift_bra* (i.e. the *j* in the above network).
- **shift_ket** (*int* / *None*) We start the *M* of the ket at site *shift_ket* (i.e. the *i* in the above network). None is deprecated, default will be changed to 0 in the future.
- **transpose** (bool) Wheter *self.matvec* acts on *RP* (False) or *LP* (True).

20.2. mps 655

- **charge_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., **assumes** the dominant eigenvector is in charge sector 0.
- **form** ('B' | 'A' | 'C' | 'G' | 'Th' | None | tuple(float, float)) In which canonical form we take the *M* and *N* matrices.

L

Number of physical sites involved in the transfer matrix, i.e. the least common multiple of bra.L and ket.L.

```
Type int
```

shift bra

We start the *N* of the bra at site *shift_bra*.

Type int

shift_ket

We start the *M* of the ket at site *shift_ket*. None defaults to *shift_bra*.

Type int | None

transpose

Wheter self.matvec acts on RP (True) or LP (False).

Type bool

qtotal

Total charge of the transfer matrix (which is gauged away in matvec).

Type charges

form

In which canonical form (all of) the M and N matrices are.

Type tuple(float, float) | None

flat_linop

Class lifting matvec() to ndarrays in order to use speigs().

Type FlatLinearOperator

pipe

Pipe corresponding to '(vL.vL*)' for transpose=False or to '(vR.vR*)' for transpose=True.

Type LegPipe

label_split

```
['vL', 'vL*'] if tranpose=False or ['vR', 'vR*'] if transpose=True.
```

_bra_N

Complex conjugated matrices of the bra, transposed for fast *matvec*.

Type list of npc.Array

_ket_M

The matrices of the ket, transposed for fast *matvec*.

Type list of npc.Array

matvec(vec)

Given *vec* as an npc.Array, apply the transfer matrix.

Parameters vec (Array) – Vector to act on with the transfermatrix. If not *transposed*, *vec* is the right part RP of an environment, with legs ' (vL.vL*)' in a pipe or splitted. If *transposed*, the left part LP of an environment with legs ' (vR*.vR)'.

Returns mat_vec – The tranfer matrix acted on *vec*, in the same form as given.

```
Return type Array
```

```
initial_guess(diag=1.0)
```

Return a diagonal matrix as initial guess for the eigenvector.

Parameters diag (float | 1D ndarray) - Should be 1. for the identity or some singular values squared.

Returns mat – A 2D array with *diag* on the diagonal such that *matvec()* can act on it.

```
Return type Array
```

```
eigenvectors (*args, **kwargs)
```

Find (dominant) eigenvector(s) of self using scipy.sparse.

For arguments see eigenvectors ().

If no charge_sector was selected, we look in *all* charge sectors. The returned eigenvectors have combined legs '(vL.vL*)' or (vR*.vR).

adjoint()

Return the hermitian conjugate of self

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

to matrix()

Contract self to a matrix.

If *self* represents an operator with very small shape, e.g. because the MPS bond dimension is very small, an algorithm might choose to contract *self* to a single tensor.

Returns matrix – Contraction of the represented operator.

```
Return type Array
```

Functions

 $build_initial_state(size, states, filling[, \dots]) \qquad \textbf{Build an "initial state" list.}$

20.2.3 build initial state

- full name: tenpy.networks.mps.build_initial_state
- parent module: tenpy.networks.mps
- type: function

tenpy.networks.mps.build_initial_state (size, states, filling, mode='random', seed=None)
Build an "initial state" list.

Uses two iterables ('states' and 'filling') to determine how to fill the state. The two lists should have the same length as every element in 'filling' gives the filling fraction for the corresponding state in 'states'.

Example

20.2. mps 657

size = 6, states = [0, 1, 2], filling = [1./3, 2./3, 0.] n_states = size * filling = [2, 4, 0] ==> Two sites will get state 0, 4 sites will get state 1, 0 sites will get state 2.

Todo: Make more general: it should be possible to specify states as strings.

Parameters

- **size** (*int*) length of state
- states (iterable) Containing the possible local states
- **filling** (*iterable*) Fraction of the total number of sites to get a certain state. If infinite fractions (e.g. 1/3) are needed, one should supply a fraction (1./3.)
- mode (str / None) State filling pattern. Only 'random' is implemented
- **seed** (*int* / *None*) Seed for random number generators

Returns initial_state (list)

Return type the initial state

Raises

- ValueError If fractonal fillings are incommensurate with system size.
- **AssertionError** If the total filling is not equal to 1, or the length of *filling* does not equal the length of *states*.

Module description

This module contains a base class for a Matrix Product State (MPS).

An MPS looks roughly like this:

```
| -- B[0] -- B[1] -- B[2] -- ...
```

We use the following label convention for the B (where arrows indicate qconj):

We store one 3-leg tensor $_B[i]$ with labels 'vL', 'vR', 'p' for each of the L sites $0 \le i \le L$. Additionally, we store L+1 singular value arrays $_S[ib]$ on each bond $0 \le ib \le L$, independent of the boundary conditions. $_S[ib]$ gives the singlur values on the bond i-1, i. However, be aware that e.g. chi returns only the dimensions of the nontrivial_bonds depending on the boundary conditions.

The matrices and singular values always represent a normalized state (i.e. np.linalg.norm(psi._S[ib]) == 1 up to roundoff errors), but (for finite MPS) we keep track of the norm in norm (which is respected by overlap(), ...).

Valid MPS boundary conditions (not to confuse with *bc_coupling* of tenpy.models.model.CouplingModel) are the following:

bc	description
'fi-	Finite MPS, G0 s1 G1 s{L-1} G{1-1}. This is acchieved by using a trivial left and right bond
nite'	s[0] = s[-1] = np.array([1.]).
'seg-	Generalization of 'finite', describes an MPS embedded in left and right environments. The left environment
ment	is described by chi[0] orthonormal states which are weighted by the singular values s[0]. Similar,
	s[L] weight some right orthonormal states. You can think of the left and right states to be generated by
	additional MPS, such that the overall structure is something like s L s L [s0 G0 s1 G1
	$s\{L-1\}$ $G\{L-1\}$ $s\{L\}$] R s R s R (where we save the part in the brackets []).
'in-	infinite MPS (iMPS): we save a 'MPS unit cell' [s0 G0 s1 G1 s{L-1} G{L-1}] which is
fi-	repeated periodically, identifying all indices modulo self.L. In particular, the last bond L is identified
nite'	with 0. (The MPS unit cell can differ from a lattice unit cell). bond is identified with the first one.

An MPS can be in different 'canonical forms' (see [[schollwoeck2011], [vidal2004]]). To take care of the different canonical forms, algorithms should use functions like $get_theta()$, $get_B()$ and $set_B()$ instead of accessing them directly, as they return the B in the desired form (which can be chosen as an argument). The values of the tuples for the form correspond to the exponent of the singular values on the left and right. To keep track of a "mixed" canonical form A A A B B, we save the tuples for each site of the MPS in MPS.form.

form	tuple	description	
'B'	(0, 1)	right canonical: _B[i] = Gamma[i] s[i+1] The default form, which algorithms	
		asssume.	
'C'	(0.5,	symmetric form: _B[i] = s[i] **0.5 Gamma[i] s[i+1] **0.5	
	0.5)		
'A'	(1, 0)	left canonical: _B[i] = s[i] Gamma[i]	
'G'	(0, 0)	Save only _B[i] = Gamma[i]	
'Th'	(1, 1)	Form of a local wave function <i>theta</i> with singular value on both sides. psi.get_B(i, 'Th')	
		is equivalent to ``psi.get_theta(i, n=1).	
None	None	General non-canoncial form. Valid form for initialization, but you need to call	
		canonical_form() (or similar) before using algorithms.	

20.3 mpo

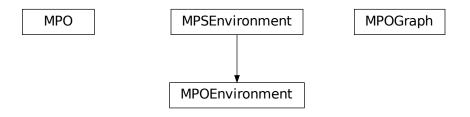
• full name: tenpy.networks.mpo

• parent module: tenpy.networks

• type: module

20.3. mpo 659

Classes



MPO(sites, Ws[, bc, IdL, IdR, max_range,])	Matrix product operator, finite (MPO) or infinite (iMPO).
MPOEnvironment(bra, H, ket[, init_LP,])	Stores partial contractions of $< bra H ket >$ for an MPO H .
<pre>MPOGraph(sites[, bc, max_range])</pre>	Representation of an MPO by a graph, based on a 'finite state machine'.

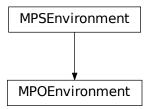
20.3.1 MPOEnvironment

• full name: tenpy.networks.mpo.MPOEnvironment

• parent module: tenpy.networks.mpo

• type: class

Inheritance Diagram



Methods

MPOEnvironmentinit(bra, H, ket[,])	Initialize self.
MPOEnvironment.del_LP(i)	Delete stored part strictly to the left of site <i>i</i> .
MPOEnvironment.del_RP(i)	Delete storde part scrictly to the right of site <i>i</i> .
MPOEnvironment.expectation_value(ops[,	(doesn't make sense)
])	
MPOEnvironment.full_contraction($f i0$)	Calculate the energy by a full contraction of the net-
	work.
$ extit{MPOEnvironment.get_LP(i[, store])}$	Calculate LP at given site from nearest available one (in-
	cluding <i>i</i>).
MPOEnvironment. $get_LP_age(i)$	Return number of physical sites in the contractions of
	get_LP(i).
$ extit{MPOEnvironment.get_RP(i[, store])}$	Calculate RP at given site from nearest available one
	(including <i>i</i>).
MPOEnvironment. $get_RP_age(i)$	Return number of physical sites in the contractions of
	get_RP(i).
MPOEnvironment.get_initialization_data	
MPOEnvironment.init_LP(i)	Build initial left part LP.
MPOEnvironment.init_RP (i)	Build initial right part RP for an
	MPS/MPOEnvironment.
MPOEnvironment.set_LP(i, LP, age)	Store part to the left of site <i>i</i> .
MPOEnvironment.set_RP(i, RP, age)	Store part to the right of site <i>i</i> .
MPOEnvironment.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

class tenpy.networks.mpo.MPOEnvironment(bra, H, ket, $init_LP=None$, $init_RP=None$, $age_LP=0$, $age_RP=0$)

Bases: tenpy.networks.mps.MPSEnvironment

Stores partial contractions of < bra|H|ket > for an MPO H.

The network for a contraction < bra|H|ket > of an MPO H bewteen two MPS looks like:

We use the following label convention (where arrows indicate *qconj*):

To avoid recalculations of the whole network e.g. in the DMRG sweeps, we store the contractions up to some site index in this class. For bc='finite', 'segment', the very left and right part LP[0] and RP[-1] are trivial and don't change in the DMRG algorithm, but for iDMRG (bc='infinite') they are also updated (by inserting another unit cell to the left/right).

20.3. mpo 661

The MPS *bra* and *ket* have to be in canonical form. All the environments are constructed without the singular values on the open bond. In other words, we contract left-canonical *A* to the left parts *LP* and right-canonical *B* to the right parts *RP*.

Parameters

- **bra** (MPS) The MPS to project on. Should be given in usual 'ket' form; we call *conj()* on the matrices directly.
- **H** (MPO) The MPO sandwiched between *bra* and *ket*. Should have 'IdL' and 'IdR' set on the first and last bond.
- **ket** (MPS) The MPS on which H acts. May be identical with bra.
- init_LP (None | Array) Initial very left part LP. If None, build trivial one with :meth`init_LP`.
- init_RP (None | Array) Initial very right part RP. If None, build trivial one with init_RP().
- age_LP (int) The number of physical sites involved into the contraction yielding firstLP.
- age_RP (int) The number of physical sites involved into the contraction yielding lastRP.

Н

The MPO sandwiched between bra and ket.

```
Type MPO
```

test sanity()

Sanity check, raises ValueErrors, if something is wrong.

$init_{LP}(i)$

Build initial left part LP.

Parameters i(int) – Build LP left of site i.

Returns init_LP – Identity contractible with the νL leg of .ket.get_B(i), multiplied with a unit vector nonzero in H.IdL[i], with labels 'vR*', 'wR', 'vR'.

```
Return type Array
```

$init_RP(i)$

Build initial right part RP for an MPS/MPOEnvironment.

```
Parameters i (int) – Build RP right of site i.
```

Returns init_RP – Identity contractible with the νR leg of self.get_B(i), multiplied with a unit vector nonzero in H.IdR[i], with labels 'vL*', 'wL', 'vL'.

```
Return type Array
```

```
get_LP (i, store=True)
```

Calculate LP at given site from nearest available one (including *i*).

The returned LP_i corresponds to the following contraction, where the M's and the N's are in the 'A' form:

Parameters

- i (int) The returned LP will contain the contraction strictly left of site i.
- **store** (bool) Wheter to store the calculated *LP* in *self* (True) or discard them (False).

Returns LP_i – Contraction of everything left of site i, with labels 'vR*', 'wR', 'vR' for bra, H, ket.

Return type Array

get_RP (i, store=True)

Calculate RP at given site from nearest available one (including *i*).

The returned RP_i corresponds to the following contraction, where the M's and the N's are in the 'B' form:

Parameters

- i (int) The returned RP will contain the contraction strictly rigth of site i.
- **store** (bool) Wheter to store the calculated RP in self (True) or discard them (False).

Returns RP_i – Contraction of everything right of site i, with labels 'vL*', 'wL', 'vL' for bra, H, ket.

Return type Array

full_contraction(i0)

Calculate the energy by a full contraction of the network.

The full contraction of the environments gives the value $\mbox{\ensuremath{\mbox{chra}\,|\,H\,|\,ket}>/}\mbox{\ensuremath{\mbox{(norm\,(|bra>)*norm\,(|ket>)), i.e.}}} i.e. if bra is ket and normalized, the total energy. For this purpose, this function contracts get_LP (i0+1, store=False) and get_RP (i0, store=False).}$

Parameters iO (int) – Site index.

expectation_value (ops, sites=None, axes=None)

(doesn't make sense)

$\mathtt{del}_\mathtt{LP}\left(i\right)$

Delete stored part strictly to the left of site *i*.

del RP(i)

Delete storde part scrictly to the right of site i.

$get_LP_age(i)$

Return number of physical sites in the contractions of get_LP(i).

Might be None.

$get_RP_age(i)$

Return number of physical sites in the contractions of get_RP(i).

Might be None.

20.3. mpo 663

get_initialization_data()

Return data for (re-)initialization.

The returned parameters are collected in a dictionary with the following names.

Returns

- init_LP, init_RP (Array) LP on the left of site 0 and RP on the right of site L-1, which can be used as $init_LP$ and $init_LP$ for the initialization of a new environment.
- age_LP, age_RP (*int*) The number of physical sites involved into the contraction yielding *init_LP* and *init_RP*, respectively.

```
set\_LP(i, LP, age)
```

Store part to the left of site i.

 $set_RP(i, RP, age)$

Store part to the right of site i.

20.3.2 MPOGraph

• full name: tenpy.networks.mpo.MPOGraph

• parent module: tenpy.networks.mpo

· type: class

Inheritance Diagram

MPOGraph

Methods

<pre>MPOGraphinit(sites[, bc, max_range])</pre>	Initialize self.
MPOGraph.add(i, keyL, keyR, opname, strength)	Insert an edge into the graph.
MPOGraph.add_missing_IdL_IdR([insert_all_id])	Add missing identity ('Id') edges connecting
	'IdL' -> 'IdL' and $``'IdR' -> 'IdR'$.
MPOGraph.add_string(i, j, key[, opname,])	Insert a bunch of edges for an 'operator string' into the
	graph.
MPOGraph.build_MPO([Ws_qtotal])	Build the MPO represented by the graph (<i>self</i>).
<pre>MPOGraph.from_term_list(term_list, sites, bc)</pre>	Initialize from a list of operator terms and prefactors.
MPOGraph.from_terms(terms, sites, bc[,])	Initialize an MPOGraph from OnsiteTerms and Cou-
	plingTerms.
MPOGraph.has_edge(i, keyL, keyR)	True if there is an edge from keyL on bond (i-1, i) to
	keyR on bond $(i, i+1)$.
MPOGraph.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

Class Attributes and Properties

MPOGraph.L	Number of physical sites; for infinite boundaries the
	length of the unit cell.

class tenpy.networks.mpo.MPOGraph (sites, bc='finite', max_range=None)
 Bases: object

Representation of an MPO by a graph, based on a 'finite state machine'.

This representation is used for building H_MPO from the interactions. The idea is to view the MPO as a kind of 'finite state machine'. The **states** or **keys** of this finite state machine life on the MPO bonds *between* the *Ws*. They label the indices of the virtul bonds of the MPOs, i.e., the indices on legs wL and wR. They can be anything hash-able like a str, int or a tuple of them.

The **edges** of the graph are the entries W[keyL], keyR], which itself are onsite operators on the local Hilbert space. The indices keyL and keyR correspond to the legs 'wL', 'wR' of the MPO. The entry W[keyL], keyR] connects the state keyL on bond (i-1, i) with the state keyR on bond (i, i+1).

The keys 'IdR' (for 'idenity left') and 'IdR' (for 'identity right') are reserved to represent only 'Id' (=identity) operators to the left and right of the bond, respectively.

Todo: might be useful to add a "cleanup" function which removes operators cancelling each other and/or unused states. Or better use a 'compress' of the MPO?

Parameters

- sites (list of Site) Local sites of the Hilbert space.
- **bc**({'finite', 'infinite'}) MPO boundary conditions.
- max_range (int | np.inf | None) Maximum range of hopping/interactions (in unit of sites) of the MPO. None for unknown.

sites

Defines the local Hilbert space for each site.

Type list of Site

chinfo

The nature of the charge.

Type ChargeInfo

bc

MPO boundary conditions.

Type {'finite', 'infinite'}

max_range

Maximum range of hopping/interactions (in unit of sites) of the MPO. None for unknown.

Type int | np.inf | None

states

states[i] gives the possible keys at the virtual bond (i-1, i) of the MPO. L+1 enries.

Type list of set of keys

20.3. mpo 665

graph

For each site i a dictionary {keyL: {keyR: [(opname, strength)]}} with keyL in states[i] and keyR in states[i+1].

Type list of dict of list of tuples

_grid_legs

The charges for the MPO

Type None | list of LegCharge

classmethod from_terms (terms, sites, bc, insert_all_id=True)

Initialize an MPOGraph from OnsiteTerms and CouplingTerms.

Parameters

- terms (iterable of tenpy.networks.terms.*Terms classes) Entries can be OnsiteTerms, CouplingTerms, MultiCouplingTerms or ExponentialCouplingTerms. All the entries get added to the new MPOGraph.
- **sites** (list of *Site*) Local sites of the Hilbert space.
- **bc**('finite' | 'infinite') MPO boundary conditions.
- insert_all_id(bool) Whether to insert identities such that *IdL* and *IdR* are defined on each bond. See add_missing_IdL_IdR().

Returns graph – Initialized with the given terms.

Return type MPOGraph

See also:

from_term_list equivalent for representation by TermList.

classmethod from_term_list (term_list, sites, bc, insert_all_id=True)

Initialize from a list of operator terms and prefactors.

Parameters

- term_list (TermList) Terms to be added to the MPOGraph.
- **sites** (list of *Site*) Local sites of the Hilbert space.
- **bc**('finite' | 'infinite') MPO boundary conditions.
- insert_all_id (bool) Whether to insert identities such that *IdL* and *IdR* are defined on each bond. See add_missing_IdL_IdR().

Returns graph – Initialized with the given terms.

Return type MPOGraph

See also:

from_terms equivalent for other representation of terms.

test_sanity()

Sanity check, raises ValueErrors, if something is wrong.

property L

Number of physical sites; for infinite boundaries the length of the unit cell.

add (i, keyL, keyR, opname, strength, check_op=True, skip_existing=False)
Insert an edge into the graph.

Parameters

- i (int) Site index at which the edge of the graph is to be inserted.
- **keyL** (hashable) The state at bond (i-1, i) to connect from.
- **keyR** (hashable) The state at bond (i, i+1) to connect to.
- opname (str) Name of the operator.
- **strength** (*str*) Prefactor of the operator to be inserted.
- **check_op** (bool) Whether to check that 'opname' exists on the given *site*.
- **skip_existing** (bool) If True, skip adding the graph node if it exists (with same keys and *opname*).

add_string (*i*, *j*, *key*, *opname='Id'*, *check_op=True*, *skip_existing=True*)

Insert a bunch of edges for an 'operator string' into the graph.

Terms like $S_i^z S_j^z$ actually stand for $S_i^z \otimes \prod_{i \le k \le j} \mathbb{1}_k \otimes S_j^z$. This function adds the $\mathbb{1}_k$ terms to the graph.

Parameters

- \mathbf{i} (int) An edge is inserted on all bonds between i and j, i < j. j can be larger than L, in which case the operators are supposed to act on different MPS unit cells.
- \mathbf{j} (*int*) An edge is inserted on all bonds between i and j, i < j. j can be larger than L, in which case the operators are supposed to act on different MPS unit cells.
- **key** (hashable) The state at bond (i-1, i) to connect from and on bond (j-1, j) to connect to. Also used for the intermediate states. No operator is inserted on a site i < k < j if has_edge (k, key, key).
- opname (str) Name of the operator to be used for the string. Useful for the Jordan-Wigner transformation to fermions.
- **skip_existing** (bool) Whether existing graph nodes should be skipped.

Returns label_j – The *key* on the left of site j to connect to. Usually the same as the parameter *key*, except if j - i > self.L, in which case we use the additional labels (key, 1), (key, 2),... to generate couplings over multiple unit cells.

Return type hashable

```
add_missing_IdL_IdR(insert_all_id=True)
```

Add missing identity ('Id') edges connecting 'IdL'->'IdL' and ``'IdR'->'IdR'.

This function should be called *after* all other operators have been inserted.

Parameters insert_all_id (bool) - If True, insert 'Id' edges on *all* bonds. If False and boundary conditions are finite, only insert 'IdL'->'IdL' to the left of the rightmost existing 'IdL' and 'IdR'->'IdR' to the right of the leftmost existing 'IdR'. The latter avoid "dead ends" in the MPO, but some functions (like *make_WI*) expect 'IdL'/'IdR' to exist on all bonds.

$has_edge(i, keyL, keyR)$

True if there is an edge from keyL on bond (i-1, i) to keyR on bond (i, i+1).

build_MPO(Ws_qtotal=None)

Build the MPO represented by the graph (self).

Parameters Ws_qtotal (None | (list of) charges) - The qtotal for each of the Ws to be generated, default (None) means 0 charge. A single qtotal holds for each site.

Returns mpo – the MPO which self represents.

20.3. mpo 667

Return type MPO

Functions

grid_insert_ops(site, grid)	Replaces entries representing operators in a grid of
	W[i] with npc.Arrays.
$make_W_{II}(t, A, B, C, D)$	W_II approx to exp(t H) from MPO parts (A, B, C, D).

20.3.3 grid_insert_ops

- full name: tenpy.networks.mpo.grid_insert_ops
- parent module: tenpy.networks.mpo
- type: function

```
tenpy.networks.mpo.grid_insert_ops (site, grid)
```

Replaces entries representing operators in a grid of W[i] with npc.Arrays.

Parameters

- **site** (*site*) The site on which the grid acts.
- grid (list of list of entries) Represents a single matrix W of an MPO, i.e. the lists correspond to the legs 'vL', 'vR', and entries to onsite operators acting on the given site. entries may be None, Array, a single string or of the form [('opname', strength), ...], where 'opname' labels an operator in the site.

Returns grid - Copy of grid with entries [('opname', strength), ...] replaced by sum([strength*site.get_op('opname') for opname, strength in entry]) and entries 'opname' replaced by site.get_op('opname').

Return type list of list of {None | Array}

20.3.4 make W II

- full name: tenpy.networks.mpo.make_W_II
- parent module: tenpy.networks.mpo
- type: function

```
tenpy.networks.mpo.make_W_{II}(t, A, B, C, D)
```

W_II approx to exp(t H) from MPO parts (A, B, C, D).

Get the W_II approximation of [[zaletel2015]].

In the paper, we have two formal parameter "phi_{r/c}" which satisfies $\phi_r^2 = phi_c^2 = 0$. To implement this, we temporarily extend the virtual Hilbert space with two hard-core bosons "br, bl". The components of Eqn (11) can be computed for each index of the virtual row/column independently The matrix exponential is done in the hard-core extended Hilbert space

Parameters

• **t** (float) – The time step per application of the propagator. Should be imaginary for real time evolution!

- A (numpy.ndarray) Blocks of the MPO tensor to be exponentiated, as defined in [[zaletel2015]]. Legs 'wL', 'wR', 'p', 'p*'; legs projected to a single IdL/IdR can be dropped.
- B (numpy.ndarray) Blocks of the MPO tensor to be exponentiated, as defined in [[zaletel2015]]. Legs 'wL', 'wR', 'p', 'p*'; legs projected to a single IdL/IdR can be dropped.
- C (numpy.ndarray) Blocks of the MPO tensor to be exponentiated, as defined in [[zaletel2015]]. Legs 'wL', 'wR', 'p', 'p*'; legs projected to a single IdL/IdR can be dropped.
- D (numpy.ndarray) Blocks of the MPO tensor to be exponentiated, as defined in [[zaletel2015]]. Legs 'wL', 'wR', 'p', 'p*'; legs projected to a single IdL/IdR can be dropped.

Module description

Matrix product operator (MPO).

An MPO is the generalization of an MPS to operators. Graphically:

So each 'matrix' has two physical legs p, p* instead of just one, i.e. the entries of the 'matrices' are local operators. Valid boundary conditions of an MPO are the same as for an MPS (i.e. 'finite' | 'segment' | 'infinite'). (In general, you can view the MPO as an MPS with larger physical space and bring it into canoncial form. However, unlike for an MPS, this doesn't simplify calculations. Thus, an MPO has no *form*.)

We use the following label convention for the W (where arrows indicate qconj):

If an MPO describes a sum of local terms (e.g. most Hamiltonians), some bond indices correspond to 'only identities to the left/right'. We store these indices in *IdL* and *IdR* (if there are such indices).

Similar as for the MPS, a bond index i is *left* of site i, i.e. between sites i-1 and i.

20.3. mpo 669

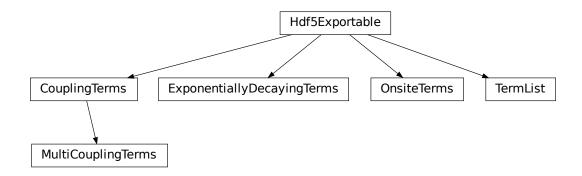
20.4 terms

• full name: tenpy.networks.terms

• parent module: tenpy.networks

• type: module

Classes



CouplingTerms(L)	Operator names, site indices and strengths representing
	two-site coupling terms.
ExponentiallyDecayingTerms(L)	Represent a sum of exponentially decaying (long-range)
	couplings.
MultiCouplingTerms(L)	Operator names, site indices and strengths representing
	general <i>M</i> -site coupling terms.
OnsiteTerms(L)	Operator names, site indices and strengths representing
	onsite terms.
TermList(terms[, strength])	A list of terms (=operator names and sites they act on)
	and associated strengths.

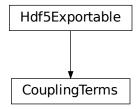
20.4.1 CouplingTerms

• full name: tenpy.networks.terms.CouplingTerms

• parent module: tenpy.networks.terms

• type: class

Inheritance Diagram



Methods

CouplingTermsinit(L)	Initialize self.
CouplingTerms.add_coupling_term(strength,	Add a two-site coupling term on given MPS sites.
i,)	
CouplingTerms.add_to_graph(graph)	Add terms from coupling_terms to an MPOGraph.
CouplingTerms.coupling_term_handle_JW(.	.Helping function to call before
])	<pre>add_coupling_term().</pre>
CouplingTerms.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
CouplingTerms.max_range()	Determine the maximal range in coupling_terms.
CouplingTerms.plot_coupling_terms(ax,	"Plot coupling terms into a given lattice.
lat[,])	
CouplingTerms.remove_zeros([tol_zero])	Remove entries close to 0 from coupling_terms.
CouplingTerms.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	
CouplingTerms.to_TermList()	Convert onsite_terms into a TermList.
CouplingTerms.to_nn_bond_Arrays(sites)	Convert the coupling_terms into Arrays on nearest
	neighbor bonds.

```
class tenpy.networks.terms.CouplingTerms (L) Bases: tenpy.tools.hdf5_io.Hdf5Exportable
```

Operator names, site indices and strengths representing two-site coupling terms.

Parameters L (int) – Number of sites.

L

Number of sites.

Type int

coupling_terms

Filled by $add_coupling_term()$. Nested dictionaries of the form {i: {('opname_i', 'opname_string'): {j: {'opname_j': strength}}}. Note that always i < j, but entries with j >= L are allowed for bc_MPS == 'infinite', in which case they indicate couplings between different iMPS unit cells.

20.4. terms 671

Type dict of dict

max range()

Determine the maximal range in coupling_terms.

Returns max_range – The maximum of j - i for the i, j occurring in a term of coupling_terms.

Return type int

add_coupling_term (strength, i, j, op_i, op_j, op_string='Id')
Add a two-site coupling term on given MPS sites.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **j** (*int*) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., *op_i* acts "left" of *op_j*. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- **op1** (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.

coupling_term_handle_JW (strength, term, sites, op_string=None)

Helping function to call before add_coupling_term().

Parameters

- **strength** (*float*) The strength of the coupling term.
- term ([(str, int), (str, int)]) List of two tuples [(op_i, i), (op_j, j)] where i is the MPS index of the site the operator named op_i acts on; we require i < j.
- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings.
- op_string (None / str) Operator name to be used as operator string between the operators, or None if the Jordan Wigner string should be figured out.

Warning: None figures out for each segment between the operators, whether a Jordan-Wigner string is needed. This is different from a plain 'JW', which just applies a string on each segment!

Returns Arguments for $MultiCouplingTerms.add_multi_coupling_term()$ such that the added term corresponds to the parameters of this function.

Return type strength, i, j, op_i, op_j, op_string

"Plot coupling terms into a given lattice.

This function plots the $coupling_terms$

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- lat (Lattice) The lattice for plotting the couplings, most probably the M.lat of the corresponding model M, see lat.
- **style_map** (function | None) Function which get's called with arguments i, j, op_i, op_string, op_j, strength for each two-site coupling and should return a keyword-dictionary with the desired plot-style for this coupling. By default (None), the *linewidth* is given by the absolute value of *strength*, and the linecolor depends on the phase of *strength* (using the *hsv* colormap).
- **common_style** (dict) Common style, which overwrites values of the dictionary returned by style_map. A 'label' is only used for the first plotted line.
- text (format_string | None) If not None, we add text labeling the couplings in the plot. Available keywords are i, j, op_i, op_string, op_j, strength as well as strength_abs, strength_angle, strength_real.
- **text_pos** (float) Specify where to put the text on the line between i (0.0) and j (1.0), e.g. 0.5 is exactly in the middle between i and j.

See also:

tenpy.models.lattice.Lattice.plot_sites plot the sites of the lattice.

add_to_graph (graph)

Add terms from coupling terms to an MPOGraph.

Parameters graph (MPOGraph) - The graph into which the terms from coupling_terms should be added.

to nn bond Arrays (sites)

Convert the coupling_terms into Arrays on nearest neighbor bonds.

Parameters sites (list of *Site*) – Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.

Returns H_bond - The coupling_terms rewritten as sum_i H_bond[i] for MPS indices i. H_bond[i] acts on sites (i-1, i), None represents 0. Legs of each H_bond[i] are ['p0', 'p0*', 'p1', 'p1*'].

Return type list of {Array | None}

remove_zeros (tol_zero=1e-15)

Remove entries close to 0 from coupling terms.

Parameters tol_zero(float) - Entries in coupling_terms with strength < tol_zero are considered to be zero and removed.

to TermList()

Convert onsite_terms into a TermList.

Returns term_list – Representation of the terms as a list of terms.

Return type TermList

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

20.4. terms 673

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

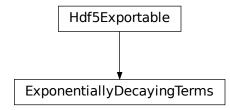
Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

20.4.2 ExponentiallyDecayingTerms

- full name: tenpy.networks.terms.ExponentiallyDecayingTerms
- parent module: tenpy.networks.terms
- type: class

Inheritance Diagram



Methods

ExponentiallyDecayingTerms.	Initialize self.
init(L)	
ExponentiallyDecayingTerms.	Add an exponentially decaying long-range coupling.
add_exponentially_decaying_coupling (\dots)	
ExponentiallyDecayingTerms.	Add terms from onsite_terms to an MPOGraph.
add_to_graph(graph)	
ExponentiallyDecayingTerms.	Load instance from a HDF5 file.
$from_hdf5()$	
ExponentiallyDecayingTerms.	Maximum range of the couplings.
max_range()	
ExponentiallyDecayingTerms.	Export self into a HDF5 file.
save_hdf5()	
ExponentiallyDecayingTerms.	Convert self into a TermList.
to_TermList([])	

class tenpy.networks.terms.ExponentiallyDecayingTerms (L)

Bases: tenpy.tools.hdf5_io.Hdf5Exportable

Represent a sum of exponentially decaying (long-range) couplings.

MPOs can represent translation invariant, exponentially decaying long-range terms of the following form with a single extra index of the virtual bonds:

$$sum_{i\neq j}lambda^{|i-j|}A_iB_i$$

For 2D cylinders (or ladders), we need a slight generalization of this, where the operators act only on a subset of the sites in each unit cell, given by a 1D array *subsites*:

$$strengthsum_{i < j} lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$$

Note that we still have |i-j|, such that this will give uniformly decaying interactions, independent of the way the MPS winds through the 2D lattice, as long as *subsites* is sorted. An easy example would be a ladder, where we want the long-range interactions on the first rung only, subsites = lat.mps_idx_fix_u(u=0), see mps_idx_fix_u().

Parameters L (int) – Number of sites.

L

Number of sites.

Type int

exp_decaying_terms

Each tuple (strength, opname_i, opname_j, lambda, subsites, opname_string) represents one of the terms as described above; see add_exponentially_decaying_coupling() for more details.

Type list of tuples

Add an exponentially decaying long-range coupling.

 $strengthsum_{i < j} lambda^{|i-j|} A_{subsites[i]} B_{subsites[j]}$

Where the operator A is given by $op_{\underline{j}}$, and B is given by $op_{\underline{j}}$. Note that the sum over i,j is long-range, for infinite systems beyond the MPS unit cell.

20.4. terms 675

Parameters

- **strength** (*float*) Overall prefactor.
- lambda (float) Decay-rate
- **op_i** (*string*) Names for the operators.
- op_j (string) Names for the operators.
- **subsites** (*None* / 1D array) Selects a subset of sites within the MPS unit cell on which the operators act. Needs to be sorted. None selects all sites.
- op_string(string) The operator to be inserted between A and B; for Fermions this should be "JW".

add_to_graph (graph, key='exp-decay')

Add terms from onsite_terms to an MPOGraph.

Parameters

- **graph** (MPOGraph) The graph into which the terms from exp_decaying_terms should be added.
- **key** (str) Key to distinguish from other states in the MPOGraph. We find integers key_nr and use (key_nr, key) as state for the different entries in exp_decaying_terms.

to_TermList (cutoff=0.01, bc='finite')

Convert self into a TermList.

Parameters

- **cutoff** (float) Drop terms where the overall prefactor is smaller then *cutoff*.
- **bc** ("finite" | "infinite") Boundary conditions to be used.

Returns term_list – Representation of the terms as a list of terms. For "infinite" *bc*, only terms starting in the first MPS unit cell are included.

```
Return type TermList
```

max_range()

Maximum range of the couplings. In this case np.inf.

classmethod from hdf5 (hdf5 loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The name of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

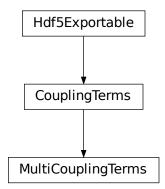
Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

20.4.3 MultiCouplingTerms

- full name: tenpy.networks.terms.MultiCouplingTerms
- parent module: tenpy.networks.terms
- type: class

Inheritance Diagram



Methods

MultiCouplingTermsinit(L)	Initialize self.
MultiCouplingTerms.	Add a two-site coupling term on given MPS sites.
$ extit{add_coupling_term}([,])$	
MultiCouplingTerms.	Add a multi-site coupling term.
add_multi_coupling_term (\dots)	
MultiCouplingTerms.add_to_graph(graph[,	Add terms from coupling_terms to an MPOGraph.
_i,])	
MultiCouplingTerms.	Helping function to call before
coupling_term_handle_J $\mathbb{W}(\dots)$	<pre>add_coupling_term().</pre>
MultiCouplingTerms.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
	continues on next page

20.4. terms 677

Table 31 – continued from previous page

MultiCouplingTerms.max_range()	Determine the maximal range in coupling_terms.
MultiCouplingTerms.	Helping function to call before
multi_coupling_term_handle_J $\mathbb{W}(\ldots)$	<pre>add_multi_coupling_term().</pre>
MultiCouplingTerms.	"Plot coupling terms into a given lattice.
<pre>plot_coupling_terms(ax, lat)</pre>	
MultiCouplingTerms.	Remove entries close to 0 from coupling_terms.
remove_zeros([tol_zero, _d0])	
MultiCouplingTerms.save_hdf5(hdf5_saver,	Export <i>self</i> into a HDF5 file.
)	
MultiCouplingTerms.to_TermList()	Convert onsite_terms into a TermList.
MultiCouplingTerms.	Convert the coupling_terms into Arrays on nearest
to_nn_bond_Arrays(sites)	neighbor bonds.

class tenpy.networks.terms.MultiCouplingTerms (L)

Bases: tenpy.networks.terms.CouplingTerms

Operator names, site indices and strengths representing general M-site coupling terms.

Generalizes the <code>coupling_terms</code> of <code>CouplingTerms</code> to <code>M</code>-site couplings. The structure of the nested dictionary <code>coupling_terms</code> is similar, but we allow an arbitrary recursion depth of the dictionary.

Parameters L (int) – Number of sites.

L

Number of sites.

Type int

coupling_terms

Nested dictionaries of the following form:

For a M-site coupling, this involves a nesting depth of 2*M dictionaries. Note that always $i < j < k < \ldots < 1$, but entries with j,k,l >= L are allowed for the case of bc_MPS == 'infinite', when they indicate couplings between different iMPS unit cells.

Type dict of dict

add_multi_coupling_term (strength, ijkl, ops_ijkl, op_string='Id')
Add a multi-site coupling term.

Parameters

- **strength** (*float*) The strength of the coupling term.
- **ijkl** (*list of int*) The MPS indices of the sites on which the operators acts. With $i, j, k, \ldots = ijkl$, we require that they are ordered ascending, $i < j < k < \ldots$ and

that $0 \le i \le N_sites$. Inidces $>= N_sites$ indicate couplings between different unit cells of an infinite MPS.

- ops_ijkl (list of str) Names of the involved operators on sites i, j, k, \ldots
- op_string ((list of) str) Names of the operator to be inserted between the operators, e.g., op_string[0] is inserted between i and j. A single name holds for all inbetween segments.

multi_coupling_term_handle_JW (strength, term, sites, op_string=None)

Helping function to call before add_multi_coupling_term().

Handle/figure out Jordan-Wigner strings if needed.

Parameters

- **strength** (*float*) The strength of the term.
- **term**(list of (str, int))—List of tuples (op_i, i) where i is the MPS index of the site the operator named op_i acts on. We **require** the operators to be sorted (strictly ascending) by sites. If necessary, call order_combine_term() beforehand.
- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings.
- op_string (None / str) Operator name to be used as operator string between the operators, or None if the Jordan Wigner string should be figured out.

Warning: None figures out for each segment between the operators, whether a Jordan-Wigner string is needed. This is different from a plain 'JW', which just applies a string on each segment!

Returns Arguments for MultiCouplingTerms.add_multi_coupling_term() such that the added term corresponds to the parameters of this function.

Return type strength, ijkl, ops ijkl, op string

max range()

Determine the maximal range in coupling_terms.

Returns max_range – The maximum of j - i for the i, j occurring in a term of coupling_terms.

Return type int

```
add_to_graph (graph, _i=None, _d1=None, _label_left=None)
```

Add terms from coupling_terms to an MPOGraph.

Parameters

- graph (MPOGraph) The graph into which the terms from coupling_terms should be added.
- _i (None) Should not be given; only needed for recursion.
- **d1** (*None*) Should not be given; only needed for recursion.
- label left (None) Should not be given; only needed for recursion.

remove zeros (tol zero=1e-15, d0=None)

Remove entries close to 0 from coupling_terms.

Parameters

20.4. terms 679

- tol_zero (float) Entries in coupling_terms with strength < tol_zero are considered to be zero and removed.
- _d0 (None) Should not be given; only needed for recursion.

to TermList()

Convert onsite terms into a TermList.

Returns term_list – Representation of the terms as a list of terms.

Return type TermList

add_coupling_term(strength, i, j, op_i, op_j, op_string='Id')

Add a two-site coupling term on given MPS sites.

Parameters

- **strength** (*float*) The strength of the coupling term.
- i (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- j (int) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N_sites and i < j, i.e., op_i acts "left" of op_j. If j >= N_sites, it indicates couplings between unit cells of an infinite MPS.
- op1 (str) Names of the involved operators.
- op2 (str) Names of the involved operators.
- op_string (str) The operator to be inserted between i and j.

coupling_term_handle_JW (strength, term, sites, op_string=None)

Helping function to call before add_coupling_term().

Parameters

- **strength** (*float*) The strength of the coupling term.
- term ([(str, int), (str, int)]) List of two tuples [(op_i, i), (op_j, j)] where i is the MPS index of the site the operator named op_i acts on; we require i < j.
- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings.
- op_string (None / str) Operator name to be used as operator string between the operators, or None if the Jordan Wigner string should be figured out.

Warning: None figures out for each segment between the operators, whether a Jordan-Wigner string is needed. This is different from a plain 'JW', which just applies a string on each segment!

Returns Arguments for MultiCouplingTerms.add_multi_coupling_term() such that the added term corresponds to the parameters of this function.

Return type strength, i, j, op_i, op_j, op_string

classmethod from hdf5 (hdf5 loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

"Plot coupling terms into a given lattice.

This function plots the coupling_terms

Parameters

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- lat (Lattice) The lattice for plotting the couplings, most probably the M.lat of the corresponding model M, see lat.
- **style_map** (function | None) Function which get's called with arguments i, j, op_i, op_string, op_j, strength for each two-site coupling and should return a keyword-dictionary with the desired plot-style for this coupling. By default (None), the *linewidth* is given by the absolute value of *strength*, and the linecolor depends on the phase of *strength* (using the *hsv* colormap).
- **common_style** (*dict*) Common style, which overwrites values of the dictionary returned by style_map. A 'label' is only used for the first plotted line.
- text (format_string | None) If not None, we add text labeling the couplings in the plot. Available keywords are i, j, op_i, op_string, op_j, strength as well as strength_abs, strength_angle, strength_real.
- **text_pos** (float) Specify where to put the text on the line between i (0.0) and j (1.0), e.g. 0.5 is exactly in the middle between i and j.

See also:

tenpy.models.lattice.Lattice.plot_sites plot the sites of the lattice.

save_hdf5 (hdf5_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

to_nn_bond_Arrays (sites)

Convert the coupling_terms into Arrays on nearest neighbor bonds.

Parameters sites (list of *Site*) – Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.

20.4. terms 681

Returns H_bond - The coupling_terms rewritten as sum_i H_bond[i] for MPS indices i. H_bond[i] acts on sites (i-1, i), None represents 0. Legs of each H_bond[i] are ['p0', 'p0*', 'p1', 'p1*'].

Return type list of {Array | None}

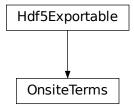
20.4.4 OnsiteTerms

• full name: tenpy.networks.terms.OnsiteTerms

• parent module: tenpy.networks.terms

• type: class

Inheritance Diagram



Methods

OnsiteTerms. $_$ init $_$ ($ m L)$	Initialize self.
OnsiteTerms.add_onsite_term(strength, i,	Add a onsite term on a given MPS site.
op)	
OnsiteTerms.add_to_graph(graph)	Add terms from onsite_terms to an MPOGraph.
OnsiteTerms.add_to_nn_bond_Arrays(H_bor	ndAdd self.onsite_terms into nearest-neighbor
)	bond arrays.
OnsiteTerms.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
OnsiteTerms.max_range()	Maximum range of the terms.
OnsiteTerms.remove_zeros([tol_zero])	Remove entries close to 0 from onsite_terms.
OnsiteTerms.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
OnsiteTerms.to_Arrays(sites)	Convert the onsite_terms into a list of
	np_conserved Arrays.
OnsiteTerms.to_TermList()	Convert onsite_terms into a TermList.

class tenpy.networks.terms.OnsiteTerms(L)

Bases: tenpy.tools.hdf5_io.Hdf5Exportable

Operator names, site indices and strengths representing onsite terms.

Represents a sum of onsite terms where the operators are only given by their name (in the form of a string). What the operator represents is later given by a list of Site with $get_op()$.

Parameters L (int) – Number of sites.

L

Number of sites.

Type int

onsite terms

Filled by meth: add_onsite_term. For each index i a dictionary { 'opname': strength} defining the onsite terms.

Type list of dict

max_range()

Maximum range of the terms. In this case 0.

add_onsite_term (strength, i, op)

Add a onsite term on a given MPS site.

Parameters

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- **op** (str) Name of the involved operator.

add_to_graph (graph)

Add terms from onsite_terms to an MPOGraph.

Parameters graph (MPOGraph) - The graph into which the terms from onsite_terms should be added.

to_Arrays (sites)

Convert the <code>onsite_terms</code> into a list of np_conserved Arrays.

Parameters sites (list of *Site*) – Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.

Returns onsite_arrays – Onsite terms represented by *self*. Entry i of the list lives on sites[i].

Return type list of Array

remove_zeros (tol_zero=1e-15)

Remove entries close to 0 from onsite terms.

Parameters tol_zero (float) - Entries in onsite_terms with strength < tol_zero are considered to be zero and removed.

$\verb"add_to_nn_bond_Arrays" (\textit{H_bond}, \textit{sites}, \textit{finite}, \textit{distribute} = (0.5, 0.5))$

Add self.onsite_terms into nearest-neighbor bond arrays.

Parameters

• H_bond (list of {Array | None}) - The coupling_terms rewritten as sum_i H_bond[i] for MPS indices i. H_bond[i] acts on sites (i-1, i), None represents 0. Legs of each H_bond[i] are ['p0', 'p0*', 'p1', 'p1*']. Modified in place.

20.4. terms 683

- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.
- **distribute**((float, float)) How to split the onsite terms (in the bulk) into the bond terms to the left (distribute[0]) and right (distribute[1]).
- **finite** (bool) Boundary conditions of the MPS, MPS.finite. If finite, we distribute the onsite term of the

to TermList()

Convert onsite_terms into a TermList.

Returns term_list – Representation of the terms as a list of terms.

Return type TermList

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save_hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

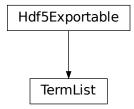
Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

20.4.5 TermList

- full name: tenpy.networks.terms.TermList
- parent module: tenpy.networks.terms
- · type: class

Inheritance Diagram



Methods

TermListinit(terms[, strength])	Initialize self.
TermList.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
TermList.from_lattice_locations(lattice,	Initialize from a list of terms given in lattice indices in-
terms)	stead of MPS indices.
TermList.limits()	Return the left-most site and right-most site any operator
	acts on.
TermList.order_combine(sites)	Order and combine operators in each term.
TermList.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
TermList.shift(i0)	Return a copy where $i0$ is added to all indices i in
	terms.
TermList.to_OnsiteTerms_CouplingTerms(sitex)nvert to OnsiteTerms and CouplingTerms	

class tenpy.networks.terms.TermList (terms, strength=1.0)
 Bases: tenpy.tools.hdf5_io.Hdf5Exportable

A list of terms (=operator names and sites they act on) and associated strengths.

A representation of terms, similar as OnsiteTerms, CouplingTerms and MultiCouplingTerms.

This class does not store operator strings between the sites. Jordan-Wigner strings of fermions are added during conversion to (Multi)CouplingTerms.

Warning: Since this class does **not** store the operator string between the sites, conversion from CouplingTerms or MultiCouplingTerms to TermList is lossy!

Parameters

- **terms** (*list* of *list* of (*str*, *int*)) List of terms where each *term* is a list of tuples (opname, i) of an operator name and a site *i* it acts on. For Fermions, the order is the order in the mathematic sense, i.e., the right-most/last operator in the list acts last.
- **strength** ((list of) float/complex) For each term in terms an associated prefactor or strength. A single number holds for all terms equally.

20.4. terms 685

terms

List of terms where each term is a tuple (opname, i) of an operator name and a site i it acts on.

```
Type list of list of (str, int)
```

strength

For each term in terms an associated prefactor or strength.

```
Type 1D ndarray
```

Examples

For fermions, the term $0.5(c_0^{\dagger}c_2 + h.c.) + 1.3 * n_1$ can be represented by:

If you have a *Lattice*, you might also want to specify the location of the operators by lattice indices insted of MPS indices. For example, you can obtain the nearest-neighbor density terms **without double counting each pair**) on a TriangularLattice:

The negative index -5 here indicates a tensor left of the current MPS unit cell.

classmethod from_lattice_locations(lattice, terms, strength=1.0, shift=None)

Initialize from a list of terms given in lattice indices instead of MPS indices.

Parameters

- lattice (Lattice) The underlying lattice to be used for conversion, e.g. *M.lat* from a *Model*.
- **terms** (*list of list of (str, tuple*)) List of terms, where each *term* is a tuple (opname, lat_idx) with *lat_idx* itself being a tuple (x, y, u) (for a 2D lattice) of the lattice coordinates.
- **strengths** ((list of) float/complex) For each term in *terms* an associated prefactor or strength. A single number holds for all terms equally.
- **shift** (*None* / tuple of int) Overall shift added to all lattice coordinates *lat_idx* in *terms* before conversion. None defaults to no shift.

Returns term_list – Representation of the terms.

```
Return type TermList
```

to_OnsiteTerms_CouplingTerms (sites)

Convert to OnsiteTerms and CouplingTerms

Performs Jordan-Wigner transformation for fermionic operators.

Parameters sites (list of Site) — Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings. The length is used as L for the *onsite_terms* and *coupling_terms*.

Returns

- onsite_terms (OnsiteTerms) Onsite terms.
- coupling_terms (CouplingTerms | MultiCouplingTerms) Coupling terms. If self contains terms involving more than two operators, a MultiCouplingTerms instance, otherwise just CouplingTerms.

order combine (sites)

Order and combine operators in each term.

Parameters sites (list of Site) – Defines the local Hilbert space for each site. Used to check whether the operators anticommute (= whether they need Jordan-Wigner strings) and for multiplication rules.

See also:

```
order_and_combine_term does it for a single term.
```

limits()

Return the left-most site and right-most site any operator acts on.

shift(i0)

Return a copy where i0 is added to all indices i in terms.

classmethod from_hdf5 (hdf5_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save_hdf5()</code>.

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

```
save hdf5 (hdf5 saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from_hdf5().

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

20.4. terms 687

Functions

order_combine_term(term, sites)

Combine operators in a term to one terms per site.

20.4.6 order_combine_term

• full name: tenpy.networks.terms.order_combine_term

• parent module: tenpy.networks.terms

• type: function

tenpy.networks.terms.order_combine_term(term, sites)

Combine operators in a term to one terms per site.

Takes in a term of operators and sites they acts on, commutes operators to order them by site and combines operators acting on the same site with multiply_op_names().

Parameters

- **term** (a list of (opname_i, i) tuples) Represents a product of onsite operators with site indices *i* they act on. Needs not to be ordered and can have multiple entries acting on the same site.
- sites (list of Site) Defines the local Hilbert space for each site. Used to check whether
 the operators anticommute (= whether they need Jordan-Wigner strings) and for multiplication rules.

Returns

- *combined_term* Equivalent to *term* but with at most one operator per site.
- **overall_sign** (+1 | -1 | 0) Comes from the (anti-)commutation relations. When the operators in *term* are multiplied from left to right, and then multiplied by *overall_sign*, the result is the same operator as the product of *combined_term* from left to right.

Module description

Classes to store a collection of operator names and sites they act on, together with prefactors.

This modules collects classes which are not strictly speaking tensor networks but represent "terms" acting on them. Each term is given by a collection of (onsite) operator names and indices of the sites it acts on. Moreover, we associate a *strength* to each term, which corresponds to the prefactor when specifying e.g. a Hamiltonian.

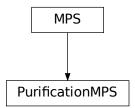
20.5 purification_mps

• full name: tenpy.networks.purification_mps

• parent module: tenpy.networks

• type: module

Classes



PurificationMPS(sites, Bs, SVs[, bc, form, norm]) An MPS representing a finite-temperature ensemble using purification.

Module description

This module contains an MPS class representing an density matrix by purification.

Usually, an MPS represents a pure state, i.e. the density matrix is $\rho = |\psi\rangle < \psi|$, describing observables as $< O > = Tr(O|\psi\rangle < \psi|) = <\psi|O|\psi\rangle$. Clearly, if $|\psi\rangle$ is the ground state of a Hamiltonian, this is the density matrix at T=0.

At finite temperatures T>0, we want to describe a non-pure density matrix $\rho=\exp(-H/T)$. This can be accived by the so-called purification: in addition to the physical space P, we introduce a second 'auxiliar' space Q and define the density matrix of the physical system as $\rho=Tr_Q(|\phi><\phi|)$, where $|\phi>$ is a pure state in the combined physical and auxiliar system.

For $T=\infty$, the density matrix ρ_{∞} is the identity matrix. In other words, expectation values are sums over all possible states $< O >= Tr_P(\rho_{\infty}O) = Tr_P(O)$. Saying that each: on top is to be connected with the corresponding: on the bottom, the trace is simply a contraction:

Clearly, we get the same result, if we insert an identity operator, written as MPO, on the top and bottom:

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We use the following label convention:

You can view the MPO as an MPS by combining the p and q leg and defining every physical operator to act trivial on the q leg. In expectation values, you would then sum over over the q legs, which is exactly what we need. In other words, the choice $B = \delta_{p,q}$ with trivial (length-1) virtual bonds yields infinite temperature expectation values for operators action only on the p legs!

Now, you go a step further and also apply imaginary time evolution (acting only on p legs) to the initial infinite temperature state. For example, the normalized state $|\psi>\propto \exp(-\beta/2H)|\phi>$ yields expectation values

$$\langle O \rangle = Tr(\exp(-\beta H)O)/Tr(\exp(-\beta H)) \propto \langle \phi | \exp(-\beta/2H)O \exp(-\beta/2H) | \phi \rangle$$
.

An additional real-time evolution allows to calculate time correlation functions:

$$< A(t)B(0) > \propto < \phi |\exp(-\beta H/2)\exp(+iHt)A\exp(-iHt)B\exp(-\beta H/2)|\phi>$$

Time evolution algorithms (TEBD and MPO application) are adjusted in the module purification.

See also [[karrasch2013]] for additional tricks! On of their crucial observations is, that one can apply arbitrary unitaries on the auxiliar space (i.e. the q) without changing the result. This can actually be used to reduce the necessary virtual bond dimensions: From the definition, it is easy to see that if we apply exp(-iHt) to the p legs of $|\phi>$, and exp(+iHt) to the q legs, they just cancel out! (They commute with $exp(-\beta H/2)...$) If the state is modified (e.g. by applying A or B to calculate correlation functions), this is not true any more. However, we still can find unitaries, which are 'optimal' in the sense of reducing the entanglement of the MPS/MPO to the minimal value. For a discussion of P is P is P in the sense of P in the sense of P is P in the sense of P in the sense of P is P in the sense of P in the sense of P is P in the sense of P in the sense of P in the sense of P is P in the sense of P in the sen

Note: The classes MPSEnvironment and TransferMatrix should also work for the PurificationMPS defined here. For example, you can use <code>expectation_value()</code> for the expectation value of operators between different PurificationMPS. However, this makes only sense if the <code>same</code> disentangler was applied to the <code>bra</code> and <code>ket</code> PurificationMPS.

Note: The literature (e.g. section 7.2 of [[schollwoeck2011]] or [[karrasch2013]]) suggests to use a *singlet* as a maximally entangled state. Here, we use instead the identity $\delta_{p,q}$, since it is easier to generalize for p running over more than two indices, and allows a simple use of charge conservation with the above qconj convention. Moreover, we don't split the physical and auxiliar space into separate sites, which makes TEBD as costly as $O(d^6\chi^3)$.

CHAPTER

TWENTYONE

SIMULATIONS

• full name: tenpy.simulations

• parent module: tenpy

• type: module

Module description

Simulation setup.

The classes provided here provide a structure for the whole setup of simulations.

Submodules

simulation	This module contains base classes for simulations.
measurement	Functions to perform measurments.
ground_state_search	Simulations for ground state searches.
time_evolution	Simulations for (real) time evolution.

21.1 simulation

• full name: tenpy.simulations.simulation

• parent module: tenpy.simulations

• type: module

Classes

Simulation Skip

<pre>Simulation(options, *[, setup_logging])</pre>	Base class for simulations.	

Exceptions

Skip Error raised if simulation output already exists.	======================================
--	--

21.1.1 Skip

- full name: tenpy.simulations.simulation.Skip
- $\bullet \ \ parent \ module: \ \textit{tenpy.simulations.simulation}$
- · type: exception

exception tenpy.simulations.simulation.**Skip**Error raised if simulation output already exists.

Functions

resume_from_checkpoint(*[, filename,])	Resume a simulation run from a given checkpoint.
<pre>run_simulation([simulation_class_name,])</pre>	Run the simulation with a simulation class.

21.1.2 resume_from_checkpoint

- full name: tenpy.simulations.simulation.resume_from_checkpoint
- parent module: tenpy.simulations.simulation
- type: function

```
tenpy.simulations.simulation.resume_from_checkpoint(*, filename=None, check-point_results=None, up-date_sim_params=None, simulation_class_kwargs=None)
```

Resume a simulation run from a given checkpoint.

(All parameters have to be given as keyword arguments.)

Parameters

- **filename** (None | str) The filename of the checkpoint to be loaded. You can either specify the *filename* or the *checkpoint_results*.
- **checkpoint_results** (*None | dict*) Alternatively to *filename* the results of the simulation so far, i.e. directly the data dicitonary saved at a simulation checkpoint.
- update_sim_params (None | dict) Allows to update specific Simulation parameters, ignored if None. Uses update_recursive() to update values, such that the keys of update_sim_params can be recursive, e.g. algorithm_params/max_sweeps.
- **simlation_class_kwargs** (*None | dict*) Further keyword arguemnts given to the simulation class, ignored if *None*.

Returns The results from running the simulation, i.e., what tenpy.simulations. Simulation.resume_run() returned.

Return type results

Notes

The *checkpoint_filename* should be relative to the current working directory. If you use the Simulation. directory, the simulation class will attempt to change to that directory during initialization. Hence, either resume the simulation from the same directory where you originally started, or update the Simulation. directory (and :cfg:option`Simulation.output filename`) parameter with *update sim params*.

21.1.3 run simulation

• full name: tenpy.simulations.simulation.run_simulation

• parent module: tenpy.simulations.simulation

· type: function

tenpy.simulation.run_simulation(simulation_class_name='GroundStateSearch', simulation_class_kwargs=None, **simulation_params) **

Run the simulation with a simulation class.

Parameters

- **simulation_class_name** (*str*) The name of a (sub)class of Simulation to be used for running the simulaiton.
- **simulation_class_kwargs** (*dict | None*) A dictionary of keyword-arguments to be used for the initializing the simulation.
- **simulation_params Further keyword arguments as documented in the corresponding simulation class, see :cfg:config`Simulation`.

Returns The results from running the simulation, i.e., what tenpy.simulations. Simulation.run() returned.

Return type results

Module description

This module contains base classes for simulations.

The Simulation class tries to put everything need for a simulation in a structured form and collects task like initializing the tensor network state, model and algorithm classes, running the actual algorithm, possibly performing measurements and saving the results.

Todo: provide examples, give user guide

21.1. simulation 693

21.2 measurement

• full name: tenpy.simulations.measurement

• parent module: tenpy.simulations

• type: module

Functions

bond_dimension(results, psi, simulation[, key])	'Measure' the bond dimension of an MPS.
bond_energies(results, psi, simulation[, key])	Measure the energy of an MPS.
correlation_length(results, psi, simulation)	Measure the correlaiton of an infinite MPS.
energy_MPO(results, psi, simulation[, key])	Measure the energy of an MPS by evaluating the MPS
	expectation value.
entropy(results, psi, simulation[, key])	Measure the entropy at all bonds of an MPS.
<pre>evolved_time(results, psi, simulation[, key])</pre>	Measure the time evolved by the engine, engine.
	evolved_time.
measurement_index(results, psi, simulation)	'Measure' the index of how many mearuements have
	been performed so far.
onsite_expectation_value(results, psi,)	Measure expectation values of an onsite operator.

21.2.1 bond_dimension

- full name: tenpy.simulations.measurement.bond_dimension
- parent module: tenpy.simulations.measurement
- type: function

tenpy.simulations.measurement.bond_dimension(results, psi, simulation, $key='bond_dimension'$)

'Measure' the bond dimension of an MPS.

Parameters

- results See measurement_index().
- **psi** See measurement_index().
- **simulation See** measurement_index().
- **key See** measurement_index().

21.2.2 bond_energies

- full name: tenpy.simulations.measurement.bond_energies
- parent module: tenpy.simulations.measurement
- · type: function

tenpy.simulations.measurement.bond_energies(results, psi, simulation, $key='bond_energies'$)

Measure the energy of an MPS.

Parameters

```
• results - See measurement_index().
```

- **psi** See measurement_index().
- **simulation See** measurement_index().
- **key See** measurement_index().

21.2.3 correlation length

- full name: tenpy.simulations.measurement.correlation_length
- parent module: tenpy.simulations.measurement
- · type: function

```
tenpy.simulations.measurement.correlation_length(results, psi, simulation, key='correlation_length', **kwargs)

Measure the correlation of an infinite MPS.
```

Parameters

- results See measurement_index().
- **psi** See measurement_index().
- **simulation See** measurement_index().
- **key See** measurement_index().
- **kwargs Further keywoard arguments given to correlation_length().

21.2.4 energy MPO

- full name: tenpy.simulations.measurement.energy_MPO
- parent module: tenpy.simulations.measurement
- · type: function

tenpy.simulations.measurement.energy_MPO (results, psi, simulation, key='energy_MPO')
Measure the energy of an MPS by evaluating the MPS expectation value.

Parameters

- results See measurement_index().
- **psi** See measurement_index().
- **simulation See** measurement_index().
- **key See** measurement_index().

21.2. measurement 695

21.2.5 entropy

- full name: tenpy.simulations.measurement.entropy
- parent module: tenpy.simulations.measurement
- · type: function

tenpy.simulations.measurement.entropy (results, psi, simulation, key='entropy') Measure the entropy at all bonds of an MPS.

Parameters

- results See measurement_index().
- psi See measurement_index().
- **simulation See** measurement_index().
- **key See** measurement_index().

21.2.6 evolved time

- full name: tenpy.simulations.measurement.evolved_time
- parent module: tenpy.simulations.measurement
- type: function

tenpy.simulations.measurement.evolved_time(results, psi, simulation, key='evolved_time')

Measure the time evolved by the engine, engine.evolved_time.

See e.g. tenpy.algorithms.tebd.TEBDEngine.evolved_time.

Parameters

- results See measurement_index().
- psi See measurement_index().
- **simulation See** measurement_index().
- **key See** measurement_index().

21.2.7 measurement index

- full name: tenpy.simulations.measurement.measurement_index
- parent module: tenpy.simulations.measurement
- · type: function

tenpy.simulations.measurement.measurement_index(results, psi, simulation, key='measurement_index')

'Measure' the index of how many mearuements have been performed so far.

The parameter description below also documents the common interface of all measurement functions, that can be registered to simulations.

Parameters

• **results** (dict) – A dictionary with measurement results performed so far. Instead of returning the result, the output should be written into this dictionary under an appropriate key (or multiple keys, if applicable).

- psi Tensor network state to be measured. Shorthand for simulation.psi.
- **simulation** (Simulation) The simulation class. This gives also access to the *model*, algorithm *engine*, etc.
- **key** (str) (Optional.) The key under which to save in *results*.
- **kwargs Other optional keyword arguments for individual measurement functions.
 Those are documented inside each measurement function.

21.2.8 onsite expectation value

- full name: tenpy.simulations.measurement.onsite_expectation_value
- parent module: tenpy.simulations.measurement
- type: function

tenpy.simulations.measurement.onsite_expectation_value(results, psi, simulation, opname, key=None)

Measure expectation values of an onsite operator.

The resulting array of measurements is indexed by *lattice* indices (x, y, u) (possibly dropping y and/or u if they are trivial), not by the MPS index. Note that this makes the result independent of the way the MPS winds through the lattice.

The key defaults to f " < { opname } > ".

Parameters

- results See measurement_index().
- **psi** See measurement_index().
- **simulation See** measurement_index().
- **key See** measurement_index().
- opname (str) The operator to be measured. Passed on to expectation_value().

Module description

Functions to perform measurments.

All measurement functions provided in this module support the interface used by the simulation class, i.e. they take the parameters documented in <code>measurement_index()</code> and write the measurement results into the <code>results</code> dictionary taken as argument.

Todo: test, provide more.

21.2. measurement 697

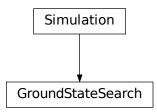
21.3 ground_state_search

• full name: tenpy.simulations.ground_state_search

• parent module: tenpy.simulations

• type: module

Classes



GroundStateSearch(options, *[, setup_logging])

Simutions for variational ground state searches.

Module description

Simulations for ground state searches.

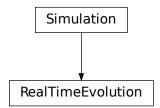
21.4 time_evolution

• full name: tenpy.simulations.time_evolution

• parent module: tenpy.simulations

• type: module

Classes



RealTimeEvolution(options)

Perform a real-time evolution on a tensor network state.

Module description

Simulations for (real) time evolution.

21.4. time_evolution 699

CHAPTER

TWENTYTWO

TOOLS

full name: tenpy.toolsparent module: tenpy

• type: module

Module description

A collection of tools: mostly short yet quite useful functions.

Some functions are explicitly imported in other parts of the library, others might just be useful when using the library. Common to all tools is that they are not just useful for a single algorithm but fairly general.

Submodules

hdf5_io	Tools to save and load data (from TeNPy) to disk.
params	Tools to handle config options/paramters for algorithms.
events	Event handler.
misc	Miscellaneous tools, somewhat random mix yet often
	helpful.
math	Different math functions needed at some point in the
	library.
fit	tools to fit to an algebraic decay.
string	Tools for handling strings.
process	Tools to read out total memory usage and get/set the
	number of threads.
optimization	Optimization options for this library.

22.1 hdf5_io

full name: tenpy.tools.hdf5_ioparent module: tenpy.tools

• type: module

Classes



Hdf5Exportable()	Interface specification for a class to be exportable to our
	HDF5 format.
Hdf5Ignored([name])	Placeholder for a dataset/group to be ignored during
	both loading and saving.
Hdf5Loader(h5group[, ignore_unknown, exclude])	Class to load and import object from a HDF5 file.
Hdf5Saver(h5group[, format_selection])	Class to save simple enough objects into a HDF5 file.

22.1.1 Hdf5Exportable

• full name: tenpy.tools.hdf5_io.Hdf5Exportable

• parent module: tenpy.tools.hdf5_io

• type: class

Inheritance Diagram

Hdf5Exportable

Methods

Hdf5Exportableinit()	Initialize self.
Hdf5Exportable.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
Hdf5Exportable.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	

class tenpy.tools.hdf5_io.Hdf5Exportable
 Bases: object

Interface specification for a class to be exportable to our HDF5 format.

To allow a class to be exported to HDF5 with <code>save_to_hdf5()</code>, it only needs to implement the <code>save_hdf5()</code> method as documented below. To allow import, a class should implement the classmethod <code>from_hdf5()</code>. During the import, the class already needs to be defined; loading can only initialize instances, not define classes.

The implementation given works for sufficiently simple (sub-)classes, for which all data is stored in __dict__. In particular, this works for python-defined classes which simply store data using self.data = data in their methods.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from_hdf5()*.

This implementation saves the content of __dict__ with save_dict_content(), storing the format under the attribute 'format'.

Parameters

- hdf5_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

Parameters

- hdf5_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

Returns obj – Newly generated class instance containing the required data.

Return type cls

22.1.2 Hdf5lgnored

• full name: tenpy.tools.hdf5_io.Hdf5Ignored

• parent module: tenpy.tools.hdf5_io

• type: class

22.1. hdf5 io 703

Inheritance Diagram

Hdf5Ignored

Methods

Hdf5Ignored.__init__([name])

Initialize self.

```
class tenpy.tools.hdf5_io.Hdf5Ignored(name='unknown')
    Bases: object
```

Placeholder for a dataset/group to be ignored during both loading and saving.

Objects of this type are not saved. Moreover, if a saved dataset/group has the *type* attribute matching *REPR_IGNORED*, instance of this class are returned instead of loading the data.

Parameters name (str) – The name of the dataset during loading; just for reference.

name

See above.

Type str

22.1.3 Hdf5Loader

• full name: tenpy.tools.hdf5_io.Hdf5Loader

 $\bullet \ \ parent \ module: \ \textit{tenpy.tools.hdf5_io}$

• type: class

Inheritance Diagram

Hdf5Loader

Methods

Hdf5Loaderinit(h5group[,])	Initialize self.
Hdf5Loader.get_attr(h5gr, attr_name)	Return attribute h5gr.attrs[attr_name], if ex-
	istent.
Hdf5Loader.load([path])	Load a Python object from the dataset.
Hdf5Loader.load_dataset(h5gr, type_info,	Load a h5py Dataset and convert it into the desired
subpath)	type.
Hdf5Loader.load_dict(h5gr, type_info, sub-	Load a dictionary in the format according to <i>type_info</i> .
path)	
Hdf5Loader.load_dtype(h5gr, type_info, sub-	Load a numpy.dtype.
path)	
<pre>Hdf5Loader.load_general_dict(h5gr,)</pre>	Load a dictionary with general keys.
Hdf5Loader.load_global(h5gr, type_info, sub-	Load a global object like a class or function from its
path)	qualified name and module.
<pre>Hdf5Loader.load_hdf5exportable(h5gr,)</pre>	Load an instance of a userdefined class.
Hdf5Loader.load_ignored(h5gr, type_info,	Ignore the group to be loaded.
subpath)	
<pre>Hdf5Loader.load_list(h5gr, type_info, sub-</pre>	Load a list.
path)	
Hdf5Loader.load_none(h5gr, type_info, sub-	Load the None object from a dataset.
path)	
Hdf5Loader.load_range(h5gr, type_info, sub-	Load a range.
path)	
Hdf5Loader.load_reduce(h5gr, type_info, sub-	Load an object where the return values of obj.
path)	reduce has been exported.
<pre>Hdf5Loader.load_set(h5gr, type_info, subpath)</pre>	Load a set.
Hdf5Loader.load_simple_dict(h5gr,	Load a dictionary with simple keys.
type_info,)	
Hdf5Loader.load_str(h5gr, type_info, subpath)	Load a string from a h5py Dataset.
Hdf5Loader.load_tuple(h5gr, type_info, sub-	Load a tuple.
path)	
Hdf5Loader.memorize_load(h5gr, obj)	Store objects already loaded in the memo_load.

Class Attributes and Properties

Hdf5Loader.dispatch_load

class tenpy.tools.hdf5_io.**Hdf5Loader**(h5group, ignore_unknown=True, exclude=None)

Bases: object

Class to load and import object from a HDF5 file.

The intended use of this class is through $load_from_hdf5()$, which is simply an alias for Hdf5Loader(h5group).load(path).

It can load data exported with <code>save_to_hdf5()</code> or the <code>Hdf5Saver</code>, respectively.

The basic structure of this class is similar as the *Unpickler* from pickle.

See Saving to disk: input/output for a specification of what can be saved and what the resulting datastructure is.

Parameters

22.1. hdf5_io 705

- h5group (Group) The HDF5 group (or file) where to save the data.
- ignore_unknown (bool) Whether to just warn (True) or raise an Error (False) if a class to be loaded is not found.
- **exclude** (list of str) List of paths (possibly relative to h5group) for objects to be excluded from loading. References to the corresponding object are replaced by an instance of Hdf5Ignored. Of course, this might break other functions expecting correctly loaded data.

h5group

The HDF5 group (or HDF5 File) where to save the data.

Type Group

ignore_unknown

Whether to just warn (True) or raise an Error (False) if a class to be loaded is not found.

Type bool

dispatch_load

Mapping from one of the global REPR_* variables to (unbound) methods f of this class. The method is called as f (self, h5gr, type_info, subpath). The call to f should load and return an object obj from the h5py Group or Dataset h5gr; and memorize the loaded obj with $memorize_load()$. subpath is just the name of h5gr with a guaranteed '/' in the end. $type_info$ is often the REPR_* variable of the type or some other information about the type, which allows to use a single dispatch_load function for different datatypes.

Type dict

memo_load

A dictionary to remember all the objects which we already loaded from h5group. The dictionary key is a h5py group- or dataset id; the value is the loaded object. See memorize_load().

Type dict

load(path=None)

Load a Python object from the dataset.

See load_from_hdf5() for more details.

Parameters path (None | str | Reference) – Path within h5group to be used for loading. Defaults to the name of h5group itself.

Returns obj – The Python object loaded from *h5group* (specified by *path*).

Return type object

memorize load (h5gr, obj)

Store objects already loaded in the memo_load.

This allows to avoid copies, if the same dataset appears multiple times in the hdf5 group of *obj*. Examples can be shared *LegCharge* objects or even shared *Array*.

To handle cyclic references correctly, this function should be called *before* loading data from subgroups with new calls of <code>load()</code>.

static get_attr (h5gr, attr_name)

Return attribute h5gr.attrs[attr_name], if existent.

Raises Hdf5ImportError – If the attribute does not exist.

load_none (h5gr, type_info, subpath)

Load the None object from a dataset.

```
load_dataset (h5gr, type_info, subpath)
     Load a h5py Dataset and convert it into the desired type.
load_str (h5gr, type_info, subpath)
     Load a string from a h5py Dataset.
load_list (h5gr, type_info, subpath)
     Load a list.
load_set (h5gr, type_info, subpath)
     Load a set.
load_tuple (h5gr, type_info, subpath)
     Load a tuple.
load_dict (h5gr, type_info, subpath)
     Load a dictionary in the format according to type_info.
load_general_dict (h5gr, type_info, subpath)
     Load a dictionary with general keys.
load_simple_dict (h5gr, type_info, subpath)
     Load a dictionary with simple keys.
load_range (h5gr, type_info, subpath)
     Load a range.
load_dtype (h5gr, type_info, subpath)
     Load a numpy.dtype.
load_hdf5exportable (h5gr, type_info, subpath)
     Load an instance of a userdefined class.
load_ignored (h5gr, type_info, subpath)
     Ignore the group to be loaded.
load_global (h5gr, type_info, subpath)
     Load a global object like a class or function from its qualified name and module.
load_reduce (h5gr, type_info, subpath)
     Load an object where the return values of obj. __reduce__ has been exported.
```

22.1.4 Hdf5Saver

• full name: tenpy.tools.hdf5_io.Hdf5Saver

• parent module: tenpy.tools.hdf5_io

• type: class

22.1. hdf5_io 707

Inheritance Diagram

Hdf5Saver

Methods

Initialize self.
Create an HDF5 group self.h5group[path] to
store <i>obj</i> .
Store objects already saved in the memo_save.
Save obj in self.h5group[path].
Save <i>obj</i> as a hdf5 dataset; in dispatch table.
Save the dictionary <i>obj</i> ; in dispatch table.
Save contents of a dictionary obj in the existing $h5gr$.
Save a dtype object; in dispatch table.
Save a global object like a function or class.
Don't save the Hdf5Ignored object; just return None.
Save an iterable <i>obj</i> like a list, tuple or set; in dispatch
table.
Save contents of an iterable obj in the existing $h5gr$.
Save the None object as a string (dataset); in dispatch
table.
Save a range object; in dispatch table.
Save the return values of objreduce follow-
ing the pickle protocol.

Class Attributes and Properties

Hdf5Saver.dispatch_save

class tenpy.tools.hdf5_io.Hdf5Saver(h5group,format_selection=None)

Bases: object

Class to save simple enough objects into a HDF5 file.

The intended use of this class is through <code>save_to_hdf5()</code>, which is simply an alias for Hdf5Saver(h5group).save(obj, path).

It exports python objects to a HDF5 file such that they can be loaded with the <code>Hdf5Loader</code>, or a call to <code>load_from_hdf5()</code>, respectively.

The basic structure of this class is similar as the *Pickler* from pickle.

See Saving to disk: input/output for a specification of what can be saved and what the resulting datastructure is.

Parameters

- h5group (Group) The HDF5 group (or HDF5 File) where to save the data.
- **format_selection** (dict) This dictionary allows to set a output format selection for user-defined Hdf5Exportable.save_hdf5() implementations. For example, LegCharge checks it for the key "LegCharge".

h5group

The HDF5 group (or HDF5 File) where to save the data.

Type Group

dispatch_save

Mapping from a type keytype to methods f of this class. The method is called as f(self, obj, path, type_repr). The call to f should save the object obj in self.h5group[path], call $memorize_save()$, and set h5gr.attr[ATTR_TYPE] = type_repr to a string $type_repr$ in order to allow loading with the dispatcher in Hdf5Loader.dispatch save[type repr].

Type dict

memo save

A dictionary to remember all the objects which we already stored to h5group. The dictionary key is the object id; the value is a two-tuple of the hdf5 group or dataset where an object was stored, and the object itself. See $memorize_save()$.

Type dict

format_selection

This dictionary allows to set a output format selection for user-defined <code>Hdf5Exportable.save_hdf5()</code> implementations. For example, <code>LegCharge checks</code> it for the key "<code>LegCharge"</code>.

Type dict

save (obj, path='/')

Save obj in self.h5group[path].

Parameters

- **obj** (object) The object (=data) to be saved.
- path (str) Path within h5group under which the obj should be saved. To avoid unwanted overwriting of important data, the group/object should not yet exist, except if path is the default '/'.

Returns h5gr – The h5py group or dataset in which *obj* was saved.

Return type Group | Dataset

create_group_for_obj (path, obj)

Create an HDF5 group self.h5group[path] to store obj.

Also handle ending of path with '/', and memorize *obj* in memo_save.

Parameters

- path (str) Path within h5group under which the obj should be saved. To avoid unwanted overwriting of important data, the group/object should not yet exist, except if path is the default '/'.
- **obj** (object) The object (=data) to be saved.

22.1. hdf5 io 709

Returns

- **h5group** (Group) Newly created h5py (sub)group self.h5group[path], unless path is '/', in which case it is simply the existing self.h5group['/'].
- **subpath** (*str*) The *group.name* ending with '/', such that other names can be appended to get the path for subgroups or datasets in the group.

:raises ValueError: if self.h5group[path]` already existed and path is not '/'.:

$memorize_save(h5gr, obj)$

Store objects already saved in the memo_save.

This allows to avoid copies, if the same python object appears multiple times in the data of *obj*. Examples can be shared *LegCharge* objects or even shared *Array*. Using the memo also avoids crashes from cyclic references, e.g., when a list contains a reference to itself.

Parameters

- h5gr (Group | Dataset) The h5py group or dataset in which obj was saved.
- obj (object) The object saved.

Save the return values of obj. __reduce__ following the pickle protocol.

```
save_none (obj, path, type_repr)
```

Save the None object as a string (dataset); in dispatch table.

```
save_dataset (obj, path, type_repr)
```

Save *obj* as a hdf5 dataset; in dispatch table.

```
save_iterable (obj, path, type_repr)
```

Save an iterable *obj* like a list, tuple or set; in dispatch table.

```
save_iterable_content (obj, h5gr, subpath)
```

Save contents of an iterable obj in the existing h5gr.

Parameters

- obj (dict) The data to be saved
- h5gr (Group) h5py Group under which the keys and values of *obj* should be saved.
- **subpath** (str) Name of h5gr with '/' in the end.

```
save_dict (obj, path, type_repr)
```

Save the dictionary *obj*; in dispatch table.

```
save_dict_content (obj, h5gr, subpath)
```

Save contents of a dictionary *obj* in the existing *h5gr*.

The format depends on whether the dictionary *obj* has simple keys valid for hdf5 path components (see *valid_hdf5_path_component()*) or not. For simple keys: directly use the keys as path. For non-simple keys: save list of keys und "keys" and list of values und "values".

Parameters

- **obj** (dict) The data to be saved
- h5gr (Group) h5py Group under which the keys and values of *obj* should be saved.
- **subpath** (str) Name of h5gr with '/' in the end.

Returns type_repr – Indicates whether the data was saved in the format for a dictionary with simple keys or general keys, see comment above.

Return type REPR_DICT_SIMPLE | REPR_DICT_GENERAL

save_range (obj, path, type_repr)

Save a range object; in dispatch table.

save_dtype (obj, path, type_repr)

Save a dtype object; in dispatch table.

save_ignored (obj, path, type_repr)

Don't save the Hdf5Ignored object; just return None.

save_global (obj, path, type_repr)

Save a global object like a function or class.

Exceptions

Hdf5ExportError	This exception is raised when something went wrong
	during export to hdf5.
Hdf5FormatError	Common base class for errors regarding our HDF5 for-
	mat.
Hdf5ImportError	This exception is raised when something went wrong
	during import from hdf5.

22.1.5 Hdf5ExportError

• full name: tenpy.tools.hdf5_io.Hdf5ExportError

• parent module: tenpy.tools.hdf5_io

• type: exception

exception tenpy.tools.hdf5_io.Hdf5ExportError

This exception is raised when something went wrong during export to hdf5.

22.1.6 Hdf5FormatError

• full name: tenpy.tools.hdf5_io.Hdf5FormatError

• parent module: tenpy.tools.hdf5_io

• type: exception

 $\textbf{exception} \ \texttt{tenpy.tools.hdf5_io.Hdf5FormatError}$

Common base class for errors regarding our HDF5 format.

22.1. hdf5_io 711

22.1.7 Hdf5ImportError

• full name: tenpy.tools.hdf5_io.Hdf5ImportError

• parent module: tenpy.tools.hdf5_io

• type: exception

exception tenpy.tools.hdf5_io.Hdf5ImportError

This exception is raised when something went wrong during import from hdf5.

Functions

<pre>find_global(module, qualified_name)</pre>	Get the object of the <i>qualified_name</i> in a given python <i>module</i> .
load(filename)	Load data from file with given filename.
load_from_hdf5(h5group[, path,])	Load an object from hdf5 file or group.
save(data, filename[, mode])	Save data to file with given filename.
save_to_hdf5(h5group, obj[, path])	Save an object <i>obj</i> into a hdf5 file or group.
<pre>valid_hdf5_path_component(name)</pre>	Determine if <i>name</i> is a valid HDF5 path component.

22.1.8 find_global

• full name: tenpy.tools.hdf5_io.find_global

• parent module: tenpy.tools.hdf5_io

• type: function

tenpy.tools.hdf5_io.find_global(module, qualified_name)

Get the object of the *qualified_name* in a given python *module*.

Parameters

- **module** (str) Name of the module containing the object. The module gets imported.
- qualified_name (str) Name of the object to be retrieved. May contain dots if the object is part of a class etc.

22.1.9 load

• full name: tenpy.tools.hdf5_io.load

• parent module: tenpy.tools.hdf5_io

• type: function

tenpy.tools.hdf5_io.load(filename)

Load data from file with given filename.

Guess the type of the file from the filename ending, see save () for possible endings.

Parameters filename (str) – The name of the file to load.

Returns data – The object loaded from the file.

Return type obj

22.1.10 load from hdf5

• full name: tenpy.tools.hdf5_io.load_from_hdf5

• parent module: tenpy.tools.hdf5_io

• type: function

```
tenpy.tools.hdf5_io.load_from_hdf5(h5group, path=None, ignore_unknown=True, ex-
clude=None)
```

Load an object from hdf5 file or group.

Roughly equivalent to obj = h5group [path] [...], but handle more complicated objects saved as hdf5 groups and/or datasets with <code>save_to_hdf5()</code>. For example, dictionaries are handled recursively. See <code>Saving to disk: input/output</code> for a specification of what can be saved/loaded and what the corresponding datastructure is.

Parameters

- h5group (Group) The HDF5 group (or h5py File) to be loaded.
- path (None | str | Reference) Path within *h5group* to be used for loading. Defaults to the *h5group* itself specified.
- ignore_unknown (bool) Whether to just warn (True) or raise an Error (False) if a class to be loaded is not found.
- **exclude** (list of str) List of paths (possibly relative to h5group) for objects to be excluded from loading. References to the corresponding object are replaced by an instance of Hdf5Ignored. For example, you could load a saved dictionary {'big_data': [...], 'small_data': small_data} with exclude=['/big_data'] to get {'big_data': Hdf5Ignored('/big_data'), 'small_data': small_data}. Of course, this might break other functions expecting correctly loaded data.

Returns obj – The Python object loaded from *h5group* (specified by *path*).

Return type object

22.1.11 save

• full name: tenpy.tools.hdf5_io.save

• parent module: tenpy.tools.hdf5_io

• type: function

```
tenpy.tools.hdf5_io.save(data, filename, mode='w')
```

Save data to file with given filename.

This function guesses the type of the file from the filename ending. Supported endings:

ending	description
.pkl	Pickle without compression
.pklz	Pickle with gzip compression.
.hdf5, .h5	HDF5 file (using $h5py$).

Parameters

• **filename** (str) – The name of the file where to save the data.

22.1. hdf5 io 713

• mode (str) – File mode for opening the file. 'w' for write (discard existing file), 'a' for append (add data to exisiting file). See open () for more details.

22.1.12 save_to_hdf5

• full name: tenpy.tools.hdf5_io.save_to_hdf5

• parent module: tenpy.tools.hdf5_io

• type: function

tenpy.tools.hdf5_io.save_to_hdf5(h5group, obj, path='/')

Save an object *obj* into a hdf5 file or group.

Roughly equivalent to h5group [path] = obj, but handle different types of *obj*. For example, dictionaries are handled recursively. See *Saving to disk: input/output* for a specification of what can be saved and what the resulting datastructure is.

Parameters

- h5group (Group) The HDF5 group (or h5py File) to which *obj* should be saved.
- **obj** (object) The object (=data) to be saved.
- **path** (str) Path within h5group under which the obj should be saved. To avoid unwanted overwriting of important data, the group/object should not yet exist, except if path is the default '/'.

Returns h5obj – The h5py group or dataset under which *obj* was saved.

Return type Group | Dataset

22.1.13 valid hdf5 path component

- full name: tenpy.tools.hdf5_io.valid_hdf5_path_component
- parent module: tenpy.tools.hdf5_io
- · type: function

tenpy.tools.hdf5_io.valid_hdf5_path_component(name)

Determine if *name* is a valid HDF5 path component.

Conditions: String, no '/', and overall name != '.'.

Module description

Tools to save and load data (from TeNPy) to disk.

Note: This module is maintained in the repository https://github.com/tenpy/hdf5_io.git

See *Saving to disk: input/output* for a motivation and specification of the HDF5 format implemented below. .. online at https://tenpy.readthedocs.io/en/latest/intro/input_output.html

The functions save() and load() are convenience functions for saving and loading quite general python objects (like dictionaries) to/from files, guessing the file type (and hence protocol for reading/writing) from the file ending.

On top of that, this function provides support for saving python objects to [HDF5] files with the <code>Hdf5Saver</code> and <code>Hdf5Loader</code> classes and the wrapper functions <code>save_to_hdf5()</code>, <code>load_from_hdf5()</code>.

Note: To use the export/import features to HDF5, you need to install the h5py python package (and hence some version of the HDF5 library).

Warning: Like loading a pickle file, loading data from a manipulated HDF5 file with the functions provided below has the potential to cause arbitrary code execution. Only load data from trusted sources!

Global module constants used for our HDF5 format

saved object represents a np.float32

```
Names of HDF5 attributes:
tenpy.tools.hdf5_io.ATTR_TYPE = 'type'
     Attribute name for type of the saved object, should be one of the REPR_*
tenpy.tools.hdf5 io.ATTR CLASS = 'class'
     Attribute name for the class name of an HDF5Exportable
tenpy.tools.hdf5_io.ATTR_MODULE = 'module'
     Attribute name for the module where ATTR_CLASS can be retrieved
tenpy.tools.hdf5_io.ATTR_LEN = 'len'
     Attribute name for the length of iterables, e.g, list, tuple
tenpy.tools.hdf5_io.ATTR_FORMAT = 'format'
     indicates the ATTR_TYPE format used by Hdf5Exportable
Names for the ATTR_TYPE attribute:
tenpy.tools.hdf5 io.REPR HDF5EXPORTABLE = 'instance'
     saved object is instance of a user-defined class following the Hdf5Exportable style.
tenpy.tools.hdf5_io.REPR_ARRAY = 'array'
     saved object represents a numpy array
tenpy.tools.hdf5 io.REPR INT = 'int'
     saved object represents a (python) int
tenpy.tools.hdf5_io.REPR_FLOAT = 'float'
     saved object represents a (python) float
tenpy.tools.hdf5_io.REPR_STR = 'str'
     saved object represents a (python unicode) string
tenpy.tools.hdf5_io.REPR_COMPLEX = 'complex'
     saved object represents a complex number
tenpy.tools.hdf5_io.REPR_INT64 = 'np.int64'
     saved object represents a np.int64
tenpy.tools.hdf5_io.REPR_FLOAT64 = 'np.float64'
     saved object represents a np.float64
tenpy.tools.hdf5_io.REPR_INT32 = 'np.int32'
     saved object represents a np.int32
tenpy.tools.hdf5 io.REPR FLOAT32 = 'np.float32'
```

22.1. hdf5_io 715

```
tenpy.tools.hdf5_io.REPR_BOOL = 'bool'
    saved object represents a boolean
tenpy.tools.hdf5_io.REPR_NONE = 'None'
    saved object is None
tenpy.tools.hdf5_io.REPR_RANGE = 'range'
    saved object is a range
tenpy.tools.hdf5_io.REPR_LIST = 'list'
    saved object represents a list
tenpy.tools.hdf5_io.REPR_TUPLE = 'tuple'
    saved object represents a tuple
tenpy.tools.hdf5_io.REPR_SET = 'set'
    saved object represents a set
tenpy.tools.hdf5_io.REPR_DICT_GENERAL = 'dict'
    saved object represents a dict with complicated keys
tenpy.tools.hdf5 io.REPR DICT SIMPLE = 'simple dict'
    saved object represents a dict with simple keys
tenpy.tools.hdf5_io.REPR_DTYPE = 'dtype'
    saved object represents a np.dtype
tenpy.tools.hdf5 io.REPR IGNORED = 'ignore'
    ignore the object/dataset during loading and saving
tenpy.tools.hdf5_io.TYPES_FOR_HDF5_DATASETS = ((<class 'numpy.ndarray'>, 'array'), (<class
    tuple of (type, type_repr) which h5py can save as datasets; one entry for each type.
```

Todo: For memory caching with big MPO environments, we need a Hdf5Cacher clearing the memo's every now and then (triggered by what?).

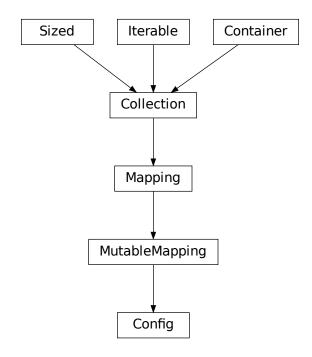
22.2 params

• full name: tenpy.tools.params

• parent module: tenpy.tools

• type: module

Classes



Config(config, name)	Dict-like wrapper class for parameter/configuration dic-
	tionaries.

Functions

asConfig(config, name)	Convert a dict-like config to a Config.
<pre>get_parameter(params, key, default, descr[,])</pre>	Read out a parameter from the dictionary and/or provide
	default values.
unused_parameters(params[, warn])	Returns a set of the parameters which have not been read
	out with get_parameters.

22.2. params 717

22.2.1 asConfig

• full name: tenpy.tools.params.asConfig

• parent module: tenpy.tools.params

· type: function

tenpy.tools.params.asConfig(config, name)

Convert a dict-like *config* to a Config.

Parameters

- config (dict | Config) If this is a Config, just return it. Otherwise, create a Config from it and return that.
- name (str) Name to be used for the Config.

Returns config – Either directly *config* or Config (config, name).

Return type Config

22.2.2 get_parameter

• full name: tenpy.tools.params.get_parameter

• parent module: tenpy.tools.params

· type: function

tenpy.tools.params.get_parameter(params, key, default, descr, asarray=False)

Read out a parameter from the dictionary and/or provide default values.

Deprecated since version 0.6.0: Use the Config instead.

This function provides a similar functionality as params.get(key, default). *Unlike dict.get* this function writes the default value into the dictionary (i.e. in other words it's more similar to params. setdefault(key, default)).

This allows the user to save the modified dictionary as meta-data, which gives a concrete record of the actually used parameters and simplifies reproducing the results and restarting simulations.

Moreover, a special entry with the key 'verbose' in the params can trigger this function to also print the used value. A higher verbose level implies more output. If verbose >= 100, it is printed every time it's used. If verbose >= 2, its printed for the first time time its used. and for verbose >= 1, non-default values are printed the first time they are used. otherwise only for the first use.

Internally, whether a parameter was used is saved in the set params ['_used_param']. This is used in unused_parameters() to print a warning if the key wasn't used at the end of the algorithm, to detect mis-spelled parameters.

Parameters

- params (dict) A dictionary of the parameters as provided by the user. If key is not a valid key, params [key] is set to default.
- **key** (string) The key for the parameter which should be read out from the dictionary.
- **default** The default value for the parameter.
- descr (str) A short description for verbose output, like 'TEBD', 'XXZ_model', 'truncation'.

• asarray (bool) - If True, convert the result to a numpy array with np.asarray (...) before returning.

Returns params [key] if the key is in params, otherwise *default*. Converted to a numpy array, if *asarray*.

Return type value

Examples

In the algorithm TEBDEngine gets a dictionary of parameters. Beside doing other stuff, it calls tenpy. models.model.NearestNeighborModel.calc_U_bond() with the dictionary as argument, which looks similar like:

```
>>> from tenpy.tools.params import get_parameter
>>> def model_calc_U(params):
...    dt = get_parameter(params, 'dt', 0.01, 'TEBD')
...    order = get_parameter(params, 'order', 1, 'TEBD')
...    print("calc U with dt =", dt, "and order =", order)
...    # ... calculate exp(-i * dt* H) ....
```

Then, when you call it without any parameters, it just uses the default value:

```
>>> model_calc_U(dict())
calc U with dt = 0.01 and order = 1
```

Of course you can also provide the parameter to use a non-default value:

```
>>> model_calc_U(dict(dt=0.02))
calc U with dt = 0.02 and order = 1
```

Increasing the special keyword 'verbose' generally prints more:

```
>>> model_calc_U(dict(dt=0.02, verbose=1))
parameter 'dt'=0.02 for TEBD
calc U with dt = 0.02 and order = 1
>>> model_calc_U(dict(dt=0.02, verbose=2))
parameter 'dt'=0.02 for TEBD
parameter 'order'=1 (default) for TEBD
calc U with dt = 0.02 and order = 1
```

22.2.3 unused_parameters

- full name: tenpy.tools.params.unused_parameters
- parent module: tenpy.tools.params
- · type: function

tenpy.tools.params.unused_parameters(params, warn=None)

Returns a set of the parameters which have not been read out with *get_parameters*.

This function might be useful to check for typos in the parameter keys.

Deprecated since version 0.6.0: Use the Config instead.

Parameters

22.2. params 719

- params (dict) A dictionary of parameters which was given to (functions using) get_parameter()
- warn (None / str) If given, print a warning "unused parameter for {warn!s}: {unused_keys!s}".

Returns unused_keys – The set of keys of the params which was not used **Return type** set

Module description

Tools to handle config options/paramters for algorithms.

See the doc-string of Config for details.

22.3 events

• full name: tenpy.tools.events

• parent module: tenpy.tools

• type: module

Classes

EventHandler

Listener

EventHandler([arg_descr])	Handler for an event represented by an instance of this
	class.
Listener(listener id callback priority)	

22.3.1 EventHandler

• full name: tenpy.tools.events.EventHandler

• parent module: tenpy.tools.events

• type: class

Inheritance Diagram

EventHandler

Methods

EventHandlerinit([arg_descr])	Initialize self.
EventHandler.connect([callback, priority])	Register a <i>callback</i> function as a listener to the event.
EventHandler.connect_by_name(module_name,	Connect to a function given by the name in a module,
)	optionally inserting arguments.
EventHandler.copy()	Make a (shallow) copy.
EventHandler.disconnect(listener_id)	De-register a listener.
EventHandler.emit(*args, **kwargs)	Call the <i>callback</i> functions of all listeners.
<pre>EventHandler.emit_until_result(*args,</pre>	Call the listeners <i>callback</i> until one returns not <i>None</i> .
**kwargs)	

Class Attributes and Properties

EventHandler.id_of_last_connected

class tenpy.tools.events.EventHandler(arg_descr=None)
 Bases: object

Handler for an event represented by an instance of this class.

All in all, events provide a flexible extension mechanism for classes to define "checkpoints" in the code where the user of a class might want to run something else, for example doing some measurements or saving intermediate results.

Parameters $arg_descr(str)$ – An informative description how the callback function is called. An empty string indicates no arguments.

arg descr

An informative description how the callback function is called.

Type str

listeners

Entries are tuples (listener_id, callback, priority).

Type list of (int, function, int)

Examples

22.3. events 721

Instances of this class typically get defined during class initialization and define an event. The event "happens" each time <code>emit()</code> or <code>emit_until_result()</code> is called, typically inside a method of the class defining the event. Example:

```
>>> class MyAlgorithm:
...    def __init__(self):
...        self.checkpoint = EventHandler("algorithm, iteration")
...        self.data = 0
...    def run(self):
...        for i in range(4):
...        self.data += i # do some complicated stuff
...        self.checkpoint.emit(self, i)
```

Other code with access to the event can then connect a *listener* to the event, i.e., give a function to the event that should be called each time the event is <code>emit()</code>-ed.

As you can see, the function $my_listener$ has been called during the MyAlgorithm.run() and had full access to the current status of the algorithm class. This is convenient to e.g. perform measurements of the state so far, print a status message of the progress or save intermediate results.

If the EventHandler is already initialized when you define the function, you can also use <code>connect()</code> as a function property like this:

```
>>> @my_alg.checkpoint.connect
... def another_one(algorithm, iteration):
      print("another_one called: iteration", iteration)
>>> @my_alg.checkpoint.connect(priority=5)
... def high_priority(algorithm, iteration):
       print("high_priority call: iteration", iteration)
>>> my_alg.run()
high_priority call: iteration 0
my_listener called: iteration 0 with data 6
another_one called: iteration 0
high_priority call: iteration 1
my_listener called: iteration 1 with data 7
another_one called: iteration 1
high_priority call: iteration 2
my listener called: iteration 2 with data 9
another_one called: iteration 2
high_priority call: iteration 3
my_listener called: iteration 3 with data 12
another_one called: iteration 3
```

copy()

Make a (shallow) copy.

connect (callback=None, priority=0)

Register a *callback* function as a listener to the event.

You can either call this function directly or use it as a function decorator, see the example in *EventHandler*.

If you ever plan to <code>disconnect()</code> again, you can read it out with <code>id_of_last_connected</code> right after connecting, i.e., right after calling this method.

Parameters

- callback (callable) A function to be called during each emit () of the event.
- **priority** (*int*) Higher priority indicates that the callback function should be called before other possibly registered callback functions.

Returns callback – The callback function exactly as given.

Return type callable

connect_by_name (module_name, func_name, kwargs=None, priority=0)

Connect to a function given by the name in a module, optionally inserting arguments.

Parameters

- **module_name** (*str*) The name of the module containing the function to be used. Gets imported.
- **func_name** (str) The (qualified) name of the function inside the module.
- **kwargs** (dict) Optional extra keyword-arguments to be given to the function.
- **priority** (*int*) Higher priority indicates that the callback function should be called before other possibly registered callback functions.

disconnect (listener_id)

De-register a listener.

Parameters listener_id (*int*) - The id of the listener, as given by id_of_last_connected right after calling *connect*().

```
emit (*args, **kwargs)
```

Call the *callback* functions of all listeners.

Returns results – List of results returned by the individual callback functions.

Return type list

```
emit_until_result(*args, **kwargs)
```

Call the listeners callback until one returns not None.

22.3.2 Listener

- full name: tenpy.tools.events.Listener
- parent module: tenpy.tools.events
- type: class

22.3. events 723

Inheritance Diagram

Listener

Methods

Listenerinit()	Initialize self.
Listener.count(value,/)	Return number of occurrences of value.
Listener.index(value[, start, stop])	Return first index of value.

Class Attributes and Properties

Listener.callback	Alias for field number 1
Listener.listener_id	Alias for field number 0
Listener.priority	Alias for field number 2

class tenpy.tools.events.Listener(listener_id, callback, priority)

Bases: tuple

property callback

Alias for field number 1

count (value,/)

Return number of occurrences of value.

index (value, start=0, stop=9223372036854775807,/)

Return first index of value.

Raises ValueError if the value is not present.

property listener_id

Alias for field number 0

property priority

Alias for field number 2

Module description

Event handler.

The *EventHandler* is basically just holds a list of functions which can get called once a certain "event" happens. Examples are given in the class doc-string.

22.4 misc

• full name: tenpy.tools.misc

• parent module: tenpy.tools

• type: module

Functions

	The state of the s
add_with_None_0(a, b)	Return a + b, treating <i>None</i> as zero.
<pre>any_nonzero(params, keys[, verbose_msg])</pre>	Check for any non-zero or non-equal entries in some
	parameters.
anynan(a)	check whether any entry of a ndarray a is 'NaN'.
argsort(a[, sort])	wrapper around np.argsort to allow sorting ascend-
	ing/descending and by magnitude.
atleast_2d_pad(a[, pad_item])	Transform a into a 2D array, filling missing places with
	pad_item.
build_initial_state(size, states, filling[,])	
chi_list(chi_max[, dchi, nsweeps, verbose])	
_ , _ , _ , _ , _ , _ , _ , _ , _ , _ ,	
find_subclass(base_class, subclass_name)	For a given base class, recursively find the subclass with
_ , _ , _ , ,	the given name.
flatten(mapping[, separator])	Obtain a flat dictionary with all key/value pairs of a
	nested data structure.
get_close(values, target[, default, eps])	Iterate through <i>values</i> and return first entry closer than
	eps.
get_recursive(nested_data, recursive_key[,])	Extract specific value from a nested data structure.
<pre>group_by_degeneracy(E, *args[, subset, cutoff])</pre>	Find groups of indices for which (energy) values are de-
	generate.
inverse_permutation(perm)	reverse sorting indices.
lexsort(a[, axis])	wrapper around np.lexsort: allow for trivial case
	a.shape[0] = 0 without sorting
list_to_dict_list(l)	Given a list l of objects, construct a lookup table.
pad(a[, w_l, v_l, w_r, v_r, axis])	Pad an array along a given axis.
set_recursive(nested_data, recursive_key, value)	Same as get_recursive(), but set the data entry to
	value.
setup_executable(mod, run_defaults[,])	Read command line arguments and turn into useable
- · · · · - · · · · · · · · · · · · · ·	dicts.
setup_logging([options, output_filename])	Configure the logging module.
to_array(a[, shape, dtype])	Convert a to an numpy array and tile to matching di-
	mension/shape.
	continues on next page
	continues on none page

22.4. misc 725

Table 18 – co	ntinued from previous page
to_iterable(a)	If a is a not iterable or a string, return [a], else return
	a.
to_iterable_of_len(a, L)	If a is a non-string iterable of length L, return a, other-
	wise return [a]*L.
transpose_list_list(D[, pad])	Returns a list of lists T, such that T[i][j] =
	D[j][i].
update_recursive(nested_data, update_	data[, Wrapper around set_recursive() to allow updat-
])	ing multiple values at once.
zero_if_close(a[, tol])	set real and/or imaginary part to 0 if their absolute value
	is smaller than tol.

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22.4.1 add_with_None_0

• full name: tenpy.tools.misc.add_with_None_0

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.add_with_None_0 (a, b)

Return a + b, treating *None* as zero.

Parameters

- a The two things to be added, or None.
- **b** The two things to be added, or None.

Returns a + b, except if a or b is *None*, in which case the other variable is returned.

Return type sum

22.4.2 any_nonzero

• full name: tenpy.tools.misc.any nonzero

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.any_nonzero(params, keys, verbose_msg=None)

Check for any non-zero or non-equal entries in some parameters.

Deprecated since version 0.8.0: This method will be removed in version 1.0.0. Use tenpy.toosl.params. Config.any_nonzero() instead.

Parameters

- \bullet params (dict / Config) A dictionary of parameters, or a Config instance.
- **keys** (list of {key | tuple of keys}) For a single key, check params[key] for non-zero entries. For a tuple of keys, all the params[key] have to be equal (as numpy arrays).
- **verbose_msg** (None / str) If params['verbose'] >= 1, we print *verbose_msg* before checking, and a short notice with the *key*, if a non-zero entry is found.

Returns match – False, if all params[key] are zero or *None* and True, if any of the params[key] for single *key* in *keys*, or if any of the entries for a tuple of *keys*

Return type bool

22.4.3 anynan

• full name: tenpy.tools.misc.anynan

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.anynan(a)

check whether any entry of a ndarray a is 'NaN'.

22.4.4 argsort

• full name: tenpy.tools.misc.argsort

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.argsort(a, sort=None, **kwargs)

wrapper around np.argsort to allow sorting ascending/descending and by magnitude.

Parameters

- a (array_like) The array to sort.
- sort ('m>', 'm<', '>', '<', None) Specify how the arguments should be sorted.

sort	order
'm>', 'LM'	Largest magnitude first
'm<', 'SM'	Smallest magnitude first
'>', 'LR', 'LA'	Largest real part first
'<', 'SR', 'SA'	Smallest real part first
'LI'	Largest imaginary part first
'SI'	Smallest imaginary part first
None	numpy default: same as '<'

• **kwargs - Further keyword arguments given directly to numpy.argsort().

Returns index_array – Same shape as *a*, such that a [index_array] is sorted in the specified way.

Return type ndarray, int

22.4. misc 727

22.4.5 atleast 2d pad

```
• full name: tenpy.tools.misc.atleast_2d_pad
```

• parent module: tenpy.tools.misc

• type: function

```
tenpy.tools.misc.atleast_2d_pad(a, pad_item=0)
```

Transform a into a 2D array, filling missing places with pad_item.

Given a list of lists, turn it to a 2D array (pad with 0), or turn a 1D list to 2D.

Parameters a (list of lists) – to be converted into ad 2D array.

Returns a_2D – a converted into a numpy array.

Return type 2D ndarray

Examples

```
>>> atleast_2d_pad([3, 4, 0])
array([[3, 4, 0]])
```

22.4.6 build_initial_state

• full name: tenpy.tools.misc.build_initial_state

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.build_initial_state(size, states, filling, mode='random', seed=None)

22.4.7 chi_list

• full name: tenpy.tools.misc.chi_list

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.chi_list(chi_max, dchi=20, nsweeps=20, verbose=0)

22.4.8 find subclass

- full name: tenpy.tools.misc.find_subclass
- parent module: tenpy.tools.misc
- · type: function

tenpy.tools.misc.find_subclass(base_class, subclass_name)

For a given base class, recursively find the subclass with the given name.

Parameters

- base_class (class) The base class of which *subclass_name* is supposed to be a subclass.
- $subclass_name(str)$ Name of the class to be found.

Returns subclass – Class with name *subclass_name* which is a subclass of the *base_class*. None, if no subclass of the given name is found.

Return type None | class

22.4.9 flatten

- full name: tenpy.tools.misc.flatten
- parent module: tenpy.tools.misc
- type: function

tenpy.tools.misc.flatten(mapping, separator='/')

Obtain a flat dictionary with all key/value pairs of a nested data structure.

Parameters separator (str) – Separator for merging keys to a single string.

Returns flat_config – A single dictionary with all key-value pairs.

Return type dict

Examples

See also:

get_recursive Useful to obtain a single entry from a nested data structure.

22.4. misc 729

22.4.10 get close

- full name: tenpy.tools.misc.get_close
- parent module: tenpy.tools.misc
- type: function

tenpy.tools.misc.get_close (values, target, default=None, eps=1e-13)

Iterate through values and return first entry closer than eps.

Parameters

- values (interable of float) Values to compare to.
- target (float) Value to find.
- **default** Returned if no value close to *target* is found.
- **eps** (*float*) Tolerance what counts as "close", namely everything with abs(val-target) < eps.

Returns value – An entry of *values*, if one close to *target* is found, otherwise *default*.

Return type float

22.4.11 get_recursive

- full name: tenpy.tools.misc.get_recursive
- parent module: tenpy.tools.misc
- type: function

tenpy.tools.misc.get_recursive (nested_data, recursive_key, separator='/') Extract specific value from a nested data structure.

Parameters

- nested_data (dict of dict (-like)) Some nested data structure supporting a dict-like interface.
- **recursive_key** (*str*) The key(-parts) to be extracted, separated by *separator*. A leading *separator* is ignored.
- **separator** (*str*) Separator for splitting *recursive_key* into subkeys.

Returns For example, recursive_key="/some/sub/key" will result in extracing nested_data["some"]["sub"]["key"].

Return type entry

See also:

set_recursive same for changing/setting a value.

flatten Get a completely flat structure.

22.4.12 group_by_degeneracy

- full name: tenpy.tools.misc.group_by_degeneracy
- parent module: tenpy.tools.misc
- type: function

tenpy.tools.misc.group_by_degeneracy (*E*, *args, subset=None, cutoff=1e-12) Find groups of indices for which (energy) values are degenerate.

Parameters

- values (1D array) Values (e.g. energies) which need to be close to count as degenerate.
- *args (1D array) Additional vectors (with same length as *values*), which also need to be close (up to cutoff) to count as degenerate.
- subset (1D array) Optionally selects a subset of the indices
- **cutoff** (float) Precision up to which values still count as degenerate.

Returns

- idx_groups (list of tuple of int) Each tuple group contains indices i, j, k, ... for which the values are closer than cutoff, i.e., |E[j, k, ...] E[i] | <= cutoff. Each index appears exactly once (if it is containted in subset).
- .. testsetup :: from tenpy.tools.misc import *
- >>> E = [2., 2.4, 1.9999, 1.8, 2.3999, 5, 1.8]
- ... #-> 0123456
- >>> k = [0, 1, 2, 2, 1, 2, 1]
- >>> group_by_degeneracy(E, cutoff=0.001)
- *[*(0, 2), (1, 4), (3, 6), (5,)]
- >>> group_by_degeneracy(E, k, cutoff=0.001) # k and E need to be close
- [(0,), (1, 4), (2,), (3,), (5,), (6,)]

22.4.13 inverse_permutation

- full name: tenpy.tools.misc.inverse_permutation
- parent module: tenpy.tools.misc
- type: function

tenpy.tools.misc.inverse_permutation(perm) reverse sorting indices.

Sort functions (as LegCharge.sort()) return a (1D) permutation perm array, such that sorted_array = old_array[perm]. This function inverts the permutation perm, such that old_array = sorted_array[inverse_permutation(perm)].

Parameters perm (1D array_like) - The permutation to be reversed.

Assumes that it is a permutation with unique indices. If it is, inverse_permutation(inverse_permutation(perm)) == perm.

22.4. misc 731

```
Returns inv_perm – The inverse permutation of perm such that inv_perm[perm[j]] = j =
              perm[inv_perm[j]].
          Return type 1D array (int)
22.4.14 lexsort
   • full name: tenpy.tools.misc.lexsort
   • parent module: tenpy.tools.misc
   • type: function
tenpy.tools.misc.lexsort(a, axis=-1)
     wrapper around np.lexsort: allow for trivial case a.shape[0] = 0 without sorting
22.4.15 list to dict list
   • full name: tenpy.tools.misc.list_to_dict_list
   • parent module: tenpy.tools.misc
   • type: function
tenpy.tools.misc.list_to_dict_list(l)
     Given a list l of objects, construct a lookup table.
     This function will handle duplicate entries in l.
          Parameters 1 (iterable of iterabele of immutable) - A list of objects that can be
              converted to tuples to be used as keys for a dictionary.
          Returns lookup - A dictionary with (key, value) pairs (key):[i1,i2,...] where i1,
              i2, ... are the indices where key is found in l: i.e. key == tuple(l[i1]) ==
              tuple(l[i2]) == ...
          Return type dict
22.4.16 pad
   · full name: tenpy.tools.misc.pad
   • parent module: tenpy.tools.misc
   • type: function
tenpy.tools.misc.pad(a, w_l = 0, v_l = 0, w_r = 0, v_r = 0, axis = 0)
     Pad an array along a given axis.
          Parameters
                • a (ndarray) – the array to be padded
```

- w_1 (int) the width to be padded in the front
- **v_1** (*dtype*) the value to be inserted before *a*
- w r (int) the width to be padded after the last index
- **v_r** (dtype) the value to be inserted after a
- axis (int) the axis along which to pad

Returns padded – a copy of a with enlarged axis, padded with the given values.

Return type ndarray

22.4.17 set recursive

• full name: tenpy.tools.misc.set recursive

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.set_recursive (nested_data, recursive_key, value, separator='/', insert_dicts=False)
Same as get_recursive(), but set the data entry to value.

22.4.18 setup_executable

• full name: tenpy.tools.misc.setup_executable

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.setup_executable (mod, run_defaults, identifier_list=None)
Read command line arguments and turn into useable dicts.

Warning: this is a deprecated interface. Use the Simulation interface in combination with $console_main()$ instead. You can invoce that from the command line as python -m tenpy ...

Uses default values defined at: - model class for model_par - here for sim_par - executable file for run_par Alternatively, a model_defaults dictionary and identifier_list can be supplied without the model

NB: for setup_executable to work with a model class, the model class needs to define two things:

- defaults, a static (class level) dictionary with (key, value) pairs that have the name of the parameter (as string) as key, and the default value as value.
- identifier, a static (class level) list or other iterable with the names of the parameters to be used in filename identifiers.

Parameters

- mod (model / dict) Model class (or instance) OR a dictionary containing model defaults
- run_defaults (dict) default values for executable file parameters
- identifier_list (ieterable) Used only if mod is a dict. Contains the identifier variables

Returns

- model_par, sim_par, run_par (dict) containing all parameters.
- args namespace with raw arguments for some backwards compatibility with executables.

22.4. misc 733

22.4.19 to array

• full name: tenpy.tools.misc.to_array

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.to_array(a, shape=(None), dtype=None)

Convert a to an numpy array and tile to matching dimension/shape.

This function provides similar functionality as numpys broadcast, but not quite the same: Only scalars are broadcasted to higher dimensions, for a non-scalar, we require the number of dimension to match. If the shape does not match, we repeat periodically, e.g. we tile $(3, 4) \rightarrow (6, 16)$, but $(4, 4) \rightarrow (6, 16)$ will raise an error.

Parameters

- **a** (scalar | array_like) The input to be converted to an array. A scalar is reshaped to the desired dimension.
- **shape** (tuple of {None | int}) The desired shape of the array. An entry None indicates arbitrary len >=1. For int entries, tile the array periodically to fit the len.
- dtype Optionally specifies the data type.

Returns a_array – A copy of a converted to a numpy ndarray of desired dimension and shape.

Return type ndarray

22.4.20 to iterable

• full name: tenpy.tools.misc.to_iterable

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.to_iterable(a)

If *a* is a not iterable or a string, return [a], else return a.

22.4.21 to iterable of len

• full name: tenpy.tools.misc.to_iterable_of_len

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.to_iterable_of_len(a, L)

If a is a non-string iterable of length L, return a, otherwise return [a]*L.

Raises ValueError if a is already an iterable of different length.

22.4.22 transpose list list

```
• full name: tenpy.tools.misc.transpose_list_list
```

```
• parent module: tenpy.tools.misc
```

• type: function

```
\texttt{tenpy.tools.misc.transpose\_list\_list}(D, pad = None)
```

```
Returns a list of lists T, such that T[i][j] = D[j][i].
```

Parameters

- **D** (list of list) to be transposed
- pad Used to fill missing places, if D is not rectangular.

```
Returns T – transposed, rectangular version of D. constructed such that T[i][j] = D[j][i] if i < len(D[j]) else pad
```

Return type list of lists

22.4.23 update_recursive

- full name: tenpy.tools.misc.update_recursive
- parent module: tenpy.tools.misc
- · type: function

```
tenpy.tools.misc.update_recursive(nested_data, update_data, separator='/', in-
sert dicts=True)
```

Wrapper around set_recursive() to allow updating multiple values at once.

It simply calls set_recursive() for each recursive_key, value in update_data.items().

22.4.24 zero_if_close

- full name: tenpy.tools.misc.zero if close
- parent module: tenpy.tools.misc
- type: function

```
tenpy.tools.misc.zero_if_close(a, tol=1e-15)
```

set real and/or imaginary part to 0 if their absolute value is smaller than tol.

Parameters

- a (ndarray) numpy array to be rounded
- tol (float) the threashold which values to consider as '0'.

22.4. misc 735

Module description

Miscellaneous tools, somewhat random mix yet often helpful.

22.5 math

• full name: tenpy.tools.math

• parent module: tenpy.tools

• type: module

Functions

	Calculate the entropy of a distribution.
entropy(p[, n])	17
gcd(a, b)	Computes the greatest common divisor (GCD) of two
	numbers.
gcd_array(a)	Return the greatest common divisor of all of entries in a
1 cm(a, b)	Returns the least common multiple (LCM) of two posi-
	tive numbers.
matvec_to_array(H)	transform an linear operator with a matvec method into
	a dense numpy array.
perm_sign(p)	Given a permutation p of numbers, returns its sign.
$qr_li(A[, cutoff])$	QR decomposition with cutoff to discard nearly linear
	dependent columns in Q .
rq_li(A[, cutoff])	RQ decomposition with cutoff to discard nearly linear
	dependent columns in Q .
speigs(A, k, *args, **kwargs)	Wrapper around scipy.sparse.linalg.
	eigs(), lifting the restriction $k < rank(A)-1$.
speigsh(A, k, *args, **kwargs)	Wrapper around scipy.sparse.linalg.
	eigsh(), lifting the restriction $k < rank(A) -1$.

22.5.1 entropy

• full name: tenpy.tools.math.entropy

• parent module: tenpy.tools.math

• type: function

tenpy.tools.math.entropy(p, n=1)

Calculate the entropy of a distribution.

Assumes that p is a normalized distribution (np.sum (p) ==1.).

Parameters

- **p** (1D array) A normalized distribution.
- n(1 / float / np.inf) Selects the entropy, see below.

Returns entropy – Shannon-entropy $-\sum_i p_i \log(p_i)$ (n=1) or Renyi-entropy $\frac{1}{1-n} \log(\sum_i p_i^n)$ (n!=1) of the distribution p.

Return type float

22.5.2 gcd

```
• full name: tenpy.tools.math.gcd
```

```
• parent module: tenpy.tools.math
```

• type: function

```
tenpy.tools.math.gcd(a, b)
```

Computes the greatest common divisor (GCD) of two numbers.

Return 0 if both a, b are zero, otherwise always return a non-negative number.

22.5.3 gcd_array

```
• full name: tenpy.tools.math.gcd_array
```

```
• parent module: tenpy.tools.math
```

• type: function

```
tenpy.tools.math.gcd_array(a)
```

Return the greatest common divisor of all of entries in a

22.5.4 lcm

```
• full name: tenpy.tools.math.lcm
```

```
• parent module: tenpy.tools.math
```

· type: function

```
tenpy.tools.math.lcm (a, b)
```

Returns the least common multiple (LCM) of two positive numbers.

22.5.5 matvec to array

```
• full name: tenpy.tools.math.matvec_to_array
```

```
• parent module: tenpy.tools.math
```

• type: function

```
tenpy.tools.math.matvec_to_array(H)
```

transform an linear operator with a *matvec* method into a dense numpy array.

Parameters H (*linear operator*) – should have *shape*, *dtype* attributes and a *matvec* method.

Returns \mathbf{H}_{-} **dense** – a dense array version of H.

Return type ndarray, shape (H.dim, H.dim)

22.5. math 737

22.5.6 perm_sign

```
full name: tenpy.tools.math.perm_sign
parent module: tenpy.tools.math
type: function
tenpy.tools.math.perm_sign(p)
Given a permutation p of numbers, returns its sign. (+1 or -1)
```

Assumes that all the elements are distinct, if not, you get crap.

Examples

22.5.7 qr_li

- full name: tenpy.tools.math.qr_li
- parent module: tenpy.tools.math
- type: function

```
tenpy.tools.math.qr_li(A, cutoff=1e-15)
```

QR decomposition with cutoff to discard nearly linear dependent columns in Q.

Perform a QR decomposition with pivoting, discard columns where R[i,i] < cuttoff, reverse the permututation from pivoting and perform another QR decomposition to ensure that R is upper right.

```
Parameters A (numpy.ndarray) – Matrix to be decomposed as A = Q.R
```

Returns Q, R – Decomposition of A into isometry $Q^{A}Q = 1$ and upper right R with diagonal entries larger than *cutoff*.

Return type numpy.ndarray

22.5.8 rq_li

- full name: tenpy.tools.math.rq_li
- parent module: tenpy.tools.math
- type: function

```
tenpy.tools.math.rq_li(A, cutoff=1e-15)
```

RQ decomposition with cutoff to discard nearly linear dependent columns in Q.

Uses qr_1i () on transpose of A. Note that R is nonzero in the lowest left corner; R has entries below the diagonal for non-square R.

```
Parameters A (numpy.ndarray) – Matrix to be decomposed as A = Q.R
```

Returns R, Q – Decomposition of A into isometry Q $Q^{\Lambda}d = 1$ and upper right R with diagonal entries larger than *cutoff*. If M, N = A.shape, then R.shape = M, K and Q.shape = K, N with K <= min(M, N).

Return type numpy.ndarray

22.5.9 speigs

- full name: tenpy.tools.math.speigs
- parent module: tenpy.tools.math
- type: function

```
tenpy.tools.math.speigs(A, k, *args, **kwargs)
```

Wrapper around scipy.sparse.linalg.eigs(), lifting the restriction k < rank(A) - 1.

Parameters

- **A** (MxM ndarray or like scipy.sparse.linalg.LinearOperator) the (square) linear operator for which the eigenvalues should be computed.
- \mathbf{k} (int) the number of eigenvalues to be computed.
- *args Further arguments directly given to scipy.sparse.linalg.eigs()
- **kwargs Further keyword arguments directly given to scipy.sparse.linalg. eigs()

Returns

- w (ndarray) array of min(k, A.shape[0]) eigenvalues
- v (*ndarray*) array of min(k, A.shape[0]) eigenvectors, v[:, i] is the *i*-th eigenvector. Only returned if kwargs['return_eigenvectors'] == True.

22.5.10 speigsh

- full name: tenpy.tools.math.speigsh
- parent module: tenpy.tools.math
- type: function

```
tenpy.tools.math.speigsh (A, k, *args, **kwargs)
```

Wrapper around scipy.sparse.linalg.eigsh(), lifting the restriction k < rank(A) - 1.

Parameters

- A (MxM ndarray or like scipy.sparse.linalg.LinearOperator) The (square) hermitian linear operator for which the eigenvalues should be computed.
- \mathbf{k} (int) The number of eigenvalues to be computed.
- *args Further arguments directly given to scipy.sparse.linalq.eiqsh().
- **kwargs Further keyword arguments directly given to scipy.sparse.linalg. eigsh().

Returns

• w (*ndarray*) – Array of min(k, A.shape[0]) eigenvalues.

22.5. math 739

• v (ndarray) - Array of min(k, A.shape[0]) eigenvectors, v[:, i] is the *i*-th eigenvector. Only returned if kwargs['return_eigenvectors'] == True.

Module description

Different math functions needed at some point in the library.

tenpy.tools.math.LeviCivita3 = array([[[0, 0, 0], [0, 0, 1], [0, -1, 0]], [[0, 0, -1], Levi-Civita Symbol of int type

22.6 fit

• full name: tenpy.tools.fit

• parent module: tenpy.tools

• type: module

Functions

$alg_decay(x, a, b, c)$	define the algebraic decay.
<pre>alg_decay_fit(x, y[, npts, power_range,])</pre>	Fit y to the form $a*x**(-b) + c$.
alg_decay_fit_res(log_b, x, y)	Returns the residue of an algebraic decay fit of the form
	$x**(-np.exp(log_b)).$
<pre>alg_decay_fits(x, ys[, npts, power_range,])</pre>	Fit arrays of y's to the form $a * x**(-b) + c$.
central_charge_from_S_profile(psi[,	Fit the entanglement entropy of a finite MPS to the ex-
exclude])	pected profile for critical models.
entropy_profile_from_CFT(size_A, L,)	Expected profile for the entanglement entropy at a criti-
	cal point.
$fit_with_sum_of_exp(f, n[, N])$	Approximate a decaying function f with a sum of expo-
	nentials.
lin_fit_res(x, y)	Returns the least-square residue of a linear fit y vs x.
linear_fit(x, y)	Perform a linear fit of y to ax + b.
<pre>plot_alg_decay_fit(plot_module, x, y, fit_par)</pre>	Given x, y, and fit_par (output from alg_decay_fit), pro-
	duces a plot of the algebraic decay fit.
<pre>sum_of_exp(lambdas, prefactors, x)</pre>	Evaluate sum_i prefactor[i] *
	lambda[i] $\star \star x$ for different x .

22.6.1 alg_decay

• full name: tenpy.tools.fit.alg_decay

• parent module: tenpy.tools.fit

• type: function

tenpy.tools.fit.alg_decay (x, a, b, c) define the algebraic decay.

22.6.2 alg decay fit

```
full name: tenpy.tools.fit.alg_decay_fit
parent module: tenpy.tools.fit
type: function
tenpy.tools.fit.alg_decay_fit (x, y, npts=5, power_range=(0.01, 4.0), power_mesh=[60, 10])
Fit y to the form a*x** (-b) + c.
```

Returns a triplet [a, b, c].

npts specifies the maximum number of points to fit. If npts < len(x), then alg_decay_fit() will only fit to the last npts points. power_range is a tuple that gives that restricts the possible ranges for b. power_mesh is a list of numbers, which specifies how fine to search for the optimal b. E.g., if power_mesh = [60,10], then it'll first divide the power_range into 60 intervals, and then divide those intervals by 10.

22.6.3 alg_decay_fit_res

```
• full name: tenpy.tools.fit.alg_decay_fit_res
```

```
• parent module: tenpy.tools.fit
```

· type: function

```
tenpy.tools.fit.alg_decay_fit_res(log_b, x, y)
```

Returns the residue of an algebraic decay fit of the form $x**(-np.exp(log_b))$.

22.6.4 alg_decay_fits

```
• full name: tenpy.tools.fit.alg_decay_fits
```

```
• parent module: tenpy.tools.fit
```

• type: function

```
tenpy.tools.fit.alg_decay_fits (x, ys, npts=5, power\_range=(0.01, 4.0), power\_mesh=[60, 10]) Fit arrays of y's to the form a * x**(-b) + c.
```

Returns arrays of [a, b, c].

22.6.5 central_charge_from_S_profile

```
• full name: tenpy.tools.fit.central charge from S profile
```

```
• parent module: tenpy.tools.fit
```

• type: function

```
tenpy.tools.fit.central_charge_from_S_profile(psi, exclude=None)
```

Fit the entanglement entropy of a finite MPS to the expected profile for critical models.

See entropy_profile_from_CFT() for the function we fit to.

Parameters

• **psi** (MPS) – Ground state of a *finite* system at a critical point (i.e. gapless!). The bond dimension should be large enough to be converged!

22.6. fit 741

• **exclude** (*int*) – How many sites at the left (and at the right) boundary to exclude from the fit (to avoid boundary effects). Defaults to psi.L // 4

Returns

- **central_charge**, **const** (*float*) Central charge and constant offset as in entropy_profile_from_CFT().
- res (float) Residuum of the error.

22.6.6 entropy profile from CFT

- full name: tenpy.tools.fit.entropy_profile_from_CFT
- parent module: tenpy.tools.fit
- type: function

tenpy.tools.fit.entropy_profile_from_CFT(size_A, L, central_charge, const)

Expected profile for the entanglement entropy at a critical point.

Conformal field theory predicts the entanglement entropy for cutting a ground state of a finite, critical (i.e. gapless) system of length L into the left l and right L-l sites to be (eq. 2 of [[calabrese2004]]):

$$S(l, L) = \frac{c}{6} \log \left(\frac{2L}{\pi a} \sin \left(\frac{\pi l}{L} \right) \right) + \text{const}$$

Here, c is the central charge of the system, and a is the lattice spacing, which we set to 1, and const is a non-universal constant.

Returns exactly that formula.

22.6.7 fit_with_sum_of_exp

- full name: tenpy.tools.fit.fit_with_sum_of_exp
- parent module: tenpy.tools.fit
- · type: function

tenpy.tools.fit.fit_with_sum_of_exp(f, n, N=50)

Approximate a decaying function f with a sum of exponentials.

MPOs can naturally represent long-range interactions with an exponential decay. A common technique for other (e.g. powerlaw) long-range interactions is to approximate them by sums of exponentials and to include them into the MPOs. This function allows to do that.

The algorithm/implementation follows the appendix of [[murg2010]].

Parameters

- **f** (function) Decaying function to be approximated. Needs to accept a 1D numpy array x
- **n** (*int*) Number of exponentials to be used.
- N (int) Number of points at which to evaluate/fit f; we evaluate and fit f at the points x = np.arange(1, N+1).

Returns lambdas, prefactors – Such that $f(k) \approx \sum_i x_i \lambda_i^k$ for (integer) $1 \le k \le N$. The function $sum_of_exp()$ evaluates this for given x.

Return type 1D arrays

22.6.8 lin fit res

```
full name: tenpy.tools.fit.lin_fit_res
parent module: tenpy.tools.fit
type: function
tenpy.tools.fit.lin_fit_res(x, y)
Returns the least-square residue of a linear fit y vs x.
```

22.6.9 linear fit

```
full name: tenpy.tools.fit.linear_fit
parent module: tenpy.tools.fit
type: function
tenpy.tools.fit.linear_fit(x, y)
Perform a linear fit of y to ax + b.
Returns a, b, res.
```

22.6.10 plot_alg_decay_fit

```
full name: tenpy.tools.fit.plot_alg_decay_fit
parent module: tenpy.tools.fit
type: function
tenpy.tools.fit.plot_alg_decay_fit (plot_module, x, y, fit_par, xfunc=None, kwargs={}, plot_fit_args={})
Given x, y, and fit_par (output from alg_decay_fit), produces a plot of the algebraic decay fit.
```

plot_module is matplotlib.pyplot, or a subplot. x, y are the data (real, 1-dimensional np.ndarray) fit_par is a triplet of numbers [a, b, c] that describes and algebraic decay (see alg_decay()). xfunc is an optional parameter that scales the x-axis in the resulting plot. kwargs is a dictionary, whoses key/items are passed to the plot function. plot_fit_args is a dictionary that controls how the fit is shown.

22.6.11 sum_of_exp

```
full name: tenpy.tools.fit.sum_of_exp
parent module: tenpy.tools.fit
type: function
tenpy.tools.fit.sum_of_exp(lambdas, prefactors, x)
Evaluate sum_i prefactor[i] * lambda[i]**x for different x.
See fit_sum_of_exp() for more details.
```

22.6. fit 743

Module description

tools to fit to an algebraic decay.

22.7 string

• full name: tenpy.tools.string

• parent module: tenpy.tools

• type: module

Functions

is_non_string_iterable(x)	Check if x is a non-string iterable, (e.g., list, tuple, dictionary, np.ndarray)
to_mathematica_lists(a)	convert nested a to string readable by mathematica using curly brackets ' $\{\}$ '.
<pre>vert_join(strlist[, valign, halign, delim])</pre>	Join strings with multilines vertically such that they appear next to each other.

22.7.1 is_non_string_iterable

• full name: tenpy.tools.string.is_non_string_iterable

• parent module: tenpy.tools.string

· type: function

tenpy.tools.string.is_non_string_iterable(x)

Check if x is a non-string iterable, (e.g., list, tuple, dictionary, np.ndarray)

22.7.2 to_mathematica_lists

• full name: tenpy.tools.string.to_mathematica_lists

• parent module: tenpy.tools.string

• type: function

tenpy.tools.string.to_mathematica_lists(a)

convert nested a to string readable by mathematica using curly brackets ' $\{\dots\}$ '.

22.7.3 vert join

```
• full name: tenpy.tools.string.vert_join
```

```
• parent module: tenpy.tools.string
```

• type: function

```
tenpy.tools.string.vert_join (strlist, valign='t', halign='l', delim='')

Join strings with multilines vertically such that they appear next to each other.
```

Parameters

```
• strlist (list of str) – the strings to be joined vertically
```

- valing ('t', 'c', 'b') vertical alignment of the strings: top, center, or bottom
- halign ('l', 'c', 'r') horizontal alignment of the strings: left, center, or right
- **delim** (str) field separator between the strings

Returns joined – a string where the strings of strlist are aligned vertically

Return type str

Examples

Module description

Tools for handling strings.

22.8 process

• full name: tenpy.tools.process

• parent module: tenpy.tools

• type: module

22.8. process 745

Functions

load_omp_library([libs, verbose])	Tries to load openMP library.
memory_usage()	Return memory usage of the running python process.
mkl_get_nthreads()	wrapper around MKL get_max_threads.
mkl_set_nthreads(n)	wrapper around MKL set_num_threads.
omp_get_nthreads()	wrapper around OpenMP get_max_threads.
omp_set_nthreads(n)	wrapper around OpenMP set_nthreads.

22.8.1 load_omp_library

• full name: tenpy.tools.process.load_omp_library

• parent module: tenpy.tools.process

• type: function

tenpy.tools.process.load_omp_library(libs=['libiomp5.so', None, 'libgomp.so.1'], verbose=True)

Tries to load openMP library.

Parameters

- libs list of possible library names we should try to load (with ctypes.CDLL).
- **verbose** (bool) wheter to print the name of the loaded library.

Returns omp – OpenMP shared libary if found, otherwise None. Once it was successfully imported, no re-imports are tried.

Return type CDLL | None

22.8.2 memory_usage

• full name: tenpy.tools.process.memory_usage

• parent module: tenpy.tools.process

· type: function

tenpy.tools.process.memory_usage()

Return memory usage of the running python process.

You can pip install psutilif you get only -1..

Returns mem – Currently used memory in megabytes. –1. if no way to read out.

Return type float

22.8.3 mkl get nthreads

```
    full name: tenpy.tools.process.mkl_get_nthreads
    parent module: tenpy.tools.process
    type: function
    tenpy.tools.process.mkl_get_nthreads()
    wrapper around MKL get_max_threads.
    Returns max threads - The maximum number
```

Returns max_threads – The maximum number of threads used by MKL. –1 if unable to read out.

Return type int

22.8.4 mkl_set_nthreads

```
• full name: tenpy.tools.process.mkl_set_nthreads
```

```
• parent module: tenpy.tools.process
```

· type: function

```
tenpy.tools.process.mkl_set_nthreads(n)
    wrapper around MKL set_num_threads.
```

Parameters n (int) – the number of threads to use

Returns success – whether the shared library was found and set.

Return type bool

22.8.5 omp get nthreads

- full name: tenpy.tools.process.omp_get_nthreads
- parent module: tenpy.tools.process
- · type: function

```
tenpy.tools.process.omp_get_nthreads()
    wrapper around OpenMP get_max_threads.
```

Returns max_threads – The maximum number of threads used by OpenMP (and thus MKL). –1 if unable to read out.

Return type int

22.8.6 omp set nthreads

- full name: tenpy.tools.process.omp_set_nthreads
- parent module: tenpy.tools.process
- · type: function

```
tenpy.tools.process.omp_set_nthreads(n)
    wrapper around OpenMP set_nthreads.
```

Parameters n(int) – the number of threads to use

Returns success – whether the shared library was found and set.

22.8. process 747

Return type bool

Module description

Tools to read out total memory usage and get/set the number of threads.

If your python is compiled against MKL (e.g. if you use *anaconda* as recommended in INSTALL), it will by default use as many threads as CPU cores are available. If you run a job on a cluster, you should limit this to the number of cores you reserved – otherwise your colleagues might get angry... A simple way to achieve this is to set a suitable enviornment variable before calling your python program, e.g. on the linux bash <code>export OMP_NUM_THREADS=4</code> for 4 threads. (MKL used OpenMP and thus respects its settings.)

Alternatively, this module provides <code>omp_get_nthreads()</code> and <code>omp_set_nthreads()</code>, which give their best to get and set the number of threads at runtime, while still being failsave if the shared OpenMP library is not found. In the latter case, you might also try the equivalent <code>mkl_get_nthreads()</code> and <code>mkl_set_nthreads()</code>.

22.9 optimization

• full name: tenpy.tools.optimization

• parent module: tenpy.tools

• type: module

Classes



OptimizationFlag(value)	Options for the global 'optimization level' used for dy-
	namical optimizations.
temporary_level(temporary_level)	Context manager to temporarily set the optimization
	level to a different value.

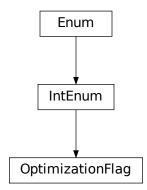
22.9.1 OptimizationFlag

• full name: tenpy.tools.optimization.OptimizationFlag

• parent module: tenpy.tools.optimization

• type: class

Inheritance Diagram



Class Attributes and Properties

```
OptimizationFlag.none
OptimizationFlag.safe
OptimizationFlag.skip_arg_checks
```

class tenpy.tools.optimization.OptimizationFlag(value)

 $Bases: \verb"enum.IntEnum"$

Options for the global 'optimization level' used for dynamical optimizations.

Whether we optimize dynamically is decided by comparison of the global "optimization level" with one of the following flags. A higher level *includes* all the previous optimizations.

22.9. optimization 749

Leve	l Flag	Description
0	none	Don't do any optimizations, i.e., run many sanity checks. Used for testing.
1	default	Skip really unnecessary sanity checks, but also don't try any optional optimizations if
		they might give an overhead.
2	safe	Activate safe optimizations in algorithms, even if they might give a small overhead. Ex-
		ample: Try to compress the MPO representing the hamiltonian.
3	skip_arg_cl	neClassafe! Skip (some) class sanity tests and (function) argument checks.

Warning: When unsafe optimizations are enabled, errors will not be detected that easily, debugging is much harder, and you might even get segmentation faults in the compiled parts. Use this kind of optimization only for code which you successfully ran before with (very) similar parmeters and disabled optimizations! Enable this optimization only during the parts of the code where it is really necessary. Check whether it actually helps - if it doesn't, keep the optimization disabled!

22.9.2 temporary_level

• full name: tenpy.tools.optimization.temporary_level

• parent module: tenpy.tools.optimization

• type: class

Inheritance Diagram

temporary_level

Methods

```
class tenpy.tools.optimization.temporary_level(temporary_level)
    Bases: object
```

Context manager to temporarily set the optimization level to a different value.

Parameters temporary_level (int | OptimizationFlag | str | None) - The optimization level to be set during the context. *None* defaults to the current value of the optimization level.

temporary_level

The optimization level to be set during the context.

Type None | OptimizationFlag

old level

Optimization level to be restored at the end of the context manager.

Type OptimizationFlag

Examples

It is recommended to use this context manager in a with statement:

```
# optimization level default
with temporary_level(OptimizationFlag.safe):
    do_some_stuff() # temporarily have Optimization level `safe`
    # you can even change the optimization level to something else:
    set_level(OptimizationFlag.skip_args_check)
    do_some_really_heavy_stuff()
# here we are back to the optimization level as before the ``with ...`` statement
```

Functions

get_level()	Return the global optimization level.
optimize([level_compare])	Called by algorithms to check whether it should (try to)
	do some optimizations.
set_level([level])	Set the global optimization level.
to_OptimizationFlag(level)	Convert strings and int to a valid OptimizationFlag.
use_cython([func, replacement, check_doc])	Decorator to replace a function with a Cython-
	equivalent from _npc_helper.pyx.

22.9.3 get_level

- full name: tenpy.tools.optimization.get_level
- parent module: tenpy.tools.optimization
- type: function

```
tenpy.tools.optimization.get_level()

Return the global optimization level.
```

22.9.4 optimize

- full name: tenpy.tools.optimization.optimize
- parent module: tenpy.tools.optimization
- type: function

tenpy.tools.optimization.optimize(level_compare=<OptimizationFlag.default: 1>)
Called by algorithms to check whether it should (try to) do some optimizations.

Parameters level_compare (OptimizationFlag) - At which level to start optimization, i.e., how safe the suggested optimization is.

Returns optimize – True if the algorithms should try to optimize, i.e., whether the global "optimization level" is equal or higher than the level to compare to.

22.9. optimization 751

Return type bool

22.9.5 set_level

```
• full name: tenpy.tools.optimization.set_level
```

```
• parent module: tenpy.tools.optimization
```

• type: function

```
tenpy.tools.optimization.set_level(level=I)
Set the global optimization level.
```

Parameters level (int | OptimizationFlag | str | None) - The new global optimization level to be set. None defaults to keeping the current level.

22.9.6 to_OptimizationFlag

• full name: tenpy.tools.optimization.to_OptimizationFlag

```
• parent module: tenpy.tools.optimization
```

• type: function

```
tenpy.tools.optimization.to_OptimizationFlag(level)
```

Convert strings and int to a valid OptimizationFlag.

None defaults to the current level.

22.9.7 use_cython

- full name: tenpy.tools.optimization.use_cython
- parent module: tenpy.tools.optimization
- type: function

tenpy.tools.optimization.use_cython (func=None, replacement=None, check_doc=True)

Decorator to replace a function with a Cython-equivalent from _npc_helper.pyx.

This is a decorator, which is supposed to be used in front of function definitions with an @ sign, for example:

This decorator indicates that there is a Cython implementation in the file tenpy/linalg/_npc_helper. pyx, which should have the same signature (i.e. same arguments and return values) as the decorated function, and can be used as a replacement for the decorated function. However, if the cython code could not be compiled on your system (or if the environment variable TENPY_OPTIMIZE is set to negative values, or the environment variable TENPY_NO_CYTHON is "true"), we just pass the previous function.

752 Chapter 22. tools

Note: In case that the decorator is used for a class method, the corresponding Cython version needs to have an @cython.binding(True).

Parameters

- func (function) The defined function
- **replacement** (*string* / *None*) The name of the function defined in tenpy/ linalg/_npc_helper.pyx which should replace the decorated function. None defaults to the name of the decorated function, e.g., in the above example *my_slow_function*.
- **check_doc** (bool) If True, we check that the cython version of the function has the exact same doc string (up to a possible first line containing the function signature) to exclude typos and inconsistent versions.

Returns replacement_func – The function replacing the decorated function *func*. If the cython code can not be loaded, this is just *func*, otherwise it's the cython version specified by *replacement*.

Return type function

Module description

Optimization options for this library.

Let me start with a quote of "Micheal Jackson" (a programmer, not the musician):

```
First rule of optimization: "Don't do it."
Second rule of optimization (for experts only): "Don't do it yet."
Third rule of optimization: "Profile before optimizing."
```

Luckily, following the third optimization rule, namely profiling code, is fairly simple in python, see the documentation. If you have a python skript running your code, you can simply call it with python -m "cProfile" -s "tottime" your_skript.py. Alternatively, save the profiling statistics with python -m "cProfile" -o "profile_data.stat" your_skript.py and run these few lines of python code:

```
import pstats
p = pstats.Pstats("profile_data.stat")
p.sort_stats('cumtime') # sort by 'cumtime' column
p.print_stats(30) # prints first 30 entries
```

That being said, I actually did profile and optimize (parts of) the library; and there are a few knobs you can turn to tweak the most out of this library, explained in the following.

- 1) Simply install the 'bottleneck' python package, which allows to optimize slow parts of numpy, most notably 'NaN' checking.
- 2) Figure out which numpy/scipy/python you are using. As explained in *Installation instructions*, we recommend to use the Python distributed provided by Intel or Anaconda. They ship with numpy and scipy which use Intels MKL library, such that e.g. np.tensordot is parallelized to use multiple cores.
- 3) In case you didn't do that yet: some parts of the library are written in both python and Cython with the same interface, so you can simply compile the Cython code, as explained in *Installation instructions*. Then everything should work the same way from a user perspective, while internally the faster, pre-compiled cython code from tenpy/linalg/_npc_helper.pyx is used. This should also be a safe thing to do. The replacement of the optimized functions is done by the decorator use_cython().

22.9. optimization 753

Note: By default, the compilation will link against the BLAS functions provided by scipy.linalg. cython_blas. Whether they use MKL depends on the scipy version you installed. However, you can explicitly link against a given MKL by providing the path during compilation, as explained in *Compile linking agains MKL*.

4) One of the great things about python is its dynamical nature - anything can be done at runtime. In that spirit, this module allows to set a global "optimization level" which can be changed *dynamically* (i.e., during runtime) with set_level(). The library will then try some extra optimiztion, most notably skip sanity checks of arguments. The possible choices for this global level are given by the OptimizationFlag. The default initial value for the global optimization level can be adjusted by the environment variable TENPY_OPTIMIZE.

Warning: When this optimizing is enabled, we skip (some) sanity checks. Thus, errors will not be detected that easily, and debugging is much harder! We recommend to use this kind of optimization only for code which you successfully have run before with (very) similar parmeters! Enable this optimization only during the parts of the code where it is really necessary. The context manager <code>temporary_level</code> can help with that. Check whether it actually helps - if it doesn't, keep the optimization disabled! Some parts of the library already do that as well (e.g. DMRG after the first sweep).

5) You might want to try some different compile time options for the cython code, set in the *setup.py* in the top directory of the repository. Since the *setup.py* reads out the *TENPY_OPTIMIZE* environment variable, you can simple use an export TENPY_OPTIMIZE=3 (in your bash/terminal) right before compilation. An export TENPY_OPTIMIZE=0 activates profiling hooks instead.

Warning: This increases the probability of getting segmentation faults and anyway might not help that much; in the crucial parts of the cython code, these optimizations are already applied. We do *not* recommend using this!

```
tenpy.tools.optimization.bottleneck = None
tenpy.tools.optimization.have_cython_functions = False
    bool whether the import of the cython file tenpy/linalg/_npc_helper.pyx succeeded.
    The value is set in the first call of use_cython().
tenpy.tools.optimization.compiled_with_MKL = False
    bool whether the cython file was compiled with HAVE_MKL.
    The value is set in the first call of use_cython().
```

754 Chapter 22. tools

CHAPTER

TWENTYTHREE

VERSION

full name: tenpy.versionparent module: tenpy

· type: module

Module description

Access to version of this library.

The version is provided in the standard python format major.minor.revision as string. Use pkg_resources.parse_version before comparing versions.

tenpy.version.version = '0.8.3'
current release version as a string

tenpy.version.released = True

whether this is a released version or modified

tenpy.version.short_version = 'v0.8.3'
same as version, but with 'v' in front

tenpy.version.git_revision = 'cc938cdcf5fba0cd24e9579b264e960264195037'
the hash of the last git commit (if available)

tenpy.version.full_version = '0.8.3'

if not released additional info with part of git revision

tenpy.version.version_summary = 'tenpy 0.8.3 (not compiled), \ngit revision cc938cdcf5fba0cd summary of the tenpy, python, numpy and scipy versions used

756 Chapter 23. version

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- [TeNPySource] https://github.com/tenpy/tenpy
- [TeNPyNotebooks] Collection of example [jupyter] notebooks using TeNPy: https://github.com/tenpy/tenpy_notebooks
- [TeNPyDoc] Online documentation, https://tenpy.readthedocs.io/
- [TeNPyForum] Community forum for discussions, FAQ and announcements, https://tenpy.johannes-hauschild.de
- [git] "git version control system", https://git-scm.com A software which we use to keep track of changes in the source code.
- [conda] "conda package manger", https://docs.conda.io/en/latest/ A package and environment management system that allows to easily install (multiple version of) various software, and in particular python packages like TeNPy.
- [pip] "pip the Python Package installer", https://pip.pypa.io/en/stable/ Traditional way to handle installed python packages with pip install ... and pip uninstall ... on the command line.
- [matplotlib] "Matplotlib", https://matplotlib.org/ A Python 2D plotting library. Some TeNPy functions expect matplotlib.axes.Axes as arguments to plot into.
- [HDF5] "Hierarchical Data Format 5 (R)", https://portal.hdfgroup.org/display/HDF5/HDF5 A file format and library for saving data (including metadata). We use it through the python interface of the h5py library, see *Saving to disk: input/output*.
- [yaml] "YAML Ain't Markup Language", https://yaml.org A human-readable file format for configuration files. TeNpy (optionally) uses it through pyyaml for reading in simulation parameters, and in some places in the documentation to keep things more readable.
- [jupyter] Jupyter notebooks, https://jupyter.org/ An amazing interface for (python) notebooks which can contain both source code, text and outputs in a single file. They provide a good way to get started with python, we use them for examples.
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758 Bibliography

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Bibliography 759

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760 Bibliography

PYTHON MODULE INDEX

```
t
                                           tenpy.simulations.simulation, 693
                                           tenpy.simulations.time_evolution,699
tenpy, 189
                                           tenpy.tools, 701
tenpy.algorithms, 193
                                           tenpy.tools.events,725
tenpy.algorithms.algorithm, 194
                                           tenpy.tools.fit,744
tenpy.algorithms.dmrg, 240
                                           tenpy.tools.hdf5 io,714
tenpy.algorithms.exact_diag, 267
                                           tenpy.tools.math, 740
tenpy.algorithms.mpo_evolution, 261
                                           tenpy.tools.misc, 736
tenpy.algorithms.mps_common, 213
                                           tenpy.tools.optimization, 753
tenpy.algorithms.network_contractor, 263
                                           tenpy.tools.params, 720
tenpy.algorithms.purification, 261
                                           tenpy.tools.process, 748
tenpy.algorithms.tdvp, 248
                                           tenpy.tools.string,745
tenpy.algorithms.tebd, 204
                                           tenpy.version,755
tenpy.algorithms.truncation, 198
tenpy.linalg, 269
tenpy.linalg.charges, 324
tenpy.linalg.lanczos, 351
tenpy.linalg.np_conserved, 302
tenpy.linalg.random matrix, 329
tenpy.linalg.sparse, 348
tenpy.linalg.svd robust, 326
tenpy.models, 353
tenpy.models.fermions_spinless, 565
tenpy.models.haldane, 596
tenpy.models.hofstadter, 595
tenpy.models.hubbard, 593
tenpy.models.lattice, 476
tenpy.models.model, 506
tenpy.models.spins, 549
tenpy.models.spins_nnn,550
tenpy.models.tf_ising, 521
tenpy.models.toric code, 607
tenpy.models.xxz_chain,535
tenpy.networks, 609
tenpy.networks.mpo,669
tenpy.networks.mps, 658
tenpy.networks.purification mps, 689
tenpy.networks.site,648
tenpy.networks.terms,688
tenpy.simulations, 691
tenpy.simulations.ground_state_search,
tenpy.simulations.measurement, 697
```

762 Python Module Index

CONFIG OPTION INDEX

Algorithm	trunc_params (mulitple definitions), ??
trunc_params (Algorithm), ??	trunc_params (MPO.apply), ??
-	trunc_params (MPO.apply_zipup), ??
ApplyMPO	trunc_params (Variational Compression), ??
<pre>chi_list (mulitple definitions), ??</pre>	trunc_params (Algorithm), ??
chi_list (Sweep.reset_stats), ??	trunc_weight (MPO.apply_zipup), ??
chi_list (VariationalApplyMPO.reset_stats), ??	Decel III John and Medel
chi_list (VariationalCompression.reset_stats), ??	BoseHubbardModel
chi_list (PurificationApplyMPO.reset_stats), 252	bc_MPS (mulitple definitions), 563
combine (Sweep), ??	bc_MPS (FermionChain.init_lattice), 563
compression_method (MPO.apply), ??	bc_MPS (FermionModel.init_lattice), ??
<pre>init_env_data (mulitple definitions), ??</pre>	bc_MPS (BosonicHaldaneModel.init_lattice), ??
init_env_data(DMRGEngine.init_env), ??	bc_MPS (FermionicHaldaneModel.init_lattice), ??
init_env_data (EngineCombine.init_env), 221	bc_MPS (<i>HofstadterBosons.init_lattice</i>), ??
init_env_data (EngineFracture.init_env), 230	bc_MPS (<i>HofstadterFermions.init_lattice</i>), ??
<pre>init_env_data (SingleSiteDMRGEngine.init_env),</pre>	bc_MPS (BoseHubbardChain.init_lattice), 578
??	bc_MPS (BoseHubbardModel.init_lattice), ??
<pre>init_env_data(TwoSiteDMRGEngine.init_env), ??</pre>	bc_MPS (FermiHubbardChain.init_lattice), 591
init_env_data(Sweep.init_env),??	bc_MPS (FermiHubbardModel.init_lattice), ??
lanczos_params (Sweep), ??	bc_MPS (CouplingMPOModel.init_lattice), ??
m_temp(MPO.apply_zipup), ??	bc_MPS (SpinChain.init_lattice), 548
N_sweeps (Variational Compression), ??	bc_MPS (SpinModel.init_lattice), ??
orthogonal_to (mulitple definitions), ??	bc_MPS (SpinChainNNN.init_lattice), ??
orthogonal_to (DMRGEngine.init_env), ??	bc_MPS (SpinChainNNN2.init_lattice), ??
orthogonal_to (EngineCombine.init_env), 221	bc_MPS (TFIChain.init_lattice), 520
orthogonal_to (EngineFracture.init_env), 230	bc_MPS (TFIModel.init_lattice), ??
orthogonal_to (SingleSiteDMRGEngine.init_env),	bc_MPS (ToricCode.init_lattice), ??
??	bc_MPS (XXZChain2.init_lattice), 534
orthogonal_to (TwoSiteDMRGEngine.init_env), ??	bc_x (mulitple definitions), 564
orthogonal_to (Sweep.init_env), ??	bc_x (FermionChain.init_lattice), 564
start_env (mulitple definitions), ??	bc_x (FermionModel.init_lattice), ??
start_env (DMRGEngine.init_env), ??	bc_x (BosonicHaldaneModel.init_lattice), ??
start_env (EngineCombine.init_env), 222	bc_x (FermionicHaldaneModel.init_lattice), ??
start_env (EngineFracture.init_env), 230	bc_x (HofstadterBosons.init_lattice), ??
<pre>start_env (SingleSiteDMRGEngine.init_env), ??</pre>	bc_x (HofstadterFermions.init_lattice), ??
<pre>start_env (TwoSiteDMRGEngine.init_env), ??</pre>	bc_x (BoseHubbardChain.init_lattice), 578
start_env (Sweep.init_env), ??	bc_x (BoseHubbardModel.init_lattice), ??
<pre>sweep_0 (mulitple definitions), ??</pre>	bc_x (FermiHubbardChain.init_lattice), 592
<pre>sweep_0 (Sweep.reset_stats), ??</pre>	bc_x (FermiHubbardModel.init_lattice), ??
<pre>sweep_0 (VariationalApplyMPO.reset_stats), ??</pre>	bc_x (CouplingMPOModel.init_lattice), ??
<pre>sweep_0 (VariationalCompression.reset_stats), ??</pre>	bc_x (SpinChain.init_lattice), 548
sweep 0 (PurificationApplyMPO.reset stats), 252	bc_x (SpinModel.init_lattice), ??

bc_x (SpinChainNNN.init_lattice), ??	<pre>lattice (HofstadterBosons.init_lattice), ??</pre>
bc_x (SpinChainNNN2.init_lattice), ??	<pre>lattice (HofstadterFermions.init_lattice), ??</pre>
bc_x (TFIChain.init_lattice), 520	<pre>lattice (BoseHubbardChain.init_lattice), 578</pre>
bc_x (TFIModel.init_lattice), ??	<pre>lattice (BoseHubbardModel.init_lattice), ??</pre>
bc_x (ToricCode.init_lattice), ??	lattice (FermiHubbardChain.init_lattice), 591
bc_x (XXZChain2.init_lattice), 535	lattice (FermiHubbardModel.init_lattice), ??
bc_y (mulitple definitions), 564	lattice (CouplingMPOModel.init_lattice), ??
bc_y (FermionChain.init_lattice), 564	lattice (SpinChain.init_lattice), 548
bc_y (FermionModel.init_lattice), ??	lattice (SpinModel.init_lattice), ??
bc_y (BosonicHaldaneModel.init_lattice), ??	lattice (SpinChainNNN.init_lattice), ??
bc_y (FermionicHaldaneModel.init_lattice), ??	lattice (SpinChainNNN2.init_lattice), ??
bc_y (HofstadterBosons.init_lattice), ??	lattice (<i>TFIChain.init_lattice</i>), 520
bc_y (HofstadterFermions.init_lattice), ??	lattice (TFIModel.init_lattice), ??
bc_y (BoseHubbardChain.init_lattice), 578	lattice (ToricCode.init_lattice), ??
bc_y (BoseHubbardModel.init_lattice), ??	lattice (XXZChain2.init_lattice), 534
bc_y (FermiHubbardChain.init_lattice), 592	Lx (mulitple definitions), 564
bc_y (FermiHubbardModel.init_lattice), ??	Lx (FermionChain.init_lattice), 564
bc_y (CouplingMPOModel.init_lattice), ??	Lx (FermionModel.init_lattice), ??
bc_y (SpinChain.init_lattice), 548	Lx (BosonicHaldaneModel.init_lattice), ??
_	Lx (FermionicHaldaneModel.init_lattice), ??
bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ??	
	Lx (HofstadterBosons.init_lattice), ??
bc_y (SpinChainNNN2.init_lattice), ??	Lx (HofstadterFermions.init_lattice), ??
bc_y (TFIMadalinit_lattice), 520	Lx (BoseHubbardChain.init_lattice), 578
bc_y (TFIModel.init_lattice), ??	Lx (BoseHubbardModel.init_lattice), ??
bc_y (ToricCode.init_lattice), ??	Lx (FermiHubbardChain.init_lattice), 592
bc_y (XXZChain2.init_lattice), 535	Lx (FermiHubbardModel.init_lattice), ??
conserve (BoseHubbardModel), ??	Lx (CouplingMPOModel.init_lattice), ??
explicit_plus_hc (CouplingMPOModel), ??	Lx (SpinChain.init_lattice), 548
filling (BoseHubbardModel), ??	Lx (SpinModel.init_lattice), ??
L (mulitple definitions), 563	Lx (SpinChainNNN.init_lattice), ??
⊥ (FermionChain.init_lattice), 563	Lx (SpinChainNNN2.init_lattice), ??
⊥ (FermionModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
□ (BosonicHaldaneModel.init_lattice), ??	Lx (TFIModel.init_lattice), ??
□ (FermionicHaldaneModel.init_lattice), ??	Lx (ToricCode.init_lattice), ??
□ (HofstadterBosons.init_lattice), ??	Lx (XXZChain2.init_lattice), 535
□ (HofstadterFermions.init_lattice), ??	Ly (mulitple definitions), 564
□ (BoseHubbardChain.init_lattice), 578	Ly (FermionChain.init_lattice), 564
□ (BoseHubbardModel.init_lattice), ??	Ly (FermionModel.init_lattice), ??
□ (FermiHubbardChain.init_lattice), 591	Ly (BosonicHaldaneModel.init_lattice), ??
□ (FermiHubbardModel.init_lattice), ??	Ly (FermionicHaldaneModel.init_lattice), ??
L (CouplingMPOModel.init_lattice), ??	Ly (HofstadterBosons.init_lattice), ??
□ (SpinChain.init_lattice), 548	Ly (HofstadterFermions.init_lattice), ??
□ (SpinModel.init_lattice), ??	Ly (BoseHubbardChain.init_lattice), 578
□ (SpinChainNNN.init_lattice), ??	Ly (BoseHubbardModel.init_lattice), ??
□ (SpinChainNNN2.init_lattice), ??	Ly (FermiHubbardChain.init_lattice), 592
⊥ (TFIChain.init_lattice), 520	Ly (FermiHubbardModel.init_lattice), ??
L (TFIModel.init_lattice), ??	Ly (CouplingMPOModel.init_lattice), ??
L (ToricCode.init_lattice), ??	Ly (SpinChain.init_lattice), 548
L (XXZChain2.init_lattice), 534	Ly (SpinModel.init_lattice), ??
lattice (mulitple definitions), 563	Ly (SpinChainNNN.init_lattice), ??
lattice (Fermion Chain.init_lattice), 563	Ly (SpinChainNNN2.init_lattice), ??
lattice (FermionModel.init_lattice), ??	Ly (TFIChain.init_lattice), 520
lattice (BosonicHaldaneModel.init_lattice), ??	Ly (TFIModel.init_lattice), ??
lattice (FermionicHaldaneModel.init_lattice), ??	Ly (ToricCode.init_lattice), ??
= (= =================================	<u> </u>

Ly (XXZChain2.init_lattice), 535	bc_x (HofstadterBosons.init_lattice), ??
mu (BoseHubbardModel), ??	bc_x (HofstadterFermions.init_lattice), ??
n_max (BoseHubbardModel), ??	bc_x (BoseHubbardChain.init_lattice), 578
order (mulitple definitions), 563	bc_x (BoseHubbardModel.init_lattice), ??
order (FermionChain.init_lattice), 563	bc_x (FermiHubbardChain.init_lattice), 592
order (FermionModel.init_lattice), ??	bc_x (FermiHubbardModel.init_lattice), ??
order (BosonicHaldaneModel.init_lattice), ??	bc_x (CouplingMPOModel.init_lattice), ??
order (FermionicHaldaneModel.init_lattice), ??	bc_x (SpinChain.init_lattice), 548
order (HofstadterBosons.init_lattice), ??	bc_x (SpinModel.init_lattice), ??
order (HofstadterFermions.init_lattice), ??	bc_x (SpinChainNNN.init_lattice), ??
order (BoseHubbardChain.init_lattice), 578	bc_x (SpinChainNNN2.init_lattice), ??
order (BoseHubbardModel.init_lattice), ??	bc_x (TFIChain.init_lattice), 520
order (FermiHubbardChain.init_lattice), 591	bc_x (TFIModel.init_lattice), ??
order (FermiHubbardModel.init_lattice), ??	bc_x (ToricCode.init_lattice), ??
order (CouplingMPOModel.init_lattice), ??	bc_x (XXZChain2.init_lattice), 535
order (SpinChain.init_lattice), 548	bc_y (mulitple definitions), 564
order (SpinModel.init_lattice), ??	bc_y (FermionChain.init_lattice), 564
order (SpinChainNNN.init_lattice), ??	bc_y (FermionModel.init_lattice), ??
order (SpinChainNNN2.init_lattice), ??	bc_y (BosonicHaldaneModel.init_lattice), ??
order (TFIChain.init_lattice), 520	bc_y (FermionicHaldaneModel.init_lattice), ??
order (TFIModel.init_lattice), ??	bc_y (HofstadterBosons.init_lattice), ??
order (ToricCode.init_lattice), ??	bc_y (HofstadterFermions.init_lattice), ??
order (XXZChain2.init_lattice), 534	bc_y (BoseHubbardChain.init_lattice), 578
<pre>sort_mpo_legs (CouplingMPOModel), ??</pre>	bc_y (BoseHubbardModel.init_lattice), ??
t (BoseHubbardModel), ??	bc_y (FermiHubbardChain.init_lattice), 592
U (BoseHubbardModel), ??	bc_y (FermiHubbardModel.init_lattice), ??
∨ (BoseHubbardModel), ??	bc_y (CouplingMPOModel.init_lattice), ??
D ' 11 11 M 11	bc_y (<i>SpinChain.init_lattice</i>), 548
BosonicHaldaneModel	bc_y (SpinChain.init_lattice), 548 bc_y (SpinModel.init_lattice), ??
BosonicHaldaneModel bc_MPS (mulitple definitions), 563	
	bc_y (SpinModel.init_lattice), ??
bc_MPS (mulitple definitions), 563	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ??
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bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (FermionicHaldaneModel.init_lattice), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinModel.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (FermionicHaldaneModel.init_lattice), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinModel.init_lattice), ?? bc_MPS (SpinModel.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (FermionicHaldaneModel.init_lattice), ?? L (HofstadterBosons.init_lattice), ?? L (HofstadterFermions.init_lattice), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (HofstadterBosons.init_lattice), ?? L (HofstadterFermions.init_lattice), ?? L (BoseHubbardChain.init_lattice), ?? L (BoseHubbardModel.init_lattice), 578 L (BoseHubbardModel.init_lattice), ?? L (FermiHubbardChain.init_lattice), 591
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), 520	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (HofstadterBosons.init_lattice), ?? L (HofstadterFermions.init_lattice), ?? L (BoseHubbardChain.init_lattice), 578 L (BoseHubbardModel.init_lattice), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (FermionicHaldaneModel.init_lattice), ?? L (HofstadterBosons.init_lattice), ?? L (BoseHubbardChain.init_lattice), 578 L (BoseHubbardModel.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (FermiHubbardModel.init_lattice), ??
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bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), 534	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (HofstadterBosons.init_lattice), ?? L (HofstadterFermions.init_lattice), ?? L (BoseHubbardChain.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (FouplingMPOModel.init_lattice), ?? L (SpinChain.init_lattice), ?? L (SpinModel.init_lattice), ??
bc_MPS (<i>FermionChain.init_lattice</i>), 563 bc_MPS (<i>FermionModel.init_lattice</i>), ?? bc_MPS (<i>FermionModel.init_lattice</i>), ?? bc_MPS (<i>BosonicHaldaneModel.init_lattice</i>), ?? bc_MPS (<i>FermionicHaldaneModel.init_lattice</i>), ?? bc_MPS (<i>HofstadterBosons.init_lattice</i>), ?? bc_MPS (<i>HofstadterFermions.init_lattice</i>), ?? bc_MPS (<i>BoseHubbardChain.init_lattice</i>), 578 bc_MPS (<i>BoseHubbardModel.init_lattice</i>), ?? bc_MPS (<i>FermiHubbardModel.init_lattice</i>), ?? bc_MPS (<i>FermiHubbardModel.init_lattice</i>), ?? bc_MPS (<i>SpinChain.init_lattice</i>), ?? bc_MPS (<i>SpinChain.init_lattice</i>), ?? bc_MPS (<i>SpinChainNNN.init_lattice</i>), ?? bc_MPS (<i>SpinChainNNN.init_lattice</i>), ?? bc_MPS (<i>TFIChain.init_lattice</i>), ?? bc_MPS (<i>TFIModel.init_lattice</i>), ?? bc_MPS (<i>TricCode.init_lattice</i>), ?? bc_MPS (<i>XXZChain2.init_lattice</i>), 534 bc_x (<i>mulitple definitions</i>), 564 bc_x (<i>FermionModel.init_lattice</i>), ??	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (HofstadterBosons.init_lattice), ?? L (HofstadterFermions.init_lattice), ?? L (BoseHubbardChain.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (SpinChain.init_lattice), ?? L (SpinChain.init_lattice), ?? L (SpinChain.init_lattice), ??
bc_MPS (mulitple definitions), 563 bc_MPS (FermionChain.init_lattice), 563 bc_MPS (FermionModel.init_lattice), ?? bc_MPS (BosonicHaldaneModel.init_lattice), ?? bc_MPS (FermionicHaldaneModel.init_lattice), ?? bc_MPS (HofstadterBosons.init_lattice), ?? bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIModel.init_lattice), ?? bc_MPS (Trimodel.init_lattice), ?? bc_MPS (TricCode.init_lattice), ?? bc_MPS (TricCode.init_lattice), ?? bc_MPS (TricCode.init_lattice), ?? bc_MPS (TricCode.init_lattice), 534 bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), 564	bc_y (SpinModel.init_lattice), ?? bc_y (SpinChainNNN.init_lattice), ?? bc_y (SpinChainNNN2.init_lattice), ?? bc_y (TFIChain.init_lattice), 520 bc_y (TFIModel.init_lattice), ?? bc_y (ToricCode.init_lattice), ?? bc_y (XXZChain2.init_lattice), 535 conserve (BosonicHaldaneModel), ?? explicit_plus_hc (CouplingMPOModel), ?? L (mulitple definitions), 563 L (FermionChain.init_lattice), 563 L (FermionModel.init_lattice), ?? L (BosonicHaldaneModel.init_lattice), ?? L (HofstadterBosons.init_lattice), ?? L (HofstadterFermions.init_lattice), ?? L (BoseHubbardChain.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (FermiHubbardModel.init_lattice), ?? L (FouplingMPOModel.init_lattice), ?? L (SpinChain.init_lattice), ?? L (SpinModel.init_lattice), ??

□ (TFIModel.init_lattice), ??	Ly (CouplingMPOModel.init_lattice), ??
□ (ToricCode.init_lattice), ??	Ly (SpinChain.init_lattice), 548
⊥ (XXZChain2.init_lattice), 534	Ly (SpinModel.init_lattice), ??
lattice (mulitple definitions), 563	Ly (SpinChainNNN.init_lattice), ??
<pre>lattice (FermionChain.init_lattice), 563</pre>	Ly (SpinChainNNN2.init_lattice), ??
<pre>lattice (FermionModel.init_lattice), ??</pre>	Ly (TFIChain.init_lattice), 520
<pre>lattice (BosonicHaldaneModel.init_lattice), ??</pre>	Ly (TFIModel.init_lattice), ??
<pre>lattice (FermionicHaldaneModel.init_lattice), ??</pre>	Ly (ToricCode.init_lattice), ??
lattice (HofstadterBosons.init_lattice), ??	Ly (XXZChain2.init_lattice), 535
<pre>lattice (HofstadterFermions.init_lattice), ??</pre>	mu (BosonicHaldaneModel), ??
lattice (BoseHubbardChain.init_lattice), 578	order (mulitple definitions), 563
lattice (BoseHubbardModel.init_lattice), ??	order (FermionChain.init_lattice), 563
lattice (FermiHubbardChain.init_lattice), 591	order (FermionModel.init_lattice), ??
lattice (FermiHubbardModel.init_lattice), ??	order (BosonicHaldaneModel.init_lattice), ??
lattice (CouplingMPOModel.init_lattice), ??	order (FermionicHaldaneModel.init_lattice), ??
lattice (SpinChain.init_lattice), 548	order (HofstadterBosons.init_lattice), ??
lattice (SpinModel.init_lattice), ??	order (HofstadterFermions.init_lattice), ??
lattice (SpinChainNNN.init_lattice), ??	order (BoseHubbardChain.init_lattice), 578
lattice (SpinChainNNN2.init_lattice), ??	order (BoseHubbardModel.init_lattice), ??
lattice (TFIChain.init_lattice), 520	order (FermiHubbardChain.init_lattice), 591
lattice (TFIModel.init_lattice), ??	order (FermiHubbardModel.init_lattice), ??
lattice (ToricCode.init_lattice), ??	order (CouplingMPOModel.init_lattice), ??
lattice (XXZChain2.init_lattice), 534	order (SpinChain.init_lattice), 548
Lx (mulitple definitions), 564	order (SpinModel.init_lattice), ??
Lx (FermionChain.init_lattice), 564	order (SpinChainNNN.init_lattice), ??
Lx (FermionModel.init_lattice), ??	order (SpinChainNNN2.init_lattice), ??
Lx (BosonicHaldaneModel.init_lattice), ??	order (TFIChain.init_lattice), 520
Lx (FermionicHaldaneModel.init_lattice), ??	order (TFIModel.init_lattice), ??
Lx (HofstadterBosons.init_lattice), ??	order (ToricCode.init_lattice), ??
Lx (HofstadterFermions.init_lattice), ??	order (XXZChain2.init_lattice), 534
Lx (BoseHubbardChain.init_lattice), 578	sort_mpo_legs(CouplingMPOModel),??
Lx (BoseHubbardModel.init_lattice), ??	t1 (BosonicHaldaneModel), ??
Lx (FermiHubbardChain.init_lattice), 592	t2 (BosonicHaldaneModel), ??
Lx (FermiHubbardModel.init_lattice), ??	∨ (BosonicHaldaneModel), ??
Lx (CouplingMPOModel.init_lattice), ??	Confin
Lx (SpinChain.init_lattice), 548	Config
Lx (SpinModel.init_lattice), ??	
Lx (SpinChainNNN.init_lattice), ??	CouplingMPOModel
Lx (SpinChainNNN2.init_lattice), ??	bc_MPS (mulitple definitions), 563
Lx (TFIChain.init_lattice), 520	bc_MPS (FermionChain.init_lattice), 563
Lx (TFIModel.init_lattice), ??	bc_MPS (FermionModel.init_lattice), ??
Lx (ToricCode.init_lattice), ??	bc_MPS (BosonicHaldaneModel.init_lattice), ??
Lx (XXZChain2.init_lattice), 535	bc_MPS (FermionicHaldaneModel.init_lattice), ??
Ly (mulitple definitions), 564	bc_MPS (HofstadterBosons.init_lattice), ??
Ly (FermionChain.init_lattice), 564	bc_MPS (HofstadterFermions.init_lattice), ??
Ly (FermionModel.init_lattice), ??	bc_MPS (BoseHubbardChain.init_lattice), 578
Ly (BosonicHaldaneModel.init_lattice), ??	bc_MPS (BoseHubbardModel.init_lattice), ??
Ly (FermionicHaldaneModel.init_lattice), ??	bc_MPS (FermiHubbardChain.init_lattice), 591
Ly (HofstadterBosons.init_lattice), ??	
by (110)statate Bosons.titt_tattiee); ••	
Ly (HofstadterFermions.init_lattice), ??	bc_MPS (FermiHubbardModel.init_lattice), ??
Ly (HofstadterFermions.init_lattice), ??	bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ??
Ly (HofstadterFermions.init_lattice), ?? Ly (BoseHubbardChain.init_lattice), 578	bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548
Ly (HofstadterFermions.init_lattice), ?? Ly (BoseHubbardChain.init_lattice), 578 Ly (BoseHubbardModel.init_lattice), ??	bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinModel.init_lattice), ??
Ly (HofstadterFermions.init_lattice), ?? Ly (BoseHubbardChain.init_lattice), 578	bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548

bc_MPS (<i>TFIChain.init_lattice</i>), 520	L (FermiHubbardChain.init_lattice), 591
bc_MPS (TFIModel.init_lattice), ??	L (FermiHubbardModel.init_lattice), ??
bc_MPS (ToricCode.init_lattice), ??	L (CouplingMPOModel.init_lattice), ??
bc_MPS (XXZChain2.init_lattice), 534	L (SpinChain.init_lattice), 548
bc_x (mulitple definitions), 564	L (SpinModel.init_lattice), ??
bc_x (FermionChain.init_lattice), 564	L (SpinChainNNN.init_lattice), ??
bc_x (FermionModel.init_lattice), ??	L (SpinChainNNN2.init_lattice), ??
bc_x (BosonicHaldaneModel.init_lattice), ??	L (TFIChain.init_lattice), 520
bc_x (FermionicHaldaneModel.init_lattice), ??	L (TFIModel.init_lattice), ??
bc_x (HofstadterBosons.init_lattice), ??	L (ToricCode.init_lattice), ??
bc_x (HofstadterFermions.init_lattice), ??	L (XXZChain2.init_lattice), 534
bc_x (BoseHubbardChain.init_lattice), 578	lattice (mulitple definitions), 563
bc_x (BoseHubbardModel.init_lattice), ??	lattice (FermionChain.init_lattice), 563
bc_x (FermiHubbardChain.init_lattice), 592	lattice (FermionModel.init_lattice), ??
bc_x (FermiHubbardModel.init_lattice), ??	lattice (BosonicHaldaneModel.init_lattice), ??
bc_x (CouplingMPOModel.init_lattice), ??	lattice (FermionicHaldaneModel.init_lattice), ??
bc_x (SpinChain.init_lattice), 548	lattice (HofstadterBosons.init_lattice), ??
bc_x (SpinModel.init_lattice), ??	lattice (HofstadterFermions.init_lattice), ??
bc_x (SpinChainNNN.init_lattice), ??	lattice (BoseHubbardChain.init_lattice), 578
bc_x (SpinChainNNN2.init_lattice), ??	lattice (BoseHubbardModel.init_lattice), ??
bc_x (TFIMadalinit_lattice), 520	lattice (FermiHubbardChain.init_lattice), 591
bc_x (TFIModel.init_lattice), ??	lattice (FermiHubbardModel.init_lattice), ??
bc_x (ToricCode.init_lattice), ??	lattice (CouplingMPOModel.init_lattice), ??
bc_x (XXZChain2.init_lattice), 535	lattice (SpinChain.init_lattice), 548
bc_y (mulitple definitions), 564	lattice (SpinModel.init_lattice), ??
bc_y (FermionChain.init_lattice), 564	lattice (SpinChainNNN.init_lattice), ??
bc_y (FermionModel.init_lattice), ??	lattice (SpinChainNNN2.init_lattice), ??
bc_y (BosonicHaldaneModel.init_lattice), ??	lattice (TFIChain.init_lattice), 520
bc_y (FermionicHaldaneModel.init_lattice), ??	lattice (TFIModel.init_lattice), ??
bc_y (HofstadterBosons.init_lattice), ??	<pre>lattice (ToricCode.init_lattice), ??</pre>
bc_y (HofstadterFermions.init_lattice), ??	lattice (XXZChain2.init_lattice), 534
bc_y (BoseHubbardChain.init_lattice), 578	Lx (mulitple definitions), 564
bc_y (BoseHubbardModel.init_lattice), ??	Lx (FermionChain.init_lattice), 564
bc_y (FermiHubbardChain.init_lattice), 592	Lx (FermionModel.init_lattice), ??
bc_y (FermiHubbardModel.init_lattice), ??	Lx (BosonicHaldaneModel.init_lattice), ??
bc_y (CouplingMPOModel.init_lattice), ??	Lx (FermionicHaldaneModel.init_lattice), ??
bc_y (SpinChain.init_lattice), 548	Lx (HofstadterBosons.init_lattice), ??
bc_y (SpinModel.init_lattice), ??	Lx (HofstadterFermions.init_lattice), ??
bc_y (SpinChainNNN.init_lattice), ??	Lx (BoseHubbardChain.init_lattice), 578
bc_y (SpinChainNNN2.init_lattice), ??	Lx (BoseHubbardModel.init_lattice), ??
bc_y (TFIChain.init_lattice), 520	Lx (FermiHubbardChain.init_lattice), 592
bc_y (TFIModel.init_lattice), ??	Lx (FermiHubbardModel.init_lattice), ??
bc_y (ToricCode.init_lattice), ??	Lx (CouplingMPOModel.init_lattice), ??
bc_y (XXZChain2.init_lattice), 535	Lx (SpinChain.init_lattice), 548
explicit_plus_hc(CouplingMPOModel), ??	Lx (SpinModel.init_lattice), ??
L (mulitple definitions), 563	Lx (SpinChainNNN.init_lattice), ??
L (FermionChain.init_lattice), 563	Lx ($SpinChainNNN2.init_lattice$), ??
L (FermionModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
L (BosonicHaldaneModel.init_lattice), ??	Lx (TFIModel.init_lattice), ??
L (FermionicHaldaneModel.init_lattice), ??	Lx (Trimodel.init_lattice), ??
L (HofstadterBosons.init_lattice), ??	Lx (XXZChain2.init_lattice), 535
L (HofstadterFermions.init_lattice), ??	Ly (mulitple definitions), 564
L (BoseHubbardChain.init_lattice), 578	Ly (FermionChain.init_lattice), 564
L (BoseHubbardModel init_lattice), 378	Ly (FermionChain.init_lattice), 304 Ly (FermionModel init_lattice), ??
TO COUSE COMPOSITION OF STREET BULLIUM BULLIUM LA LA	TIV TEELIHUUUVUULELUULE KUULEEL LUULEEL

Ly (BosonicHaldaneModel.init_lattice), ??	<pre>diag_method(DMRGEngine.diag), ??</pre>
Ly (FermionicHaldaneModel.init_lattice), ??	diag_method(<i>EngineCombine.diag</i>), 220
Ly (HofstadterBosons.init_lattice), ??	diag_method(<i>EngineCombine.run</i>), 224
Ly (HofstadterFermions.init_lattice), ??	diag_method(<i>EngineFracture.diag</i>), 228
Ly (BoseHubbardChain.init_lattice), 578	diag_method(EngineFracture.run), 233
Ly (BoseHubbardModel.init_lattice), ??	diag_method(SingleSiteDMRGEngine.diag), ??
Ly (FermiHubbardChain.init_lattice), 592	diag_method (SingleSiteDMRGEngine.run), ??
Ly (FermiHubbardModel.init_lattice), ??	diag_method(TwoSiteDMRGEngine.diag),??
Ly (CouplingMPOModel.init_lattice), ??	<pre>diag_method(TwoSiteDMRGEngine.run), ??</pre>
Ly (SpinChain.init_lattice), 548	<pre>E_tol_max (mulitple definitions), ??</pre>
Ly (SpinModel.init_lattice), ??	<pre>E_tol_max (DMRGEngine.run), ??</pre>
Ly (SpinChainNNN.init_lattice), ??	E_tol_max (EngineCombine.run), 224
Ly (SpinChainNNN2.init_lattice), ??	E_tol_max (EngineFracture.run), 233
Ly (TFIChain.init_lattice), 520	<pre>E_tol_max (SingleSiteDMRGEngine.run), ??</pre>
Ly (TFIModel.init_lattice), ??	<pre>E_tol_max (TwoSiteDMRGEngine.run), ??</pre>
Ly (ToricCode.init_lattice), ??	<pre>E_tol_min (mulitple definitions), ??</pre>
Ly (XXZChain2.init_lattice), 535	E_tol_min (DMRGEngine.run), ??
order (mulitple definitions), 563	E_tol_min(<i>EngineCombine.run</i>), 224
order (FermionChain.init_lattice), 563	E_tol_min (EngineFracture.run), 233
order (FermionModel.init_lattice), ??	<pre>E_tol_min (SingleSiteDMRGEngine.run), ??</pre>
order (BosonicHaldaneModel.init_lattice), ??	<pre>E_tol_min (TwoSiteDMRGEngine.run), ??</pre>
order (FermionicHaldaneModel.init_lattice), ??	<pre>E_tol_to_trunc (mulitple definitions), ??</pre>
order (HofstadterBosons.init_lattice), ??	E_tol_to_trunc (DMRGEngine.run), ??
order (HofstadterFermions.init_lattice), ??	E_tol_to_trunc (EngineCombine.run), 224
order (BoseHubbardChain.init_lattice), 578	E_tol_to_trunc (EngineFracture.run), 233
order (BoseHubbardModel.init_lattice), ??	<pre>E_tol_to_trunc (SingleSiteDMRGEngine.run), ??</pre>
order (FermiHubbardChain.init_lattice), 591	<pre>E_tol_to_trunc (TwoSiteDMRGEngine.run), ??</pre>
order (FermiHubbardModel.init_lattice), ??	<pre>init_env_data (mulitple definitions), ??</pre>
order (CouplingMPOModel.init_lattice), ??	<pre>init_env_data(DMRGEngine.init_env), ??</pre>
order (SpinChain.init_lattice), 548	<pre>init_env_data(EngineCombine.init_env), 221</pre>
order (SpinModel.init_lattice), ??	<pre>init_env_data(EngineFracture.init_env), 230</pre>
order (SpinChainNNN.init_lattice), ??	<pre>init_env_data (SingleSiteDMRGEngine.init_env),</pre>
order (SpinChainNNN2.init_lattice), ??	??
order (TFIChain.init_lattice), 520	<pre>init_env_data(TwoSiteDMRGEngine.init_env), ??</pre>
order (TFIModel.init_lattice), ??	<pre>init_env_data (Sweep.init_env), ??</pre>
order (ToricCode.init_lattice), ??	lanczos_params(Sweep),??
order (XXZChain2.init_lattice), 534	max_E_err (mulitple definitions), ??
sort_mpo_legs(CouplingMPOModel),??	<pre>max_E_err (DMRGEngine.run), ??</pre>
	max_E_err(<i>EngineCombine.run</i>), 224
DMRG	max_E_err(<i>EngineFracture.run</i>), 233
active_sites(run),??	<pre>max_E_err (SingleSiteDMRGEngine.run), ??</pre>
chi_list (mulitple definitions), ??	<pre>max_E_err (TwoSiteDMRGEngine.run), ??</pre>
chi_list (DMRGEngine.reset_stats), ??	max_hours (mulitple definitions), ??
chi_list (EngineCombine.reset_stats), 223	max_hours (DMRGEngine.run), ??
chi_list (EngineFracture.reset_stats), 232	max_hours (EngineCombine.run), 224
chi_list (SingleSiteDMRGEngine.reset_stats), ??	max_hours (EngineFracture.run), 233
chi_list (TwoSiteDMRGEngine.reset_stats), ??	<pre>max_hours (SingleSiteDMRGEngine.run), ??</pre>
chi_list (Sweep.reset_stats), ??	<pre>max_hours (TwoSiteDMRGEngine.run), ??</pre>
chi_list (VariationalApplyMPO.reset_stats), ??	max_N_for_ED (mulitple definitions), ??
chi_list (VariationalCompression.reset_stats), ??	max_N_for_ED (DMRGEngine.diag), ??
chi_list (<i>PurificationApplyMPO.reset_stats</i>), 252	max_N_for_ED (EngineCombine.diag), 220
combine (Sweep), ??	max_N_for_ED (EngineFracture.diag), 228
diag_method (mulitple definitions), ??	max_N_for_ED (SingleSiteDMRGEngine.diag), ??
diag method (DMRGEngine run) ??	max N for ED (TwoSiteDMRGEngine.diag), ??

max_S_err (mulitple definitions), ??	orthogonal_to (SingleSiteDMRGEngine.init_env)
max_S_err(<i>DMRGEngine.run</i>), ??	??
max_S_err(<i>EngineCombine.run</i>), 224	orthogonal_to (TwoSiteDMRGEngine.init_env), ??
max_S_err (EngineFracture.run), 233	orthogonal_to(Sweep.init_env),??
max_S_err (SingleSiteDMRGEngine.run), ??	P_tol_max (mulitple definitions), ??
max_S_err(TwoSiteDMRGEngine.run), ??	P_tol_max (DMRGEngine.run), ??
max_sweeps (mulitple definitions), ??	P_tol_max (EngineCombine.run), 225
max_sweeps (DMRGEngine.run), ??	P_tol_max (EngineFracture.run), 233
max_sweeps (EngineCombine.run), 224	P_tol_max (SingleSiteDMRGEngine.run), ??
max_sweeps (EngineFracture.run), 233	P_tol_max (TwoSiteDMRGEngine.run), ??
max_sweeps (SingleSiteDMRGEngine.run), ??	P_tol_min (mulitple definitions), ??
max_sweeps (TwoSiteDMRGEngine.run), ??	P_tol_min (DMRGEngine.run), ??
min_sweeps (mulitple definitions), ??	P_tol_min (EngineCombine.run), 225
min_sweeps (DMRGEngine.run), ??	P_tol_min (EngineFracture.run), 233
min_sweeps (EngineCombine.run), 224	P_tol_min (SingleSiteDMRGEngine.run),??
min_sweeps (EngineFracture.run), 233	P_tol_min (TwoSiteDMRGEngine.run), ??
min_sweeps (SingleSiteDMRGEngine.run), ??	P_tol_to_trunc (mulitple definitions), ??
min_sweeps (TwoSiteDMRGEngine.run), ??	P_tol_to_trunc (DMRGEngine.run), ??
mixer (mulitple definitions), ??	P_tol_to_trunc (EngineCombine.run), 225
mixer (DMRGEngine.mixer_activate), ??	P_tol_to_trunc (EngineFracture.run), 233
mixer (EngineCombine.mixer_activate), 222	P_tol_to_trunc (SingleSiteDMRGEngine.run), ??
mixer (EngineFracture.mixer_activate), 231	P_tol_to_trunc(TwoSiteDMRGEngine.run), ??
mixer (SingleSiteDMRGEngine.mixer_activate), ??	start_env (mulitple definitions), ??
mixer (TwoSiteDMRGEngine.mixer_activate), ??	start_env (DMRGEngine.init_env), ??
mixer_params (mulitple definitions), ??	start_env (EngineCombine.init_env), 222
mixer_params (DMRGEngine.mixer_activate),??	start_env (EngineFracture.init_env), 230
mixer_params (EngineCombine.mixer_activate), 222	start_env (SingleSiteDMRGEngine.init_env),??
mixer_params (EngineFracture.mixer_activate), 231	start_env (TwoSiteDMRGEngine.init_env), ??
mixer_params (SingleSiteDMR-	start_env (Sweep.init_env), ??
GEngine.mixer_activate), ??	<pre>sweep_0 (mulitple definitions), ??</pre>
mixer_params (TwoSiteDMR-	<pre>sweep_0 (DMRGEngine.reset_stats), ??</pre>
GEngine.mixer_activate), ??	sweep_0 (EngineCombine.reset_stats), 224
N_sweeps_check (mulitple definitions), ??	sweep_0 (EngineFracture.reset_stats), 232
N_sweeps_check (DMRGEngine.run), ??	<pre>sweep_0 (SingleSiteDMRGEngine.reset_stats), ??</pre>
N_sweeps_check (EngineCombine.run), 224	<pre>sweep_0 (TwoSiteDMRGEngine.reset_stats), ??</pre>
N_sweeps_check (EngineFracture.run), 233	<pre>sweep_0 (Sweep.reset_stats), ??</pre>
N_sweeps_check (SingleSiteDMRGEngine.run), ??	<pre>sweep_0 (VariationalApplyMPO.reset_stats), ??</pre>
N_sweeps_check (TwoSiteDMRGEngine.run), ??	<pre>sweep_0 (VariationalCompression.reset_stats), ??</pre>
norm_tol (mulitple definitions), ??	sweep_0 (PurificationApplyMPO.reset_stats), 252
norm_tol (DMRGEngine.run), ??	trunc_params (Algorithm), ??
norm_tol (EngineCombine.run), 225	update_env (mulitple definitions),??
norm_tol (EngineFracture.run), 233	update_env (DMRGEngine.run), ??
norm_tol(SingleSiteDMRGEngine.run), ??	update_env (EngineCombine.run), 225
norm_tol (TwoSiteDMRGEngine.run), ??	update_env (EngineFracture.run), 233
norm_tol_iter (mulitple definitions), ??	update_env (SingleSiteDMRGEngine.run), ??
norm_tol_iter(<i>DMRGEngine.run</i>),??	update_env (TwoSiteDMRGEngine.run), ??
norm_tol_iter(<i>EngineCombine.run</i>), 225	DMDCEngine
norm_tol_iter(EngineFracture.run), 233	DMRGEngine
norm_tol_iter(SingleSiteDMRGEngine.run),??	<pre>chi_list (mulitple definitions), ??</pre>
norm_tol_iter(TwoSiteDMRGEngine.run), ??	<pre>chi_list (DMRGEngine.reset_stats), ??</pre>
orthogonal_to (mulitple definitions), ??	<pre>chi_list (EngineCombine.reset_stats), 223</pre>
orthogonal_to (DMRGEngine.init_env), ??	chi_list (EngineFracture.reset_stats), 232
orthogonal_to (EngineCombine.init_env), 221	<pre>chi_list (SingleSiteDMRGEngine.reset_stats), ??</pre>
orthogonal to (EngineFracture.init env), 230	chi list (Two Site DMR G. Engine reset state) ??

<pre>chi_list (Sweep.reset_stats), ??</pre>	<pre>max_hours (TwoSiteDMRGEngine.run), ??</pre>
<pre>chi_list (VariationalApplyMPO.reset_stats), ??</pre>	<pre>max_N_for_ED (mulitple definitions), ??</pre>
<pre>chi_list (VariationalCompression.reset_stats), ??</pre>	<pre>max_N_for_ED (DMRGEngine.diag), ??</pre>
chi_list (PurificationApplyMPO.reset_stats), 252	max_N_for_ED (EngineCombine.diag), 220
combine (Sweep), ??	max_N_for_ED (EngineFracture.diag), 228
diag_method(mulitple definitions),??	<pre>max_N_for_ED (SingleSiteDMRGEngine.diag), ??</pre>
diag_method(DMRGEngine.run),??	max_N_for_ED (TwoSiteDMRGEngine.diag), ??
diag_method(DMRGEngine.diag),??	max_S_err (mulitple definitions), ??
diag_method (EngineCombine.diag), 220	max_S_err (DMRGEngine.run), ??
diag_method(<i>EngineCombine.run</i>), 224	max_S_err (EngineCombine.run), 224
diag_method (EngineFracture.diag), 228	max_S_err (EngineFracture.run), 233
diag_method (EngineFracture.run), 233	max_S_err (SingleSiteDMRGEngine.run), ??
diag_method (SingleSiteDMRGEngine.diag), ??	max_S_err (TwoSiteDMRGEngine.run), ??
diag_method (SingleSiteDMRGEngine.run), ??	max_sweeps (mulitple definitions), ??
diag_method(<i>TwoSiteDMRGEngine.diag</i>),??	max_sweeps (multiple definitions), max_sweeps (DMRGEngine.run), ??
diag_method (TwoSiteDMRGEngine.run), ??	max_sweeps (EngineCombine.run), 224
E_tol_max (mulitple definitions), ??	max_sweeps (EngineFracture.run), 233
E_tol_max (DMRGEngine.run), ??	max_sweeps (SingleSiteDMRGEngine.run), ??
E_tol_max (EngineCombine.run), 224	max_sweeps (TwoSiteDMRGEngine.run), ??
E_tol_max (EngineFracture.run), 233	min_sweeps (mulitple definitions), ??
E_tol_max (SingleSiteDMRGEngine.run), ??	min_sweeps (DMRGEngine.run), ??
E_tol_max (TwoSiteDMRGEngine.run), ??	min_sweeps (EngineCombine.run), 224
E_tol_min (mulitple definitions), ??	min_sweeps (EngineFracture.run), 233
E_tol_min(<i>DMRGEngine.run</i>), ??	min_sweeps (SingleSiteDMRGEngine.run), ??
E_tol_min (<i>EngineCombine.run</i>), 224	min_sweeps (TwoSiteDMRGEngine.run), ??
E_tol_min (<i>EngineFracture.run</i>), 233	N_sweeps_check (mulitple definitions), ??
E_tol_min (SingleSiteDMRGEngine.run), ??	N_sweeps_check (DMRGEngine.run), ??
E_tol_min (<i>TwoSiteDMRGEngine.run</i>), ??	N_sweeps_check (EngineCombine.run), 224
E_tol_to_trunc (mulitple definitions), ??	N_sweeps_check (EngineFracture.run), 233
E_tol_to_trunc (<i>DMRGEngine.run</i>), ??	N_sweeps_check (SingleSiteDMRGEngine.run), ??
E_tol_to_trunc (<i>EngineCombine.run</i>), 224	N_sweeps_check (TwoSiteDMRGEngine.run), ??
E_tol_to_trunc (<i>EngineFracture.run</i>), 233	<pre>norm_tol (mulitple definitions), ??</pre>
<pre>E_tol_to_trunc (SingleSiteDMRGEngine.run), ??</pre>	<pre>norm_tol (DMRGEngine.run), ??</pre>
<pre>E_tol_to_trunc (TwoSiteDMRGEngine.run), ??</pre>	norm_tol (EngineCombine.run), 225
<pre>init_env_data (mulitple definitions), ??</pre>	norm_tol (EngineFracture.run), 233
<pre>init_env_data (DMRGEngine.init_env), ??</pre>	norm_tol (SingleSiteDMRGEngine.run), ??
init_env_data (EngineCombine.init_env), 221	norm_tol (TwoSiteDMRGEngine.run), ??
<pre>init_env_data(EngineFracture.init_env), 230</pre>	norm_tol_iter (mulitple definitions), ??
<pre>init_env_data (SingleSiteDMRGEngine.init_env),</pre>	norm_tol_iter (DMRGEngine.run), ??
??	norm_tol_iter (EngineCombine.run), 225
<pre>init_env_data (TwoSiteDMRGEngine.init_env), ??</pre>	norm_tol_iter (EngineFracture.run), 233
init_env_data(Sweep.init_env), ??	norm_tol_iter(SingleSiteDMRGEngine.run), ??
lanczos_params (Sweep), ??	norm_tol_iter(TwoSiteDMRGEngine.run), ??
max_E_err (mulitple definitions), ??	orthogonal_to (mulitple definitions), ??
max_E_err (DMRGEngine.run), ??	orthogonal_to (DMRGEngine.init_env), ??
max_E_err(EngineCombine.run), 224	orthogonal_to (EngineCombine.init_env), 221
max_E_err (EngineFracture.run), 233	orthogonal_to (EngineFracture.init_env), 230
max_E_err(SingleSiteDMRGEngine.run), ??	orthogonal_to (SingleSiteDMRGEngine.init_env)
max_E_err(TwoSiteDMRGEngine.run), ??	??
max_hours (mulitple definitions), ??	orthogonal_to (TwoSiteDMRGEngine.init_env), ??
max_hours (DMRGEngine.run), ??	orthogonal_to(Sweep.init_env),??
max_hours (<i>EngineCombine.run</i>), 224	P_tol_max (mulitple definitions), ??
max_hours (EngineFracture.run), 233	P_tol_max (<i>DMRGEngine.run</i>), ??
max_hours (SingleSiteDMRGEngine.run), ??	P_tol_max (EngineCombine.run), 225

P_tol_max (<i>EngineFracture.run</i>), 233	<pre>init_env_data (SingleSiteDMRGEngine.init_env),</pre>
P_tol_max (SingleSiteDMRGEngine.run), ??	??
P_tol_max(<i>TwoSiteDMRGEngine.run</i>), ??	<pre>init_env_data(TwoSiteDMRGEngine.init_env), ??</pre>
P_tol_min (mulitple definitions), ??	<pre>init_env_data (Sweep.init_env), ??</pre>
P_tol_min(<i>DMRGEngine.run</i>), ??	lanczos_params (Sweep), ??
P_tol_min(<i>EngineCombine.run</i>), 225	m_temp (MPO.apply_zipup), ??
P_tol_min (<i>EngineFracture.run</i>), 233	<pre>N_steps (TimeEvolutionAlgorithm), ??</pre>
P_tol_min (SingleSiteDMRGEngine.run), ??	N_sweeps (Variational Compression), ??
P_tol_min (TwoSiteDMRGEngine.run), ??	order (ExpMPOEvolution), ??
P_tol_to_trunc (<i>mulitple definitions</i>), ??	<pre>orthogonal_to (mulitple definitions), ??</pre>
P_tol_to_trunc (DMRGEngine.run), ??	orthogonal_to (DMRGEngine.init_env), ??
P_tol_to_trunc (EngineCombine.run), 225	orthogonal_to (EngineCombine.init_env), 221
P_tol_to_trunc (EngineFracture.run), 233	orthogonal_to (EngineFracture.init_env), 230
P_tol_to_trunc (SingleSiteDMRGEngine.run), ??	orthogonal_to (SingleSiteDMRGEngine.init_env),
P_tol_to_trunc(TwoSiteDMRGEngine.run),??	??
start_env (<i>mulitple definitions</i>), ??	orthogonal_to (TwoSiteDMRGEngine.init_env), ??
start_env (<i>DMRGEngine.init_env</i>), ??	orthogonal_to (Sweep.init_env), ??
start_env (EngineCombine.init_env), 222	start_env (mulitple definitions), ??
start_env (EngineFracture.init_env), 230	start_env (DMRGEngine.init_env), ??
start_env (SingleSiteDMRGEngine.init_env), ??	start_env (EngineCombine.init_env), 222
start_env (TwoSiteDMRGEngine.init_env), ??	start_env (EngineFracture.init_env), 230
start_env (Sweep.init_env), ??	start_env (SingleSiteDMRGEngine.init_env), ??
sweep_0 (mulitple definitions), ??	start_env (TwoSiteDMRGEngine.init_env), ??
	_ ,
sweep_0 (DMRGEngine.reset_stats), ?? sweep_0 (EngineCombine.reset_stats), 224	start_env (Sweep.init_env), ??
- /	start_time (TimeEvolutionAlgorithm), ??
sweep_0 (EngineFracture.reset_stats), 232	start_trunc_err (ExpMPOEvolution), ??
sweep_0 (SingleSiteDMRGEngine.reset_stats), ??	sweep_0 (mulitple definitions), ??
sweep_0 (TwoSiteDMRGEngine.reset_stats), ??	sweep_0 (Sweep.reset_stats), ??
sweep_0 (Sweep.reset_stats), ??	sweep_0 (VariationalApplyMPO.reset_stats), ??
sweep_0 (VariationalApplyMPO.reset_stats), ??	<pre>sweep_0 (VariationalCompression.reset_stats), ??</pre>
sweep_0 (VariationalCompression.reset_stats), ??	sweep_0 (PurificationApplyMPO.reset_stats), 252
sweep_0 (PurificationApplyMPO.reset_stats), 252	trunc_params (mulitple definitions), ??
trunc_params (Algorithm), ??	trunc_params (MPO.apply), ??
update_env (mulitple definitions), ??	trunc_params (MPO.apply_zipup), ??
update_env (<i>DMRGEngine.run</i>), ??	trunc_params (Variational Compression), ??
update_env (<i>EngineCombine.run</i>), 225	trunc_params (Algorithm), ??
update_env (<i>EngineFracture.run</i>), 233	trunc_weight (MPO.apply_zipup), ??
update_env (SingleSiteDMRGEngine.run), ??	Cawaaii li Jalaayal Maalal
update_env (TwoSiteDMRGEngine.run), ??	FermiHubbardModel
E MDOE Latina	bc_MPS (mulitple definitions), 563
ExpMPOEvolution	bc_MPS (FermionChain.init_lattice), 563
approximation (ExpMPOEvolution), ??	bc_MPS (FermionModel.init_lattice), ??
chi_list (mulitple definitions), ??	bc_MPS (BosonicHaldaneModel.init_lattice), ??
chi_list (Sweep.reset_stats), ??	bc_MPS (FermionicHaldaneModel.init_lattice), ??
chi_list (VariationalApplyMPO.reset_stats), ??	bc_MPS (HofstadterBosons.init_lattice), ??
chi_list (VariationalCompression.reset_stats), ??	bc_MPS (HofstadterFermions.init_lattice), ??
chi_list (PurificationApplyMPO.reset_stats), 252	bc_MPS (BoseHubbardChain.init_lattice), 578
combine (Sweep), ??	bc_MPS (BoseHubbardModel.init_lattice), ??
compression_method (MPO.apply), ??	bc_MPS (FermiHubbardChain.init_lattice), 591
dt (TimeEvolutionAlgorithm), ??	bc_MPS (FermiHubbardModel.init_lattice), ??
init_env_data (mulitple definitions), ??	bc_MPS (CouplingMPOModel.init_lattice), ??
init_env_data (<i>DMRGEngine.init_env</i>), ??	bc_MPS (SpinChain.init_lattice), 548
init_env_data(<i>BMRGEngine.init_env</i>), :: init_env_data(<i>EngineCombine.init_env</i>), 221	bc_MPS (SpinModel.init_lattice), ??
init env data (Engine Combine.init env), 221	bc_MPS (SpinChainNNN init_lattice) ??

bc_MPS (SpinChainNNN2.init_lattice), ??	□ (HofstadterFermions.init_lattice), ??
bc_MPS (TFIChain.init_lattice), 520	L (BoseHubbardChain.init_lattice), 578
bc_MPS (TFIModel.init_lattice), ??	L (BoseHubbardModel.init_lattice), ??
bc_MPS (ToricCode.init_lattice), ??	L (FermiHubbardChain.init_lattice), 591
bc_MPS (XXZChain2.init_lattice), 534	L (FermiHubbardModel.init_lattice), ??
bc_x (mulitple definitions), 564	L (CouplingMPOModel.init_lattice), ??
bc_x (FermionChain.init_lattice), 564	L (SpinChain.init_lattice), 548
bc_x (FermionModel.init_lattice), ??	L (SpinModel.init_lattice), ??
bc_x (BosonicHaldaneModel.init_lattice), ??	L (SpinChainNNN.init_lattice), ??
bc_x (FermionicHaldaneModel.init_lattice), ??	L (SpinChainNNN2.init_lattice), ??
bc_x (HofstadterBosons.init_lattice), ??	L (TFIChain.init_lattice), 520
bc_x (HofstadterFermions.init_lattice), ??	L (TFIModel.init_lattice), ??
bc_x (BoseHubbardChain.init_lattice), 578	L (ToricCode.init_lattice), ??
bc_x (BoseHubbardModel.init_lattice), ??	L (XXZChain2.init_lattice), 534
bc_x (FermiHubbardChain.init_lattice), 592	lattice (mulitple definitions), 563
bc_x (FermiHubbardModel.init_lattice), ??	lattice (FermionChain.init_lattice), 563
bc_x (CouplingMPOModel.init_lattice), ??	<pre>lattice (FermionModel.init_lattice), ??</pre>
bc_x (SpinChain.init_lattice), 548	lattice (BosonicHaldaneModel.init_lattice), ??
bc_x (SpinModel.init_lattice), ??	lattice (FermionicHaldaneModel.init_lattice), ??
bc_x (SpinChainNNN.init_lattice), ??	lattice (HofstadterBosons.init_lattice), ??
bc_x (SpinChainNNN2.init_lattice), ??	lattice (HofstadterFermions.init_lattice), ??
bc_x (TFIChain.init_lattice), 520	lattice (BoseHubbardChain.init_lattice), 578
bc_x (TFIModel.init_lattice), ??	lattice (BoseHubbardModel.init_lattice), ??
bc_x (ToricCode.init_lattice), ??	lattice (FermiHubbardChain.init_lattice), 591
bc_x (XXZChain2.init_lattice), 535	<pre>lattice (FermiHubbardModel.init_lattice), ??</pre>
bc_y (mulitple definitions), 564	lattice (CouplingMPOModel.init_lattice), ??
bc_y (FermionChain.init_lattice), 564	lattice (SpinChain.init_lattice), 548
bc_y (FermionModel.init_lattice), ??	<pre>lattice (SpinModel.init_lattice), ??</pre>
bc_y (BosonicHaldaneModel.init_lattice), ??	<pre>lattice (SpinChainNNN.init_lattice), ??</pre>
bc_y (FermionicHaldaneModel.init_lattice), ??	<pre>lattice (SpinChainNNN2.init_lattice), ??</pre>
bc_y (HofstadterBosons.init_lattice), ??	lattice (TFIChain.init_lattice), 520
bc_y (HofstadterFermions.init_lattice), ??	<pre>lattice (TFIModel.init_lattice), ??</pre>
bc_y (BoseHubbardChain.init_lattice), 578	<pre>lattice (ToricCode.init_lattice), ??</pre>
bc_y (BoseHubbardModel.init_lattice), ??	lattice (XXZChain2.init_lattice), 534
bc_y (FermiHubbardChain.init_lattice), 592	Lx (mulitple definitions), 564
bc_y (FermiHubbardModel.init_lattice), ??	Lx (FermionChain.init_lattice), 564
bc_y (CouplingMPOModel.init_lattice), ??	Lx (FermionModel.init_lattice), ??
bc_y (SpinChain.init_lattice), 548	Lx (BosonicHaldaneModel.init_lattice), ??
bc_y (SpinModel.init_lattice), ??	Lx (FermionicHaldaneModel.init_lattice), ??
bc_y (SpinChainNNN.init_lattice), ??	Lx (HofstadterBosons.init_lattice), ??
bc_y (SpinChainNNN2.init_lattice), ??	Lx (HofstadterFermions.init_lattice), ??
bc_y (TFIChain.init_lattice), 520	Lx (BoseHubbardChain.init_lattice), 578
bc_y (TFIModel.init_lattice), ??	Lx (BoseHubbardModel.init_lattice), ??
bc_y (ToricCode.init_lattice), ??	Lx (FermiHubbardChain.init_lattice), 592
bc_y (XXZChain2.init_lattice), 535	Lx (FermiHubbardModel.init_lattice), ??
cons_N (FermiHubbardModel), ??	Lx (CouplingMPOModel.init_lattice), ??
cons_Sz (<i>FermiHubbardModel</i>), ??	Lx (SpinChain.init_lattice), 548
explicit_plus_hc(CouplingMPOModel), ??	Lx (SpinModel.init_lattice), ??
L (mulitple definitions), 563	Lx (SpinChainNNN.init_lattice), ??
L (FermionChain.init_lattice), 563	Lx (SpinChainNNN2.init_lattice), ??
L (FermionModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
L (BosonicHaldaneModel.init_lattice), ??	Lx (TFIModel.init_lattice), ??
L (FermionicHaldaneModel.init_lattice), ??	Lx (ToricCode.init_lattice), ??
□ (HofstadterBosons.init_lattice), ??	Lx (XXZChain2.init_lattice), 535

Ly (mulitple definitions), 564	bc_MPS (BoseHubbardModel.init_lattice), ??
Ly (FermionChain.init_lattice), 564	bc_MPS (FermiHubbardChain.init_lattice), 591
Ly (FermionModel.init_lattice), ??	bc_MPS (FermiHubbardModel.init_lattice), ??
Ly (BosonicHaldaneModel.init_lattice), ??	bc_MPS (CouplingMPOModel.init_lattice), ??
Ly (FermionicHaldaneModel.init_lattice), ??	bc_MPS (SpinChain.init_lattice), 548
Ly (HofstadterBosons.init_lattice), ??	bc_MPS (SpinModel.init_lattice), ??
Ly (HofstadterFermions.init_lattice), ??	bc_MPS (SpinChainNNN.init_lattice), ??
Ly (BoseHubbardChain.init_lattice), 578	bc_MPS (SpinChainNNN2.init_lattice), ??
Ly (BoseHubbardModel.init_lattice), ??	bc_MPS (<i>TFIChain.init_lattice</i>), 520
Ly (FermiHubbardChain.init_lattice), 592	bc_MPS (TFIModel.init_lattice), ??
Ly (FermiHubbardModel.init_lattice), ??	bc_MPS (ToricCode.init_lattice), ??
Ly (CouplingMPOModel.init_lattice), ??	bc_MPS (XXZChain2.init_lattice), 534
Ly (SpinChain.init_lattice), 548	bc_x (mulitple definitions), 564
Ly (SpinModel.init_lattice), ??	bc_x (FermionChain.init_lattice), 564
Ly (SpinChainNNN.init_lattice), ??	bc_x (FermionModel.init_lattice), ??
Ly (SpinChainNNN2.init_lattice), ??	bc_x (BosonicHaldaneModel.init_lattice), ??
Ly (TFIChain.init_lattice), 520	bc_x (FermionicHaldaneModel.init_lattice), ??
Ly (TFIModel.init_lattice), ??	bc_x (HofstadterBosons.init_lattice), ??
Ly (ToricCode.init_lattice), ??	bc_x (HofstadterFermions.init_lattice), ??
Ly (XXZChain2.init_lattice), 535	bc_x (BoseHubbardChain.init_lattice), 578
mu (FermiHubbardModel), ??	bc_x (BoseHubbardModel.init_lattice), ??
order (mulitple definitions), 563	bc_x (FermiHubbardChain.init_lattice), 592
order (FermionChain.init_lattice), 563	bc_x (FermiHubbardModel.init_lattice), ??
order (FermionModel.init_lattice), ??	bc_x (CouplingMPOModel.init_lattice), ??
order (BosonicHaldaneModel.init_lattice), ??	bc_x (SpinChain.init_lattice), 548
order (FermionicHaldaneModel.init_lattice), ??	bc_x (SpinChain.init_lattice), ??
order (HofstadterBosons.init_lattice), ??	bc_x (SpinChainNNN.init_lattice), ??
order (HofstadterFermions.init_lattice), ??	bc_x (SpinChainNNN2.init_lattice), ??
order (BoseHubbardChain.init_lattice), 578	bc_x (TFIChain.init_lattice), 520
order (BoseHubbardModel.init_lattice), ??	bc_x (TFIModel.init_lattice), ??
order (FermiHubbardChain.init_lattice), 591	bc_x (ToricCode.init_lattice), ??
order (FermiHubbardModel.init_lattice), ??	bc_x (XXZChain2.init_lattice), 535
order (CouplingMPOModel.init_lattice), ??	bc_y (mulitple definitions), 564
order (SpinChain.init_lattice), 548	bc_y (FermionChain.init_lattice), 564
order (SpinModel.init_lattice), ??	bc_y (FermionModel.init_lattice), ??
order (SpinChainNNN.init_lattice), ??	bc_y (BosonicHaldaneModel.init_lattice), ??
order (SpinChainNNN2.init_lattice), ??	bc_y (FermionicHaldaneModel.init_lattice), ??
order (TFIChain.init_lattice), 520	bc_y (HofstadterBosons.init_lattice), ??
order (TFIModel.init_lattice), ??	bc_y (HofstadterFermions.init_lattice), ??
order (ToricCode.init_lattice), ??	bc_y (BoseHubbardChain.init_lattice), 578
order (XXZChain2.init_lattice), 534	bc_y (BoseHubbardModel.init_lattice), ??
sort_mpo_legs(CouplingMPOModel),??	bc_y (FermiHubbardChain.init_lattice), 592
t (FermiHubbardModel), ??	bc_y (FermiHubbardModel.init_lattice), ??
U (FermiHubbardModel), ??	bc_y (CouplingMPOModel.init_lattice), ??
Formionial Inlana Madal	bc_y (SpinChain.init_lattice), 548
FermionicHaldaneModel	bc_y (SpinModel.init_lattice), ??
bc_MPS (mulitple definitions), 563	bc_y (SpinChainNNN.init_lattice), ??
bc_MPS (FermionChain.init_lattice), 563	bc_y (SpinChainNNN2.init_lattice), ??
bc_MPS (FermionModel.init_lattice), ??	bc_y (<i>TFIChain.init_lattice</i>), 520
bc_MPS (BosonicHaldaneModel.init_lattice), ??	bc_y (TFIModel.init_lattice), ??
bc_MPS (FermionicHaldaneModel.init_lattice), ??	bc_y (ToricCode.init_lattice), ??
bc_MPS (HofstadterBosons.init_lattice), ??	bc_y (XXZChain2.init_lattice), 535
bc_MPS (HofstadterFermions.init_lattice), ??	conserve (FermionicHaldaneModel), ??
bc_MPS (BoseHubbardChain.init_lattice), 578	explicit_plus_hc(CouplingMPOModel),?

□ (mulitple definitions), 563	Lx (SpinChainNNN.init_lattice), ??
□ (FermionChain.init_lattice), 563	Lx (SpinChainNNN2.init_lattice), ??
L (FermionModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
□ (BosonicHaldaneModel.init_lattice), ??	Lx (TFIModel.init_lattice), ??
□ (FermionicHaldaneModel.init_lattice), ??	Lx (ToricCode.init_lattice), ??
L (HofstadterBosons.init_lattice), ??	Lx (XXZChain2.init_lattice), 535
L (HofstadterFermions.init_lattice), ??	Ly (mulitple definitions), 564
L (BoseHubbardChain.init_lattice), 578	Ly (FermionChain.init_lattice), 564
L (BoseHubbardModel.init_lattice), ??	Ly (FermionModel.init_lattice), ??
L (FermiHubbardChain.init_lattice), 591	Ly (BosonicHaldaneModel.init_lattice), ??
□ (FermiHubbardModel.init_lattice), ??	Ly (FermionicHaldaneModel.init_lattice), ??
L (CouplingMPOModel.init_lattice), ??	Ly (HofstadterBosons.init_lattice), ??
L (SpinChain.init_lattice), 548	Ly (HofstadterFermions.init_lattice), ??
L (SpinModel.init_lattice), ??	Ly (BoseHubbardChain.init_lattice), 578
L (SpinChainNNN.init_lattice), ??	Ly (BoseHubbardModel.init_lattice), ??
L (SpinChainNNN2.init_lattice), ??	Ly (FermiHubbardChain.init_lattice), 592
L (TFIChain.init_lattice), 520	Ly (FermiHubbardModel.init_lattice), ??
L (TFIModel.init_lattice), ??	Ly (CouplingMPOModel.init_lattice), ??
L (ToricCode.init_lattice), ??	Ly (SpinChain.init_lattice), 548
L (XXZChain2.init_lattice), 534	Ly (SpinModel.init_lattice), ??
lattice (mulitple definitions), 563	Ly (SpinChainNNN.init_lattice), ??
lattice (Fermion Chain.init_lattice), 563	Ly (SpinChainNNN2.init_lattice), ??
lattice (Fermion Model.init_lattice), ??	Ly (TFIChain.init_lattice), 520
	- ·
lattice (BosonicHaldaneModel.init_lattice), ??	Ly (TFIModel.init_lattice), ??
lattice (FermionicHaldaneModel.init_lattice), ??	Ly (ToricCode.init_lattice), ??
lattice (HofstadterBosons.init_lattice), ??	Ly (XXZChain2.init_lattice), 535
lattice (HofstadterFermions.init_lattice), ??	mu (FermionicHaldaneModel), ??
lattice (BoseHubbardChain.init_lattice), 578	order (mulitple definitions), 563
lattice (BoseHubbardModel.init_lattice), ??	order (FermionChain.init_lattice), 563
lattice (FermiHubbardChain.init_lattice), 591	order (FermionModel.init_lattice), ??
lattice (FermiHubbardModel.init_lattice), ??	order (BosonicHaldaneModel.init_lattice), ??
lattice (CouplingMPOModel.init_lattice), ??	order (FermionicHaldaneModel.init_lattice), ??
lattice (SpinChain.init_lattice), 548	order (HofstadterBosons.init_lattice), ??
lattice (SpinModel.init_lattice), ??	order (HofstadterFermions.init_lattice), ??
lattice (SpinChainNNN.init_lattice), ??	order (BoseHubbardChain.init_lattice), 578
lattice (SpinChainNNN2.init_lattice), ??	order (BoseHubbardModel.init_lattice), ??
lattice (TFIChain.init_lattice), 520	order (FermiHubbardChain.init_lattice), 591
lattice (TFIModel.init_lattice), ??	order (FermiHubbardModel.init_lattice), ??
<pre>lattice (ToricCode.init_lattice), ??</pre>	order (CouplingMPOModel.init_lattice), ??
lattice (XXZChain2.init_lattice), 534	order (SpinChain.init_lattice), 548
Lx (mulitple definitions), 564	order (SpinModel.init_lattice), ??
Lx (FermionChain.init_lattice), 564	order (SpinChainNNN.init_lattice), ??
Lx (FermionModel.init_lattice), ??	order (SpinChainNNN2.init_lattice), ??
Lx (BosonicHaldaneModel.init_lattice), ??	order (TFIChain.init_lattice), 520
Lx (FermionicHaldaneModel.init_lattice), ??	order (TFIModel.init_lattice), ??
Lx (HofstadterBosons.init_lattice), ??	order (ToricCode.init_lattice), ??
Lx (HofstadterFermions.init_lattice), ??	order (XXZChain2.init_lattice), 534
Lx (BoseHubbardChain.init_lattice), 578	sort_mpo_legs(CouplingMPOModel), ??
Lx (BoseHubbardModel.init_lattice), ??	t1 (FermionicHaldaneModel), ??
Lx (FermiHubbardChain.init_lattice), 592	t2 (FermionicHaldaneModel), ??
Lx (FermiHubbardModel.init_lattice), ??	∨ (FermionicHaldaneModel), ??
Lx (CouplingMPOModel.init_lattice), ??	
Lx (SpinChain.init_lattice), 548	FermionModel
Lx (SpinModel.init_lattice), ??	bc_MPS (mulitple definitions), 563

bc_MPS (FermionChain.init_lattice), 563	bc_y (SpinChainNNN2.init_lattice), ??
bc_MPS (FermionModel.init_lattice), ??	bc_y (<i>TFIChain.init_lattice</i>), 520
bc_MPS (BosonicHaldaneModel.init_lattice), ??	bc_y (TFIModel.init_lattice), ??
bc_MPS (FermionicHaldaneModel.init_lattice), ??	bc_y (ToricCode.init_lattice), ??
bc_MPS (HofstadterBosons.init_lattice), ??	bc_y (XXZChain2.init_lattice), 535
<pre>bc_MPS (HofstadterFermions.init_lattice), ??</pre>	conserve (FermionModel), ??
bc_MPS (BoseHubbardChain.init_lattice), 578	<pre>explicit_plus_hc(CouplingMPOModel),??</pre>
bc_MPS (BoseHubbardModel.init_lattice), ??	J (FermionModel), ??
bc_MPS (FermiHubbardChain.init_lattice), 591	L (mulitple definitions), 563
bc_MPS (FermiHubbardModel.init_lattice), ??	L (FermionChain.init_lattice), 563
bc_MPS (CouplingMPOModel.init_lattice), ??	⊥ (FermionModel.init_lattice), ??
bc_MPS (SpinChain.init_lattice), 548	L (BosonicHaldaneModel.init_lattice), ??
bc_MPS (SpinModel.init_lattice), ??	L (FermionicHaldaneModel.init_lattice), ??
bc_MPS (SpinChainNNN.init_lattice), ??	L (HofstadterBosons.init_lattice), ??
bc_MPS (SpinChainNNN2.init_lattice), ??	L (HofstadterFermions.init_lattice), ??
bc_MPS (TFIChain.init_lattice), 520	L (BoseHubbardChain.init_lattice), 578
bc_MPS (TFIModel.init_lattice), ??	L (BoseHubbardModel.init_lattice), ??
bc_MPS (ToricCode.init_lattice), ??	L (FermiHubbardChain.init_lattice), 591
bc_MPS (XXZChain2.init_lattice), 534	L (FermiHubbardModel.init_lattice), ??
bc_x (mulitple definitions), 564	L (CouplingMPOModel.init_lattice), ??
bc_x (FermionChain.init_lattice), 564	L (SpinChain.init_lattice), 548
bc_x (FermionModel.init_lattice), ??	L (SpinModel.init_lattice), ??
bc_x (BosonicHaldaneModel.init_lattice), ??	L (SpinChainNNN.init_lattice), ??
bc_x (FermionicHaldaneModel.init_lattice), ??	L (SpinChainNNN2.init_lattice), ??
bc_x (HofstadterBosons.init_lattice), ??	L (TFIChain.init_lattice), 520
bc_x (HofstadterFermions.init_lattice), ??	L (TFIModel.init_lattice), ??
bc_x (BoseHubbardChain.init_lattice), 578	L (ToricCode.init_lattice), ??
bc_x (BoseHubbardModel.init_lattice), ??	L (XXZChain2.init_lattice), 534
bc_x (FermiHubbardChain.init_lattice), 592	lattice (mulitple definitions), 563
bc_x (FermiHubbardModel.init_lattice), ??	lattice (FermionChain.init_lattice), 563
bc_x (CouplingMPOModel.init_lattice), ??	<pre>lattice (FermionModel.init_lattice), ??</pre>
bc_x (SpinChain.init_lattice), 548	<pre>lattice (BosonicHaldaneModel.init_lattice), ??</pre>
bc_x (SpinModel.init_lattice), ??	<pre>lattice (FermionicHaldaneModel.init_lattice), ??</pre>
bc_x (SpinChainNNN.init_lattice), ??	<pre>lattice (HofstadterBosons.init_lattice), ??</pre>
bc_x (SpinChainNNN2.init_lattice), ??	<pre>lattice (HofstadterFermions.init_lattice), ??</pre>
bc_x (TFIChain.init_lattice), 520	lattice (BoseHubbardChain.init_lattice), 578
bc_x (TFIModel.init_lattice), ??	<pre>lattice (BoseHubbardModel.init_lattice), ??</pre>
bc_x (ToricCode.init_lattice), ??	<pre>lattice (FermiHubbardChain.init_lattice), 591</pre>
bc_x (XXZChain2.init_lattice), 535	<pre>lattice (FermiHubbardModel.init_lattice), ??</pre>
bc_y (mulitple definitions), 564	<pre>lattice (CouplingMPOModel.init_lattice), ??</pre>
bc_y (FermionChain.init_lattice), 564	lattice (SpinChain.init_lattice), 548
bc_y (FermionModel.init_lattice), ??	<pre>lattice (SpinModel.init_lattice), ??</pre>
bc_y (BosonicHaldaneModel.init_lattice), ??	<pre>lattice (SpinChainNNN.init_lattice), ??</pre>
bc_y (FermionicHaldaneModel.init_lattice), ??	<pre>lattice (SpinChainNNN2.init_lattice), ??</pre>
bc_y (HofstadterBosons.init_lattice), ??	lattice (TFIChain.init_lattice), 520
bc_y (HofstadterFermions.init_lattice), ??	<pre>lattice (TFIModel.init_lattice), ??</pre>
bc_y (BoseHubbardChain.init_lattice), 578	<pre>lattice (ToricCode.init_lattice), ??</pre>
bc_y (BoseHubbardModel.init_lattice), ??	lattice (XXZChain2.init_lattice), 534
bc_y (FermiHubbardChain.init_lattice), 592	Lx (mulitple definitions), 564
bc_y (FermiHubbardModel.init_lattice), ??	Lx (FermionChain.init_lattice), 564
bc_y (CouplingMPOModel.init_lattice), ??	Lx (FermionModel.init_lattice), ??
bc_y (SpinChain.init_lattice), 548	Lx (BosonicHaldaneModel.init_lattice), ??
bc_y (SpinModel.init_lattice), ??	Lx (FermionicHaldaneModel.init_lattice), ??
bc v (SpinChainNNN.init lattice). ??	Lx (HofstadterBosons init_lattice), ??

Lx (HofstadterFermions.init_lattice), ??	order (XXZChain2.init_lattice), 534
Lx (BoseHubbardChain.init_lattice), 578	<pre>sort_mpo_legs (CouplingMPOModel), ??</pre>
Lx (BoseHubbardModel.init_lattice), ??	∨ (FermionModel), ??
Lx (FermiHubbardChain.init_lattice), 592	0 10: : 0 1
Lx (FermiHubbardModel.init_lattice), ??	GroundStateSearch
Lx (CouplingMPOModel.init_lattice), ??	algorithm_class (mulitple definitions), ??
Lx (SpinChain.init_lattice), 548	algorithm_class (Simulation.init_algorithm), ??
Lx (SpinModel.init_lattice), ??	algorithm_class (RealTimeEvolu-
Lx (SpinChainNNN.init_lattice), ??	tion.init_algorithm), ??
Lx (SpinChainNNN2.init_lattice), ??	algorithm_params (mulitple definitions), ??
Lx (TFIChain.init_lattice), 520	algorithm_params (Simulation.init_algorithm), ??
Lx (TFIModel.init_lattice), ??	algorithm_params (RealTimeEvolu-
Lx (ToricCode.init_lattice), ??	tion.init_algorithm), ??
Lx (XXZChain2.init_lattice), 535	connect_algorithm_checkpoint (mulitple defi-
Ly (mulitple definitions), 564	nitions), ??
Ly (FermionChain.init_lattice), 564	connect_algorithm_checkpoint (Simula-
Ly (FermionModel.init_lattice), ??	tion.init_algorithm), ??
Ly (BosonicHaldaneModel.init_lattice), ??	connect_algorithm_checkpoint (Real-
Ly (FermionicHaldaneModel.init_lattice), ??	TimeEvolution.init_algorithm), ??
Ly (HofstadterBosons.init_lattice), ??	connect_measurements (mulitple definitions), ??
Ly (HofstadterFermions.init_lattice), ??	connect_measurements (GroundState-
Ly (BoseHubbardChain.init_lattice), 578	Search.init_measurements), ??
Ly (BoseHubbardModel.init_lattice), ??	connect_measurements (Simula-
Ly (FermiHubbardChain.init_lattice), 592	tion.init_measurements), ??
Ly (FermiHubbardModel.init_lattice), ??	connect_measurements (RealTimeEvolu-
Ly (CouplingMPOModel.init_lattice), ??	tion.init_measurements), ??
Ly (SpinChain.init_lattice), 548	directory (Simulation), ??
Ly (SpinModel.init_lattice), ??	initial_state_builder_class (mulitple defini-
Ly (SpinChainNNN.init_lattice), ??	tions), ??
Ly (SpinChainNNN2.init_lattice), ??	initial_state_builder_class (GroundState-
Ly (TFIChain.init_lattice), 520	Search.init_state), ??
Ly (TFIModel.init_lattice), ??	initial_state_builder_class (Simula-
Ly (ToricCode.init_lattice), ??	tion.init_state), ??
Ly (XXZChain2.init_lattice), 535	initial_state_builder_class (RealTimeEvo-
mu (FermionModel), ??	lution.init_state), ??
order (mulitple definitions), 563	<pre>initial_state_params (mulitple definitions), ??</pre>
order (FermionChain.init_lattice), 563	initial_state_params (GroundState-
order (FermionModel.init_lattice), ??	Search.init_state), ??
order (BosonicHaldaneModel.init_lattice), ??	<pre>initial_state_params (Simulation.init_state), ??</pre>
order (FermionicHaldaneModel.init_lattice), ??	initial_state_params (RealTimeEvolu-
order (HofstadterBosons.init_lattice), ??	tion.init_state), ??
order (HofstadterFermions.init_lattice), ??	logging_params (Simulation), ??
order (BoseHubbardChain.init_lattice), 578	model_class (mulitple definitions), ??
order (BoseHubbardModel.init_lattice), ??	<pre>model_class (GroundStateSearch.init_model), ??</pre>
order (FermiHubbardChain.init_lattice), 591	<pre>model_class (Simulation.init_model), ??</pre>
order (FermiHubbardModel.init_lattice), ??	<pre>model_class (RealTimeEvolution.init_model), ??</pre>
order (CouplingMPOModel.init_lattice), ??	model_params (mulitple definitions), ??
order (SpinChain.init_lattice), 548	<pre>model_params (GroundStateSearch.init_model), ??</pre>
order (SpinModel.init_lattice), ??	model_params (Simulation.init_model), ??
order (SpinChainNNN.init_lattice), ??	model_params (RealTimeEvolution.init_model), ??
order (SpinChainNNN2.init_lattice), ??	output_filename (mulitple definitions), ??
order (<i>TFIChain.init_lattice</i>), 520	output_filename (GroundState-
order (TFIModel.init_lattice), ??	Search.fix_output_filenames), ??
order (ToricCode.init_lattice), ??	output_filename (Simulation), ??

<pre>output_filename (Simulation.fix_output_filenames),</pre>	bc_MPS (HofstadterFermions.init_lattice), ??
??	bc_MPS (BoseHubbardChain.init_lattice), 578
output_filename (RealTimeEvolu-	bc_MPS (BoseHubbardModel.init_lattice), ??
tion.fix_output_filenames), ??	bc_MPS (FermiHubbardChain.init_lattice), 591
overwrite_output (mulitple definitions), ??	<pre>bc_MPS (FermiHubbardModel.init_lattice), ??</pre>
overwrite_output (GroundState-	<pre>bc_MPS (CouplingMPOModel.init_lattice), ??</pre>
Search.fix_output_filenames), ??	bc_MPS (SpinChain.init_lattice), 548
<pre>overwrite_output (Simulation), ??</pre>	bc_MPS (SpinModel.init_lattice), ??
overwrite_output (Simula-	bc_MPS (SpinChainNNN.init_lattice), ??
tion.fix_output_filenames), ??	bc_MPS (SpinChainNNN2.init_lattice), ??
overwrite_output (RealTimeEvolu-	bc_MPS (TFIChain.init_lattice), 520
tion.fix_output_filenames), ??	bc_MPS (TFIModel.init_lattice), ??
random_seed (Simulation), ??	bc_MPS (ToricCode.init_lattice), ??
safe_write (mulitple definitions), ??	bc_MPS (XXZChain2.init_lattice), 534
safe_write (GroundState-	bc_x (mulitple definitions), 564
Search.fix_output_filenames), ??	bc_x (FermionChain.init_lattice), 564
safe_write (Simulation.fix_output_filenames), ??	bc_x (FermionModel.init_lattice), ??
safe_write (RealTimeEvolu-	bc_x (BosonicHaldaneModel.init_lattice), ??
tion.fix_output_filenames), ??	bc_x (FermionicHaldaneModel.init_lattice), ??
save_every_x_seconds (mulitple definitions), ??	bc_x (HofstadterBosons.init_lattice), ??
save_every_x_seconds (GroundState-	bc_x (HofstadterFermions.init_lattice), ??
Search.save_at_checkpoint), ??	bc_x (BoseHubbardChain.init_lattice), 578
save_every_x_seconds (Simula-	bc_x (BoseHubbardModel.init_lattice), ??
tion.save_at_checkpoint), ??	bc_x (FermiHubbardChain.init_lattice), 592
save_every_x_seconds (RealTimeEvolu-	bc_x (FermiHubbardModel.init_lattice), ??
tion.save_at_checkpoint), ??	bc_x (<i>CouplingMPOModel.init_lattice</i>), ??
save_psi (mulitple definitions), ??	bc_x (SpinChain.init_lattice), 548
save_psi (GroundStateSearch.init_state), ??	bc_x (SpinModel.init_lattice), ??
save_psi (Simulation.init_state), ??	bc_x (SpinChainNNN.init_lattice), ??
save_psi (RealTimeEvolution.init_state), ??	bc_x (SpinChainNNN2.init_lattice), ??
save_stats (GroundStateSearch.init_algorithm), ??	bc_x (TFIM Living 1920
skip_if_output_exists (mulitple definitions), ??	bc_x (TFIModel.init_lattice), ??
skip_if_output_exists (GroundState-	bc_x (ToricCode.init_lattice), ??
Search.fix_output_filenames), ??	bc_x (XXZChain2.init_lattice), 535
skip_if_output_exists (Simula-	bc_y (mulitple definitions), 564
tion.fix_output_filenames), ??	bc_y (FermionChain.init_lattice), 564
skip_if_output_exists (RealTimeEvolu-	bc_y (FermionModel.init_lattice), ??
tion.fix_output_filenames), ??	bc_y (BosonicHaldaneModel.init_lattice), ??
use_default_measurements (mulitple defini-	bc_y (FermionicHaldaneModel.init_lattice), ??
tions), ??	bc_y (HofstadterBosons.init_lattice), ??
use_default_measurements (GroundState-	bc_y (HofstadterFermions.init_lattice), ??
Search.init_measurements), ??	bc_y (BoseHubbardChain.init_lattice), 578
use_default_measurements (Simula-	bc_y (BoseHubbardModel.init_lattice), ??
tion.init_measurements), ??	bc_y (FermiHubbardChain.init_lattice), 592
${\tt use_default_measurements} \qquad \textit{(RealTimeEvolu-}$	bc_y (FermiHubbardModel.init_lattice), ??
tion.init_measurements), ??	<pre>bc_y (CouplingMPOModel.init_lattice), ??</pre>
	bc_y (SpinChain.init_lattice), 548
HofstadterBosons	bc_y (SpinModel.init_lattice), ??
bc_MPS (mulitple definitions), 563	bc_y (SpinChainNNN.init_lattice), ??
bc_MPS (FermionChain.init_lattice), 563	bc_y (SpinChainNNN2.init_lattice), ??
bc_MPS (FermionModel.init_lattice), ??	bc_y (TFIChain.init_lattice), 520
bc_MPS (BosonicHaldaneModel.init_lattice), ??	bc_y (TFIModel.init_lattice), ??
bc_MPS (FermionicHaldaneModel.init_lattice), ??	bc_y (ToricCode.init_lattice), ??
bc_MPS (HofstadterBosons.init_lattice), ??	bc_y (XXZChain2.init_lattice), 535
_ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	

conserve (HofstadterBosons), ??	Lx (BoseHubbardChain.init_lattice), 578
explicit_plus_hc (CouplingMPOModel), ??	Lx (BoseHubbardModel.init_lattice), ??
filling (HofstadterBosons), ??	Lx (FermiHubbardChain.init_lattice), 592
gauge (HofstadterBosons), ??	Lx (FermiHubbardModel.init_lattice), ??
Jx (HofstadterBosons), ??	Lx (CouplingMPOModel.init_lattice), ??
Jy (HofstadterBosons), ??	Lx (SpinChain.init_lattice), 548
L (mulitple definitions), 563	Lx (SpinModel.init_lattice), ??
L (FermionChain.init_lattice), 563	Lx (SpinChainNNN.init_lattice), ??
L (FermionModel.init_lattice), ??	Lx (SpinChainNNN2.init_lattice), ??
L (BosonicHaldaneModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
L (FermionicHaldaneModel.init_lattice), ??	Lx (TFIModel.init_lattice), ??
L (HofstadterBosons.init_lattice), ??	Lx (ToricCode.init_lattice), ??
L (HofstadterFermions.init_lattice), ??	Lx (XXZChain2.init_lattice), 535
L (BoseHubbardChain.init_lattice), 578	Ly (mulitple definitions), ??
L (BoseHubbardModel.init_lattice), ??	Ly (HofstadterBosons), ??
L (FermiHubbardChain.init_lattice), 591	Ly (FermionChain.init_lattice), 564
L (FermiHubbardModel.init_lattice), ??	Ly (FermionModel.init_lattice), ??
L (CouplingMPOModel.init_lattice), ??	Ly (BosonicHaldaneModel.init_lattice), ??
L (SpinChain.init_lattice), 548	Ly (FermionicHaldaneModel.init_lattice), ??
L (SpinModel.init_lattice), ??	Ly (HofstadterBosons.init_lattice), ??
L (SpinChainNNN.init_lattice), ??	Ly (HofstadterFermions.init_lattice), ??
L (SpinChainNNN2.init_lattice), ??	Ly (BoseHubbardChain.init_lattice), 578
L (TFIChain.init_lattice), 520	Ly (BoseHubbardModel.init_lattice), ??
L (TFIModel.init_lattice), ??	Ly (FermiHubbardChain.init_lattice), 592
□ (ToricCode.init_lattice), ??	Ly (FermiHubbardModel.init_lattice), ??
L (XXZChain2.init_lattice), 534	Ly (CouplingMPOModel.init_lattice), ??
lattice (mulitple definitions), 563	Ly (SpinChain.init_lattice), 548
<pre>lattice (FermionChain.init_lattice), 563</pre>	Ly (SpinModel.init_lattice), ??
<pre>lattice (FermionModel.init_lattice), ??</pre>	Ly (SpinChainNNN.init_lattice), ??
<pre>lattice (BosonicHaldaneModel.init_lattice), ??</pre>	Ly (SpinChainNNN2.init_lattice), ??
<pre>lattice (FermionicHaldaneModel.init_lattice), ??</pre>	Ly (TFIChain.init_lattice), 520
<pre>lattice (HofstadterBosons.init_lattice), ??</pre>	Ly (TFIModel.init_lattice), ??
<pre>lattice (HofstadterFermions.init_lattice), ??</pre>	Ly (ToricCode.init_lattice), ??
<pre>lattice (BoseHubbardChain.init_lattice), 578</pre>	Ly (XXZChain2.init_lattice), 535
<pre>lattice (BoseHubbardModel.init_lattice), ??</pre>	mu (HofstadterBosons), ??
lattice (FermiHubbardChain.init_lattice), 591	mx (HofstadterBosons), ??
<pre>lattice (FermiHubbardModel.init_lattice), ??</pre>	my (HofstadterBosons), ??
lattice (CouplingMPOModel.init_lattice), ??	Nmax (HofstadterBosons), ??
lattice (SpinChain.init_lattice), 548	order (mulitple definitions), 563
lattice (SpinModel.init_lattice), ??	order (FermionChain.init_lattice), 563
lattice (SpinChainNNN.init_lattice), ??	order (FermionModel.init_lattice), ??
lattice (SpinChainNNN2.init_lattice), ??	order (BosonicHaldaneModel.init_lattice), ??
lattice (TFIChain.init_lattice), 520	order (FermionicHaldaneModel.init_lattice), ??
lattice (TFIModel.init_lattice), ??	order (HofstadterBosons.init_lattice), ??
lattice (Trinductimi_tattice), ??	order (HofstadterFermions.init_lattice), ??
lattice (XXZChain2.init_lattice), 534	order (BoseHubbardChain.init_lattice), 578
Lx (mulitple definitions), ??	order (BoseHubbardModel.init_lattice), ??
Lx (HofstadterBosons), ??	order (FermiHubbardChain.init_lattice), 591
Lx (FermionChain.init_lattice), 564	order (FermiHubbardModel.init_lattice), ??
Lx (FermionModel.init_lattice), ??	order (CouplingMPOModel.init_lattice), ??
Lx (BosonicHaldaneModel.init_lattice), ??	order (SpinChain.init_lattice), 548
Lx (FermionicHaldaneModel.init_lattice), ??	order (SpinModel.init_lattice), ??
Lx (HofstadterBosons.init_lattice), ??	order (SpinChainNNN.init_lattice), ??
Lx (HofstadterFermions.init_lattice), ??	order (SpinChainNNN2.init_lattice), ??

order (TFIChain.init_lattice), 520	bc_y (FermionicHaldaneModel.init_lattice), ??
order (TFIModel.init_lattice), ??	bc_y (HofstadterBosons.init_lattice), ??
order (ToricCode.init_lattice), ??	bc_y (HofstadterFermions.init_lattice), ??
order (XXZChain2.init_lattice), 534	bc_y (BoseHubbardChain.init_lattice), 578
phi (HofstadterBosons), ??	bc_y (BoseHubbardModel.init_lattice), ??
phi_ext (HofstadterBosons), ??	bc_y (FermiHubbardChain.init_lattice), 592
<pre>sort_mpo_legs (CouplingMPOModel), ??</pre>	bc_y (FermiHubbardModel.init_lattice), ??
U (HofstadterBosons), ??	bc_y (CouplingMPOModel.init_lattice), ??
	bc_y (SpinChain.init_lattice), 548
HofstadterFermions	bc_y (SpinModel.init_lattice), ??
bc_MPS (mulitple definitions), 563	bc_y (SpinChainNNN.init_lattice), ??
bc_MPS (FermionChain.init_lattice), 563	bc_y (SpinChainNNN2.init_lattice), ??
bc_MPS (FermionModel.init_lattice), ??	bc_y (TFIChain.init_lattice), 520
bc_MPS (BosonicHaldaneModel.init_lattice), ??	bc_y (TFIModel.init_lattice), ??
bc_MPS (FermionicHaldaneModel.init_lattice), ??	bc_y (ToricCode.init_lattice), ??
bc_MPS (HofstadterBosons.init_lattice), ??	bc_y (XXZChain2.init_lattice), 535
bc_MPS (HofstadterFermions.init_lattice), ??	conserve (HofstadterFermions), ??
bc_MPS (BoseHubbardChain.init_lattice), 578	explicit_plus_hc(CouplingMPOModel),??
bc_MPS (BoseHubbardModel.init_lattice), ??	filling (HofstadterFermions), ??
bc_MPS (FermiHubbardChain.init_lattice), 591	gauge (HofstadterFermions), ??
bc_MPS (FermiHubbardModel.init_lattice), ??	Jx (HofstadterFermions), ??
bc_MPS (CouplingMPOModel.init_lattice), ??	Jy (HofstadterFermions), ??
bc_MPS (SpinChain.init_lattice), 548	□ (mulitple definitions), 563
bc_MPS (SpinModel.init_lattice), ??	□ (FermionChain.init_lattice), 563
bc_MPS (SpinChainNNN.init_lattice), ??	L (FermionModel.init_lattice), ??
bc_MPS (SpinChainNNN2.init_lattice), ??	L (BosonicHaldaneModel.init_lattice), ??
bc_MPS (TFIChain.init_lattice), 520	L (FermionicHaldaneModel.init_lattice), ??
bc_MPS (TFIModel.init_lattice), ??	\bot (HofstadterBosons.init_lattice), ??
bc_MPS (ToricCode.init_lattice), ??	\bot (HofstadterFermions.init_lattice), ??
bc_MPS (XXZChain2.init_lattice), 534	\bot (BoseHubbardChain.init_lattice), 578
bc_x (mulitple definitions), 564	\bot (BoseHubbardModel.init_lattice), ??
bc_x (FermionChain.init_lattice), 564	\bot (FermiHubbardChain.init_lattice), 591
bc_x (FermionModel.init_lattice), ??	\bot (FermiHubbardModel.init_lattice), ??
bc_x (BosonicHaldaneModel.init_lattice), ??	\bot (CouplingMPOModel.init_lattice), ??
bc_x (FermionicHaldaneModel.init_lattice), ??	\bot (SpinChain.init_lattice), 548
bc_x (HofstadterBosons.init_lattice), ??	□ (SpinModel.init_lattice), ??
bc_x (HofstadterFermions.init_lattice), ??	□ (SpinChainNNN.init_lattice), ??
bc_x (BoseHubbardChain.init_lattice), 578	□ (SpinChainNNN2.init_lattice), ??
bc_x (BoseHubbardModel.init_lattice), ??	\bot (TFIChain.init_lattice), 520
bc_x (FermiHubbardChain.init_lattice), 592	L (TFIModel.init_lattice), ??
bc_x (FermiHubbardModel.init_lattice), ??	⊥ (ToricCode.init_lattice), ??
bc_x (CouplingMPOModel.init_lattice), ??	\bot (XXZChain2.init_lattice), 534
bc_x (SpinChain.init_lattice), 548	lattice (mulitple definitions), 563
bc_x (SpinModel.init_lattice), ??	lattice (FermionChain.init_lattice), 563
bc_x (SpinChainNNN.init_lattice), ??	<pre>lattice (FermionModel.init_lattice), ??</pre>
bc_x (SpinChainNNN2.init_lattice), ??	lattice (BosonicHaldaneModel.init_lattice), ??
bc_x (TFIChain.init_lattice), 520	lattice (FermionicHaldaneModel.init_lattice), ??
bc_x (TFIModel.init_lattice), ??	<pre>lattice (HofstadterBosons.init_lattice), ??</pre>
bc_x (ToricCode.init_lattice), ??	<pre>lattice (HofstadterFermions.init_lattice), ??</pre>
bc_x (XXZChain2.init_lattice), 535	lattice (BoseHubbardChain.init_lattice), 578
bc_y (mulitple definitions), 564	lattice (BoseHubbardModel.init_lattice), ??
bc_y (FermionChain.init_lattice), 564	lattice (FermiHubbardChain.init_lattice), 591
bc_y (FermionModel.init_lattice), ??	lattice (FermiHubbardModel.init_lattice), ??
bc_y (BosonicHaldaneModel.init_lattice), ??	<pre>lattice (CouplingMPOModel.init_lattice), ??</pre>

lattice (SpinChain.init_lattice), 548	order (FermionChain.init_lattice), 563
<pre>lattice (SpinModel.init_lattice), ??</pre>	order (FermionModel.init_lattice), ??
<pre>lattice (SpinChainNNN.init_lattice), ??</pre>	order (BosonicHaldaneModel.init_lattice), ??
<pre>lattice (SpinChainNNN2.init_lattice), ??</pre>	order (FermionicHaldaneModel.init_lattice), ??
lattice (TFIChain.init_lattice), 520	order (HofstadterBosons.init_lattice), ??
lattice (TFIModel.init_lattice), ??	order (HofstadterFermions.init_lattice), ??
<pre>lattice (ToricCode.init_lattice), ??</pre>	order (BoseHubbardChain.init_lattice), 578
<pre>lattice (XXZChain2.init_lattice), 534</pre>	order (BoseHubbardModel.init_lattice), ??
Lx (mulitple definitions), ??	order (FermiHubbardChain.init_lattice), 591
Lx (HofstadterFermions), ??	order (FermiHubbardModel.init_lattice), ??
Lx (FermionChain.init_lattice), 564	order (CouplingMPOModel.init_lattice), ??
Lx (FermionModel.init_lattice), ??	order (SpinChain.init_lattice), 548
Lx (BosonicHaldaneModel.init_lattice), ??	order (SpinModel.init_lattice), ??
Lx (FermionicHaldaneModel.init_lattice), ??	order (SpinChainNNN.init_lattice), ??
Lx (HofstadterBosons.init_lattice), ??	order (SpinChainNNN2.init_lattice), ??
Lx (HofstadterFermions.init_lattice), ??	order (TFIChain.init_lattice), 520
Lx (BoseHubbardChain.init_lattice), 578	order (TFIModel.init_lattice), ??
Lx (BoseHubbardModel.init_lattice), ??	order (ToricCode.init_lattice), ??
Lx (FermiHubbardChain.init_lattice), 592	order (XXZChain2.init_lattice), 534
Lx (FermiHubbardModel.init_lattice), ??	phi (HofstadterFermions), ??
Lx (CouplingMPOModel.init_lattice), ??	<pre>phi_ext (HofstadterFermions), ??</pre>
Lx (SpinChain.init_lattice), 548	sort_mpo_legs(CouplingMPOModel), ??
Lx (SpinModel.init_lattice), ??	v (HofstadterFermions), ??
Lx (SpinChainNNN.init_lattice), ??	Letter Oracle Destruction
Lx (SpinChainNNN2.init_lattice), ??	InitialStateBuilder
Lx (TFIChain.init_lattice), 520	<pre>check_filling (InitialStateBuilder.check_filling), ??</pre>
Lx (TFIModel.init_lattice), ??	data_key (InitialStateBuilder.from_file), ??
Lx (ToricCode.init_lattice), ??	filename (InitialStateBuilder.from_file), ??
Lx (XXZChain2.init_lattice), 535	fill_where (InitialStateBuilder.fill_where), ??
Ly (mulitple definitions), ??	full_empty (mulitple definitions), ??
Ly (HofstadterFermions), ??	<pre>full_empty (InitialStateBuilder.check_filling), ??</pre>
Ly (FermionChain.init_lattice), 564	<pre>full_empty (InitialStateBuilder.fill_where), ??</pre>
Ly (FermionModel.init_lattice), ??	method (InitialStateBuilder), ??
Ly (BosonicHaldaneModel.init_lattice), ??	<pre>product_state (mulitple definitions), ??</pre>
Ly (FermionicHaldaneModel.init_lattice), ??	product_state (InitialState-
Ly (HofstadterBosons.init_lattice), ??	Builder.lat_product_state), ??
Ly (HofstadterFermions.init_lattice), ??	product_state (InitialState-
Ly (BoseHubbardChain.init_lattice), 578	Builder.mps_product_state), ??
Ly (BoseHubbardModel.init_lattice), ??	randomize_canonicalize (InitialState-
Ly (FermiHubbardChain.init_lattice), 592	Builder.randomized), ??
Ly (FermiHubbardModel.init_lattice), ??	randomize_params (InitialState-
Ly (CouplingMPOModel.init_lattice), ??	Builder.randomized), ??
Ly (SpinChain.init_lattice), 548	randomized_from_method (InitialState-
Ly (SpinModel.init_lattice), ??	Builder.randomized), ??
Ly (SpinChainNNN.init_lattice), ??	1
Ly (SpinChainNNN2.init_lattice), ??	Lanczos
Ly (TFIChain.init_lattice), 520	cutoff (LanczosGroundState), ??
Ly (TFIModel.init_lattice), ??	E_shift (LanczosGroundState), ??
Ly (ToricCode.init_lattice), ??	E_tol (LanczosGroundState), ??
Ly (XXZChain2.init_lattice), 535	min_gap (LanczosGroundState), ??
mu (HofstadterFermions), ??	N_cache (LanczosGroundState), ??
mx (HofstadterFermions), ??	N_max (LanczosGroundState), ??
my (HofstadterFermions), ??	N_min (LanczosGroundState), ??
order (mulitple definitions), 563	P tol (LanczosGroundState), ??

```
reortho (Lanczos Ground State), ??
                                                    orthogonal to (mulitple definitions), ??
                                                    orthogonal_to (DMRGEngine.init_env), ??
LanczosEvolution
                                                   orthogonal to (EngineCombine.init env), 221
                                                   orthogonal_to (EngineFracture.init_env), 230
cutoff (LanczosGroundState), ??
                                                    orthogonal to (SingleSiteDMRGEngine.init env),
E_shift (LanczosGroundState), ??
                                                            ??
E_tol (mulitple definitions), ??
                                                   orthogonal to (TwoSiteDMRGEngine.init env), ??
E_tol (LanczosEvolution), ??
                                                   orthogonal to (Sweep.init env), ??
E_tol (LanczosGroundState), ??
                                                    start env (mulitple definitions), ??
min_gap (mulitple definitions), ??
min_gap (LanczosEvolution), ??
                                                    start_env (DMRGEngine.init_env), ??
                                                    start_env (EngineCombine.init_env), 222
min gap (Lanczos Ground State), ??
                                                    start_env (EngineFracture.init_env), 230
N_cache (LanczosGroundState), ??
                                                    start_env (SingleSiteDMRGEngine.init_env), ??
N max (LanczosGroundState), ??
                                                    start_env (TwoSiteDMRGEngine.init_env), ??
N_min (LanczosGroundState), ??
                                                    start_env (Sweep.init_env), ??
P tol (LanczosGroundState), ??
                                                    sweep_0 (mulitple definitions), ??
reortho (Lanczos Ground State), ??
                                                    sweep_0 (Sweep.reset_stats), ??
logging
                                                    sweep 0 (Variational Apply MPO. reset stats), ??
                                                    sweep_0 (VariationalCompression.reset_stats), ??
capture_warnings (setup_logging), ??
                                                    sweep 0 (PurificationApplyMPO.reset stats), 252
dict_config (setup_logging), ??
                                                   trunc_params (mulitple definitions), ??
filename (setup logging), ??
                                                    trunc params (MPS.compress), ??
format (setup_logging), ??
                                                   trunc_params (PurificationMPS.compress), ??
logger_levels (setup_logging), ??
                                                    trunc params (Variational Compression), ??
skip_setup (setup_logging), ??
                                                    trunc params (Algorithm), ??
to file (setup logging), ??
to_stdout (setup_logging), ??
                                                   PurificationTEBD
Mixer
                                                   delta tau list (mulitple definitions), ??
                                                   delta_tau_list (PurificationTEBD.run_GS), ??
amplitude (Mixer), ??
                                                    delta_tau_list (PurificationTEBD2.run_GS), 257
decay (Mixer), ??
                                                    delta_tau_list (Engine.run_GS), 202
disable_after (Mixer), ??
                                                    delta_tau_list
                                                                                 (RandomUnitaryEvolu-
MPS compress
                                                            tion.run_GS), ??
                                                   delta_tau_list (TEBDEngine.run_GS), ??
chi list (mulitple definitions), ??
                                                    disentangle (PurificationTEBD), ??
chi list (Sweep.reset stats), ??
                                                   dt (TimeEvolutionAlgorithm), ??
chi_list (VariationalApplyMPO.reset_stats), ??
                                                   N_steps (mulitple definitions), ??
chi list (Variational Compression. reset stats), ??
                                                   N_steps (PurificationTEBD.run_GS), ??
chi list (PurificationApplyMPO.reset stats), 252
                                                   N steps (PurificationTEBD2.run GS), 257
combine (Sweep), ??
                                                   N_steps (Engine.run_GS), 202
compression_method (mulitple definitions), ??
                                                   N steps (RandomUnitaryEvolution.run GS), ??
compression_method (MPS.compress), ??
                                                   N_steps (TEBDEngine.run_GS), ??
compression_method (PurificationMPS.compress),
                                                   N_steps (TimeEvolutionAlgorithm), ??
                                                   order (mulitple definitions), ??
init_env_data (mulitple definitions), ??
                                                   order (PurificationTEBD.run GS), ??
init_env_data (DMRGEngine.init_env), ??
                                                    order (PurificationTEBD2.run GS), 257
init_env_data (EngineCombine.init_env), 221
                                                   order (Engine.run GS), 202
init_env_data (EngineFracture.init_env), 230
                                                    order (RandomUnitaryEvolution.run_GS), ??
init_env_data (SingleSiteDMRGEngine.init_env),
                                                    order (TEBDEngine), ??
                                                    order (TEBDEngine.run_GS), ??
init env data (TwoSiteDMRGEngine.init env), ??
                                                    start_time (TimeEvolutionAlgorithm), ??
init_env_data (Sweep.init_env), ??
                                                    start_trunc_err (TEBDEngine), ??
lanczos params (Sweep), ??
                                                   trunc_params (Algorithm), ??
```

N_sweeps (Variational Compression), ??

RandomUnitaryEvolution	initial_state_builder_class (mulitple defini-
delta_tau_list (mulitple definitions), ??	tions), ??
delta_tau_list(<i>PurificationTEBD.run_GS</i>), ??	<pre>initial_state_builder_class (GroundState-</pre>
delta_tau_list (PurificationTEBD2.run_GS), 257	Search.init_state), ??
delta_tau_list (<i>Engine.run_GS</i>), 202	initial_state_builder_class (Simula-
delta_tau_list (RandomUnitaryEvolu-	tion.init_state), ??
tion.run_GS), ??	initial_state_builder_class (RealTimeEvo-
delta_tau_list(<i>TEBDEngine.run_GS</i>),??	lution.init_state), ??
dt (TimeEvolutionAlgorithm), ??	<pre>initial_state_params (mulitple definitions),??</pre>
N_steps (mulitple definitions), ??	initial_state_params (GroundState-
N_steps (RandomUnitaryEvolution), ??	Search.init_state), ??
N_steps (PurificationTEBD.run_GS), ??	<pre>initial_state_params (Simulation.init_state), ??</pre>
N_steps (<i>PurificationTEBD2.run_GS</i>), 257	initial_state_params (RealTimeEvolu-
N_steps (<i>Engine.run_GS</i>), 202	tion.init_state), ??
$N_steps(RandomUnitaryEvolution.run_GS), \ref{steps}$	logging_params (Simulation), ??
N_steps (<i>TEBDEngine.run_GS</i>), ??	model_class (mulitple definitions), ??
N_steps (<i>TimeEvolutionAlgorithm</i>), ??	model_class (GroundStateSearch.init_model), ??
order (mulitple definitions), ??	model_class(Simulation.init_model), ??
order (PurificationTEBD.run_GS), ??	model_class (RealTimeEvolution.init_model), ??
order (PurificationTEBD2.run_GS), 257	model_params (mulitple definitions), ??
order (<i>Engine.run_GS</i>), 202	model_params (GroundStateSearch.init_model), ??
order (RandomUnitaryEvolution.run_GS), ??	model_params (Simulation.init_model), ??
order (TEBDEngine), ??	model_params (RealTimeEvolution.init_model), ??
order (TEBDEngine.run_GS), ??	output_filename (mulitple definitions), ??
<pre>start_time (TimeEvolutionAlgorithm), ??</pre>	output_filename (GroundState-
start_trunc_err(<i>TEBDEngine</i>), ??	Search.fix_output_filenames), ??
trunc_params (mulitple definitions), ??	output_filename (Simulation), ??
trunc_params (RandomUnitaryEvolution), ??	<pre>output_filename (Simulation.fix_output_filenames),</pre>
trunc_params (Algorithm), ??	
Cimulation	<pre>output_filename</pre>
Simulation	
algorithm_class (mulitple definitions), ??	<pre>overwrite_output (mulitple definitions), ?? overwrite_output (GroundState-</pre>
algorithm_class(Simulation.init_algorithm), ??	overwrite_output (GroundState- Search.fix_output_filenames), ??
algorithm_class (RealTimeEvolu-	
tion.init_algorithm), ??	<pre>overwrite_output (Simulation), ?? overwrite_output (Simula-</pre>
algorithm_params (mulitple definitions), ??	tion.fix_output_filenames), ??
algorithm_params (Simulation.init_algorithm), ??	(5. 10)
algorithm_params (RealTimeEvolu-	overwrite_output (RealTimeEvolution.fix_output_filenames), ??
tion.init_algorithm), ??	random_seed (Simulation), ??
connect_algorithm_checkpoint (mulitple defi-	safe_write (mulitple definitions), ??
nitions), ??	safe_write (maniple definitions), (GroundState-
connect_algorithm_checkpoint (Simula-	Search.fix_output_filenames), ??
tion.init_algorithm), ??	safe_write (Simulation.fix_output_filenames), ??
connect_algorithm_checkpoint (Real-	safe_write (Simulation.jtx_output_juenames), (RealTimeEvolu-
TimeEvolution.init_algorithm), ??	tion.fix_output_filenames), ??
connect_measurements (mulitple definitions), ??	save_every_x_seconds (mulitple definitions), ??
connect_measurements (GroundState-	save_every_x_seconds (GroundState-
Search.init_measurements), ??	Search.save_at_checkpoint), ??
connect_measurements (Simula-	save_every_x_seconds (Simula-
tion.init_measurements), ??	tion.save_at_checkpoint), ??
connect_measurements (RealTimeEvolu-	save_every_x_seconds (RealTimeEvolu-
tion.init_measurements), ??	tion.save_at_checkpoint), ??
directory (Simulation), ??	save_psi (mulitple definitions), ??

<pre>save_psi (GroundStateSearch.init_state), ??</pre>	<pre>E_tol_to_trunc (mulitple definitions), ??</pre>
<pre>save_psi (Simulation.init_state), ??</pre>	<pre>E_tol_to_trunc (DMRGEngine.run), ??</pre>
<pre>save_psi (RealTimeEvolution.init_state), ??</pre>	E_tol_to_trunc (EngineCombine.run), 224
<pre>skip_if_output_exists (mulitple definitions), ??</pre>	E_tol_to_trunc (EngineFracture.run), 233
skip_if_output_exists (GroundState-	<pre>E_tol_to_trunc (SingleSiteDMRGEngine.run), ??</pre>
Search.fix_output_filenames), ??	<pre>E_tol_to_trunc (TwoSiteDMRGEngine.run), ??</pre>
skip_if_output_exists (Simula-	<pre>init_env_data (mulitple definitions), ??</pre>
tion.fix_output_filenames), ??	<pre>init_env_data(DMRGEngine.init_env), ??</pre>
skip_if_output_exists (RealTimeEvolu-	<pre>init_env_data (EngineCombine.init_env), 221</pre>
tion.fix_output_filenames), ??	<pre>init_env_data (EngineFracture.init_env), 230</pre>
<pre>use_default_measurements (mulitple defini- tions), ??</pre>	<pre>init_env_data (SingleSiteDMRGEngine.init_env),</pre>
use_default_measurements (GroundState-	<pre>init_env_data (TwoSiteDMRGEngine.init_env), ??</pre>
Search.init_measurements), ??	<pre>init_env_data (Sweep.init_env), ??</pre>
use_default_measurements (Simula-	lanczos_params (Sweep), ??
tion.init_measurements), ??	<pre>max_E_err (mulitple definitions), ??</pre>
use_default_measurements (RealTimeEvolu-	<pre>max_E_err (DMRGEngine.run), ??</pre>
tion.init_measurements), ??	max_E_err (EngineCombine.run), 224
	max_E_err (EngineFracture.run), 233
SingleSiteDMRGEngine	<pre>max_E_err (SingleSiteDMRGEngine.run), ??</pre>
<pre>chi_list (mulitple definitions), ??</pre>	<pre>max_E_err (TwoSiteDMRGEngine.run), ??</pre>
<pre>chi_list (DMRGEngine.reset_stats), ??</pre>	<pre>max_hours (mulitple definitions), ??</pre>
chi_list (EngineCombine.reset_stats), 223	max_hours (<i>DMRGEngine.run</i>), ??
chi_list (EngineFracture.reset_stats), 232	max_hours (EngineCombine.run), 224
<pre>chi_list (SingleSiteDMRGEngine.reset_stats), ??</pre>	max_hours (EngineFracture.run), 233
<pre>chi_list (TwoSiteDMRGEngine.reset_stats), ??</pre>	<pre>max_hours (SingleSiteDMRGEngine.run), ??</pre>
<pre>chi_list (Sweep.reset_stats), ??</pre>	<pre>max_hours (TwoSiteDMRGEngine.run), ??</pre>
<pre>chi_list (VariationalApplyMPO.reset_stats), ??</pre>	max_N_for_ED (mulitple definitions), ??
<pre>chi_list (VariationalCompression.reset_stats), ??</pre>	max_N_for_ED (<i>DMRGEngine.diag</i>), ??
<pre>chi_list (PurificationApplyMPO.reset_stats), 252</pre>	max_N_for_ED (<i>EngineCombine.diag</i>), 220
combine (Sweep), ??	max_N_for_ED (<i>EngineFracture.diag</i>), 228
diag_method (mulitple definitions), ??	<pre>max_N_for_ED (SingleSiteDMRGEngine.diag), ??</pre>
diag_method(DMRGEngine.run), ??	max_N_for_ED (TwoSiteDMRGEngine.diag), ??
<pre>diag_method(DMRGEngine.diag), ??</pre>	max_S_err (mulitple definitions), ??
diag_method(<i>EngineCombine.diag</i>), 220	max_S_err(DMRGEngine.run), ??
diag_method(EngineCombine.run), 224	max_S_err (EngineCombine.run), 224
diag_method(<i>EngineFracture.diag</i>), 228	max_S_err (EngineFracture.run), 233
diag_method(EngineFracture.run), 233	max_S_err (SingleSiteDMRGEngine.run), ??
<pre>diag_method (SingleSiteDMRGEngine.diag), ??</pre>	max_S_err (TwoSiteDMRGEngine.run), ??
<pre>diag_method (SingleSiteDMRGEngine.run), ??</pre>	max_sweeps (mulitple definitions), ??
<pre>diag_method(TwoSiteDMRGEngine.diag), ??</pre>	max_sweeps (DMRGEngine.run), ??
diag_method(TwoSiteDMRGEngine.run), ??	max_sweeps (EngineCombine.run), 224
E_tol_max (mulitple definitions), ??	max_sweeps (EngineFracture.run), 233
E_tol_max (DMRGEngine.run), ??	max_sweeps (SingleSiteDMRGEngine.run), ??
E_tol_max (EngineCombine.run), 224	max_sweeps (TwoSiteDMRGEngine.run), ??
E_tol_max (EngineFracture.run), 233	min_sweeps (mulitple definitions), ??
E_tol_max (SingleSiteDMRGEngine.run), ??	min_sweeps (DMRGEngine.run), ??
E_tol_max (TwoSiteDMRGEngine.run), ??	min_sweeps (EngineCombine.run), 224 min_sweeps (EngineFracture.run), 233
E_tol_min (mulitple definitions), ??	
E_tol_min (DMRGEngine.run), ??	min_sweeps (SingleSiteDMRGEngine.run), ?? min_sweeps (TwoSiteDMRGEngine.run), ??
E_tol_min (EngineCombine.run), 224	N_sweeps_check (mulitple definitions), ??
E_tol_min (EngineFracture.run), 233	N_sweeps_check (Multiple definitions), :: N_sweeps_check (DMRGEngine.run), ??
E_tol_min (SingleSiteDMRGEngine.run), ??	N_sweeps_check (EngineCombine.run), 224
<pre>E_tol_min (TwoSiteDMRGEngine.run), ??</pre>	N_Sweeps_check (Engine Comoine.run), 224

N_sweeps_check (EngineFracture.run), 233	<pre>sweep_0 (Sweep.reset_stats), ??</pre>
N_sweeps_check (SingleSiteDMRGEngine.run), ??	<pre>sweep_0 (VariationalApplyMPO.reset_stats), ??</pre>
N_sweeps_check (TwoSiteDMRGEngine.run), ??	<pre>sweep_0 (VariationalCompression.reset_stats), ??</pre>
<pre>norm_tol (mulitple definitions), ??</pre>	<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252</pre>
norm_tol (DMRGEngine.run), ??	trunc_params (Algorithm), ??
norm_tol (EngineCombine.run), 225	update_env (mulitple definitions), ??
norm_tol (EngineFracture.run), 233	update_env (DMRGEngine.run), ??
norm_tol (SingleSiteDMRGEngine.run), ??	update_env (EngineCombine.run), 225
norm_tol (TwoSiteDMRGEngine.run), ??	update_env (EngineFracture.run), 233
norm_tol_iter (mulitple definitions), ??	update_env (SingleSiteDMRGEngine.run), ??
norm_tol_iter(DMRGEngine.run),??	update_env (TwoSiteDMRGEngine.run), ??
norm_tol_iter(EngineCombine.run), 225	3,,
norm_tol_iter (EngineFracture.run), 233	SpinChainNNN
norm_tol_iter(SingleSiteDMRGEngine.run), ??	
norm_tol_iter(TwoSiteDMRGEngine.run), ??	bc_MPS (mulitple definitions), ??
orthogonal_to (mulitple definitions), ??	bc_MPS (SpinChainNNN), ??
orthogonal_to (DMRGEngine.init_env), ??	bc_MPS (FermionChain.init_lattice), 563
orthogonal_to (EngineCombine.init_env), 221	bc_MPS (FermionModel.init_lattice), ??
orthogonal_to (Engine Fracture.init_env), 221	bc_MPS (BosonicHaldaneModel.init_lattice), ??
orthogonal_to (SingleSiteDMRGEngine.init_env),	bc_MPS (FermionicHaldaneModel.init_lattice), ??
??	bc_MPS (HofstadterBosons.init_lattice), ??
orthogonal_to (TwoSiteDMRGEngine.init_env), ??	bc_MPS (HofstadterFermions.init_lattice), ??
	bc_MPS (BoseHubbardChain.init_lattice), 578
orthogonal_to (Sweep.init_env), ??	bc_MPS (BoseHubbardModel.init_lattice), ??
P_tol_max (mulitple definitions), ??	bc_MPS (FermiHubbardChain.init_lattice), 591
P_tol_max (DMRGEngine.run), ??	bc_MPS (FermiHubbardModel.init_lattice), ??
P_tol_max (EngineCombine.run), 225	bc_MPS (CouplingMPOModel.init_lattice), ??
P_tol_max (EngineFracture.run), 233	bc_MPS (SpinChain.init_lattice), 548
P_tol_max (SingleSiteDMRGEngine.run), ??	bc_MPS (SpinModel.init_lattice), ??
P_tol_max (TwoSiteDMRGEngine.run), ??	bc_MPS (SpinChainNNN.init_lattice), ??
P_tol_min (mulitple definitions), ??	bc_MPS (SpinChainNNN2.init_lattice), ??
P_tol_min (DMRGEngine.run), ??	bc_MPS (<i>TFIChain.init_lattice</i>), 520
P_tol_min (EngineCombine.run), 225	bc_MPS (<i>TFIModel.init_lattice</i>), ??
P_tol_min (EngineFracture.run), 233	bc_MPS (<i>ToricCode.init_lattice</i>), ??
P_tol_min (SingleSiteDMRGEngine.run), ??	bc_MPS (XXZChain2.init_lattice), 534
P_tol_min (TwoSiteDMRGEngine.run), ??	bc_x (mulitple definitions), 564
P_tol_to_trunc (mulitple definitions), ??	bc_x (FermionChain.init_lattice), 564
P_tol_to_trunc (DMRGEngine.run), ??	bc_x (FermionModel.init_lattice), ??
P_tol_to_trunc (EngineCombine.run), 225	bc_x (BosonicHaldaneModel.init_lattice), ??
P_tol_to_trunc (EngineFracture.run), 233	bc_x (FermionicHaldaneModel.init_lattice), ??
P_tol_to_trunc (SingleSiteDMRGEngine.run), ??	bc_x (HofstadterBosons.init_lattice), ??
P_tol_to_trunc (TwoSiteDMRGEngine.run), ??	bc_x (HofstadterFermions.init_lattice), ??
start_env (mulitple definitions), ??	bc_x (BoseHubbardChain.init_lattice), 578
start_env (<i>DMRGEngine.init_env</i>), ??	bc_x (BoseHubbardModel.init_lattice), ??
start_env (EngineCombine.init_env), 222	bc_x (FermiHubbardChain.init_lattice), 592
start_env (EngineFracture.init_env), 230	bc_x (FermiHubbardModel.init_lattice), ??
<pre>start_env (SingleSiteDMRGEngine.init_env), ??</pre>	bc_x (CouplingMPOModel.init_lattice), ??
<pre>start_env (TwoSiteDMRGEngine.init_env), ??</pre>	bc_x (SpinChain.init_lattice), 548
start_env (Sweep.init_env), ??	bc_x (SpinModel.init_lattice), ??
<pre>sweep_0 (mulitple definitions), ??</pre>	bc_x (SpinChainNNN.init_lattice), ??
sweep_0 (DMRGEngine.reset_stats), ??	bc_x (SpinChainNNN2.init_lattice), ??
sweep_0 (EngineCombine.reset_stats), 224	bc_x (TFIChain.init_lattice), 520
sweep_0 (EngineFracture.reset_stats), 232	bc_x (TFIModel.init_lattice), ??
<pre>sweep_0 (SingleSiteDMRGEngine.reset_stats), ??</pre>	bc_x (Trinoaet.init_tattice), ??
sweep_0 (TwoSiteDMRGEngine.reset_stats), ??	bc_x (XXZChain2.init_lattice), 535
	~ ~

bc_y (mulitple definitions), 564	<pre>lattice (FermionModel.init_lattice), ??</pre>
bc_y (FermionChain.init_lattice), 564	<pre>lattice (BosonicHaldaneModel.init_lattice), ??</pre>
bc_y (FermionModel.init_lattice), ??	<pre>lattice (FermionicHaldaneModel.init_lattice), ??</pre>
bc_y (BosonicHaldaneModel.init_lattice), ??	<pre>lattice (HofstadterBosons.init_lattice), ??</pre>
bc_y (FermionicHaldaneModel.init_lattice), ??	<pre>lattice (HofstadterFermions.init_lattice), ??</pre>
bc_y (HofstadterBosons.init_lattice), ??	lattice (BoseHubbardChain.init_lattice), 578
bc_y (HofstadterFermions.init_lattice), ??	<pre>lattice (BoseHubbardModel.init_lattice), ??</pre>
bc_y (BoseHubbardChain.init_lattice), 578	<pre>lattice (FermiHubbardChain.init_lattice), 591</pre>
bc_y (BoseHubbardModel.init_lattice), ??	<pre>lattice (FermiHubbardModel.init_lattice), ??</pre>
bc_y (FermiHubbardChain.init_lattice), 592	<pre>lattice (CouplingMPOModel.init_lattice), ??</pre>
bc_y (FermiHubbardModel.init_lattice), ??	lattice (SpinChain.init_lattice), 548
bc_y (CouplingMPOModel.init_lattice), ??	lattice (SpinModel.init_lattice), ??
bc_y (SpinChain.init_lattice), 548	lattice (SpinChainNNN.init_lattice), ??
bc_y (SpinModel.init_lattice), ??	lattice (SpinChainNNN2.init_lattice), ??
bc_y (SpinChainNNN.init_lattice), ??	lattice (TFIChain.init_lattice), 520
bc_y (SpinChainNNN2.init_lattice), ??	lattice (TFIModel.init_lattice), ??
bc_y (TFIChain.init_lattice), 520	lattice (ToricCode.init_lattice), ??
bc_y (TFIModel.init_lattice), ??	lattice (XXZChain2.init_lattice), 534
bc_y (ToricCode.init_lattice), ??	Lx (mulitple definitions), 564
bc_y (XXZChain2.init_lattice), 535	Lx (FermionChain.init_lattice), 564
conserve (SpinChainNNN), ??	Lx (FermionModel.init_lattice), ??
explicit_plus_hc (CouplingMPOModel), ??	Lx (BosonicHaldaneModel.init_lattice), ??
hx (SpinChainNNN), ??	Lx (FermionicHaldaneModel.init_lattice), ??
hy (SpinChainNNN), ??	Lx (HofstadterBosons.init_lattice), ??
hz (SpinChainNNN), ??	Lx (HofstadterFermions.init_lattice), ??
Jx (SpinChainNNN), ??	Lx (BoseHubbardChain.init_lattice), 578
Jxp (SpinChainNNN), ??	Lx (BoseHubbardModel.init_lattice), ??
Jy (SpinChainNNN), ??	Lx (FermiHubbardChain.init_lattice), 592
Jyp (SpinChainNNN), ??	Lx (FermiHubbardModel.init_lattice), ??
Jz (SpinChainNNN), ??	Lx (CouplingMPOModel.init_lattice), ??
Jzp (SpinChainNNN), ??	Lx (SpinChain.init_lattice), 548
L (mulitple definitions), ??	Lx (SpinModel.init_lattice), ??
L (SpinChainNNN), ??	Lx (SpinChainNNN.init_lattice), ??
L (FermionChain.init_lattice), 563	Lx (SpinChainNNN2.init_lattice), ??
L (FermionModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
L (BosonicHaldaneModel.init_lattice), ??	Lx (TFIModel.init_lattice), ??
L (FermionicHaldaneModel.init_lattice), ??	Lx (ToricCode.init_lattice), ??
L (HofstadterBosons.init_lattice), ??	Lx (XXZChain2.init_lattice), 535
L (HofstadterFermions.init_lattice), ?? L (PageHubbardChain init_lattice), 579	Ly (mulitple definitions), 564
L (BoseHubbardChain.init_lattice), 578 L (BoseHubbardModel.init_lattice), 22	Ly (FermionChain.init_lattice), 564
L (BoseHubbardModel.init_lattice), ??	Ly (FermionModel.init_lattice), ?? Ly (BosonicHaldaneModel.init_lattice), ??
L (FermiHubbardChain.init_lattice), 591	Ly (FermionicHaldaneModel.init_lattice), ??
L (FermiHubbardModel.init_lattice), ?? L (CouplingMPOModel.init_lattice), ??	- ·
L (CouplingMPOModel.init_lattice), ??	Ly (HofstadterBosons.init_lattice), ??
L (SpinChain.init_lattice), 548	Ly (HofstadterFermions.init_lattice), ??
L (SpinModel.init_lattice), ??	Ly (BoseHubbardChain.init_lattice), 578
L (SpinChainNNN.init_lattice), ??	Ly (BoseHubbardModel.init_lattice), ??
L (SpinChainNNN2.init_lattice), ??	Ly (FermiHubbardChain.init_lattice), 592
L (TFIChain.init_lattice), 520	Ly (FermiHubbardModel.init_lattice), ??
L (TFIModel.init_lattice), ??	Ly (CouplingMPOModel.init_lattice), ??
L (ToricCode.init_lattice), ??	Ly (SpinChain.init_lattice), 548
L (XXZChain2.init_lattice), 534	Ly (SpinModel.init_lattice), ??
lattice (mulitple definitions), 563	Ly (SpinChainNNN.init_lattice), ??
lattice (FermionChain.init_lattice), 563	Ly (SpinChainNNN2.init_lattice), ??

Ly (TFIChain.init_lattice), 520	bc_x (HofstadterFermions.init_lattice), ??
Ly (TFIModel.init_lattice), ??	bc_x (BoseHubbardChain.init_lattice), 578
Ly (ToricCode.init_lattice), ??	bc_x (BoseHubbardModel.init_lattice), ??
Ly (XXZChain2.init_lattice), 535	bc_x (FermiHubbardChain.init_lattice), 592
order (mulitple definitions), 563	bc_x (FermiHubbardModel.init_lattice), ??
order (FermionChain.init_lattice), 563	bc_x (CouplingMPOModel.init_lattice), ??
order (FermionModel.init_lattice), ??	bc_x (SpinChain.init_lattice), 548
order (BosonicHaldaneModel.init_lattice), ??	bc_x (SpinModel.init_lattice), ??
order (FermionicHaldaneModel.init_lattice), ??	bc_x (SpinChainNNN.init_lattice), ??
order (HofstadterBosons.init_lattice), ??	bc_x (SpinChainNNN2.init_lattice), ??
order (HofstadterFermions.init_lattice), ??	bc_x (TFIChain.init_lattice), 520
order (BoseHubbardChain.init_lattice), 578	bc_x (TFIModel.init_lattice), ??
order (BoseHubbardModel.init_lattice), ??	bc_x (ToricCode.init_lattice), ??
order (FermiHubbardChain.init_lattice), 591	bc_x (XXZChain2.init_lattice), 535
order (FermiHubbardModel.init_lattice), ??	bc_y (mulitple definitions), 564
order (CouplingMPOModel.init_lattice), ??	bc_y (FermionChain.init_lattice), 564
order (SpinChain.init_lattice), 548	bc_y (FermionModel.init_lattice), ??
order (SpinModel.init_lattice), ??	bc_y (BosonicHaldaneModel.init_lattice), ??
order (SpinChainNNN.init_lattice), ??	bc_y (FermionicHaldaneModel.init_lattice), ??
order (SpinChainNNN2.init_lattice), ??	bc_y (HofstadterBosons.init_lattice), ??
order (TFIChain.init_lattice), 520	bc_y (HofstadterFermions.init_lattice), ??
order (TFIModel.init_lattice), ??	bc_y (BoseHubbardChain.init_lattice), 578
order (ToricCode.init_lattice), ??	bc_y (BoseHubbardModel.init_lattice), ??
order (XXZChain2.init_lattice), 534	bc_y (FermiHubbardChain.init_lattice), 592
S (SpinChainNNN), ??	bc_y (FermiHubbardModel.init_lattice), ??
sort_mpo_legs(CouplingMPOModel), ??	bc_y (CouplingMPOModel.init_lattice), ??
	bc_y (SpinChain.init_lattice), 548
SpinChainNNN2	bc_y (SpinModel.init_lattice), ??
bc_MPS (mulitple definitions), 563	bc_y (SpinChainNNN.init_lattice), ??
bc_MPS (FermionChain.init_lattice), 563	bc_y (SpinChainNNN2.init_lattice), ??
bc_MPS (FermionModel.init_lattice), ??	bc_y (TFIChain.init_lattice), 520
bc_MPS (BosonicHaldaneModel.init_lattice), ??	bc_y (TFIModel.init_lattice), ??
bc_MPS (FermionicHaldaneModel.init_lattice), ??	bc_y (ToricCode.init_lattice), ??
bc_MPS (HofstadterBosons.init_lattice), ??	bc_y (XXZChain2.init_lattice), 535
bc_MPS (HofstadterFermions.init_lattice), ??	conserve (SpinChainNNN2), ??
bc_MPS (BoseHubbardChain.init_lattice), 578	explicit_plus_hc(CouplingMPOModel), ??
bc_MPS (BoseHubbardModel.init_lattice), ??	hx (SpinChainNNN2), ??
bc_MPS (FermiHubbardChain.init_lattice), 591	hy (SpinChainNNN2), ??
bc_MPS (FermiHubbardModel.init_lattice), ??	hz (SpinChainNNN2), ??
bc_MPS (CouplingMPOModel.init_lattice), ??	Jx (SpinChainNNN2), ??
bc_MPS (SpinChain.init_lattice), 548	Jxp (SpinChainNNN2), ??
bc_MPS (SpinModel.init_lattice), ??	Jy (SpinChainNNN2), ??
bc_MPS (SpinChainNNN.init_lattice), ??	Jyp (SpinChainNNN2), ??
bc_MPS (SpinChainNNN2.init_lattice), ??	Jz (SpinChainNNN2), ??
bc_MPS (<i>TFIChain.init_lattice</i>), 520	Jzp (SpinChainNNN2), ??
bc_MPS (TFIModel.init_lattice), ??	L (mulitple definitions), 563
bc_MPS (ToricCode.init_lattice), ??	L (FermionChain.init_lattice), 563
bc_MPS (XXZChain2.init_lattice), 534	L (FermionModel.init_lattice), ??
	L (BosonicHaldaneModel.init_lattice), ??
bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), 564	L (FermionicHaldaneModel.init_lattice), ??
bc_x (FermionChain.inti_tattice), 304 bc_x (FermionModel.init_lattice), ??	L (HofstadterBosons.init_lattice), ??
bc_x (BosonicHaldaneModel.init_lattice), ??	L (HofstadterFermions.init_lattice), ??
bc_x (FermionicHaldaneModel.init_lattice), ??	L (BoseHubbardChain.init_lattice), 578
bc_x (FermionicHataaneModel.init_lattice), ?? bc_x (HofstadterBosons.init_lattice), ??	L (BoseHubbardModel.init_lattice), ??
A CHUMBING DOMINAMIN BULLEY	· · · · · · · · · · · · · · · · · · ·

□ (FermiHubbardChain.init_lattice), 591	Ly (BosonicHaldaneModel.init_lattice), ??
L (FermiHubbardModel.init_lattice), ??	Ly (FermionicHaldaneModel.init_lattice), ??
L (CouplingMPOModel.init_lattice), ??	Ly (HofstadterBosons.init_lattice), ??
L (SpinChain.init_lattice), 548	Ly (HofstadterFermions.init_lattice), ??
L (SpinModel.init_lattice), ??	Ly (BoseHubbardChain.init_lattice), 578
<pre> L (SpinChainNNN.init_lattice), ?? </pre>	Ly (BoseHubbardModel.init_lattice), ??
L (SpinChainNNN2.init_lattice), ??	Ly (FermiHubbardChain.init_lattice), 592
L (TFIChain.init_lattice), 520	Ly (FermiHubbardModel.init_lattice), ??
L (TFIModel.init_lattice), ??	Ly (CouplingMPOModel.init_lattice), ??
L (ToricCode.init_lattice), ??	Ly (SpinChain.init_lattice), 548
L (XXZChain2.init_lattice), 534	Ly (SpinModel.init_lattice), ??
lattice (mulitple definitions), 563	Ly (SpinChainNNN.init_lattice), ??
lattice (FermionChain.init_lattice), 563	Ly (SpinChainNNN2.init_lattice), ??
lattice (FermionModel.init_lattice), ??	Ly (TFIChain.init_lattice), 520
lattice (BosonicHaldaneModel.init_lattice), ??	Ly (TFIModel.init_lattice), ??
lattice (FermionicHaldaneModel.init_lattice), ??	Ly (ToricCode.init_lattice), ??
lattice (HofstadterBosons.init_lattice), ??	Ly (XXZChain2.init_lattice), 535
lattice (HofstadterFermions.init_lattice), ??	order (mulitple definitions), 563
lattice (BoseHubbardChain.init_lattice), 578	order (FermionChain.init_lattice), 563
lattice (BoseHubbardModel.init_lattice), ??	order (FermionModel.init_lattice), ??
lattice (FermiHubbardChain.init_lattice), 591	order (BosonicHaldaneModel.init_lattice), ??
lattice (FermiHubbardModel.init_lattice), ??	order (FermionicHaldaneModel.init_lattice), ??
lattice (CouplingMPOModel.init_lattice), ??	order (HofstadterBosons.init_lattice), ??
lattice (SpinChain.init_lattice), 548	order (HofstadterFermions.init_lattice), ??
lattice (SpinModel.init_lattice), ??	order (BoseHubbardChain.init_lattice), 578
lattice (SpinChainNNN.init_lattice), ??	order (BoseHubbardModel.init_lattice), ??
lattice (SpinChainNNN2.init_lattice), ??	order (FermiHubbardChain.init_lattice), 591
lattice (TFIChain.init_lattice), 520	order (FermiHubbardModel.init_lattice), ??
lattice (TFIModel.init_lattice), ??	order (CouplingMPOModel.init_lattice), ??
lattice (Trimout:.mi_tante), ::	order (SpinChain.init_lattice), 548
lattice (XXZChain2.init_lattice), 534	order (SpinModel.init_lattice), ??
Lx (mulitple definitions), 564	order (SpinChainNNN.init_lattice), ??
Lx (FermionChain.init_lattice), 564	order (SpinChainNNN2.init_lattice), ??
Lx (FermionModel.init_lattice), ??	order (<i>TFIChain.init_lattice</i>), 520
Lx (BosonicHaldaneModel.init_lattice), ??	order (TFIModel.init_lattice), ??
Lx (FermionicHaldaneModel.init_lattice), ??	order (Trimodel.init_lattice), ??
Lx (HofstadterBosons.init_lattice), ??	order (XXZChain2.init_lattice), 534
Lx (HofstadterFermions.init_lattice), ??	S (SpinChainNNN2), ??
Lx (BoseHubbardChain.init_lattice), 578	sort_mpo_legs(CouplingMPOModel), ??
Lx (BoseHubbardModel.init_lattice), ??	sort_mpo_regs (Coupungmi Omodei), ::
_ /	SpinModel
Lx (FermiHubbardChain.init_lattice), 592	•
Lx (FermiHubbardModel.init_lattice), ??	bc_MPS (mulitple definitions), 563
Lx (CouplingMPOModel.init_lattice), ??	bc_MPS (FermionChain.init_lattice), 563
Lx (SpinChain.init_lattice), 548	bc_MPS (FermionModel.init_lattice), ??
Lx (SpinModel.init_lattice), ??	bc_MPS (BosonicHaldaneModel.init_lattice), ??
Lx (SpinChainNNN.init_lattice), ??	bc_MPS (FermionicHaldaneModel.init_lattice), ??
Lx (SpinChainNNN2.init_lattice), ??	bc_MPS (HofstadterBosons.init_lattice), ??
Lx (TFIChain.init_lattice), 520	bc_MPS (HofstadterFermions.init_lattice), ??
Lx (TFIModel.init_lattice), ??	bc_MPS (BoseHubbardChain.init_lattice), 578
Lx (ToricCode.init_lattice), ??	bc_MPS (BoseHubbardModel.init_lattice), ??
Lx (XXZChain2.init_lattice), 535	bc_MPS (FermiHubbardChain.init_lattice), 591
Ly (mulitple definitions), 564	bc_MPS (FermiHubbardModel.init_lattice), ??
Ly (FermionChain.init_lattice), 564	bc_MPS (CouplingMPOModel.init_lattice), ??
Ly (FermionModel.init_lattice), ??	bc_MPS (SpinChain.init_lattice), 548

bc_MPS (SpinModel.init_lattice), ??	Jx (SpinModel), ??
bc_MPS (SpinChainNNN.init_lattice), ??	Jy (SpinModel), ??
bc_MPS (SpinChainNNN2.init_lattice), ??	Jz (SpinModel), ??
bc_MPS (<i>TFIChain.init_lattice</i>), 520	L (mulitple definitions), 563
bc_MPS (TFIModel.init_lattice), ??	L (FermionChain.init_lattice), 563
bc_MPS (ToricCode.init_lattice), ??	L (FermionModel.init_lattice), ??
bc_MPS (XXZChain2.init_lattice), 534	L (BosonicHaldaneModel.init_lattice), ??
bc_x (mulitple definitions), 564	L (FermionicHaldaneModel.init_lattice), ??
bc_x (FermionChain.init_lattice), 564	L (HofstadterBosons.init_lattice), ??
bc_x (FermionModel.init_lattice), ??	L (HofstadterFermions.init_lattice), ??
bc_x (BosonicHaldaneModel.init_lattice), ??	L (BoseHubbardChain.init_lattice), 578
bc_x (FermionicHaldaneModel.init_lattice), ??	L (BoseHubbardModel.init_lattice), ??
bc_x (HofstadterBosons.init_lattice), ??	L (FermiHubbardChain.init_lattice), 591
bc_x (HofstadterFermions.init_lattice), ??	L (FermiHubbardModel.init_lattice), ??
bc_x (BoseHubbardChain.init_lattice), 578	L (CouplingMPOModel.init_lattice), ??
bc_x (BoseHubbardModel.init_lattice), ??	L (SpinChain.init_lattice), 548
bc_x (FermiHubbardChain.init_lattice), 592	L (SpinModel.init_lattice), ??
bc_x (FermiHubbardModel.init_lattice), ??	L (SpinChainNNN.init_lattice), ??
bc_x (CouplingMPOModel.init_lattice), ??	L (SpinChainNNN2.init_lattice), ??
bc_x (SpinChain.init_lattice), 548	L (TFIChain.init_lattice), 520
bc_x (SpinModel.init_lattice), ??	L (TFIModel.init_lattice), ??
bc_x (SpinChainNNN.init_lattice), ??	L (ToricCode.init_lattice), ??
bc_x (SpinChainNNN2.init_lattice), ??	\bot (XXZChain2.init_lattice), 534
bc_x (TFIChain.init_lattice), 520	lattice (mulitple definitions), 563
bc_x (TFIModel.init_lattice), ??	<pre>lattice (FermionChain.init_lattice), 563</pre>
bc_x (ToricCode.init_lattice), ??	<pre>lattice (FermionModel.init_lattice), ??</pre>
bc_x (XXZChain2.init_lattice), 535	<pre>lattice (BosonicHaldaneModel.init_lattice), ??</pre>
bc_y (mulitple definitions), 564	<pre>lattice (FermionicHaldaneModel.init_lattice), ??</pre>
bc_y (FermionChain.init_lattice), 564	<pre>lattice (HofstadterBosons.init_lattice), ??</pre>
bc_y (FermionModel.init_lattice), ??	<pre>lattice (HofstadterFermions.init_lattice), ??</pre>
bc_y (BosonicHaldaneModel.init_lattice), ??	<pre>lattice (BoseHubbardChain.init_lattice), 578</pre>
bc_y (FermionicHaldaneModel.init_lattice), ??	<pre>lattice (BoseHubbardModel.init_lattice), ??</pre>
bc_y (HofstadterBosons.init_lattice), ??	<pre>lattice (FermiHubbardChain.init_lattice), 591</pre>
bc_y (HofstadterFermions.init_lattice), ??	<pre>lattice (FermiHubbardModel.init_lattice), ??</pre>
bc_y (BoseHubbardChain.init_lattice), 578	<pre>lattice (CouplingMPOModel.init_lattice), ??</pre>
bc_y (BoseHubbardModel.init_lattice), ??	lattice (SpinChain.init_lattice), 548
bc_y (FermiHubbardChain.init_lattice), 592	lattice (SpinModel.init_lattice), ??
bc_y (FermiHubbardModel.init_lattice), ??	lattice (SpinChainNNN.init_lattice), ??
bc_y (CouplingMPOModel.init_lattice), ??	lattice (SpinChainNNN2.init_lattice), ??
bc_y (SpinChain.init_lattice), 548	lattice (<i>TFIChain.init_lattice</i>), 520
bc_y (SpinModel.init_lattice), ??	lattice (TFIModel.init_lattice), ??
bc_y (SpinChainNNN.init_lattice), ??	lattice (ToricCode.init_lattice), ??
bc_y (SpinChainNNN2.init_lattice), ??	lattice (XXZChain2.init_lattice), 534
bc_y (<i>TFIChain.init_lattice</i>), 520	Lx (mulitple definitions), 564
bc_y (TFIModel.init_lattice), ??	Lx (FermionChain.init_lattice), 564
bc_y (ToricCode.init_lattice), ??	Lx (FermionModel.init_lattice), ??
bc_y (XXZChain2.init_lattice), 535	Lx (BosonicHaldaneModel.init_lattice), ??
conserve (SpinModel), ??	Lx (FermionicHaldaneModel.init_lattice), ??
D (SpinModel), ??	Lx (Fermionic Hadanemode Little, Lx (Hofstadter Bosons.init_lattice), ??
E (SpinModel), ??	Lx (HofstadterFormions.init_lattice), ??
· ·	
explicit_plus_hc (CouplingMPOModel), ??	Lx (BoseHubbardChain.init_lattice), 578
hx (SpinModel), ??	Lx (BoseHubbardModel.init_lattice), ??
hy (SpinModel), ??	Lx (FermiHubbardChain.init_lattice), 592
hz (SpinModel), ??	Lx (FermiHubbardModel.init_lattice), ??

Lx (CouplingMPOModel.init_lattice), ??	Sweep
Lx (SpinChain.init_lattice), 548	chi_list (mulitple definitions), ??
Lx (SpinModel.init_lattice), ??	chi_list (Sweep.reset_stats), ??
Lx (SpinChainNNN.init_lattice), ??	chi_list (VariationalApplyMPO.reset_stats), ??
Lx (SpinChainNNN2.init_lattice), ??	chi_list (Variational Compression.reset_stats), ??
Lx (TFIChain.init_lattice), 520	chi_list (PurificationApplyMPO.reset_stats), 252
Lx (TFIModel.init_lattice), ??	combine (Sweep), ??
Lx (ToricCode.init_lattice), ??	init_env_data (mulitple definitions), ??
Lx (XXZChain2.init_lattice), 535	init_env_data (DMRGEngine.init_env), ??
Ly (mulitple definitions), 564	init_env_data (EngineCombine.init_env), 221
Ly (FermionChain.init_lattice), 564	init_env_data (EngineFracture.init_env), 221
Ly (FermionModel.init_lattice), ??	init_env_data (SingleSiteDMRGEngine.init_env), 250
Ly (BosonicHaldaneModel.init_lattice), ??	??
Ly (FermionicHaldaneModel.init_lattice), ??	<pre>init_env_data (TwoSiteDMRGEngine.init_env), ??</pre>
Ly (HofstadterBosons.init_lattice), ??	init_env_data (Sweep.init_env), ??
Ly (HofstadterFermions.init_lattice), ??	lanczos_params (Sweep), ??
Ly (BoseHubbardChain.init_lattice), 578	orthogonal_to (mulitple definitions), ??
Ly (BoseHubbardModel.init_lattice), ??	orthogonal_to (DMRGEngine.init_env), ??
Ly (FermiHubbardChain.init_lattice), 592	orthogonal_to (EngineCombine.init_env), 221
Ly (FermiHubbardModel.init_lattice), ??	
Ly (CouplingMPOModel.init_lattice), ??	orthogonal_to (EngineFracture.init_env), 230 orthogonal_to (SingleSiteDMRGEngine.init_env),
Ly (SpinChain.init_lattice), 548	??
Ly (SpinModel.init_lattice), ??	orthogonal_to (TwoSiteDMRGEngine.init_env), ??
Ly (SpinChainNNN.init_lattice), ??	orthogonal_to (Sweep.init_env), ??
Ly (SpinChainNNN2.init_lattice), ??	start_env (mulitple definitions), ??
Ly (TFIChain.init_lattice), 520	start_env (<i>DMRGEngine.init_env</i>), ??
Ly (TFIModel.init_lattice), ??	start_env (EngineCombine.init_env), 222
Ly (ToricCode.init_lattice), ??	start_env (EngineFracture.init_env), 222
Ly (XXZChain2.init_lattice), 535	start_env (SingleSiteDMRGEngine.init_env), ??
muJ (SpinModel), ??	start_env (TwoSiteDMRGEngine.init_env), ??
order (mulitple definitions), 563	start_env (Sweep.init_env), ??
order (FermionChain.init_lattice), 563	sweep_0 (mulitple definitions), ??
order (FermionModel.init_lattice), ??	sweep_0 (Sweep.reset_stats), ??
order (BosonicHaldaneModel.init_lattice), ??	<pre>sweep_0 (VariationalApplyMPO.reset_stats), ??</pre>
order (FermionicHaldaneModel.init_lattice), ??	sweep_0 (Variational Compression. reset_stats), ??
order (HofstadterBosons.init_lattice), ??	sweep_0 (PurificationApplyMPO.reset_stats), 252
order (HofstadterFermions.init_lattice), ??	trunc_params (Algorithm), ??
order (BoseHubbardChain.init_lattice), 578	erano_paramo (ingermini), v
order (BoseHubbardModel.init_lattice), ??	TDVP
order (FermiHubbardChain.init_lattice), 591	<pre>active_sites (TDVPEngine), ??</pre>
order (FermiHubbardModel.init_lattice), ??	dt (TimeEvolutionAlgorithm), ??
order (CouplingMPOModel.init_lattice), ??	lanczos_options (TDVPEngine), ??
order (SpinChain.init_lattice), 548	N_steps (TimeEvolutionAlgorithm), ??
order (SpinModel.init_lattice), ??	start_time (TimeEvolutionAlgorithm), ??
order (SpinChainNNN.init_lattice), ??	trunc_params (mulitple definitions), ??
order (SpinChainNNN2.init_lattice), ??	trunc_params (TDVPEngine), ??
order (TFIChain.init_lattice), 520	trunc_params (Algorithm), ??
order (TFIModel.init_lattice), ??	crano_paramo (nigorium),
order (ToricCode.init_lattice), ??	TEBDEngine
order (XXZChain2.init_lattice), 534	delta_tau_list (mulitple definitions), ??
S (SpinModel), ??	delta_tau_list (<i>Muniple definitions</i>), :: delta_tau_list (<i>PurificationTEBD.run_GS</i>), ??
<pre>sort_mpo_legs (CouplingMPOModel), ??</pre>	delta_tau_list (PurificationTEBD2.run_GS), 257
	delta_tau_list (Figine run GS) 202

delta_tau_list	(Random Unitary Evolu-	bc_x (CouplingMPOModel.init_lattice), ??
$tion.run_GS), eq:simple_$		bc_x (SpinChain.init_lattice), 548
delta_tau_list(TEBDEng		bc_x (SpinModel.init_lattice), ??
dt (TimeEvolutionAlgorithm), ?	?	bc_x (SpinChainNNN.init_lattice), ??
N_steps (mulitple definitions),		bc_x (SpinChainNNN2.init_lattice), ??
${\tt N_steps}$ (PurificationTEBD.ru	un_GS), ??	bc_x (<i>TFIChain.init_lattice</i>), 520
$N_steps(PurificationTEBD2.r)$	run_GS), 257	bc_x (TFIModel.init_lattice), ??
N_steps (Engine.run_GS), 202	2	bc_x (ToricCode.init_lattice), ??
${\tt N_steps}$ (RandomUnitaryEvol	ution.run_GS), ??	bc_x (XXZChain2.init_lattice), 535
N_steps(TEBDEngine.run_G	S), ??	bc_y (mulitple definitions), 564
${\tt N_steps}$ (${\it TimeEvolutionAlgority}$	ithm), ??	bc_y (FermionChain.init_lattice), 564
order (mulitple definitions), ??		bc_y (FermionModel.init_lattice), ??
order (PurificationTEBD.run_	GS), ??	bc_y (BosonicHaldaneModel.init_lattice), ??
order (PurificationTEBD2.run	_GS), 257	bc_y (FermionicHaldaneModel.init_lattice), ??
order (Engine.run_GS), 202		bc_y (HofstadterBosons.init_lattice), ??
order (RandomUnitaryEvolution	on.run_GS), ??	bc_y (HofstadterFermions.init_lattice), ??
order (TEBDEngine), ??		bc_y (BoseHubbardChain.init_lattice), 578
order (TEBDEngine.run_GS),	??	bc_y (BoseHubbardModel.init_lattice), ??
start_time (TimeEvolutionA	lgorithm), ??	bc_y (FermiHubbardChain.init_lattice), 592
start_trunc_err(TEBDEn	_	bc_y (FermiHubbardModel.init_lattice), ??
trunc_params (Algorithm),?	_	bc_y (CouplingMPOModel.init_lattice), ??
_		bc_y (SpinChain.init_lattice), 548
TFIModel		bc_y (SpinModel.init_lattice), ??
bc_MPS (mulitple definitions), 5	663	bc_y (SpinChainNNN.init_lattice), ??
bc_MPS (FermionChain.init_lat		bc_y (SpinChainNNN2.init_lattice), ??
bc_MPS (FermionModel.init_lat		bc_y (TFIChain.init_lattice), 520
bc_MPS (BosonicHaldaneMode		bc_y (TFIModel.init_lattice), ??
bc_MPS (FermionicHaldaneMod		bc_y (ToricCode.init_lattice), ??
bc_MPS (HofstadterBosons.init_		bc_y (XXZChain2.init_lattice), 535
bc_MPS (HofstadterFermions.in		conserve (TFIModel), ??
bc_MPS (BoseHubbardChain.in		explicit_plus_hc(CouplingMPOModel),??
bc_MPS (BoseHubbardModel.in		g (TFIModel), ??
bc_MPS (FermiHubbardChain.i.		J (TFIModel), ??
bc_MPS (FermiHubbardModel.i		L (mulitple definitions), 563
bc_MPS (CouplingMPOModel.i		L (FermionChain.init_lattice), 563
bc_MPS (SpinChain.init_lattice)		L (FermionModel.init_lattice), ??
bc_MPS (SpinModel.init_lattice)		L (BosonicHaldaneModel.init_lattice), ??
		L (FermionicHaldaneModel.init_lattice), ??
bc_MPS (SpinChainNNN.init_labc_MPS (SpinChainNNN2.init_labc_MPS (SpinChainNN).init_labc_MPS (SpinChainNN).init_labc_		L (HofstadterBosons.init_lattice), ??
bc_MPS (<i>TFIChain.init_lattice</i>).	, · ·	L (HofstadterFermions.init_lattice), ??
bc_MPS (TFIModel.init_lattice)		L (BoseHubbardChain.init_lattice), 578
bc_MPS (ToricCode.init_lattice)		L (BoseHubbardModel.init_lattice), ??
bc_MPS (XXZChain2.init_lattice)		L (FermiHubbardChain.init_lattice), 591
bc_x (mulitple definitions), 564		L (FermiHubbardModel.init_lattice), ??
bc_x (FermionChain.init_lattice		L (CouplingMPOModel.init_lattice), ??
bc_x (FermionModel.init_lattice		L (SpinChain.init_lattice), 548
bc_x (<i>BosonicHaldaneModel.im</i>	* *	L (SpinModel.init_lattice), ??
		L (SpinChainNNN.init_lattice), ??
bc_x (FermionicHaldaneModel.bc_x (HofstadterBosons.init_landameModel.bc_x)		L (SpinChainNNN2.init_lattice), ??
		L (TFIChain.init_lattice), 520
bc_x (HofstadterFermions.init_ bc_x (BoseHubbardChain.init_		L (TFIModel.init_lattice), ??
bc_x (BoseHubbardModel.init_		L (ToricCode.init_lattice), ??
bc_x (FermiHubbardChain.init_		L (XXZChain2.init_lattice), 534
bc × (FermiHubbardModel.init		lattice (mulitple definitions), 563

<pre>lattice (FermionChain.init_lattice), 563</pre>	Ly (SpinChainNNN2.init_lattice), ??
lattice (FermionModel.init_lattice), ??	Ly (TFIChain.init_lattice), 520
lattice (BosonicHaldaneModel.init_lattice), ??	Ly (TFIModel.init_lattice), ??
lattice (FermionicHaldaneModel.init_lattice), ??	Ly (ToricCode.init_lattice), ??
lattice (HofstadterBosons.init_lattice), ??	Ly (XXZChain2.init_lattice), 535
lattice (HofstadterFermions.init_lattice), ??	order (mulitple definitions), 563
lattice (BoseHubbardChain.init_lattice), 578	order (FermionChain.init_lattice), 563
lattice (BoseHubbardModel.init_lattice), ??	order (FermionModel.init_lattice), ??
lattice (FermiHubbardChain.init_lattice), 591	order (BosonicHaldaneModel.init_lattice), ??
lattice (FermiHubbardModel.init_lattice), ??	order (FermionicHaldaneModel.init_lattice), ??
<pre>lattice (CouplingMPOModel.init_lattice), ??</pre>	order (HofstadterBosons.init_lattice), ??
lattice (SpinChain.init_lattice), 548	order (HofstadterFermions.init_lattice), ??
lattice (SpinModel.init_lattice), ??	order (BoseHubbardChain.init_lattice), 578
<pre>lattice (SpinChainNNN.init_lattice), ??</pre>	order (BoseHubbardModel.init_lattice), ??
lattice (SpinChainNNN2.init_lattice), ??	order (FermiHubbardChain.init_lattice), 591
lattice (TFIChain.init_lattice), 520	order (FermiHubbardModel.init_lattice), ??
lattice (TFIModel.init_lattice), ??	order (CouplingMPOModel.init_lattice), ??
lattice (ToricCode.init_lattice), ??	order (SpinChain.init_lattice), 548
lattice (XXZChain2.init_lattice), 534	order (SpinModel.init_lattice), ??
Lx (mulitple definitions), 564	order (SpinChainNNN.init_lattice), ??
Lx (FermionChain.init_lattice), 564	order (SpinChainNNN2.init_lattice), ??
Lx (FermionModel.init_lattice), ??	order (TFIChain.init_lattice), 520
Lx (BosonicHaldaneModel.init_lattice), ??	order (TFIModel.init_lattice), ??
Lx (FermionicHaldaneModel.init_lattice), ??	order (ToricCode.init_lattice), ??
Lx (HofstadterBosons.init_lattice), ??	order (XXZChain2.init_lattice), 534
Lx (HofstadterFermions.init_lattice), ??	sort_mpo_legs (CouplingMPOModel), ??
Lx (BoseHubbardChain.init_lattice), 578	
Lx (BoseHubbardModel.init_lattice), ??	TimeEvolution
Lx (FermiHubbardChain.init_lattice), 592	algorithm_class (mulitple definitions), ??
Lx (FermiHubbardModel.init_lattice), ??	algorithm_class (Simulation.init_algorithm), ??
Lx (CouplingMPOModel.init_lattice), ??	algorithm_class (RealTimeEvolu-
Lx (SpinChain.init_lattice), 548	tion.init_algorithm), ??
Lx (SpinModel.init_lattice), ??	algorithm_params (mulitple definitions), ??
Lx (SpinChainNNN.init_lattice), ??	algorithm_params (Simulation.init_algorithm), ??
Lx (SpinChainNNN2.init_lattice), ??	algorithm_params (RealTimeEvolu-
Lx (TFIChain.init_lattice), 520	tion.init_algorithm), ??
Lx (TFIModel.init_lattice), ??	connect_algorithm_checkpoint (mulitple defi-
Lx (ToricCode.init_lattice), ??	nitions), ??
Lx (XXZChain2.init_lattice), 535	connect_algorithm_checkpoint (Simula-
Ly (mulitple definitions), 564	tion.init_algorithm), ??
Ly (FermionChain.init_lattice), 564	connect_algorithm_checkpoint (Real-
Ly (FermionModel.init_lattice), ??	TimeEvolution.init_algorithm), ??
Ly (BosonicHaldaneModel.init_lattice), ??	connect_measurements (mulitple definitions), ??
Ly (FermionicHaldaneModel.init_lattice), ??	connect_measurements (GroundState-
Ly (HofstadterBosons.init_lattice), ??	Search.init_measurements), ??
Ly (HofstadterFermions.init_lattice), ??	connect_measurements (Simula-
Ly (BoseHubbardChain.init_lattice), 578	tion.init_measurements), ??
Ly (BoseHubbardModel.init_lattice), ??	connect_measurements (RealTimeEvolu-
Ly (FermiHubbardChain.init_lattice), 592	tion.init_measurements), ??
Ly (FermiHubbardModel.init_lattice), ??	directory (Simulation), ??
Ly (CouplingMPOModel.init_lattice), ??	final_time (RealTimeEvolution), ??
Ly (SpinChain.init_lattice), 548	initial_state_builder_class (mulitple defini-
Ly (SpinModel.init_lattice), ??	tions), ??
Ly (SpinChainNNN.init_lattice), ??	,,

<pre>initial_state_builder_class (GroundState-</pre>	<pre>save_psi (RealTimeEvolution.init_state), ??</pre>
Search.init_state), ??	<pre>skip_if_output_exists (mulitple definitions), ??</pre>
initial_state_builder_class (Simula-	skip_if_output_exists (GroundState
tion.init_state), ??	Search.fix_output_filenames), ??
<pre>initial_state_builder_class (RealTimeEvo-</pre>	skip_if_output_exists (Simula
lution.init_state), ??	tion.fix_output_filenames), ??
<pre>initial_state_params (mulitple definitions), ??</pre>	skip_if_output_exists (RealTimeEvolu
initial_state_params (GroundState-	tion.fix_output_filenames), ??
Search.init_state), ??	use_default_measurements (mulitple definit
<pre>initial_state_params (Simulation.init_state), ??</pre>	tions), ??
initial_state_params (RealTimeEvolu-	use_default_measurements (GroundState
tion.init_state), ??	Search.init_measurements), ??
<pre>logging_params (Simulation), ??</pre>	use_default_measurements (Simula
<pre>model_class (mulitple definitions), ??</pre>	tion.init_measurements), ??
<pre>model_class (GroundStateSearch.init_model), ??</pre>	use_default_measurements (RealTimeEvolu
<pre>model_class (Simulation.init_model), ??</pre>	tion.init_measurements), ??
<pre>model_class (RealTimeEvolution.init_model), ??</pre>	
model_params (mulitple definitions), ??	TimeEvolutionAlgorithm
model_params (GroundStateSearch.init_model), ??	dt (TimeEvolutionAlgorithm), ??
<pre>model_params (Simulation.init_model), ??</pre>	N_steps (TimeEvolutionAlgorithm), ??
model_params (RealTimeEvolution.init_model), ??	start_time (TimeEvolutionAlgorithm), ??
output_filename (mulitple definitions), ??	trunc_params (Algorithm), ??
output_filename (GroundState-	crane_paramo (mgorman),
Search.fix_output_filenames), ??	ToricCode
output_filename (Simulation), ??	
output_filename(Simulation.fix_output_filenames),	bc_MPS (mulitple definitions), 563
??	bc_MPS (FermionChain.init_lattice), 563
output_filename (RealTimeEvolu-	bc_MPS (FermionModel.init_lattice), ??
tion.fix_output_filenames), ??	bc_MPS (BosonicHaldaneModel.init_lattice), ??
overwrite_output (mulitple definitions), ??	bc_MPS (FermionicHaldaneModel.init_lattice), ??
overwrite_output (GroundState-	bc_MPS (HofstadterBosons.init_lattice), ??
Search.fix_output_filenames), ??	bc_MPS (HofstadterFermions.init_lattice), ??
overwrite_output (Simulation), ??	bc_MPS (BoseHubbardChain.init_lattice), 578
overwrite_output (Simula-	bc_MPS (BoseHubbardModel.init_lattice), ??
tion.fix_output_filenames), ??	bc_MPS (FermiHubbardChain.init_lattice), 591
overwrite_output (RealTimeEvolu-	bc_MPS (FermiHubbardModel.init_lattice), ??
tion.fix_output_filenames), ??	bc_MPS (CouplingMPOModel.init_lattice), ??
random_seed (Simulation), ??	bc_MPS (SpinChain.init_lattice), 548
safe_write (mulitple definitions), ??	bc_MPS (SpinModel.init_lattice), ??
safe_write (GroundState-	bc_MPS (SpinChainNNN.init_lattice), ??
Search.fix_output_filenames), ??	bc_MPS (SpinChainNNN2.init_lattice), ??
safe_write (Simulation.fix_output_filenames), ??	bc_MPS (TFIChain.init_lattice), 520
safe_write (RealTimeEvolu-	bc_MPS (TFIModel.init_lattice), ??
tion.fix_output_filenames), ??	bc_MPS (ToricCode.init_lattice), ??
save_every_x_seconds (mulitple definitions), ??	bc_MPS (XXZChain2.init_lattice), 534
save_every_x_seconds (GroundState-	bc_x (mulitple definitions), ??
Search.save_at_checkpoint), ??	bc_x (ToricCode), ??
save_every_x_seconds (Simula-	bc_x (FermionChain.init_lattice), 564
tion.save_at_checkpoint), ??	bc_x (FermionModel.init_lattice), ??
save_every_x_seconds (RealTimeEvolu-	bc_x (BosonicHaldaneModel.init_lattice), ??
tion.save_at_checkpoint), ??	bc_x (FermionicHaldaneModel.init_lattice), ??
save_psi (mulitple definitions), ??	bc_x (HofstadterBosons.init_lattice), ??
save_psi (GroundStateSearch.init_state), ??	bc_x (HofstadterFermions.init_lattice), ??
save_psi(Simulation.init_state), ??	bc_x (BoseHubbardChain.init_lattice), 578
	DO STEROGORIUNDAVANIOAALIMIT LATTICAL 77

bc_x (FermiHubbardChain.init_lattice), 592	
bc_x (FermiHubbardModel.init_lattice), ??	□ (XXZChain2.init_lattice), 534
bc_x (CouplingMPOModel.init_lattice), ??	lattice (mulitple definitions), 563
bc_x (SpinChain.init_lattice), 548	<pre>lattice (FermionChain.init_lattice), 563</pre>
bc_x (SpinModel.init_lattice), ??	<pre>lattice (FermionModel.init_lattice), ??</pre>
bc_x (SpinChainNNN.init_lattice), ??	<pre>lattice (BosonicHaldaneModel.init_lattice), ??</pre>
bc_x (SpinChainNNN2.init_lattice), ??	<pre>lattice (FermionicHaldaneModel.init_lattice), ??</pre>
bc_x (TFIChain.init_lattice), 520	lattice (HofstadterBosons.init_lattice), ??
bc_x (TFIModel.init_lattice), ??	lattice (HofstadterFermions.init_lattice), ??
bc_x (ToricCode.init_lattice), ??	lattice (BoseHubbardChain.init_lattice), 578
bc_x (XXZChain2.init_lattice), 535	lattice (BoseHubbardModel.init_lattice), ??
bc_y (mulitple definitions), ??	lattice (FermiHubbardChain.init_lattice), 591
bc_y (ToricCode), ??	lattice (FermiHubbardModel.init_lattice), ??
bc_y (FermionChain.init_lattice), 564	lattice (CouplingMPOModel.init_lattice), ??
bc_y (FermionModel.init_lattice), ??	lattice (SpinChain.init_lattice), 548
bc_y (BosonicHaldaneModel.init_lattice), ??	lattice (SpinModel.init_lattice), ??
bc_y (FermionicHaldaneModel.init_lattice), ??	lattice (SpinChainNNN.init_lattice), ??
bc_y (HofstadterBosons.init_lattice), ??	lattice (SpinChainNNN2.init_lattice), ??
bc_y (HofstadterFermions.init_lattice), ??	lattice (<i>TFIChain.init_lattice</i>), 520
bc_y (BoseHubbardChain.init_lattice), 578	lattice (TFIModel.init_lattice), ??
bc_y (BoseHubbardModel.init_lattice), ??	lattice (ToricCode.init_lattice), ??
bc_y (FermiHubbardChain.init_lattice), 592	lattice (XXZChain2.init_lattice), 534
bc_y (FermiHubbardModel.init_lattice), ??	Lx (mulitple definitions), ??
bc_y (CouplingMPOModel.init_lattice), ??	Lx (ToricCode), ??
bc_y (SpinChain.init_lattice), 548	Lx (FermionChain.init_lattice), 564
bc_y (SpinModel.init_lattice), ??	Lx (FermionModel.init_lattice), ??
bc_y (SpinChainNNN.init_lattice), ??	Lx (BosonicHaldaneModel.init_lattice), ??
bc_y (SpinChainNNN2.init_lattice), ??	Lx (FermionicHaldaneModel.init_lattice), ??
bc_y (<i>TFIChain.init_lattice</i>), 520	Lx (HofstadterBosons.init_lattice), ??
bc_y (TFIModel.init_lattice), ??	Lx (HofstadterFermions.init_lattice), ??
bc_y (Trinoaet.init_lattice), ??	Lx (BoseHubbardChain.init_lattice), 578
bc_y (XXZChain2.init_lattice), 535	Lx (BoseHubbardModel.init_lattice), ??
conserve (ToricCode), ??	Lx (FermiHubbardChain.init_lattice), 592
explicit_plus_hc (CouplingMPOModel), ??	Lx (FermiHubbardModel.init_lattice), ??
Jp (ToricCode), ??	Lx (CouplingMPOModel.init_lattice), ??
Jv (ToricCode), ??	Lx (SpinChain.init_lattice), 548
L (mulitple definitions), 563	Lx (SpinModel.init_lattice), ??
	=
L (Fermion Model init lattice), 563	Lx (SpinChainNNN.init_lattice), ??
L (FermionModel.init_lattice), ?? L (PaganiaHaldaneModel.init_lattice), ??	Lx (SpinChainNNN2.init_lattice), ??
L (BosonicHaldaneModel.init_lattice), ?? L (FermionicHaldaneModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
	Lx (TFIModel.init_lattice), ??
L (HofstadterBosons.init_lattice), ??	Lx (ToricCode.init_lattice), ??
L (HofstadterFermions.init_lattice), ??	Lx (XXZChain2.init_lattice), 535
L (BoseHubbardChain.init_lattice), 578	Ly (mulitple definitions), ??
L (BoseHubbardModel.init_lattice), ??	Ly (ToricCode), ??
L (FermiHubbardChain.init_lattice), 591	Ly (FermionChain.init_lattice), 564
L (FermiHubbardModel.init_lattice), ??	Ly (FermionModel.init_lattice), ??
L (CouplingMPOModel.init_lattice), ??	Ly (BosonicHaldaneModel.init_lattice), ??
L (SpinChain.init_lattice), 548	Ly (FermionicHaldaneModel.init_lattice), ??
L (SpinModel.init_lattice), ??	Ly (HofstadterBosons.init_lattice), ??
L (SpinChainNNN.init_lattice), ??	Ly (HofstadterFermions.init_lattice), ??
L (SpinChainNNN2.init_lattice), ??	Ly (BoseHubbardChain.init_lattice), 578
L (TFIChain.init_lattice), 520	Ly (BoseHubbardModel.init_lattice), ??
□ (TFIModel.init_lattice), ??	Ly (FermiHubbardChain.init_lattice), 592

Ly (FermiHubbardModel.init_lattice), ??	<pre>diag_method(DMRGEngine.diag), ??</pre>
Ly (CouplingMPOModel.init_lattice), ??	diag_method(EngineCombine.diag), 220
Ly (SpinChain.init_lattice), 548	diag_method(EngineCombine.run), 224
Ly (SpinModel.init_lattice), ??	diag_method(<i>EngineFracture.diag</i>), 228
Ly (SpinChainNNN.init_lattice), ??	diag_method(EngineFracture.run), 233
Ly (SpinChainNNN2.init_lattice), ??	<pre>diag_method(SingleSiteDMRGEngine.diag), ??</pre>
Ly (TFIChain.init_lattice), 520	<pre>diag_method(SingleSiteDMRGEngine.run), ??</pre>
Ly (TFIModel.init_lattice), ??	<pre>diag_method(TwoSiteDMRGEngine.diag), ??</pre>
Ly (ToricCode.init_lattice), ??	<pre>diag_method(TwoSiteDMRGEngine.run), ??</pre>
Ly (XXZChain2.init_lattice), 535	<pre>E_tol_max (mulitple definitions), ??</pre>
order (mulitple definitions), ??	<pre>E_tol_max (DMRGEngine.run), ??</pre>
order (<i>ToricCode</i>), ??	<pre>E_tol_max (EngineCombine.run), 224</pre>
order (FermionChain.init_lattice), 563	<pre>E_tol_max (EngineFracture.run), 233</pre>
order (FermionModel.init_lattice), ??	<pre>E_tol_max (SingleSiteDMRGEngine.run), ??</pre>
order (BosonicHaldaneModel.init_lattice), ??	<pre>E_tol_max (TwoSiteDMRGEngine.run), ??</pre>
order (FermionicHaldaneModel.init_lattice), ??	<pre>E_tol_min (mulitple definitions), ??</pre>
order (HofstadterBosons.init_lattice), ??	E_tol_min (DMRGEngine.run), ??
order (HofstadterFermions.init_lattice), ??	E_tol_min (EngineCombine.run), 224
order (BoseHubbardChain.init_lattice), 578	<pre>E_tol_min (EngineFracture.run), 233</pre>
order (BoseHubbardModel.init_lattice), ??	<pre>E_tol_min (SingleSiteDMRGEngine.run), ??</pre>
order (FermiHubbardChain.init_lattice), 591	<pre>E_tol_min (TwoSiteDMRGEngine.run), ??</pre>
order (FermiHubbardModel.init_lattice), ??	<pre>E_tol_to_trunc (mulitple definitions), ??</pre>
order (CouplingMPOModel.init_lattice), ??	<pre>E_tol_to_trunc (DMRGEngine.run), ??</pre>
order (SpinChain.init_lattice), 548	<pre>E_tol_to_trunc (EngineCombine.run), 224</pre>
order (SpinModel.init_lattice), ??	E_tol_to_trunc (EngineFracture.run), 233
order (SpinChainNNN.init_lattice), ??	<pre>E_tol_to_trunc (SingleSiteDMRGEngine.run), ??</pre>
order (SpinChainNNN2.init_lattice), ??	<pre>E_tol_to_trunc (TwoSiteDMRGEngine.run), ??</pre>
order (<i>TFIChain.init_lattice</i>), 520	<pre>init_env_data (mulitple definitions), ??</pre>
order (TFIModel.init_lattice), ??	<pre>init_env_data(DMRGEngine.init_env), ??</pre>
order (ToricCode.init_lattice), ??	<pre>init_env_data (EngineCombine.init_env), 221</pre>
order (XXZChain2.init_lattice), 534	<pre>init_env_data (EngineFracture.init_env), 230</pre>
sort_mpo_legs(CouplingMPOModel),??	<pre>init_env_data (SingleSiteDMRGEngine.init_env),</pre>
truncation	<pre>init_env_data(TwoSiteDMRGEngine.init_env), ??</pre>
chi_max(truncate),??	init_env_data(Sweep.init_env),??
chi_min (truncate), ??	lanczos_params (Sweep), ??
degeneracy_tol (truncate), ??	max_E_err (mulitple definitions), ??
svd_min (truncate), ??	max_E_err (DMRGEngine.run), ??
trunc_cut (truncate), ??	max_E_err (EngineCombine.run), 224
51 and _6 as (max_E_err (EngineFracture.run), 233
TwoSiteDMRGEngine	<pre>max_E_err (SingleSiteDMRGEngine.run), ??</pre>
chi_list (mulitple definitions), ??	<pre>max_E_err (TwoSiteDMRGEngine.run), ??</pre>
chi_list (MMRGEngine.reset_stats), ??	<pre>max_hours (mulitple definitions), ??</pre>
chi_list (EngineCombine.reset_stats), 223	max_hours (DMRGEngine.run), ??
chi_list (EngineFracture.reset_stats), 223	max_hours (EngineCombine.run), 224
chi_list (EnginePracture.reset_stats), 232 chi_list (SingleSiteDMRGEngine.reset_stats), ??	max_hours (EngineFracture.run), 233
chi_list (TwoSiteDMRGEngine.reset_stats), ??	<pre>max_hours (SingleSiteDMRGEngine.run), ??</pre>
chi_list (Sweep.reset_stats), ??	<pre>max_hours (TwoSiteDMRGEngine.run), ??</pre>
chi_list (Sweep.reset_stats), :: chi_list (VariationalApplyMPO.reset_stats), ??	max_N_for_ED (mulitple definitions), ??
chi_list (VariationalCompression.reset_stats), ??	max_N_for_ED (DMRGEngine.diag), ??
chi_list (<i>PurificationApplyMPO.reset_stats</i>), 252	max_N_for_ED (EngineCombine.diag), 220
combine (Sweep), ??	max_N_for_ED (EngineFracture.diag), 228
diag_method (mulitple definitions), ??	<pre>max_N_for_ED (SingleSiteDMRGEngine.diag), ??</pre>
diag_method(<i>DMRGEngine run</i>) ??	max_N_for_ED (TwoSiteDMRGEngine.diag), ??
araa meriloa <i>aamaanenena.</i>	

<pre>max_S_err (mulitple definitions), ??</pre>	orthogonal_to (SingleSiteDMRGEngine.init_env)
max_S_err (DMRGEngine.run), ??	??
max_S_err(<i>EngineCombine.run</i>), 224	orthogonal_to (TwoSiteDMRGEngine.init_env), ??
max_S_err(<i>EngineFracture.run</i>), 233	orthogonal_to(<i>Sweep.init_env</i>),??
max_S_err (SingleSiteDMRGEngine.run), ??	P_tol_max (mulitple definitions), ??
max_S_err (TwoSiteDMRGEngine.run), ??	P_tol_max (DMRGEngine.run), ??
<pre>max_sweeps (mulitple definitions), ??</pre>	P_tol_max (EngineCombine.run), 225
<pre>max_sweeps (DMRGEngine.run), ??</pre>	P_tol_max (EngineFracture.run), 233
max_sweeps (EngineCombine.run), 224	P_tol_max (SingleSiteDMRGEngine.run), ??
max_sweeps (EngineFracture.run), 233	P_tol_max (<i>TwoSiteDMRGEngine.run</i>), ??
<pre>max_sweeps (SingleSiteDMRGEngine.run), ??</pre>	P_tol_min (mulitple definitions), ??
<pre>max_sweeps (TwoSiteDMRGEngine.run), ??</pre>	P_tol_min (DMRGEngine.run), ??
min_sweeps (mulitple definitions), ??	P_tol_min (EngineCombine.run), 225
min_sweeps (DMRGEngine.run), ??	P_tol_min (EngineFracture.run), 233
min_sweeps (EngineCombine.run), 224	P_tol_min (SingleSiteDMRGEngine.run), ??
min_sweeps (EngineFracture.run), 233	P_tol_min (TwoSiteDMRGEngine.run), ??
min_sweeps (SingleSiteDMRGEngine.run), ??	P_tol_to_trunc (mulitple definitions), ??
min_sweeps (TwoSiteDMRGEngine.run), ??	P_tol_to_trunc (DMRGEngine.run), ??
mixer (mulitple definitions), ??	P_tol_to_trunc (EngineCombine.run), 225
mixer (DMRGEngine.mixer_activate), ??	P_tol_to_trunc (EngineFracture.run), 233
mixer (EngineCombine.mixer_activate), 222	P_tol_to_trunc (SingleSiteDMRGEngine.run), ??
mixer (EngineFracture.mixer_activate), 231	P_tol_to_trunc (TwoSiteDMRGEngine.run), ??
mixer (SingleSiteDMRGEngine.mixer_activate), ??	start_env (mulitple definitions), ??
mixer (TwoSiteDMRGEngine.mixer_activate), ??	start_env (DMRGEngine.init_env), ??
mixer_params (mulitple definitions), ??	start_env (EngineCombine.init_env), 222
mixer_params (DMRGEngine.mixer_activate), ??	start_env (EngineFracture.init_env), 230
mixer_params (EngineCombine.mixer_activate), 222	start_env (SingleSiteDMRGEngine.init_env), ??
mixer_params (EngineFracture.mixer_activate), 231	<pre>start_env (TwoSiteDMRGEngine.init_env), ??</pre>
mixer_params (SingleSiteDMR-	start_env (Sweep.init_env), ??
GEngine.mixer_activate), ??	<pre>sweep_0 (mulitple definitions), ??</pre>
mixer_params (TwoSiteDMR-	<pre>sweep_0 (DMRGEngine.reset_stats), ??</pre>
GEngine.mixer_activate), ??	sweep_0 (EngineCombine.reset_stats), 224
N_sweeps_check (mulitple definitions), ??	sweep_0 (EngineFracture.reset_stats), 232
N_sweeps_check (DMRGEngine.run), ??	<pre>sweep_0 (SingleSiteDMRGEngine.reset_stats), ??</pre>
N_sweeps_check (EngineCombine.run), 224	<pre>sweep_0 (TwoSiteDMRGEngine.reset_stats), ??</pre>
N_sweeps_check (EngineFracture.run), 233	sweep_0 (Sweep.reset_stats), ??
N_sweeps_check (SingleSiteDMRGEngine.run), ??	<pre>sweep_0 (VariationalApplyMPO.reset_stats), ??</pre>
N_sweeps_check (TwoSiteDMRGEngine.run), ??	<pre>sweep_0 (VariationalCompression.reset_stats), ??</pre>
norm_tol (mulitple definitions), ??	<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252</pre>
<pre>norm_tol (DMRGEngine.run), ??</pre>	trunc_params (Algorithm), ??
norm_tol (EngineCombine.run), 225	update_env (mulitple definitions), ??
norm_tol (EngineFracture.run), 233	update_env (DMRGEngine.run), ??
<pre>norm_tol (SingleSiteDMRGEngine.run), ??</pre>	update_env (EngineCombine.run), 225
<pre>norm_tol (TwoSiteDMRGEngine.run), ??</pre>	update_env (EngineFracture.run), 233
<pre>norm_tol_iter (mulitple definitions), ??</pre>	update_env (SingleSiteDMRGEngine.run), ??
norm_tol_iter(DMRGEngine.run),??	update_env (TwoSiteDMRGEngine.run), ??
norm_tol_iter(EngineCombine.run), 225	
norm_tol_iter(EngineFracture.run), 233	VariationalApplyMPO
<pre>norm_tol_iter (SingleSiteDMRGEngine.run), ??</pre>	chi_list (mulitple definitions), ??
norm_tol_iter(TwoSiteDMRGEngine.run),??	chi_list (Sweep.reset_stats), ??
orthogonal_to (mulitple definitions), ??	chi_list (VariationalApplyMPO.reset_stats), ??
orthogonal_to (DMRGEngine.init_env), ??	chi_list (VariationalCompression.reset_stats), ??
orthogonal_to (EngineCombine.init_env), 221	chi_list (PurificationApplyMPO.reset_stats), 252
orthogonal_to (EngineFracture.init_env), 230	combine (Sweep), ??
	\ 1 //

<pre>init_env_data (mulitple definitions), ??</pre>	orthogonal_to (EngineFracture.init_env), 230
<pre>init_env_data (DMRGEngine.init_env), ??</pre>	orthogonal_to (SingleSiteDMRGEngine.init_env),
<pre>init_env_data(EngineCombine.init_env), 221</pre>	??
<pre>init_env_data (EngineFracture.init_env), 230</pre>	orthogonal_to (TwoSiteDMRGEngine.init_env), ??
$\verb init_env_data (Single Site DMRGEngine.init_env),$	orthogonal_to (Sweep.init_env), ??
??	start_env (mulitple definitions), ??
<pre>init_env_data(TwoSiteDMRGEngine.init_env), ??</pre>	start_env (DMRGEngine.init_env), ??
<pre>init_env_data (Sweep.init_env), ??</pre>	start_env (EngineCombine.init_env), 222
lanczos_params (Sweep), ??	start_env (EngineFracture.init_env), 230
N_sweeps (Variational Compression), ??	start_env (SingleSiteDMRGEngine.init_env), ??
orthogonal_to (mulitple definitions), ??	start_env (TwoSiteDMRGEngine.init_env), ??
orthogonal_to (DMRGEngine.init_env), ??	start_env (Sweep.init_env), ??
orthogonal_to (EngineCombine.init_env), 221	<pre>sweep_0 (mulitple definitions), ??</pre>
orthogonal_to (EngineFracture.init_env), 230	<pre>sweep_0 (Sweep.reset_stats), ??</pre>
orthogonal_to (SingleSiteDMRGEngine.init_env),	<pre>sweep_0 (VariationalApplyMPO.reset_stats), ??</pre>
??	<pre>sweep_0 (VariationalCompression.reset_stats), ??</pre>
orthogonal_to (TwoSiteDMRGEngine.init_env), ??	<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252</pre>
orthogonal_to (Sweep.init_env), ??	trunc_params (mulitple definitions), ??
start_env (mulitple definitions), ??	trunc_params (Variational Compression), ??
start_env (DMRGEngine.init_env), ??	trunc_params (Algorithm), ??
start_env (EngineCombine.init_env), 222	VV/701 '
start_env (EngineFracture.init_env), 230	XXZChain
<pre>start_env (SingleSiteDMRGEngine.init_env), ??</pre>	bc_MPS (mulitple definitions), ??
<pre>start_env (TwoSiteDMRGEngine.init_env), ??</pre>	bc_MPS (XXZChain), ??
start_env (Sweep.init_env), ??	bc_MPS (FermionChain.init_lattice), 563
<pre>sweep_0 (mulitple definitions), ??</pre>	bc_MPS (FermionModel.init_lattice), ??
<pre>sweep_0 (Sweep.reset_stats), ??</pre>	bc_MPS (BosonicHaldaneModel.init_lattice), ??
<pre>sweep_0 (VariationalApplyMPO.reset_stats), ??</pre>	bc_MPS (FermionicHaldaneModel.init_lattice), ??
<pre>sweep_0 (VariationalCompression.reset_stats), ??</pre>	
sweep_0 (variational compression. reset_stats), ::	be MPS (HofstadterBosons.init lattice), ??
sweep_0 (Variational Compression. reset_stats), 252	bc_MPS (HofstadterBosons.init_lattice), ?? bc MPS (HofstadterFermions.init_lattice), ??
	bc_MPS (HofstadterFermions.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ??</pre> VariationalCompression	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinModel.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinModel.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinModel.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (TFIChain.init_lattice), 520
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ?? init_env_data (mulitple definitions), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), 520 bc_MPS (TFIModel.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ?? init_env_data (mulitple definitions), ?? init_env_data (DMRGEngine.init_env), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.NNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIModel.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? init_env_data (mulitple definitions), ?? init_env_data (EngineCombine.init_env), ?? init_env_data (EngineCombine.init_env), 221</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.NNN.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TriCode.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), 534
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ?? init_env_data (mulitple definitions), ?? init_env_data (DMRGEngine.init_env), ??</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), 534 bc_x (mulitple definitions), 564
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? init_env_data (mulitple definitions), ?? init_env_data (DMRGEngine.init_env), ?? init_env_data (EngineCombine.init_env), 221 init_env_data (EngineFracture.init_env), 230</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIModel.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), 534 bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), 564
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? init_env_data (mulitple definitions), ?? init_env_data (EngineCombine.init_env), ?? init_env_data (EngineFracture.init_env), 221 init_env_data (SingleSiteDMRGEngine.init_env),</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), 534 bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? init_env_data (mulitple definitions), ?? init_env_data (EngineCombine.init_env), ?? init_env_data (EngineFracture.init_env), 230 init_env_data (SingleSiteDMRGEngine.init_env),</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIModel.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), 534 bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), ?? bc_x (BosonicHaldaneModel.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ?? init_env_data (mulitple definitions), ?? init_env_data (EngineCombine.init_env), ?? init_env_data (EngineFracture.init_env), 230 init_env_data (SingleSiteDMRGEngine.init_env),</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.NNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), ?? bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), ?? bc_x (FermionModel.init_lattice), ?? bc_x (FermionHaldaneModel.init_lattice), ?? bc_x (FermionicHaldaneModel.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ?? init_env_data (mulitple definitions), ?? init_env_data (EngineCombine.init_env), ?? init_env_data (EngineFracture.init_env), 230 init_env_data (SingleSiteDMRGEngine.init_env),</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIModel.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), ?? bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), ?? bc_x (BosonicHaldaneModel.init_lattice), ?? bc_x (FermionicHaldaneModel.init_lattice), ?? bc_x (FermionicHaldaneModel.init_lattice), ?? bc_x (HofstadterBosons.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ?? init_env_data (mulitple definitions), ?? init_env_data (EngineCombine.init_env), ?? init_env_data (EngineFracture.init_env), 230 init_env_data (SingleSiteDMRGEngine.init_env),</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIModel.init_lattice), ?? bc_MPS (ToricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), 534 bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), ?? bc_x (FermionModel.init_lattice), ?? bc_x (FermionHaldaneModel.init_lattice), ?? bc_x (HofstadterBosons.init_lattice), ?? bc_x (HofstadterFermions.init_lattice), ??
<pre>sweep_0 (PurificationApplyMPO.reset_stats), 252 trunc_params (mulitple definitions), ?? trunc_params (VariationalCompression), ?? trunc_params (Algorithm), ?? VariationalCompression chi_list (mulitple definitions), ?? chi_list (Sweep.reset_stats), ?? chi_list (VariationalApplyMPO.reset_stats), ?? chi_list (VariationalCompression.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), ?? chi_list (PurificationApplyMPO.reset_stats), 252 combine (Sweep), ?? init_env_data (mulitple definitions), ?? init_env_data (EngineCombine.init_env), ?? init_env_data (EngineFracture.init_env), 230 init_env_data (SingleSiteDMRGEngine.init_env),</pre>	bc_MPS (HofstadterFermions.init_lattice), ?? bc_MPS (BoseHubbardChain.init_lattice), 578 bc_MPS (BoseHubbardModel.init_lattice), ?? bc_MPS (FermiHubbardChain.init_lattice), 591 bc_MPS (FermiHubbardModel.init_lattice), ?? bc_MPS (CouplingMPOModel.init_lattice), ?? bc_MPS (SpinChain.init_lattice), 548 bc_MPS (SpinChain.init_lattice), ?? bc_MPS (SpinChainNNN.init_lattice), ?? bc_MPS (SpinChainNNN2.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TFIChain.init_lattice), ?? bc_MPS (TricCode.init_lattice), ?? bc_MPS (XXZChain2.init_lattice), ?? bc_x (mulitple definitions), 564 bc_x (FermionChain.init_lattice), ?? bc_x (FermionModel.init_lattice), ?? bc_x (FermionicHaldaneModel.init_lattice), ?? bc_x (HofstadterBosons.init_lattice), ?? bc_x (HofstadterFermions.init_lattice), ?? bc_x (BoseHubbardChain.init_lattice), ??

bc_x (CouplingMPOModel.init_lattice), ??	lattice (mulitple definitions), 563
bc_x (SpinChain.init_lattice), 548	<pre>lattice (FermionChain.init_lattice), 563</pre>
bc_x (SpinModel.init_lattice), ??	<pre>lattice (FermionModel.init_lattice), ??</pre>
bc_x (SpinChainNNN.init_lattice), ??	<pre>lattice (BosonicHaldaneModel.init_lattice), ??</pre>
bc_x (SpinChainNNN2.init_lattice), ??	<pre>lattice (FermionicHaldaneModel.init_lattice), ??</pre>
bc_x (TFIChain.init_lattice), 520	lattice (HofstadterBosons.init_lattice), ??
bc_x (TFIModel.init_lattice), ??	lattice (HofstadterFermions.init_lattice), ??
bc_x (ToricCode.init_lattice), ??	lattice (BoseHubbardChain.init_lattice), 578
bc_x (XXZChain2.init_lattice), 535	lattice (BoseHubbardModel.init_lattice), ??
bc_y (mulitple definitions), 564	lattice (FermiHubbardChain.init_lattice), 591
bc_y (FermionChain.init_lattice), 564	lattice (FermiHubbardModel.init_lattice), ??
bc_y (FermionModel.init_lattice), ??	lattice (CouplingMPOModel.init_lattice), ??
bc_y (BosonicHaldaneModel.init_lattice), ??	lattice (SpinChain.init_lattice), 548
bc_y (FermionicHaldaneModel.init_lattice), ??	lattice (SpinModel.init_lattice), ??
bc_y (HofstadterBosons.init_lattice), ??	lattice (SpinChainNNN.init_lattice), ??
bc_y (HofstadterFermions.init_lattice), ??	lattice (SpinChainNNN2.init_lattice), ??
bc_y (BoseHubbardChain.init_lattice), 578	lattice (TFIChain.init_lattice), 520
bc_y (BoseHubbardModel.init_lattice), ??	lattice (TFIModel.init_lattice), ??
bc_y (FermiHubbardChain.init_lattice), 592	lattice (Trimoati.mi_tattice), ??
bc_y (FermiHubbardModel.init_lattice), ??	lattice (XXZChain2.init_lattice), 534
bc_y (CouplingMPOModel.init_lattice), ??	Lx (mulitple definitions), 564
bc_y (SpinChain.init_lattice), 548	Lx (FermionChain.init_lattice), 564
bc_y (SpinModel.init_lattice), ??	Lx (FermionModel.init_lattice), ??
bc_y (SpinChainNNN.init_lattice), ??	Lx (BosonicHaldaneModel.init_lattice), ??
bc_y (SpinChainNNN2.init_lattice), ??	Lx (FermionicHaldaneModel.init_lattice), ??
bc_y (TFIMadalinit_lattice), 520	Lx (HofstadterBosons.init_lattice), ??
bc_y (TFIModel.init_lattice), ??	Lx (HofstadterFermions.init_lattice), ??
bc_y (ToricCode.init_lattice), ??	Lx (BoseHubbardChain.init_lattice), 578
bc_y (XXZChain2.init_lattice), 535	Lx (BoseHubbardModel.init_lattice), ??
explicit_plus_hc (CouplingMPOModel), ??	Lx (FermiHubbardChain.init_lattice), 592
hz (XXZChain), ??	Lx (FermiHubbardModel.init_lattice), ??
Jxx (XXZChain), ??	Lx (CouplingMPOModel.init_lattice), ??
Jz (XXZChain), ??	Lx (SpinChain.init_lattice), 548
L (mulitple definitions), ??	Lx (SpinModel.init_lattice), ??
L (XXZChain), ??	Lx (SpinChainNNN.init_lattice), ??
L (FermionChain.init_lattice), 563	Lx (SpinChainNNN2.init_lattice), ??
□ (FermionModel.init_lattice), ??	Lx (TFIChain.init_lattice), 520
□ (BosonicHaldaneModel.init_lattice), ??	Lx (TFIModel.init_lattice), ??
□ (FermionicHaldaneModel.init_lattice), ??	Lx (ToricCode.init_lattice), ??
□ (HofstadterBosons.init_lattice), ??	Lx (XXZChain2.init_lattice), 535
□ (HofstadterFermions.init_lattice), ??	Ly (mulitple definitions), 564
\bot (BoseHubbardChain.init_lattice), 578	Ly (FermionChain.init_lattice), 564
□ (BoseHubbardModel.init_lattice), ??	Ly (FermionModel.init_lattice), ??
□ (FermiHubbardChain.init_lattice), 591	Ly (BosonicHaldaneModel.init_lattice), ??
□ (FermiHubbardModel.init_lattice), ??	Ly (FermionicHaldaneModel.init_lattice), ??
L (CouplingMPOModel.init_lattice), ??	Ly (HofstadterBosons.init_lattice), ??
□ (SpinChain.init_lattice), 548	Ly (HofstadterFermions.init_lattice), ??
□ (SpinModel.init_lattice), ??	Ly (BoseHubbardChain.init_lattice), 578
□ (SpinChainNNN.init_lattice), ??	Ly (BoseHubbardModel.init_lattice), ??
L (SpinChainNNN2.init_lattice), ??	Ly (FermiHubbardChain.init_lattice), 592
L (TFIChain.init_lattice), 520	Ly (FermiHubbardModel.init_lattice), ??
L (TFIModel.init_lattice), ??	Ly (CouplingMPOModel.init_lattice), ??
L (ToricCode.init_lattice), ??	Ly (SpinChain.init_lattice), 548
L (XXZChain2.init_lattice), 534	Ly (SpinModel.init_lattice), ??

```
Ly (SpinChainNNN.init lattice), ??
Ly (SpinChainNNN2.init_lattice), ??
Ly (TFIChain.init_lattice), 520
Ly (TFIModel.init_lattice), ??
Ly (ToricCode.init_lattice), ??
Ly (XXZChain2.init_lattice), 535
order (mulitple definitions), 563
order (FermionChain.init_lattice), 563
order (FermionModel.init lattice), ??
order (BosonicHaldaneModel.init_lattice), ??
order (FermionicHaldaneModel.init_lattice), ??
order (HofstadterBosons.init_lattice), ??
order (HofstadterFermions.init_lattice), ??
order (BoseHubbardChain.init_lattice), 578
order (BoseHubbardModel.init_lattice), ??
order (FermiHubbardChain.init_lattice), 591
order (FermiHubbardModel.init_lattice), ??
order (CouplingMPOModel.init lattice), ??
order (SpinChain.init_lattice), 548
order (SpinModel.init lattice), ??
order (SpinChainNNN.init_lattice), ??
order (SpinChainNNN2.init_lattice), ??
order (TFIChain.init_lattice), 520
order (TFIModel.init lattice), ??
order (ToricCode.init_lattice), ??
order (XXZChain2.init_lattice), 534
sort_mpo_legs (CouplingMPOModel), ??
ZipUpApplyMPO
```

```
m_temp (MPO.apply_zipup), ??
trunc_params (MPO.apply_zipup), ??
trunc_weight (MPO.apply_zipup), ??
```

CONFIG INDEX

Algorithm, ?? ApplyMPO, ??	MPS_compress(master), ?? MPS_compress, ?? MPS_compress, ??
BoseHubbardModel, ?? BosonicHaldaneModel, ??	PurificationTEBD, ??
Config,??	RandomUnitaryEvolution,??
CouplingMPOModel, ?? D	Simulation, ?? SingleSiteDMRGEngine, ??
DMRG, ?? DMRGEngine, ??	SpinChainNNN, ?? SpinChainNNN2, ?? SpinModel, ??
ExpMPOEvolution, ??	Sweep, ??
FermiHubbardModel, ?? FermionicHaldaneModel, ?? FermionModel, ??	TDVP, ?? TEBDEngine, ?? TFIModel, ?? TimeEvolution, ?? TimeEvolutionAlgorithm, ??
GroundStateSearch, ??	ToricCode, ?? truncation, ?? TwoSiteDMRGEngine, ??
H HofstadterBosons, ?? HofstadterFermions, ??	V VariationalApplyMPO,?? VariationalCompression,??
InitialStateBuilder, ??	XXXZChain, ??
L Lanczos, ?? LanczosEvolution, ?? logging, ??	ZipUpApplyMPO,??
M	

Mixer,??

800 Config Index

INDEX

Symbols	(tenpy. linalg. sparse. Flat Linear Operator
_LP (tenpy.networks.mps.MPSEnvironment attribute),	attribute), 338
651	_old_level (tenpy.tools.optimization.temporary_level
_LP_age (tenpy.networks.mps.MPSEnvironment at-	attribute), 750
tribute), 651	_order (tenpy.models.lattice.Lattice attribute), 421
_RP (tenpy.networks.mps.MPSEnvironment attribute),	_perm (tenpy.linalg.charges.LegPipe attribute), 318
651	_perm (tenpy.models.lattice.Lattice attribute), 422
_RP_age (tenpy.networks.mps.MPSEnvironment at- tribute), 651	_pipe (tenpy.algorithms.exact_diag.ExactDiag at- tribute), 266
full_version (in module tenpy), 192	_pipe_conj (tenpy.algorithms.exact_diag.ExactDiag
version (in module tenpy), 192	attribute), 266
_bra_N (tenpy.networks.mps.TransferMatrix attribute),	_qdata (tenpy.linalg.np_conserved.Array attribute),
656	273
_data (tenpy.linalg.np_conserved.Array attribute), 273	_qdata_sorted (tenpy.linalg.np_conserved.Array at-
finite (tenpy.networks.mps.MPSEnvironment at-	tribute), 273
tribute), 651	_sites (tenpy.algorithms.exact_diag.ExactDiag at-
_grid_legs (tenpy.networks.mpo.MPOGraph at-	tribute), 265
<i>tribute</i>), 666	_strides (tenpy.linalg.charges.LegPipe attribute), 318
_ket_M (tenpy.networks.mps.TransferMatrix attribute),	_strides (tenpy.models.lattice.Lattice attribute), 421
656	٨
_labels (tenpy.linalg.np_conserved.Array attribute),	A
273	acts_on (tenpy.algorithms.mps_common.EffectiveH at-
_labels_p (tenpy.algorithms.exact_diag.ExactDiag	tribute), 207
attribute), 265	<pre>acts_on (tenpy.algorithms.mps_common.OneSiteH at-</pre>
_labels_pconj(tenpy.algorithms.exact_diag.ExactDia	
attribute), 266	<pre>acts_on (tenpy.algorithms.mps_common.TwoSiteH at-</pre>
$\verb _labels_split (tenpy. linalg. sparse. Flat Linear Operator) $	
attribute), 338	<pre>acts_on (tenpy.linalg.sparse.NpcLinearOperator at-</pre>
_mask (tenpy.algorithms.exact_diag.ExactDiag at-	tribute), 343
tribute), 266	add() (tenpy.linalg.charges.ChargeInfo class method),
_mask (tenpy.linalg.charges.ChargeInfo attribute), 305	306
_mask (tenpy.linalg.sparse.FlatLinearOperator at-	add() (tenpy.networks.mpo.MPOGraph method), 666
tribute), 338	add_charge() (tenpy.linalg.np_conserved.Array
_mod_masked (tenpy.linalg.charges.ChargeInfo at-	method), 280
tribute), 305	add_coupling()(tenpy.models.fermions_spinless.FermionChair
_mps2lat_vals_idx (tenpy.models.lattice.Lattice at-	method), 554
tribute), 422	add_coupling() (tenpy.models.hubbard.BoseHubbardChain
_mps2lat_vals_idx_fix_u	method), 568
(tenpy.models.lattice.Lattice attribute), 422	add_coupling() (tenpy.models.hubbard.FermiHubbardChain
_mps_fix_u (tenpy.models.lattice.Lattice attribute),	method), 582
422	add_coupling() (tenpy.models.model.CouplingModel
_npc_matvec_multileg	method), 481

$\verb"add_coupling"()" (\textit{tenpy.models.model.MultiCouplingM}) \\$	
method), 494	<pre>add_exponentially_decaying_coupling()</pre>
add_coupling() (tenpy.models.spins.SpinChain	(tenpy.models.tf_ising.TFIChain method), 513
method), 538	<pre>add_exponentially_decaying_coupling()</pre>
<pre>add_coupling() (tenpy.models.tf_ising.TFIChain</pre>	(tenpy.models.xxz_chain.XXZChain2 method),
method), 510	527
<pre>add_coupling() (tenpy.models.xxz_chain.XXZChain2</pre>	<pre>add_exponentially_decaying_coupling()</pre>
method), 525	(tenpy.networks.terms.ExponentiallyDecayingTerms
<pre>add_coupling_term()</pre>	method), 675
(tenpy.models.fermions_spinless.FermionChain method), 556	<pre>add_leg() (tenpy.linalg.np_conserved.Array method),</pre>
<pre>add_coupling_term()</pre>	<pre>add_local_term() (tenpy.models.fermions_spinless.FermionChain</pre>
(tenpy.models.hubbard.BoseHubbardChain	method), 557
method), 570	<pre>add_local_term() (tenpy.models.hubbard.BoseHubbardChain</pre>
<pre>add_coupling_term()</pre>	method), 571
(tenpy.models.hubbard.FermiHubbardChain	<pre>add_local_term() (tenpy.models.hubbard.FermiHubbardChain</pre>
method), 583	method), 585
add_coupling_term()	add_local_term() (tenpy.models.model.CouplingModel
(tenpy.models.model.CouplingModel method),	method), 480
483	add_local_term() (tenpy.models.model.MultiCouplingModel
add_coupling_term()	method), 497
(tenpy.models.model.MultiCouplingModel	add_local_term() (tenpy.models.spins.SpinChain
method), 496	method), 542
add_coupling_term()	add_local_term() (tenpy.models.tf_ising.TFIChain
(tenpy.models.spins.SpinChain method),	method), 514
540	add_local_term() (tenpy.models.xxz_chain.XXZChain2
add_coupling_term()	method), 528
(tenpy.models.tf_ising.TFIChain method),	add_missing_IdL_IdR()
512	(tenpy.networks.mpo.MPOGraph method),
add_coupling_term()	667
(tenpy.models.xxz_chain.XXZChain2 method),	add_multi_coupling()
527	(tenpy.models.fermions_spinless.FermionChain
add_coupling_term()	method), 557
(tenpy.networks.terms.CouplingTerms method),	add_multi_coupling()
672	(tenpy.models.hubbard.BoseHubbardChain
add_coupling_term()	method), 571
(tenpy.networks.terms.MultiCouplingTerms	add_multi_coupling()
method), 680	(tenpy.models.hubbard.FermiHubbardChain
add_exponentially_decaying_coupling()	method), 585
(tenpy.models.fermions_spinless.FermionChain	add_multi_coupling()
method), 556	(tenpy.models.model.CouplingModel method),
add_exponentially_decaying_coupling()	483
(tenpy.models.hubbard.BoseHubbardChain	add_multi_coupling()
method), 570	(tenpy.models.model.MultiCouplingModel
add_exponentially_decaying_coupling()	method), 498
(tenpy.models.hubbard.FermiHubbardChain	add_multi_coupling()
method), 584	(tenpy.models.spins.SpinChain method),
add_exponentially_decaying_coupling()	542
(tenpy.models.model.CouplingModel method),	add_multi_coupling()
485	(tenpy.models.tf_ising.TFIChain method),
add_exponentially_decaying_coupling()	(tenpy.moders.tj_ising.1F1Cnain memoa), 514
(tenpy.models.model.MultiCouplingModel	add_multi_coupling()
method), 497	(tenpy.models.xxz_chain.XXZChain2 method),
add exponentially decaying coupling()	528

add_multi_coupling_term()	480
$(tenpy. models. fermions_spinless. Fermion Chain$	<pre>add_onsite_term()</pre>
method), 559	(tenpy. models. Multi Coupling Model
<pre>add_multi_coupling_term()</pre>	method), 500
(tenpy.models.hubbard.BoseHubbardChain	add_onsite_term() (tenpy.models.spins.SpinChain
method), 573	method), 544
add_multi_coupling_term()	add_onsite_term()
(tenpy.models.hubbard.FermiHubbardChain method), 586	(tenpy.models.tf_ising.TFIChain method), 516
add_multi_coupling_term()	add_onsite_term()
(tenpy.models.model.CouplingModel method), 484	(tenpy.models.xxz_chain.XXZChain2 method), 530
add_multi_coupling_term()	<pre>add_onsite_term()</pre>
(tenpy.models.model.MultiCouplingModel method), 499	(tenpy.networks.terms.OnsiteTerms method), 683
<pre>add_multi_coupling_term()</pre>	add_op() (tenpy.networks.site.BosonSite method), 612
(tenpy.models.spins.SpinChain method), 543	add_op() (tenpy.networks.site.FermionSite method), 617
<pre>add_multi_coupling_term() (tenpy.models.tf_ising.TFIChain method),</pre>	add_op() (tenpy.networks.site.GroupedSite method), 622
515	add_op() (tenpy.networks.site.Site method), 627
<pre>add_multi_coupling_term()</pre>	add_op() (tenpy.networks.site.SpinHalfFermionSite
(tenpy.models.xxz_chain.XXZChain2 method),	method), 632
529	add_op() (tenpy.networks.site.SpinHalfSite method),
add_multi_coupling_term()	636
(tenpy.networks.terms.MultiCouplingTerms method), 678	add_op() (tenpy.networks.site.SpinSite method), 641 add_string() (tenpy.networks.mpo.MPOGraph
add_onsite()(tenpy.models.fermions_spinless.Fermio	
<pre>method), 559 add_onsite() (tenpy.models.hubbard.BoseHubbardCh</pre>	add_to_graph() (tenpy.networks.terms.CouplingTerms ain method), 673
method), 573	<pre>add_to_graph() (tenpy.networks.terms.ExponentiallyDecayingTerms</pre>
add_onsite()(tenpy.models.hubbard.FermiHubbardC	
method), 587	add_to_graph() (tenpy.networks.terms.MultiCouplingTerms
<pre>add_onsite() (tenpy.models.model.CouplingModel</pre>	method), 679
method), 480	<pre>add_to_graph() (tenpy.networks.terms.OnsiteTerms</pre>
$\verb"add_onsite" () \textit{ (tenpy.models.model.MultiCouplingModels.} \\$	
method), 499	add_to_nn_bond_Arrays()
add_onsite() (tenpy.models.spins.SpinChain method), 544	(tenpy.networks.terms.OnsiteTerms method), 683
add_onsite() (tenpy.models.tf_ising.TFIChain	
method), 516	(tenpy.linalg.np_conserved.Array method),
add_onsite() (tenpy.models.xxz_chain.XXZChain2	278
method), 530	add_with_None_0() (in module tenpy.tools.misc),
<pre>add_onsite_term() (tenpy.models.fermions_spinless.FermionChain</pre>	726
method), 559	adjoint() (tenpy.algorithms.mps_common.EffectiveH method), 207
add_onsite_term()	<pre>adjoint() (tenpy.algorithms.mps_common.OneSiteH</pre>
(tenpy.models.hubbard.BoseHubbardChain	method), 210
method), 573	adjoint() (tenpy.algorithms.mps_common.TwoSiteH
<pre>add_onsite_term() (tenpy.models.hubbard.FermiHubbardChain</pre>	method), 213 adjoint() (tenpy.algorithms.purification.PurificationTwoSiteU
method), 587	method), 260
<pre>add_onsite_term() (tenpy.models.model.CouplingModel method),</pre>	adjoint() (tenpy.linalg.sparse.FlatHermitianOperator method), 332

adjoint() (tenpy.linalg.sparse.FlatLinearOperator	481
method), 340	all_onsite_terms()
<pre>adjoint() (tenpy.linalg.sparse.NpcLinearOperator</pre>	(tenpy.models.model.MultiCouplingModel method), 500
adjoint()(tenpy.linalg.sparse.NpcLinearOperatorWrap	ppdrl_onsite_terms()
method), 345	(tenpy.models.spins.SpinChain method),
adjoint()(tenpy.linalg.sparse.OrthogonalNpcLinearOp	
method), 346	all_onsite_terms()
<pre>adjoint() (tenpy.linalg.sparse.ShiftNpcLinearOperator</pre>	(tenpy.models.tf_ising.TFIChain method), 516
adjoint() (tenpy.linalg.sparse.SumNpcLinearOperator	all onsite terms()
method), 347	(tenpy.models.xxz_chain.XXZChain2 method),
adjoint() (tenpy.networks.mps.TransferMatrix	531
method), 657	any_nonzero() (in module tenpy.tools.misc), 726
alg_decay() (in module tenpy.tools.fit), 740	anynan () (in module tenpy.tools.misc), 727
alg_decay_fit() (in module tenpy.tools.fit), 741	arg_descr (tenpy.tools.events.EventHandler at-
alg_decay_fit_res() (in module tenpy.tools.fit),	tribute), 721
741	argsort () (in module tenpy.tools.misc), 727
alg_decay_fits() (in module tenpy.tools.fit), 741	Array (class in tenpy.linalg.np_conserved), 272
all_coupling_terms()	as_completely_blocked()
(tenpy.models.fermions_spinless.FermionChain	(tenpy.linalg.np_conserved.Array method),
method), 560	(tenpy.tinuig.np_conserved.Array method), 283
all_coupling_terms()	
(tenpy.models.hubbard.BoseHubbardChain	asConfig() (in module tenpy.tools.params), 718
	astype() (tenpy.linalg.np_conserved.Array method), 283
method), 574	
all_coupling_terms()	atleast_2d_pad() (in module tenpy.tools.misc), 728
(tenpy.models.hubbard.FermiHubbardChain	ATTR_CLASS (in module tenpy.tools.hdf5_io), 715
method), 588	ATTR_FORMAT (in module tenpy.tools.hdf5_io), 715
all_coupling_terms()	ATTR_LEN (in module tenpy.tools.hdf5_io), 715
(tenpy.models.model.CouplingModel method),	ATTR_MODULE (in module tenpy.tools.hdf5_io), 715
483	ATTR_TYPE (in module tenpy.tools.hdf5_io), 715
all_coupling_terms()	В
(tenpy.models.model.MultiCouplingModel	
method), 500	basis (tenpy.models.lattice.Lattice attribute), 421
all_coupling_terms()	bc (tenpy.models.lattice.Lattice attribute), 421
(tenpy.models.spins.SpinChain method),	bc (tenpy.networks.mpo.MPOGraph attribute), 665
544	bc_MPS (tenpy.models.lattice.Lattice attribute), 421
all_coupling_terms()	bc_shift (tenpy.models.lattice.Lattice attribute), 421
(tenpy.models.tf_ising.TFIChain method),	<pre>binary_blockwise()</pre>
516	(tenpy.linalg.np_conserved.Array method),
all_coupling_terms()	287
(tenpy.models.xxz_chain.XXZChain2 method),	block_number (tenpy.linalg.charges.LegCharge at-
531	tribute), 309
all_onsite_terms()	bond_dimension() (in module
(tenpy.models.fermions_spinless.FermionChain	tenpy.simulations.measurement), 694
method), 560	bond_energies() (in module
all_onsite_terms()	tenpy.simulations.measurement), 694
(tenpy. models. hubbard. Bose Hubbard Chain	bond_energies()(tenpy.models.fermions_spinless.FermionChain
method), 574	method), 560
all_onsite_terms()	bond_energies() (tenpy.models.hubbard.BoseHubbardChain
(tenpy. models. hubbard. Fermi Hubbard Chain	method), 574
method), 588	bond_energies() (tenpy.models.hubbard.FermiHubbardChain
all_onsite_terms()	method), 588
(tenny models model Counling Model method)	

bond_energies()(tenpy.models.model.NearestNeight	borModel tenpy.tools.misc), 728
method), 505	<pre>build_MPO() (tenpy.networks.mpo.MPOGraph</pre>
bond_energies() (tenpy.models.spins.SpinChain	method), 667
method), 544	bunch () (tenpy.linalg.charges.LegCharge method), 314
<pre>bond_energies() (tenpy.models.tf_ising.TFIChain</pre>	bunch () (tenpy.linalg.charges.LegPipe method), 319
method), 516	bunched (tenpy.linalg.charges.LegCharge attribute),
bond_energies()(tenpy.models.xxz_chain.XXZChain	2 309
method), 531	С
BoseHubbardChain (class in tenpy.models.hubbard),	
568	<pre>calc_H_bond() (tenpy.models.fermions_spinless.FermionChain</pre>
BosonSite (class in tenpy.networks.site), 612	method), 560
bottleneck (in module tenpy.tools.optimization), 754	calc_H_bond() (tenpy.models.hubbard.BoseHubbardChain
boundary_conditions()	method), 574
(tenpy.models.lattice.Chain property), 357	calc_H_bond() (tenpy.models.hubbard.FermiHubbardChain
boundary_conditions()	method), 588
(tenpy.models.lattice.HelicalLattice prop-	calc_H_bond() (tenpy.models.model.CouplingModel
erty), 370	method), 486
<pre>boundary_conditions() (tenpy.models.lattice.Honeycomb property),</pre>	<pre>calc_H_bond() (tenpy.models.model.MultiCouplingModel method), 500</pre>
375	calc_H_bond() (tenpy.models.spins.SpinChain
boundary_conditions()	method), 545
(tenpy.models.lattice.IrregularLattice prop-	calc_H_bond() (tenpy.models.tf_ising.TFIChain
erty), 389	method), 517
boundary_conditions()	calc_H_bond() (tenpy.models.xxz_chain.XXZChain2
(tenpy.models.lattice.Kagome property),	method), 531
397	calc_H_bond_from_MPO()
boundary_conditions()	(tenpy.models.fermions_spinless.FermionChain
(tenpy.models.lattice.Ladder property), 409	method), 560
<pre>boundary_conditions()</pre>	calc_H_bond_from_MPO()
(tenpy.models.lattice.Lattice property), 424	(tenpy.models.hubbard.BoseHubbardChain
<pre>boundary_conditions()</pre>	method), 575
(tenpy.models.lattice.SimpleLattice property),	<pre>calc_H_bond_from_MPO()</pre>
433	(tenpy.models.hubbard.FermiHubbardChain
<pre>boundary_conditions()</pre>	method), 588
(tenpy.models.lattice.Square property), 442	<pre>calc_H_bond_from_MPO()</pre>
boundary_conditions()	(tenpy.models.model.MPOModel method),
(tenpy.models.lattice.Triangular property),	490
455	<pre>calc_H_bond_from_MPO()</pre>
boundary_conditions()	(tenpy.models.spins.SpinChain method),
(tenpy.models.lattice.TrivialLattice property),	545
464	calc_H_bond_from_MPO()
boundary_conditions()	(tenpy.models.tf_ising.TFIChain method),
(tenpy.models.toric_code.DualSquare prop-	517
erty), 600 box() (in module tenpy.linalg.random_matrix), 329	calc_H_bond_from_MPO()
build_full_H_from_bonds()	(tenpy.models.xxz_chain.XXZChain2 method), 531
(tenpy.algorithms.exact_diag.ExactDiag	calc_H_MPO() (tenpy.models.fermions_spinless.FermionChain
method), 266	method), 560
build_full_H_from_mpo()	calc_H_MPO() (tenpy.models.hubbard.BoseHubbardChain
(tenpy.algorithms.exact_diag.ExactDiag	method), 574
method), 266	calc_H_MPO() (tenpy.models.hubbard.FermiHubbardChain
build_initial_state() (in module	method), 588
tenpy.networks.mps), 657	calc_H_MPO() (tenpy.models.model.CouplingModel
<pre>build_initial_state() (in module</pre>	method), 486

calc_H_MPO() (tenpy.models.model.MultiCouplingMode	
method), 500	method), 280
calc_H_MPO() (tenpy.models.spins.SpinChain	
method), 545	method), 613
calc_H_MPO() (tenpy.models.tf_ising.TFIChain	<pre>change_charge() (tenpy.networks.site.FermionSite</pre>
method), 517	method), 617
calc_H_MPO() (tenpy.models.xxz_chain.XXZChain2	change_charge() (tenpy.networks.site.GroupedSite
method), 531	method), 622
calc_H_MPO_from_bond()	change_charge() (tenpy.networks.site.Site method),
(tenpy.models.fermions_spinless.FermionChain	627
method), 560	change_charge() (tenpy.networks.site.SpinHalfFermionSite
calc_H_MPO_from_bond()	method), 632
(tenpy.models.hubbard.Bose Hubbard Chain	change_charge() (tenpy.networks.site.SpinHalfSite
method), 574	method), 637
calc_H_MPO_from_bond()	change_charge() (tenpy.networks.site.SpinSite
(tenpy.models.hubbard.FermiHubbardChain	method), 641
method), 588	charge_sector(tenpy.algorithms.exact_diag.ExactDiag
calc_H_MPO_from_bond()	attribute), 265
	charge_sector() (tenpy.linalg.sparse.FlatHermitianOperator
(tenpy.models.model.NearestNeighborModel	
method), 505	property), 332
calc_H_MPO_from_bond()	charge_sector() (tenpy.linalg.sparse.FlatLinearOperator
(tenpy.models.spins.SpinChain method),	property), 338
545	<pre>charge_sectors() (tenpy.linalg.charges.LegCharge</pre>
calc_H_MPO_from_bond()	method), 315
(tenpy.models.tf_ising.TFIChain method),	<pre>charge_sectors() (tenpy.linalg.charges.LegPipe</pre>
517	method), 320
calc_H_MPO_from_bond()	ChargeInfo (class in tenpy.linalg.charges), 305
(tenpy.models.xxz_chain.XXZChain2 method),	charges (tenpy.linalg.charges.LegCharge attribute),
531	309
calc_H_onsite() (tenpy.models.fermions_spinless.Fer	
method), 561	method), 307
calc_H_onsite()(tenpy.models.hubbard.BoseHubbard	
method), 575	chi_list() (in module tenpy.tools.misc), 728
calc_H_onsite()(<i>tenpy.models.hubbard.FermiHubba</i>	
method), 589	tribute), 265
calc_H_onsite()(tenpy.models.model.CouplingMode	lchinfo (tenpy.linalg.charges.LegCharge attribute), 309
method), 486	chinfo (tenpy.linalg.np_conserved.Array attribute),
calc_H_onsite()(<i>tenpy.models.model.MultiCouplingl</i>	
method), 500	chinfo (tenpy.networks.mpo.MPOGraph attribute), 665
calc_H_onsite() (tenpy.models.spins.SpinChain	COE () (in module tenpy.linalg.random_matrix), 327
method), 545	combine (tenpy.algorithms.mps_common.EffectiveH at-
calc_H_onsite() (tenpy.models.tf_ising.TFIChain	tribute), 207
method), 517	combine (tenpy.algorithms.mps_common.TwoSiteH at-
calc_H_onsite()(<i>tenpy.models.xxz_chain.XXZChain2</i>	
method), 532	<pre>combine_Heff() (tenpy.algorithms.mps_common.OneSiteH</pre>
calc_U() (tenpy.algorithms.purification.PurificationTEB	D2 method), 210
method), 256	<pre>combine_Heff() (tenpy.algorithms.mps_common.TwoSiteH</pre>
calc_U() (tenpy.algorithms.tebd.Engine method), 201	method), 212
callback () (tenpy.tools.events.Listener property), 724	${\tt combine_Heff()}$ (tenpy.algorithms.purification.PurificationTwoSiteU
central_charge_from_S_profile() (in mod-	method), 260
ule tenpy.tools.fit), 741	combine_legs() (tenpy.linalg.np_conserved.Array
Chain (class in tenpy.models.lattice), 356	method), 281
change() (tenpy.linalg.charges.ChargeInfo class	combine_theta() (tenpy.algorithms.mps_common.EffectiveH method) 207
memuu w()	memora. Zali

<pre>combine_theta() (tenpy.algorithms.mps_common.One</pre>	eSiteHht_neighbors() (tenpy.models.lattice.Kagome method), 397
<pre>combine_theta() (tenpy.algorithms.mps_common.Two method), 212</pre>	oSiteHht_neighbors() (tenpy.models.lattice.Ladder method), 409
<pre>combine_theta() (tenpy.algorithms.purification.Purifi</pre>	icationTvoStiveGhbors() (tenpy.models.lattice.Lattice method), 427
compiled_with_MKL (in module	<pre>count_neighbors()</pre>
tenpy.tools.optimization), 754	(tenpy.models.lattice.SimpleLattice method),
complex_conj() (tenpy.linalg.np_conserved.Array	433
method), 286 concatenate() (in module	<pre>count_neighbors() (tenpy.models.lattice.Square method), 444</pre>
concatenate() (in module tenpy.linalg.np_conserved), 288	count_neighbors()
conj() (tenpy.linalg.charges.LegCharge method), 312	(tenpy.models.lattice.Triangular method),
conj () (tenpy.linalg.charges.LegPipe method), 319	455
conj () (tenpy.linalg.np_conserved.Array method), 286	count_neighbors()
connect () (tenpy.tools.events.EventHandler method), 722	(tenpy.models.lattice.TrivialLattice method), 464
<pre>connect_by_name()</pre>	<pre>count_neighbors()</pre>
(tenpy.tools.events.EventHandler method), 723	(tenpy.models.toric_code.DualSquare method), 600
cons_N (tenpy.networks.site.SpinHalfFermionSite attribute), 632	coupling_shape() (tenpy.models.lattice.Chain method), 357
cons_Sz (tenpy.networks.site.SpinHalfFermionSite attribute), 632	coupling_shape() (tenpy.models.lattice.HelicalLattice method), 370
conserve (tenpy.networks.site.BosonSite attribute), 612	coupling_shape() (tenpy.models.lattice.Honeycomb method), 378
conserve (tenpy.networks.site.FermionSite attribute), 617	<pre>coupling_shape() (tenpy.models.lattice.IrregularLattice</pre>
conserve (tenpy.networks.site.SpinHalfSite attribute), 636	coupling_shape() (tenpy.models.lattice.Kagome method), 398
conserve (tenpy.networks.site.SpinSite attribute), 641 console_main() (in module tenpy), 190	coupling_shape() (tenpy.models.lattice.Ladder method), 409
contract() (in module	<pre>coupling_shape() (tenpy.models.lattice.Lattice</pre>
tenpy.algorithms.network_contractor), 262	method), 428
copy () (tenpy.algorithms.truncation.TruncationError method), 196	<pre>coupling_shape() (tenpy.models.lattice.SimpleLattice</pre>
copy () (tenpy.linalg.charges.LegCharge method), 310 copy () (tenpy.linalg.charges.LegPipe method), 319	coupling_shape() (tenpy.models.lattice.Square method), 444
copy () (tenpy.linalg.np_conserved.Array method), 273 copy () (tenpy.tools.events.EventHandler method), 722	coupling_shape() (tenpy.models.lattice.Triangular method), 455
correlation_length() (in module tenpy.simulations.measurement), 695	coupling_shape()(tenpy.models.lattice.TrivialLattice method), 465
count () (tenpy.tools.events.Listener method), 724	coupling_shape()(tenpy.models.toric_code.DualSquare
<pre>count_neighbors() (tenpy.models.lattice.Chain</pre>	<pre>method), 600 coupling_strength_add_ext_flux()</pre>
<pre>count_neighbors()</pre>	$(tenpy. models. fermions_spinless. Fermion Chain$
(tenpy.models.lattice.HelicalLattice method),	method), 561
370	coupling_strength_add_ext_flux()
count_neighbors() (tanny models lattice Honovcomb method)	(tenpy.models.hubbard.BoseHubbardChain
(tenpy.models.lattice.Honeycomb method), 378	<pre>method), 575 coupling_strength_add_ext_flux()</pre>
count_neighbors()	(tenpy.models.hubbard.FermiHubbardChain
(tenpy.models.lattice.IrregularLattice method),	method), 589
389	coupling_strength_add_ext_flux()

(tenpy.models.model.CouplingModel method), 486	<pre>del_RP() (tenpy.networks.mps.MPSEnvironment method), 653</pre>
<pre>coupling_strength_add_ext_flux() (tenpy.models.model.MultiCouplingModel</pre>	DensityMatrixMixer (class in tenpy.algorithms.dmrg), 215
method), 501	detect_grid_outer_legcharge() (in module
coupling_strength_add_ext_flux()	tenpy.linalg.np_conserved), 289
(tenpy.models.spins.SpinChain method),	detect_legcharge() (in module
546	tenpy.linalg.np_conserved), 289
coupling_strength_add_ext_flux()	detect_qtotal() (in module
(tenpy.models.tf_ising.TFIChain method),	tenpy.linalg.np_conserved), 290
(tenpy.modeis.tj_isting.TFTChain method), 518	diag() (in module tenpy.linalg.np_conserved), 290
coupling_strength_add_ext_flux()	
(tenpy.models.xxz_chain.XXZChain2 method),	method), 220
532	diag() (tenpy.algorithms.dmrg.EngineFracture
coupling_term_handle_JW()	method), 228
(tenpy.networks.terms.CouplingTerms method),	dim (tenpy.models.lattice.Chain attribute), 357
672	dim (tenpy.models.lattice.Honeycomb attribute), 375
coupling_term_handle_JW()	dim (tenpy.models.lattice.Kagome attribute), 406
(tenpy. networks. terms. Multi Coupling Terms	dim (tenpy.models.lattice.Ladder attribute), 409
method), 680	dim (tenpy.models.lattice.Square attribute), 442
$\verb coupling_terms (tenpy.models.model.CouplingModel $	
attribute), 479	dim(tenpy.models.toric_code.DualSquare attribute), 600
coupling_terms (tenpy.networks.terms.CouplingTerms attribute), 671	s dim() (tenpy.models.lattice.HelicalLattice property), 370
coupling_terms (tenpy.networks.terms.MultiCoupling attribute), 678	Terims() (tenpy.models.lattice.IrregularLattice property), 389
CouplingModel (class in tenpy.models.model), 479	dim() (tenpy.models.lattice.Lattice property), 422
CouplingTerms (class in tenpy.networks.terms), 671	dim() (tenpy.models.lattice.SimpleLattice property),
CRE () (in module tenpy.linalg.random_matrix), 327	433
<pre>create_group_for_obj()</pre>	dim() (tenpy.models.lattice.TrivialLattice property),
(tenpy.tools.hdf5_io.Hdf5Saver method),	465
709	dim() (tenpy.networks.site.BosonSite property), 613
CUE () (in module tenpy.linalg.random_matrix), 327	dim() (tenpy.networks.site.FermionSite property), 617
	dim() (tenpy.networks.site.GroupedSite property), 622
D	dim() (tenpy.networks.site.Site property), 627
default_lattice(tenpy.models.fermions_spinless.Fer	.dimeth. steppy networks site SpinHalfFermionSite prop-
attribute) 554	erty), 633
default_lattice(tenpy.models.hubbard.FermiHubba	rachdin (tenpy.networks.site.SpinHalfSite property), 637
attribute), 582	dim() (tenpy.networks.site.SpinSite property), 641
<pre>default_lattice (tenpy.models.spins.SpinChain at- tribute), 538</pre>	disconnect() (tenpy.tools.events.EventHandler method), 723
<pre>default_lattice (tenpy.models.tf_ising.TFIChain</pre>	<pre>disent_iterations()</pre>
attribute), 510	(tenpy. algorithms. purification. Purification TEBD2
DefaultMixer(tenpy.algorithms.dmrg.EngineCombine	property), 256
attribute), 220	$\verb disentangle() \textit{(tenpy.algorithms.purification.PurificationTEBD2} $
DefaultMixer(tenpy.algorithms.dmrg.EngineFracture	method), 256
attribute), 228	disentangle_global()
del_LP() (tenpy.networks.mpo.MPOEnvironment	(tenpy.algorithms.purification.PurificationTEBD2
method), 663	method), 256
del_LP() (tenpy.networks.mps.MPSEnvironment	disentangle_global_nsite()
method), 653	(tenpy.algorithms.purification.PurificationTEBD2
del_RP() (tenpy.networks.mpo.MPOEnvironment	method), 256
method), 663	disentangle_n_site()
momowy, odd	(tenpy.algorithms.purification.PurificationTEBD2
	- · · · · · · · · · · · · · · · · · · ·

method), 256	${\tt EffectiveH}~(\textit{tenpy.algorithms.purification.PurificationApplyMPO}$
dispatch_load (tenpy.tools.hdf5_io.Hdf5Loader at-	attribute), 251
tribute), 706	eig() (in module tenpy.linalg.np_conserved), 291
dispatch_save (tenpy.tools.hdf5_io.Hdf5Saver at- tribute), 709	eigenvectors() (tenpy.linalg.sparse.FlatHermitianOperator method), 335
distance() (tenpy.models.lattice.Chain method), 357	eigenvectors()(tenpy.linalg.sparse.FlatLinearOperator
distance() (tenpy.models.lattice.HelicalLattice	method), 339
method), 370	eigenvectors () (tenpy.networks.mps.TransferMatrix
distance() (tenpy.models.lattice.Honeycomb method), 378	method), 657 eigh() (in module tenpy.linalg.np_conserved), 291
distance() (tenpy.models.lattice.IrregularLattice	eigvals() (in module tenpy.linalg.np_conserved), 292
method), 389	eigvalsh() (in module tenpy.linalg.np_conserved),
distance() (tenpy.models.lattice.Kagome method),	292
398	<pre>emit() (tenpy.tools.events.EventHandler method), 723</pre>
distance() (tenpy.models.lattice.Ladder method),	<pre>emit_until_result()</pre>
409	(tenpy.tools.events.EventHandler method),
distance() (tenpy.models.lattice.Lattice method), 427	723
distance() (tenpy.models.lattice.SimpleLattice method), 433	energy_MPO() (in module tenpy.simulations.measurement), 695
distance() (tenpy.models.lattice.Square method), 444	Engine (class in tenpy.algorithms.tdvp), 243
distance() (tenpy.models.lattice.Triangular method),	Engine (class in tempy.algorithms.tebd), 201
455	EngineCombine (class in tenpy.algorithms.dmrg), 219
distance() (tenpy.models.lattice.TrivialLattice	EngineFracture (class in tenpy.algorithms.dmrg),
method), 465	228
distance() (tenpy.models.toric_code.DualSquare	enlarge_mps_unit_cell()
method), 600 dot() (tenpy.linalg.sparse.FlatHermitianOperator	(tenpy.models.fermions_spinless.FermionChain method), 562
dot () (tenpy.linalg.sparse.FlatHermitianOperator method), 332	enlarge_mps_unit_cell()
dot() (tenpy.linalg.sparse.FlatLinearOperator	(tenpy.models.hubbard.BoseHubbardChain
method), 340	method), 576
drop() (tenpy.linalg.charges.ChargeInfo class method),	enlarge_mps_unit_cell()
306	(tenpy. models. hubbard. Fermi Hubbard Chain
drop_charge() (tenpy.linalg.np_conserved.Array	method), 590
method), 280	enlarge_mps_unit_cell()
dtype (tenpy.algorithms.mps_common.EffectiveH attribute), 207	(tenpy.models.lattice.Chain method), 358 enlarge_mps_unit_cell()
dtype (tenpy.linalg.np_conserved.Array attribute), 273	(tenpy.models.lattice.HelicalLattice method),
dtype (tenpy.linalg.sparse.FlatLinearOperator at-	368
tribute), 337	enlarge_mps_unit_cell()
dtype (tenpy.linalg.sparse.NpcLinearOperator at-	(tenpy.models.lattice.Honeycomb method),
tribute), 343	378
dtype (tenpy.models.model.Model attribute), 491	<pre>enlarge_mps_unit_cell()</pre>
dtype (tenpy.networks.mps.MPSEnvironment attribute), 651	(tenpy.models.lattice.IrregularLattice method), 390
DualSquare (class in tenpy.models.toric_code), 598	<pre>enlarge_mps_unit_cell()</pre>
E	(tenpy.models.lattice.Kagome method), 399
	enlarge_mps_unit_cell()
E (tenpy.algorithms.exact_diag.ExactDiag attribute), 265	(tenpy.models.lattice.Ladder method), 410 enlarge_mps_unit_cell()
EffectiveH (class in tenpy.algorithms.mps_common), 206	(tenpy.models.lattice.Lattice method), 424
EffectiveH (tenpy.algorithms.dmrg.EngineCombine	enlarge_mps_unit_cell()
attribute), 220	(tenpy.models.lattice.SimpleLattice method),
EffectiveH (tenpy.algorithms.dmrg.EngineFracture	434
attribute), 228	<pre>enlarge_mps_unit_cell()</pre>

(tenpy.models.lattice.Square method), 444	$(ten py. models. model. Coupling Model\ attribute),$
enlarge_mps_unit_cell()	479
(tenpy.models.lattice.Triangular method), 456	<pre>exp_decaying_terms (tenpy.networks.terms.ExponentiallyDecayingTerms</pre>
enlarge_mps_unit_cell()	attribute), 675
(tenpy.models.lattice.TrivialLattice method), 465	exp_H() (tenpy.algorithms.exact_diag.ExactDiag method), 266
enlarge_mps_unit_cell()	expectation_value()
(tenpy.models.model.CouplingModel method), 487	(tenpy.networks.mpo.MPOEnvironment method), 663
enlarge_mps_unit_cell()	expectation_value()
(tenpy.models.model.Model method), 491	(tenpy.networks.mps.MPSEnvironment
enlarge_mps_unit_cell()	method), 653
(tenpy.models.model.MPOModel method), 489	explicit_plus_hc(tenpy.models.model.CouplingModel attribute), 479
enlarge_mps_unit_cell()	expm() (in module tenpy.linalg.np_conserved), 293
(tenpy.models.model.MultiCouplingModel method), 502	ExponentiallyDecayingTerms (class in tenpy.networks.terms), 675
enlarge_mps_unit_cell() (tenpy.models.model.NearestNeighborModel	extend() (tenpy.linalg.charges.LegCharge method), 315
method), 505	extend() (tenpy.linalg.charges.LegPipe method), 320
enlarge_mps_unit_cell()	extend() (tenpy.linalg.np_conserved.Array method),
(tenpy.models.spins.SpinChain method),	279
546	<pre>eye_like() (in module tenpy.linalg.np_conserved),</pre>
enlarge_mps_unit_cell()	293
(tenpy.models.tf_ising.TFIChain method), 518	F
enlarge_mps_unit_cell()	FermiHubbardChain (class in
(tenpy.models.toric_code.DualSquare method),	tenpy.models.hubbard), 581
601	FermionChain (class in
enlarge_mps_unit_cell()	tenpy.models.fermions_spinless), 554
(tenpy.models.xxz_chain.XXZChain2 method),	FermionSite (class in tenpy.networks.site), 616
533	filling (tenpy.networks.site.BosonSite attribute), 612
entropy() (in module	filling (tenpy.networks.site.FermionSite attribute),
tenpy.simulations.measurement), 696 entropy() (in module tenpy.tools.math), 736	617
entropy() (in module tenpy.tools.math), 730 entropy_profile_from_CFT() (in module	filling (tenpy.networks.site.SpinHalfFermionSite at-
tenpy.tools.fit), 742	<pre>tribute), 632 find_coupling_pairs()</pre>
environment_sweeps()	(tenpy.models.lattice.Chain method), 358
(tenpy.algorithms.dmrg.EngineCombine	find_coupling_pairs()
method), 220	(tenpy.models.lattice.HelicalLattice method),
environment_sweeps()	371
(tenpy.algorithms.dmrg.EngineFracture	<pre>find_coupling_pairs()</pre>
method), 229	(tenpy.models.lattice.Honeycomb method),
environment_sweeps()	379
(tenpy. algorithms. purification. Purification Apply and the property of the	<i>MPO</i> nd_coupling_pairs()
method), 251	(tenpy.models.lattice.IrregularLattice method),
eps (tenpy.algorithms.truncation.TruncationError at-	390
tribute), 196	<pre>find_coupling_pairs()</pre>
EventHandler (class in tenpy.tools.events), 721	(tenpy.models.lattice.Kagome method), 399
evolved_time() (in module	<pre>find_coupling_pairs()</pre>
tenpy.simulations.measurement), 696	(tenpy.models.lattice.Ladder method), 410
ExactDiag (class in tenpy.algorithms.exact_diag), 265	find_coupling_pairs()
exp_decaying_terms	(tenpy.models.lattice.Lattice method), 427

<pre>find_coupling_pairs()</pre>	class method), 320
(tenpy.models.lattice.SimpleLattice method),	<pre>from_change_charge()</pre>
434	(tenpy.linalg.charges.LegCharge class method),
<pre>find_coupling_pairs()</pre>	311
(tenpy.models.lattice.Square method), 445	<pre>from_change_charge()</pre>
<pre>find_coupling_pairs()</pre>	(tenpy.linalg.charges.LegPipe class method),
(tenpy.models.lattice.Triangular method),	321
456	<pre>from_drop_charge()</pre>
<pre>find_coupling_pairs()</pre>	(tenpy.linalg.charges.LegCharge class method),
(tenpy.models.lattice.TrivialLattice method),	311
466	<pre>from_drop_charge() (tenpy.linalg.charges.LegPipe</pre>
<pre>find_coupling_pairs()</pre>	class method), 321
(tenpy.models.toric_code.DualSquare method),	<pre>from_func() (tenpy.linalg.np_conserved.Array class</pre>
601	method), 275
find_global() (in module tenpy.tools.hdf5_io), 712	<pre>from_func_square()</pre>
find_subclass() (in module tenpy.tools.misc), 729	(tenpy.linalg.np_conserved.Array class
fit_with_sum_of_exp() (in module	method), 276
tenpy.tools.fit), 742	<pre>from_guess_with_pipe()</pre>
flat_linop (tenpy.networks.mps.TransferMatrix at-	(tenpy.linalg.sparse.FlatHermitianOperator
tribute), 656	class method), 333
flat_to_npc() (tenpy.linalg.sparse.FlatHermitianOper	
method), 333	(tenpy.linalg.sparse.FlatLinearOperator
flat_to_npc() (tenpy.linalg.sparse.FlatLinearOperato	
method), 338	from_H_mpo() (tenpy.algorithms.exact_diag.ExactDiag
flat_to_npc_all_sectors()	class method), 266
(tenpy.linalg.sparse.FlatHermitianOperator method), 333	from_hdf5() (tenpy.algorithms.truncation.TruncationError class method), 196
<pre>flat_to_npc_all_sectors()</pre>	<pre>from_hdf5() (tenpy.linalg.charges.ChargeInfo class</pre>
(tenpy. linalg. sparse. Flat Linear Operator	method), 306
method), 339	<pre>from_hdf5() (tenpy.linalg.charges.LegCharge class</pre>
<pre>flat_to_npc_None_sector()</pre>	method), 310
(tenpy. linalg. sparse. Flat Hermitian Operator	from_hdf5() (tenpy.linalg.charges.LegPipe class
method), 333	method), 319
<pre>flat_to_npc_None_sector()</pre>	<pre>from_hdf5() (tenpy.linalg.np_conserved.Array class</pre>
(tenpy.linalg.sparse.FlatLinearOperator	method), 274
method), 339	from_hdf5() (tenpy.models.fermions_spinless.FermionChain
FlatHermitianOperator (class in	class method), 562
tenpy.linalg.sparse), 332	from_hdf5() (tenpy.models.hubbard.BoseHubbardChain
FlatLinearOperator (class in tenpy.linalg.sparse),	class method), 576
337	from_hdf5() (tenpy.models.hubbard.FermiHubbardChain
flatten() (in module tenpy.tools.misc), 729	class method), 590
flip_charges_qconj()	from_hdf5() (tenpy.models.lattice.Chain class
(tenpy.linalg.charges.LegCharge method),	method), 358
312	from_hdf5() (tenpy.models.lattice.HelicalLattice
flip_charges_qconj()	class method), 367
(tenpy.linalg.charges.LegPipe method), 320	from_hdf5() (tenpy.models.lattice.Honeycomb class
form (tenpy.networks.mps.TransferMatrix attribute),	method), 379
656	from_hdf5() (tenpy.models.lattice.IrregularLattice
format_selection (tenpy.tools.hdf5_io.Hdf5Saver	class method), 388
attribute), 709	from_hdf5() (tenpy.models.lattice.Kagome class
<pre>from_add_charge() (tenpy.linalg.charges.LegCharge class method),</pre>	method), 399 from_hdf5() (tenpy.models.lattice.Ladder class
(tenpy.tinatg.charges.LegCharge class method), 311	method), 410
from add charge () (tenny linals charges Les Pine	

method), 422	(tenpy.networks.terms.TermList class method),
from_hdf5() (tenpy.models.lattice.SimpleLattice	686
class method), 434	from_MPOModel() (tenpy.models.fermions_spinless.FermionChain
from_hdf5() (tenpy.models.lattice.Square class method), 445	class method), 562 from_MPOModel() (tenpy.models.hubbard.BoseHubbardChain
from_hdf5() (tenpy.models.lattice.Triangular class	class method), 576
method), 456	from_MPOModel() (tenpy.models.hubbard.FermiHubbardChain
from_hdf5() (tenpy.models.lattice.TrivialLattice class	class method), 590
method), 466	<pre>from_MPOModel() (tenpy.models.model.NearestNeighborModel</pre>
${\tt from_hdf5()} \qquad \textit{(tenpy.models.model.CouplingModel)}$	class method), 504
class method), 487	<pre>from_MPOModel() (tenpy.models.spins.SpinChain</pre>
from_hdf5() (tenpy.models.model.Model class	class method), 546
method), 492	from_MPOModel() (tenpy.models.tf_ising.TFIChain
from_hdf5() (tenpy.models.model.MPOModel class	class method), 518
method), 490 from_hdf5() (tenpy.models.model.MultiCouplingMode.	from_MPOModel() (tenpy.models.xxz_chain.XXZChain2 l class method), 533
class method), 502	from_ndarray() (tenpy.linalg.np_conserved.Array
from_hdf5() (tenpy.models.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.model.NearestNeighborModels.mode	- ' ** ' * *
class method), 505	from_ndarray_trivial()
<pre>from_hdf5() (tenpy.models.spins.SpinChain class</pre>	(tenpy.linalg.np_conserved.Array class
method), 547	method), 274
<pre>from_hdf5() (tenpy.models.tf_ising.TFIChain class</pre>	$from_norm()$ (tenpy.algorithms.truncation.TruncationError
method), 519	class method), 196
from_hdf5() (tenpy.models.toric_code.DualSquare	from_NpcArray() (tenpy.linalg.sparse.FlatHermitianOperator
class method), 601	class method), 333
from_hdf5() (tenpy.models.xxz_chain.XXZChain2 class method), 533	from_NpcArray() (tenpy.linalg.sparse.FlatLinearOperator class method), 338
from_hdf5() (tenpy.networks.site.BosonSite class	from_qdict() (tenpy.linalg.charges.LegCharge class
method), 613	method), 311
<pre>from_hdf5() (tenpy.networks.site.FermionSite class</pre>	<pre>from_qdict() (tenpy.linalg.charges.LegPipe class</pre>
method), 617	method), 321
<pre>from_hdf5() (tenpy.networks.site.GroupedSite class</pre>	<pre>from_qflat() (tenpy.linalg.charges.LegCharge class</pre>
method), 622	method), 310
from_hdf5() (tenpy.networks.site.Site class method),	from_qflat() (tenpy.linalg.charges.LegPipe class
629 from hdf5 () (tenny networks site SpinHalfFermion Site	method), 321 from_qind() (tenpy.linalg.charges.LegCharge class
class method), 633	method), 311
from_hdf5() (tenpy.networks.site.SpinHalfSite class	
method), 637	method), 322
<pre>from_hdf5() (tenpy.networks.site.SpinSite class</pre>	from_S() (tenpy.algorithms.truncation.TruncationError
method), 641	class method), 196
<pre>from_hdf5() (tenpy.networks.terms.CouplingTerms</pre>	<pre>from_term_list() (tenpy.networks.mpo.MPOGraph</pre>
class method), 673	class method), 666
	ying Teamtserms () (tenpy.networks.mpo.MPOGraph class
class method), 676	method), 666
from_hdf5() (tenpy.networks.terms.MultiCouplingTerm class method), 680	nsfrom_trivial() (tenpy.linalg.charges.LegCharge class method), 310
	from_trivial() (tenpy.linalg.charges.LegPipe class
class method), 684	method), 322
from_hdf5() (tenpy.networks.terms.TermList class	full_contraction()
method), 687	(tenpy.networks.mpo.MPOEnvironment
<pre>from_hdf5() (tenpy.tools.hdf5_io.Hdf5Exportable</pre>	method), 663
class method), 703	full_contraction()
<pre>from_lattice_locations()</pre>	(tenpy.networks.mps.MPSEnvironment

method), 653	<pre>get_lattice() (in module tenpy.models.lattice), 474</pre>
<pre>full_diag_effH() (in module tenpy.algorithms.dmrg), 239</pre>	<pre>get_leg() (tenpy.linalg.np_conserved.Array method),</pre>
<pre>full_diagonalization()</pre>	<pre>get_leg_index() (tenpy.linalg.np_conserved.Array</pre>
(tenpy.algorithms.exact_diag.ExactDiag	method), 277
method), 266	get_leg_indices()
full_H (tenpy.algorithms.exact_diag.ExactDiag attribute), 265	(tenpy.linalg.np_conserved.Array method), 277
<pre>full_to_mps() (tenpy.algorithms.exact_diag.ExactDia</pre>	<pre>gget_leg_labels() (tenpy.linalg.np_conserved.Array</pre>
full_version (in module tenpy.version), 755	<pre>get_level() (in module tenpy.tools.optimization),</pre>
G	<pre>get_LP() (tenpy.networks.mpo.MPOEnvironment</pre>
<pre>gauge_hopping()</pre>	method), 662
tenpy.models.hofstadter), 594 gauge_total_charge()	<pre>get_LP() (tenpy.networks.mps.MPSEnvironment method), 652</pre>
(tenpy.linalg.np_conserved.Array method), 279	<pre>get_LP_age() (tenpy.networks.mpo.MPOEnvironment method), 663</pre>
gcd() (in module tenpy.tools.math), 737	get_LP_age() (tenpy.networks.mps.MPSEnvironment method), 652
gcd_array() (in module tenpy.tools.math), 737	get_op() (tenpy.networks.site.BosonSite method), 613
<pre>get_attr() (tenpy.tools.hdf5_io.Hdf5Loader static</pre>	<pre>get_op() (tenpy.networks.site.FermionSite method),</pre>
<pre>get_block() (tenpy.linalg.np_conserved.Array</pre>	618
method), 278	get_op() (tenpy.networks.site.GroupedSite method),
<pre>get_block_sizes()</pre>	623
(tenpy.linalg.charges.LegCharge method), 313	get_op() (tenpy.networks.site.Site method), 628 get_op() (tenpy.networks.site.SpinHalfFermionSite
<pre>get_block_sizes() (tenpy.linalg.charges.LegPipe</pre>	method), 633
method), 322	get_op() (tenpy.networks.site.SpinHalfSite method),
<pre>get_charge() (tenpy.linalg.charges.LegCharge</pre>	637
method), 314	get_op() (tenpy.networks.site.SpinSite method), 642
<pre>get_charge() (tenpy.linalg.charges.LegPipe</pre>	get_order() (in module tenpy.models.lattice), 474
method), 322	<pre>get_order_grouped()</pre>
<i>memoa)</i> , 322	
get_close() (in module tenpy.tools.misc), 730	tenpy.models.lattice), 475
	<pre>tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718</pre>
<pre>get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite</pre>	<pre>tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params),</pre>
<pre>get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite</pre>	<pre>tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params),</pre>
<pre>get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite</pre>	<pre>tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params),</pre>
<pre>get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite</pre>	<pre>tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params),</pre>
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site method), 628	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method),
<pre>get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite</pre>	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method),
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site method), 628	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges()
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method), ionSite 314 get_qindex_of_charges()
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 637	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges()
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 637 get_hc_op_name() (tenpy.networks.site.SpinSite	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method), ionSite 314 get_qindex_of_charges() (tenpy.linalg.charges.LegPipe method), 322 get_recursive() (in module tenpy.tools.misc), 730
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 637 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 641	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method), ionSite 314 get_qindex_of_charges() (tenpy.linalg.charges.LegPipe method), 322 get_recursive() (in module tenpy.tools.misc), 730 get_resume_data()
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 637 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 641 get_initialization_data()	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method), ionSite 314 get_qindex_of_charges() (tenpy.linalg.charges.LegPipe method), 322 get_recursive() (in module tenpy.tools.misc), 730 get_resume_data() (tenpy.algorithms.dmrg.EngineCombine
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 637 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 641 get_initialization_data() (tenpy.networks.mpo.MPOEnvironment	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method), ionSite 314 get_qindex_of_charges() (tenpy.linalg.charges.LegPipe method), 322 get_recursive() (in module tenpy.tools.misc), 730 get_resume_data() (tenpy.algorithms.dmrg.EngineCombine method), 220
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 637 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 641 get_initialization_data() (tenpy.networks.mpo.MPOEnvironment method), 663	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges() (tenpy.linalg.charges.LegCharge method), ionSite 314 get_qindex_of_charges() (tenpy.linalg.charges.LegPipe method), 322 get_reqursive() (in module tenpy.tools.misc), 730 get_resume_data() (tenpy.algorithms.dmrg.EngineCombine method), 220 get_resume_data()
get_close() (in module tenpy.tools.misc), 730 get_hc_op_name() (tenpy.networks.site.BosonSite method), 613 get_hc_op_name() (tenpy.networks.site.FermionSite method), 618 get_hc_op_name() (tenpy.networks.site.GroupedSite method), 622 get_hc_op_name() (tenpy.networks.site.Site method), 628 get_hc_op_name() (tenpy.networks.site.SpinHalfFerm method), 633 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 637 get_hc_op_name() (tenpy.networks.site.SpinHalfSite method), 641 get_initialization_data() (tenpy.networks.mpo.MPOEnvironment	tenpy.models.lattice), 475 get_parameter() (in module tenpy.tools.params), 718 get_qindex() (tenpy.linalg.charges.LegCharge method), 313 get_qindex() (tenpy.linalg.charges.LegPipe method), 322 get_qindex_of_charges()

method), 251	method), 591
get_resume_data()	<pre>group_sites() (tenpy.models.model.CouplingModel</pre>
(tenpy. algorithms. purification. Purification TEBD.	2 <i>method</i>), 487
method), 256	<pre>group_sites() (tenpy.models.model.Model method),</pre>
<pre>get_resume_data() (tenpy.algorithms.tdvp.Engine</pre>	492
method), 243	group_sites() (tenpy.models.model.MPOModel
<pre>get_resume_data() (tenpy.algorithms.tebd.Engine</pre>	method), 489
method), 201	<pre>group_sites() (tenpy.models.model.MultiCouplingModel</pre>
get_RP() (tenpy.networks.mpo.MPOEnvironment	method), 502
method), 663	$\verb group_sites() \textit{(tenpy.models.model.NearestNeighborModel} $
get_RP() (tenpy.networks.mps.MPSEnvironment	method), 505
method), 652	group_sites() (tenpy.models.spins.SpinChain
<pre>get_RP_age() (tenpy.networks.mpo.MPOEnvironment</pre>	method), 547
method), 663	group_sites() (tenpy.models.tf_ising.TFIChain
get_RP_age() (tenpy.networks.mps.MPSEnvironment	method), 519
method), 653	group_sites() (tenpy.models.xxz_chain.XXZChain2
<pre>get_slice() (tenpy.linalg.charges.LegCharge</pre>	method), 534
method), 313	GroupedSite (class in tenpy.networks.site), 621
get_slice() (tenpy.linalg.charges.LegPipe method),	GUE () (in module tenpy.linalg.random_matrix), 328
322	H
get_sweep_schedule()	
(tenpy.algorithms.dmrg.EngineCombine	H (tenpy.networks.mpo.MPOEnvironment attribute), 662
method), 221	H() (tenpy.linalg.sparse.FlatHermitianOperator prop-
get_sweep_schedule()	erty), 332
(tenpy.algorithms.dmrg.EngineFracture	H() (tenpy.linalg.sparse.FlatLinearOperator property),
<pre>method), 229 get_sweep_schedule()</pre>	340
	H0_mixed (class in tenpy.algorithms.tdvp), 245
method), 251	MPQ _mixed (class in tenpy.algorithms.tdvp), 246
get_xL() (tenpy.algorithms.dmrg.DensityMatrixMixer	H2_mixed (class in tenpy.algorithms.tdvp), 247
method), 217	h5group (tenpy.tools.hdf5_io.Hdf5Loader attribute),
get_xR() (tenpy.algorithms.dmrg.DensityMatrixMixer	706
method), 217	h5group (tenpy.tools.hdf5_io.Hdf5Saver attribute), 709 H_bond (tenpy.models.model.NearestNeighborModel at-
git_revision (in module tenpy.version), 755	tribute), 504
GOE () (in module tenpy.linalg.random_matrix), 328	H_MPO (tenpy.models.model.MPOModel attribute), 489
gram_schmidt() (in module tenpy.linalg.lanczos),	has_edge() (tenpy.networks.mpo.MPOGraph
349	method), 667
graph (tenpy.networks.mpo.MPOGraph attribute), 665	has_label() (tenpy.linalg.np_conserved.Array
grid_concat() (in module	method), 277
tenpy.linalg.np_conserved), 293	have_cython_functions (in module
grid_insert_ops() (in module	tenpy.tools.optimization), 754
tenpy.networks.mpo), 668	hc_ops (tenpy.networks.site.Site attribute), 626
<pre>grid_outer() (in module tenpy.linalg.np_conserved),</pre>	Hdf5Exportable (class in tenpy.tools.hdf5_io), 702
295	Hdf5ExportError, 711
groundstate() (tenpy.algorithms.exact_diag.ExactDia	8Hdf5FormatError,711
method), 266	Hdf5Ignored (class in tenpy.tools.hdf5_io), 704
<pre>group_by_degeneracy() (in module</pre>	Hdf5ImportError, 712
tenpy.tools.misc), 731	Hdf5Loader (class in tenpy.tools.hdf5_io), 705
group_sites() (in module tenpy.networks.site), 644	Hdf5Saver (class in tenpy.tools.hdf5 io), 708
group_sites()(<i>tenpy.models.fermions_spinless.Fermi</i>	on Chain al Lattice (class in tenpy models lattice), 365
method), 563	Honeycomb (class in tenpy.models.lattice), 375
<pre>group_sites() (tenpy.models.hubbard.BoseHubbardC</pre>	
group_sites()(tenpy.models.hubbard.FermiHubbard	· Chain ladd prefactor other()
	TANA PIETACIOI OUNEI //

```
(tenpy.linalg.np_conserved.Array
                                          method),
                                                   init_RP()
                                                                   (tenpy.networks.mpo.MPOEnvironment
        287
                                                            method), 662
ibinary blockwise()
                                                    init RP()
                                                                    (tenpy.networks.mps.MPSEnvironment
        (tenpy.linalg.np_conserved.Array
                                                            method), 652
                                          method),
                                                    init_sites() (tenpy.models.fermions_spinless.FermionChain
iconj() (tenpy.linalg.np conserved.Array method),
                                                            method), 564
        286
                                                    init sites()(tenpy.models.hubbard.BoseHubbardChain
idrop_labels() (tenpy.linalg.np_conserved.Array
                                                            method), 578
        method), 278
                                                    init sites()(tenpy.models.hubbard.FermiHubbardChain
ignore_unknown(tenpy.tools.hdf5_io.Hdf5Loader at-
                                                            method), 592
        tribute), 706
                                                   init_sites()
                                                                           (tenpy.models.spins.SpinChain
ind_len (tenpy.linalg.charges.LegCharge attribute),
                                                            method), 549
                                                                         (tenpy.models.tf_ising.TFIChain
                                                    init_sites()
index () (tenpy.tools.events.Listener method), 724
                                                            method), 521
init_env() (tenpy.algorithms.dmrg.EngineCombine
                                                                     (tenpy.models.xxz_chain.XXZChain2
                                                   init_sites()
        method), 221
                                                            method), 524
               (tenpy.algorithms.dmrg.EngineFracture
                                                   init_terms()(tenpy.models.fermions_spinless.FermionChain
init_env()
        method), 230
                                                            method), 564
init_env() (tenpy.algorithms.purification.PurificationApplyMPOerms() (tenpy.models.hubbard.BoseHubbardChain
        method), 252
                                                            method), 578
init_H_from_terms()
                                                    init_terms()(tenpy.models.hubbard.FermiHubbardChain
        (tenpy.models.fermions_spinless.FermionChain
                                                            method), 592
        method), 563
                                                                           (tenpy.models.spins.SpinChain
                                                    init terms()
init H from terms()
                                                            method), 549
        (tenpy.models.hubbard.BoseHubbardChain
                                                    init_terms()
                                                                         (tenpy.models.tf_ising.TFIChain
        method), 577
                                                            method), 521
init_H_from_terms()
                                                    init_terms() (tenpy.models.xxz_chain.XXZChain2
        (tenpy.models.hubbard.FermiHubbardChain
                                                            method), 525
        method), 591
                                                    initial_guess() (tenpy.networks.mps.TransferMatrix
init_H_from_terms()
                                                            method), 657
        (tenpy.models.spins.SpinChain
                                          method),
                                                   inner() (in module tenpy.linalg.np_conserved), 296
        548
                                                    inverse_permutation()
                                                                                      (in
                                                                                               module
init_H_from_terms()
                                                            tenpy.tools.misc), 731
        (tenpy.models.tf_ising.TFIChain
                                          method),
                                                   iproject()
                                                                         (tenpy.linalg.np_conserved.Array
        520
                                                            method), 284
                                                                        (tenpy.linalg.np_conserved.Array
init H from terms()
                                                    ipurge_zeros()
        (tenpy.models.xxz chain.XXZChain2 method),
                                                            method), 284
                                                    ireplace_label() (tenpy.linalg.np_conserved.Array
init_lattice() (tenpy.models.fermions_spinless.FermionChain method), 277
        method), 563
                                                    ireplace_labels()
init lattice()(tenpy.models.hubbard.BoseHubbardChain
                                                            (tenpy.linalg.np conserved.Array
                                                                                              method),
        method), 577
                                                            2.77
init lattice() (tenpy.models.hubbard.FermiHubbardChaireqularLattice (class in tenpy.models.lattice),
        method), 591
                                                            386
                       (tenpy.models.spins.SpinChain is_blocked()
                                                                         (tenpy.linalg.charges.LegCharge
init_lattice()
        method), 548
                                                            method), 312
                                                                           (tenpy.linalg.charges.LegPipe
init_lattice()
                     (tenpy.models.tf_ising.TFIChain is_blocked()
        method), 520
                                                            method), 322
init_lattice() (tenpy.models.xxz_chain.XXZChain2 is_bunched()
                                                                         (tenpy.linalg.charges.LegCharge
        method), 534
                                                            method), 312
               (tenpy.networks.mpo.MPOEnvironment
                                                                            (tenpy.linalg.charges.LegPipe
                                                   is_bunched()
init_LP()
        method), 662
                                                            method), 322
init_LP()
                (tenpy.networks.mps.MPSEnvironment is_completely_blocked()
        method), 651
                                                            (tenpy.linalg.np conserved.Array
                                                                                              method),
```

280	<pre>lat2mps_idx() (tenpy.models.lattice.HelicalLattice</pre>
is_non_string_iterable() (in module	method), 367
tenpy.tools.string), 744	lat2mps_idx() (tenpy.models.lattice.Honeycomb
is_sorted() (tenpy.linalg.charges.LegCharge method), 312	method), 379 lat2mps_idx() (tenpy.models.lattice.IrregularLattice
is_sorted() (tenpy.linalg.charges.LegPipe method),	method), 390
322	lat2mps_idx() (tenpy.models.lattice.Kagome
<pre>iscale_axis() (tenpy.linalg.np_conserved.Array</pre>	method), 400
method), 285	<pre>lat2mps_idx() (tenpy.models.lattice.Ladder</pre>
<pre>iscale_prefactor()</pre>	method), 410
(tenpy.linalg.np_conserved.Array method),	lat2mps_idx() (tenpy.models.lattice.Lattice
287	method), 425
<pre>iset_leg_labels() (tenpy.linalg.np_conserved.Array method),</pre>	lat2mps_idx() (tenpy.models.lattice.SimpleLattice
(tenpy.linalg.np_conserved.Array method), 277	method), 435 lat2mps_idx() (tenpy.models.lattice.Square
isort_qdata() (tenpy.linalg.np_conserved.Array	method), 445
method), 281	lat2mps_idx() (tenpy.models.lattice.Triangular
<pre>iswapaxes() (tenpy.linalg.np_conserved.Array</pre>	method), 456
method), 285	<pre>lat2mps_idx() (tenpy.models.lattice.TrivialLattice</pre>
itranspose() (tenpy.linalg.np_conserved.Array	method), 466
method), 285	lat2mps_idx() (tenpy.models.toric_code.DualSquare
<pre>iunary_blockwise()</pre>	method), 602
(tenpy.linalg.np_conserved.Array method), 285	Lattice (class in tenpy.models.lattice), 419 lcm() (in module tenpy.tools.math), 737
283	leg (tenpy.linalg.sparse.FlatLinearOperator attribute),
J	337
JW_exponent (tenpy.networks.site.Site attribute), 626	leg (tenpy.networks.site.Site attribute), 626
	LegCharge (class in tenpy.linalg.charges), 309
K	LegPipe (class in tenpy.linalg.charges), 317
Kagome (class in tenpy.models.lattice), 397	legs (tenpy.linalg.charges.LegPipe attribute), 317
kroneckerproduct()	legs (tenpy.linalg.np_conserved.Array attribute), 273
(tenpy.networks.site.GroupedSite method),	length (tenpy.algorithms.mps_common.EffectiveH at-
621	tribute), 207 length (tenpy.algorithms.mps_common.OneSiteH at-
1	tribute), 209
L (tenpy.networks.mps.MPSEnvironment attribute), 651	length (tenpy.algorithms.mps_common.TwoSiteH at-
L (tenpy.networks.mps.TransferMatrix attribute), 656	tribute), 212
L (tenpy.networks.terms.CouplingTerms attribute), 671	LeviCivita3 (in module tenpy.tools.math), 740
L (tenpy.networks.terms.ExponentiallyDecayingTerms at-	lexsort() (in module tenpy.tools.misc), 732
tribute), 675	LHeff (tenpy.algorithms.mps_common.TwoSiteH
$\verb L (tenpy.networks.terms.MultiCouplingTerms \ attribute),$	attribute), 212
678	<pre>limits() (tenpy.networks.terms.TermList method), 687 lin_fit_res() (in module tenpy.tools.fit), 743</pre>
L (tenpy.networks.terms.OnsiteTerms attribute), 683	linear_fit() (in module tenpy.tools.fit), 743
L() (tenpy.networks.mpo.MPOGraph property), 666	list_to_dict_list() (in module
label_split (tenpy.networks.mps.TransferMatrix attribute), 656	tenpy.tools.misc), 732
labels (tenpy.networks.site.GroupedSite attribute), 621	Listener (class in tenpy.tools.events), 724
Ladder (class in tenpy.models.lattice), 408	listener_id() (tenpy.tools.events.Listener prop-
lanczos () (in module tenpy.linalg.lanczos), 349	erty), 724
lanczos_arpack() (in module tenpy.linalg.lanczos),	listeners (tenpy.tools.events.EventHandler at-
350	tribute), 721
lat (tenpy.models.model.Model attribute), 491	load() (in module tenpy.tools.hdf5_io), 712 load() (tenpy.tools.hdf5_io.Hdf5Loader method), 706
<pre>lat2mps_idx() (tenpy.models.lattice.Chain method), 359</pre>	load_dataset() (tenpy.tools.hdf5_io.Hdf5Loader

method), 706	<pre>make_pipe() (tenpy.linalg.np_conserved.Array</pre>
load_dict() (tenpy.tools.hdf5_io.Hdf5Loader	method), 281
method), 707	make_valid() (tenpy.linalg.charges.ChargeInfo
load_dtype() (tenpy.tools.hdf5_io.Hdf5Loader	method), 307
method), 707	<pre>make_W_II() (in module tenpy.networks.mpo), 668 map_incoming_flat()</pre>
load_from_hdf5() (in module tenpy.tools.hdf5_io), 713	(tenpy.linalg.charges.LegPipe method), 320
load_general_dict()	matmat() (tenpy.linalg.sparse.FlatHermitianOperator
(tenpy.tools.hdf5_io.Hdf5Loader method),	method), 334
707	matmat() (tenpy.linalg.sparse.FlatLinearOperator
<pre>load_global() (tenpy.tools.hdf5_io.Hdf5Loader</pre>	method), 341
method), 707	matvec() (tenpy.algorithms.exact_diag.ExactDiag
<pre>load_hdf5exportable()</pre>	method), 267
(tenpy.tools.hdf5_io.Hdf5Loader method),	<pre>matvec() (tenpy.algorithms.mps_common.EffectiveH</pre>
707	method), 207
<pre>load_ignored() (tenpy.tools.hdf5_io.Hdf5Loader</pre>	<pre>matvec() (tenpy.algorithms.mps_common.OneSiteH</pre>
method), 707	method), 210
load_list() (tenpy.tools.hdf5_io.Hdf5Loader	<pre>matvec() (tenpy.algorithms.mps_common.TwoSiteH</pre>
method), 707	method), 212
load_none() (tenpy.tools.hdf5_io.Hdf5Loader	matvec() (tenpy.algorithms.purification.PurificationTwoSiteU
method), 706	method), 260
load_omp_library() (in module	matvec() (tenpy.linalg.np_conserved.Array method),
tenpy.tools.process), 746	287
<pre>load_range() (tenpy.tools.hdf5_io.Hdf5Loader method), 707</pre>	matvec() (tenpy.linalg.sparse.FlatHermitianOperator method), 334
load_reduce() (tenpy.tools.hdf5_io.Hdf5Loader	matvec() (tenpy.linalg.sparse.FlatLinearOperator
method), 707	method), 341
load_set() (tenpy.tools.hdf5_io.Hdf5Loader	matvec() (tenpy.linalg.sparse.NpcLinearOperator
method), 707	method), 343
<pre>load_simple_dict()</pre>	matvec() (tenpy.networks.mps.TransferMatrix
(tenpy.tools.hdf5_io.Hdf5Loader method),	method), 656
707	<pre>matvec_count (tenpy.linalg.sparse.FlatLinearOperator</pre>
<pre>load_str() (tenpy.tools.hdf5_io.Hdf5Loader</pre>	attribute), 337
method), 707	<pre>matvec_to_array() (in module tenpy.tools.math),</pre>
load_tuple() (tenpy.tools.hdf5_io.Hdf5Loader	737
method), 707	max_range (tenpy.networks.mpo.MPOGraph at-
logger (tenpy.models.model.Model attribute), 491	tribute), 665
Lp (tenpy.algorithms.tdvp.H0_mixed attribute), 245	max_range() (tenpy.networks.terms.CouplingTerms
Lp (tenpy.algorithms.tdvp.H1_mixed attribute), 246	method), 672 max_range() (tenpy.networks.terms.ExponentiallyDecayingTerms
Lp (tenpy.algorithms.tdvp.H2_mixed attribute), 247 Ls (tenpy.models.lattice.Lattice attribute), 420	method), 676
Lu (tenpy.models.lattice.Honeycomb attribute), 375	max_range() (tenpy.networks.terms.MultiCouplingTerms
Lu (tenpy.models.lattice.Kagome attribute), 406	method), 679
Lu (tenpy.models.lattice.Ladder attribute), 409	max_range() (tenpy.networks.terms.OnsiteTerms
Lu (tenpy.models.lattice.Lattice attribute), 422	method), 683
Lu (tenpy.models.lattice.SimpleLattice attribute), 433	max_size (tenpy.algorithms.exact_diag.ExactDiag at-
	tribute), 265
M	<pre>measurement_index()</pre>
<pre>make_eff_H() (tenpy.algorithms.dmrg.EngineCombine</pre>	tenpy.simulations.measurement), 696
method), 222	<pre>memo_load (tenpy.tools.hdf5_io.Hdf5Loader attribute),</pre>
<pre>make_eff_H() (tenpy.algorithms.dmrg.EngineFracture</pre>	706
method), 230	memo_save (tenpy.tools.hdf5_io.Hdf5Saver attribute),
$\verb make_eff_H() (\textit{tenpy.algorithms.purification.Purification} \textit{tenpy.algorithms.purification} tenpy$	onApplyMPO'9
method), 252	<pre>memorize_load() (tenpy.tools.hdf5_io.Hdf5Loader</pre>

method), 706	tenpy.models.lattice,476
memorize_save() (tenpy.tools.hdf5_io.Hdf5Saver	tenpy.models.model,506
method), 710	tenpy.models.spins,549
memory_usage() (in module tenpy.tools.process), 746	tenpy.models.spins_nnn,550
<pre>mix_rho_L() (tenpy.algorithms.dmrg.DensityMatrixMixer</pre>	tenpy.models.tf_ising,521
method), 216	tenpy.models.toric_code, 607
<pre>mix_rho_R() (tenpy.algorithms.dmrg.DensityMatrixMixer</pre>	tenpy.models.xxz_chain,535
method), 216	tenpy.networks,609
<pre>mixed_svd() (tenpy.algorithms.dmrg.EngineCombine</pre>	tenpy.networks.mpo,669
method), 222	tenpy.networks.mps,658
<pre>mixed_svd() (tenpy.algorithms.dmrg.EngineFracture</pre>	tenpy.networks.purification_mps,689
method), 230	tenpy.networks.site,648
<pre>mixer_activate() (tenpy.algorithms.dmrg.EngineCombin</pre>	netenpy.networks.terms,688
method), 222	tenpy.simulations,691
<pre>mixer_activate() (tenpy.algorithms.dmrg.EngineFracture</pre>	<pre>re tenpy.simulations.ground_state_search, 698</pre>
mixer_cleanup()(<i>tenpy.algorithms.dmrg.EngineCombine</i>	e tenpy.simulations.measurement,697
method), 222	tenpy.simulations.simulation, 693
<pre>mixer_cleanup() (tenpy.algorithms.dmrg.EngineFracture</pre>	
mkl_get_nthreads() (in module	tenpy.tools,701
tenpy.tools.process), 747	tenpy.tools.events,725
mkl_set_nthreads() (in module	tenpy.tools.fit,744
tenpy.tools.process), 747	tenpy.tools.hdf5_io,714
mod () (tenpy.linalg.charges.ChargeInfo property), 307	tenpy.tools.math,740
Model (class in tenpy.models.model), 491	tenpy.tools.misc, 736
model (tenpy.algorithms.exact_diag.ExactDiag at-	tenpy.tools.optimization, 753
tribute), 265	tenpy.tools.params, 720
module	tenpy.tools.process,748
tenpy, 189	tenpy.tools.string,745
tenpy.algorithms, 193	tenpy.version,755
	ove_right (tenpy.algorithms.mps_common.EffectiveH
tenpy.algorithms.dmrg, 240	attribute), 207
	POEnvironment (class in tenpy.networks.mpo), 661
	POGraph (class in tenpy.networks.mpo), 665
	PoModel (class in tenpy.models.model), 489
tenpy.algorithms.network_contractor, mp	
263	359
	ps2lat_idx() (tenpy.models.lattice.HelicalLattice
tenpy.algorithms.tdvp, 248	method), 367
	ps2lat_idx() (tenpy.models.lattice.Honeycomb
tenpy.algorithms.truncation, 198	method), 379
	ps2lat_idx() (tenpy.models.lattice.IrregularLattice
tenpy.linalg.charges,324	method), 391
	os2lat_idx() (tenpy.models.lattice.Kagome
tenpy.linalg.np_conserved,302	method), 400
	os2lat_idx() (tenpy.models.lattice.Ladder
tenpy.linalg.sparse, 348	method), 411
	os2lat_idx() (tenpy.models.lattice.Lattice
tenpy.models, 353	method), 424
	ps2lat_idx() (tenpy.models.lattice.SimpleLattice
tenpy.models.haldane, 596	method), 435
	os2lat_idx() (tenpy.models.lattice.Square
tenpy.models.hubbard,593	method), 445

<pre>mps2lat_idx() (tenpy.models.lattice.Triangular</pre>	457
method), 457	<pre>mps2lat_values_masked()</pre>
<pre>mps2lat_idx() (tenpy.models.lattice.TrivialLattice</pre>	(tenpy.models.lattice.TrivialLattice method), 468
	mps2lat_values_masked()
method), 602	(tenpy.models.toric_code.DualSquare method),
mps2lat_values() (tenpy.models.lattice.Chain	603
method), 359	mps_idx_fix_u() (tenpy.models.lattice.Chain
mps2lat_values()(tenpy.models.lattice.HelicalLattic	- · · · · · · · · · · · · · · · · · · ·
method), 368	mps_idx_fix_u() (tenpy.models.lattice.HelicalLattice
	method), 371
mps2lat_values() (tenpy.models.lattice.Honeycomb	
method), 380	mps_idx_fix_u() (tenpy.models.lattice.Honeycomb
mps2lat_values() (tenpy.models.lattice.IrregularLatt	
method), 391	mps_idx_fix_u() (tenpy.models.lattice.IrregularLattice
mps2lat_values() (tenpy.models.lattice.Kagome	method), 389
method), 400	mps_idx_fix_u() (tenpy.models.lattice.Kagome
mps2lat_values() (tenpy.models.lattice.Ladder	method), 401
method), 411	mps_idx_fix_u() (tenpy.models.lattice.Ladder
mps2lat_values() (tenpy.models.lattice.Lattice	method), 412
method), 425	mps_idx_fix_u() (tenpy.models.lattice.Lattice
<pre>mps2lat_values() (tenpy.models.lattice.SimpleLattice</pre>	
method), 433	<pre>mps_idx_fix_u() (tenpy.models.lattice.SimpleLattice</pre>
<pre>mps2lat_values() (tenpy.models.lattice.Square</pre>	method), 435
method), 446	mps_idx_fix_u() (tenpy.models.lattice.Square
<pre>mps2lat_values() (tenpy.models.lattice.Triangular</pre>	method), 446
method), 457	<pre>mps_idx_fix_u() (tenpy.models.lattice.Triangular</pre>
<pre>mps2lat_values() (tenpy.models.lattice.TrivialLattice</pre>	method), 457
method), 467	<pre>mps_idx_fix_u() (tenpy.models.lattice.TrivialLattice</pre>
<pre>method), 467 mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	= :
	= :
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.HelicalLattice method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method),
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.HelicalLattice method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method), 392
<pre>mps2lat_values() (tenpy.models.toric_code.DualSqu</pre>	are method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSquemethod), 602 mps2lat_values_masked()</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSquemethod), 602 mps2lat_values_masked()</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.HelicalLattice method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method), 392 mps_lat_idx_fix_u() (tenpy.models.lattice.Kagome method), 401 mps_lat_idx_fix_u()
<pre>mps2lat_values() (tenpy.models.toric_code.DualSque</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.HelicalLattice method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method), 392 mps_lat_idx_fix_u() (tenpy.models.lattice.Kagome method), 401 mps_lat_idx_fix_u() (tenpy.models.lattice.Ladder method), 412
mps2lat_values() (tenpy.models.toric_code.DualSquemethod), 602 mps2lat_values_masked()	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.HelicalLattice method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method), 392 mps_lat_idx_fix_u() (tenpy.models.lattice.Kagome method), 401 mps_lat_idx_fix_u() (tenpy.models.lattice.Ladder method), 412 mps_lat_idx_fix_u() (tenpy.models.lattice.Lattice
<pre>mps2lat_values() (tenpy.models.toric_code.DualSquemethod), 602 mps2lat_values_masked()</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method), 392 mps_lat_idx_fix_u() (tenpy.models.lattice.Kagome method), 401 mps_lat_idx_fix_u() (tenpy.models.lattice.Ladder method), 412 mps_lat_idx_fix_u() (tenpy.models.lattice.Lattice method), 425
mps2lat_values() (tenpy.models.toric_code.DualSquemethod), 602 mps2lat_values_masked()	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
mps2lat_values() (tenpy.models.toric_code.DualSquemethod), 602 mps2lat_values_masked()	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.HelicalLattice method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method), 392 mps_lat_idx_fix_u() (tenpy.models.lattice.Kagome method), 401 mps_lat_idx_fix_u() (tenpy.models.lattice.Ladder method), 412 mps_lat_idx_fix_u() (tenpy.models.lattice.Lattice method), 425 mps_lat_idx_fix_u() (tenpy.models.lattice.Lattice method), 425 mps_lat_idx_fix_u() (tenpy.models.lattice.Method), 425
<pre>mps2lat_values() (tenpy.models.toric_code.DualSque</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()
mps2lat_values() (tenpy.models.toric_code.DualSquemethod), 602 mps2lat_values_masked()	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u() (tenpy.models.lattice method), 371 mps_lat_idx_fix_u() (tenpy.models.lattice.HelicalLattice method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.Honeycomb method), 381 mps_lat_idx_fix_u() (tenpy.models.lattice.IrregularLattice method), 392 mps_lat_idx_fix_u() (tenpy.models.lattice.Kagome method), 401 mps_lat_idx_fix_u() (tenpy.models.lattice.Lattice method), 425 mps_lat_idx_fix_u() (tenpy.models.lattice.Lattice method), 425 mps_lat_idx_fix_u() (tenpy.models.lattice method), 436 mps_lat_idx_fix_u() (tenpy.models.lattice.Square
<pre>mps2lat_values() (tenpy.models.toric_code.DualSque</pre>	method), 468 mps_idx_fix_u() (tenpy.models.toric_code.DualSquare method), 603 mps_lat_idx_fix_u() (tenpy.models.lattice.Chain method), 360 mps_lat_idx_fix_u()

(tenpy.models.lattice.Triangular method),	436
457	multi_coupling_shape()
mps_lat_idx_fix_u() (toppy_models_lattice_TrivialLatticemethod)	(tenpy.models.lattice.Square method), 447
(tenpy.models.lattice.TrivialLattice method), 468	<pre>multi_coupling_shape()</pre>
mps_lat_idx_fix_u()	(tenpy.modeis.tatice.11tanguar method), 458
(tenpy.models.toric_code.DualSquare method),	
603	(tenpy.models.lattice.TrivialLattice method),
<pre>mps_sites() (tenpy.models.lattice.Chain method),</pre>	468
360 (tenpy.modeis.tatice.Chain method),	multi_coupling_shape()
mps_sites() (tenpy.models.lattice.HelicalLattice	(tenpy.models.toric_code.DualSquare method),
method), 371	604
	multi_coupling_term_handle_JW()
method), 381	(tenpy.networks.terms.MultiCouplingTerms
mps_sites() (tenpy.models.lattice.IrregularLattice	method), 679
method), 392	multi_sites_combine_charges() (in module
mps_sites() (tenpy.models.lattice.Kagome method),	tenpy.networks.site), 644
402	MultiCouplingModel (class in tenpy.models.model),
mps_sites() (tenpy.models.lattice.Ladder method),	494
413	MultiCouplingTerms (class in
	tenpy.networks.terms), 678
<pre>mps_sites() (tenpy.models.lattice.Lattice method),</pre>	multiply_op_names()
	(tenpy.networks.site.BosonSite method),
<pre>mps_sites() (tenpy.models.lattice.SimpleLattice method), 436</pre>	(tenpy.networks.sue.Bosonsue method), 613
<pre>mps_sites() (tenpy.models.lattice.Square method),</pre>	multiply_op_names() (tanyunatuswka sita Farmian Sita mathad)
	(tenpy.networks.site.FermionSite method), 618
mps_sites() (tenpy.models.lattice.Triangular	
method), 458	<pre>multiply_op_names() (tenpy.networks.site.GroupedSite method),</pre>
mps_sites() (tenpy.models.lattice.TrivialLattice	(tenpy.networks.site.GroupedSite method), 623
<pre>method), 468 mps_sites() (tenpy.models.toric_code.DualSquare</pre>	
<pre>mps_sites() (tenpy.models.toric_code.DualSquare</pre>	multiply_op_names() (tenpy.networks.site.Site method), 629
mps_to_full()(tenpy.algorithms.exact_diag.ExactDia	
method), 266	(tenpy.networks.site.SpinHalfFermionSite
MPSEnvironment (class in tenpy.networks.mps), 650	method), 633
multi_coupling_shape()	multiply_op_names()
(tenpy.models.lattice.Chain method), 360	(tenpy.networks.site.SpinHalfSite method),
multi_coupling_shape()	637
	multiply_op_names()
371	(tenpy.networks.site.SpinSite method), 642
<pre>multi_coupling_shape()</pre>	multiply_operators()
(tenpy.models.lattice.Honeycomb method),	(tenpy.networks.site.BosonSite method),
381	614
multi_coupling_shape()	<pre>multiply_operators()</pre>
(tenpy.models.lattice.IrregularLattice method),	(tenpy.networks.site.FermionSite method),
392	618
multi_coupling_shape()	<pre>multiply_operators()</pre>
(tenpy.models.lattice.Kagome method), 402	(tenpy.networks.site.GroupedSite method),
multi_coupling_shape()	623
(tenpy.models.lattice.Ladder method), 413	<pre>multiply_operators() (tenpy.networks.site.Site</pre>
multi_coupling_shape()	method), 629
(tenpy.models.lattice.Lattice method), 429	multiply_operators()
multi_coupling_shape()	(tenpy.networks.site.SpinHalfFermionSite
(tenpy.models.lattice.SimpleLattice method),	method), 633
* **	·

<pre>multiply_operators() (tenpy.networks.site.SpinHalfSite method), 638</pre>	<pre>number_nearest_neighbors() (tenpy.models.lattice.HelicalLattice method),</pre>
<pre>multiply_operators() (tenpy.networks.site.SpinSite method), 642</pre>	<pre>number_nearest_neighbors() (tenpy.models.lattice.Honeycomb method),</pre>
N	number_nearest_neighbors()
N (tenpy.algorithms.mps_common.EffectiveH attribute), 207	(tenpy.models.lattice.IrregularLattice method), 393
N_cells (tenpy.models.lattice.Lattice attribute), 421 n_optimize() (tenpy.algorithms.dmrg.EngineCombine property), 222	<pre>number_nearest_neighbors() (tenpy.models.lattice.Kagome method), 402 number_nearest_neighbors()</pre>
$\verb"n_optimize"()" (\textit{tenpy.algorithms.dmrg.EngineFracture}"$	(tenpy.models.lattice.Ladder method), 413 number_nearest_neighbors()
<pre>property), 231 n_optimize() (tenpy.algorithms.purification.Purification</pre>	onApplyMP(cenpy.models.lattice.Lattice method), 427
property), 252	<pre>number_nearest_neighbors()</pre>
N_rings (tenpy.models.lattice.Lattice attribute), 421	(tenpy.models.lattice.SimpleLattice method), 436
N_sites (tenpy.models.lattice.Lattice attribute), 421	number_nearest_neighbors()
n_sites (tenpy.networks.site.GroupedSite attribute),	(tenpy.models.lattice.Square method), 447
N_sites_per_ring (tenpy.models.lattice.Lattice at-	number_nearest_neighbors()
tribute), 421	(tenpy.models.lattice.Triangular method),
name (tenpy.tools.hdf5_io.Hdf5Ignored attribute), 704	458
names (tenpy.linalg.charges.ChargeInfo attribute), 305	<pre>number_nearest_neighbors()</pre>
ncon() (in module tenpy.algorithms.network_contractor), 262	(tenpy.models.lattice.TrivialLattice method), 469
ndim() (tenpy.linalg.np_conserved.Array property), 276	<pre>number_nearest_neighbors() (tenpy.models.toric_code.DualSquare method), 604</pre>
NearestNeighborModel (class in	number_next_nearest_neighbors()
tenpy.models.model), 504	(tenpy.models.lattice.Chain method), 360
need_JW_string (tenpy.networks.site.Site attribute), 626	number_next_nearest_neighbors()
nlegs (tenpy.linalg.charges.LegPipe attribute), 317	(tenpy.models.lattice.HelicalLattice method),
norm() (in module tenpy.linalg.np_conserved), 296	372
norm() (tenpy.linalg.np_conserved.Array method), 286	<pre>number_next_nearest_neighbors()</pre>
npc_matvec (tenpy.linalg.sparse.FlatLinearOperator attribute), 337	(tenpy.models.lattice.Honeycomb method), 382
npc_to_flat() (tenpy.linalg.sparse.FlatHermitianOpen	ranamber_next_nearest_neighbors()
method), 334	(tenpy.models.lattice.IrregularLattice method),
$\verb"npc_to_flat"() \textit{ (tenpy.linalg.sparse.FlatLinearOperato)} \\$	r 393
method), 339	<pre>number_next_nearest_neighbors()</pre>
<pre>npc_to_flat_all_sectors()</pre>	(tenpy.models.lattice.Kagome method), 402
(tenpy.linalg.sparse.FlatHermitianOperator	<pre>number_next_nearest_neighbors() (tenpy.models.lattice.Ladder method), 413</pre>
method), 334	number_next_nearest_neighbors()
npc_to_flat_all_sectors()	(tenpy.models.lattice.Lattice method), 427
(tenpy.linalg.sparse.FlatLinearOperator method), 339	number_next_nearest_neighbors()
NpcLinearOperator (class in tenpy.linalg.sparse),	(tenpy.models.lattice.SimpleLattice method), 436
343	number_next_nearest_neighbors()
NpcLinearOperatorWrapper (class in tenpy.linalg.sparse), 344	(tenpy.models.lattice.Square method), 447
number_nearest_neighbors()	number_next_nearest_neighbors()
(tenpy.models.lattice.Chain method), 360	(tenpy.models.lattice.Triangular method), 458

<pre>number_next_nearest_neighbors() (tenpy.models.lattice.TrivialLattice method), 469</pre>	OptimizationFlag (class in tenpy.tools.optimization), 749 optimize() (in module tenpy.tools.optimization), 751
<pre>number_next_nearest_neighbors() (tenpy.models.toric_code.DualSquare method), 604</pre>	order() (tenpy.models.lattice.Chain property), 360 order() (tenpy.models.lattice.HelicalLattice property), 367
0	order() (tenpy.models.lattice.Honeycomb property), 382
O_close_1 () (in module tenpy.linalg.random_matrix), 328	order() (tenpy.models.lattice.IrregularLattice property), 388
<pre>omp_get_nthreads() (in module</pre>	order() (tenpy.models.lattice.Kagome property), 402 order() (tenpy.models.lattice.Ladder property), 413
<pre>omp_set_nthreads() (in module tenpy.tools.process), 747</pre>	order() (tenpy.models.lattice.Lattice property), 423 order() (tenpy.models.lattice.SimpleLattice property),
ones () (in module tenpy.linalg.np_conserved), 297	436
OneSiteH (class in tenpy.algorithms.mps_common), 209	order() (tenpy.models.lattice.Square property), 447 order() (tenpy.models.lattice.Triangular property),
<pre>onsite_expectation_value() (in module</pre>	458
<pre>tenpy.simulations.measurement), 697 onsite_ops() (tenpy.networks.site.BosonSite prop-</pre>	order() (tenpy.models.lattice.TrivialLattice property), 469
erty), 614	order() (tenpy.models.toric_code.DualSquare prop-
onsite_ops() (tenpy.networks.site.FermionSite prop-	erty), 604
erty), 618	order_combine() (tenpy.networks.terms.TermList method), 687
onsite_ops() (tenpy.networks.site.GroupedSite property), 623	order_combine_term() (in module
onsite_ops() (tenpy.networks.site.Site property), 627	tenpy.networks.terms), 688
onsite_ops() (tenpy.networks.site.SpinHalfFermionSit	ordering() (tenpy.models.lattice.HelicalLattice
<pre>property), 634 onsite_ops() (tenpy.networks.site.SpinHalfSite</pre>	method), 367
onsite_ops() (tenpy.networks.site.SpinHalfSite property), 638	ordering() (tenpy.models.lattice.Honeycomb
onsite_ops() (tenpy.networks.site.SpinSite prop-	method), 375
erty), 642	ordering() (tenpy.models.lattice.IrregularLattice
onsite_terms (tenpy.models.model.CouplingModel	method), 388
attribute), 479	ordering() (tenpy.models.lattice.Kagome method),
onsite_terms (tenpy.networks.terms.OnsiteTerms at-	402
tribute), 683	ordering() (tenpy.models.lattice.Ladder method),
OnsiteTerms (class in tenpy.networks.terms), 682	413
op_needs_JW() (tenpy.networks.site.BosonSite	ordering() (tenpy.models.lattice.Lattice method), 423
method), 614	ordering() (tenpy.models.lattice.SimpleLattice
op_needs_JW() (tenpy.networks.site.FermionSite	method), 436 ordering() (tenpy.models.lattice.Square method), 447
method), 618 op_needs_JW() (tenpy.networks.site.GroupedSite	ordering() (tenpy.models.lattice.Triangular method),
method), 623	458
op_needs_JW() (tenpy.networks.site.Site method), 628	ordering() (tenpy.models.lattice.TrivialLattice method), 469
op_needs_JW() (tenpy.networks.site.SpinHalfFermionS method), 634	
op_needs_JW() (tenpy.networks.site.SpinHalfSite method), 638	orig_operator(tenpy.linalg.sparse.NpcLinearOperatorWrapper attribute), 344
op_needs_JW() (tenpy.networks.site.SpinSite	OrthogonalNpcLinearOperator (class in
method), 642	tenpy.linalg.sparse), 345
opnames (tenpy.networks.site.Site attribute), 626	outer() (in module tenpy.linalg.np_conserved), 297
ops (tenpy.networks.site.Site attribute), 626	outer_conj() (tenpy.linalg.charges.LegPipe method), 319

outer_product (in module tenpy.algorithms.network_contractor), 263	plot_basis() (tenpy.models.lattice.Triangular method), 460
ov (tenpy.algorithms.truncation.TruncationError at- tribute), 196	
	plot_basis() (tenpy.models.toric_code.DualSquare method), 604
D	<pre>plot_bc_identified()</pre>
P	(tenpy.models.lattice.Chain method), 361
pad() (in module tenpy.tools.misc), 732	plot_bc_identified()
pairs (tenpy.models.lattice.Lattice attribute), 421 perm (tenpy.networks.site.Site attribute), 626	(tenpy.models.lattice.HelicalLattice method), 372
perm_flat_from_perm_qind()	<pre>plot_bc_identified()</pre>
(tenpy.linalg.charges.LegCharge method), 315	(tenpy.models.lattice.Honeycomb method), 382
perm_flat_from_perm_qind()	<pre>plot_bc_identified()</pre>
(tenpy.linalg.charges.LegPipe method), 323 perm_qind_from_perm_flat()	(tenpy.models.lattice.IrregularLattice method), 393
(tenpy.linalg.charges.LegCharge method), 315	<pre>plot_bc_identified() (tenpy.models.lattice.Kagome method), 404</pre>
perm_qind_from_perm_flat()	<pre>plot_bc_identified()</pre>
(tenpy.linalg.charges.LegPipe method), 323	(tenpy.models.lattice.Ladder method), 415
perm_sign() (in module tenpy.tools.math), 738	<pre>plot_bc_identified()</pre>
<pre>permute() (tenpy.linalg.np_conserved.Array method),</pre>	(tenpy.models.lattice.Lattice method), 430
284	plot_bc_identified()
perturb_svd() (tenpy.algorithms.dmrg.DensityMatrix method), 215	438
<pre>perturb_svd() (tenpy.algorithms.dmrg.SingleSiteMixe</pre>	(tenpy.models.lattice.Square method), 449
perturb_svd() (tenpy.algorithms.dmrg.TwoSiteMixer method), 238	(tenpy.models.lattice.Triangular method),
pinv() (in module tenpy.linalg.np_conserved), 298	460
pipe (tenpy.networks.mps.TransferMatrix attribute), 656	<pre>plot_bc_identified() (tenpy.models.lattice.TrivialLattice method),</pre>
<pre>plot_alg_decay_fit() (in module tenpy.tools.fit),</pre>	471 plot_bc_identified()
plot_basis() (tenpy.models.lattice.Chain method), 360	(tenpy.models.toric_code.DualSquare method), 604
plot_basis() (tenpy.models.lattice.HelicalLattice method), 372	plot_coupling() (tenpy.models.lattice.Chain method), 361
plot_basis() (tenpy.models.lattice.Honeycomb method), 382	<pre>plot_coupling() (tenpy.models.lattice.HelicalLattice</pre>
plot_basis() (tenpy.models.lattice.IrregularLattice method), 393	plot_coupling() (tenpy.models.lattice.Honeycomb method), 382
plot_basis() (tenpy.models.lattice.Kagome method), 404	plot_coupling() (tenpy.models.lattice.IrregularLattice method), 393
plot_basis() (tenpy.models.lattice.Ladder method), 415	plot_coupling() (tenpy.models.lattice.Kagome method), 404
plot_basis() (tenpy.models.lattice.Lattice method), 430	plot_coupling() (tenpy.models.lattice.Ladder method), 415
plot_basis() (tenpy.models.lattice.SimpleLattice method), 438	plot_coupling() (tenpy.models.lattice.Lattice method), 430
plot_basis() (tenpy.models.lattice.Square method), 449	<pre>plot_coupling() (tenpy.models.lattice.SimpleLattice</pre>
	<pre>plot_coupling() (tenpy.models.lattice.Square</pre>

method), 449	450
plot_coupling() (tenpy.models.lattice.Triangular method), 460	plot_sites() (tenpy.models.lattice.Triangular method), 461
<pre>plot_coupling() (tenpy.models.lattice.TrivialLattice</pre>	<pre>plot_sites() (tenpy.models.lattice.TrivialLattice</pre>
plot_coupling() (tenpy.models.toric_code.DualSqua method), 605	<pre>replot_sites() (tenpy.models.toric_code.DualSquare</pre>
plot_coupling_terms()	plot_stats() (in module tenpy.linalg.lanczos), 350
(tenpy.networks.terms.CouplingTerms method), 672	<pre>plot_sweep_stats() (tenpy.algorithms.dmrg.EngineCombine</pre>
<pre>plot_coupling_terms()</pre>	method), 222
(tenpy. networks. terms. Multi Coupling Terms	<pre>plot_sweep_stats()</pre>
method), 681	(tenpy.algorithms.dmrg.EngineFracture
plot_order() (tenpy.models.lattice.Chain method),	method), 231
361	plot_update_stats()
plot_order() (tenpy.models.lattice.HelicalLattice method), 372	(tenpy.algorithms.dmrg.EngineCombine method), 223
plot_order() (tenpy.models.lattice.Honeycomb	plot_update_stats()
method), 383	(tenpy.algorithms.dmrg.EngineFracture
<pre>plot_order() (tenpy.models.lattice.IrregularLattice</pre>	method), 231
method), 394	position() (tenpy.models.lattice.Chain method), 362
plot_order() (tenpy.models.lattice.Kagome method), 404	position() (tenpy.models.lattice.HelicalLattice method), 373
<pre>plot_order() (tenpy.models.lattice.Ladder method),</pre>	position() (tenpy.models.lattice.Honeycomb method), 383
plot_order() (tenpy.models.lattice.Lattice method), 429	position() (tenpy.models.lattice.IrregularLattice method), 394
<pre>plot_order() (tenpy.models.lattice.SimpleLattice</pre>	position() (tenpy.models.lattice.Kagome method), 405
plot_order() (tenpy.models.lattice.Square method), 449	position() (tenpy.models.lattice.Ladder method), 416
plot_order() (tenpy.models.lattice.Triangular method), 460	position() (tenpy.models.lattice.Lattice method), 424 position() (tenpy.models.lattice.SimpleLattice
<pre>plot_order() (tenpy.models.lattice.TrivialLattice</pre>	method), 439 position() (tenpy.models.lattice.Square method), 450
plot_order() (tenpy.models.toric_code.DualSquare method), 605	position() (tenpy.models.lattice.Triangular method), 461
plot_sites() (tenpy.models.lattice.Chain method), 361	position() (tenpy.models.lattice.TrivialLattice method), 472
plot_sites() (tenpy.models.lattice.HelicalLattice method), 372	position() (tenpy.models.toric_code.DualSquare method), 605
plot_sites() (tenpy.models.lattice.Honeycomb method), 383	possible_charge_sectors (tenpy.linalg.sparse.FlatLinearOperator
<pre>plot_sites() (tenpy.models.lattice.IrregularLattice</pre>	attribute), 337 possible_couplings()
plot_sites() (tenpy.models.lattice.Kagome method), 405	(tenpy.models.lattice.Chain method), 362 possible_couplings()
plot_sites() (tenpy.models.lattice.Ladder method), 416	(tenpy.models.lattice.HelicalLattice method), 368
<pre>plot_sites() (tenpy.models.lattice.Lattice method),</pre>	possible_couplings()
429	(tenpy.models.lattice.Honeycomb method),
plot_sites() (tenpy.models.lattice.SimpleLattice	383
method), 439	possible_couplings()
<pre>plot_sites() (tenpy.models.lattice.Square method),</pre>	(tenpy.models.lattice.IrregularLattice method),

394	method), 223
possible_couplings()	<pre>post_update_local()</pre>
(tenpy.models.lattice.Kagome method), 405	(tenpy.algorithms.dmrg.EngineFracture
possible_couplings()	method), 232
(tenpy.models.lattice.Ladder method), 416	<pre>post_update_local()</pre>
possible_couplings()	(tenpy. algorithms. purification. Purification Apply MPO
(tenpy.models.lattice.Lattice method), 428	method), 252
possible_couplings()	prepare_svd() (tenpy.algorithms.dmrg.EngineCombine
(tenpy.models.lattice.SimpleLattice method),	method), 223
439	<pre>prepare_svd() (tenpy.algorithms.dmrg.EngineFracture</pre>
possible_couplings() (tenpy.models.lattice.Square method), 450	prepare_update() (tenpy.algorithms.dmrg.EngineCombine
possible_couplings()	method), 223
(tenpy.models.lattice.Triangular method),	prepare_update() (tenpy.algorithms.dmrg.EngineFracture
461	method), 232
possible_couplings()	prepare_update()(tenpy.algorithms.purification.PurificationApplyMa
(tenpy.models.lattice.TrivialLattice method),	method), 252
472	priority()(tenpy.tools.events.Listener property), 724
possible_couplings()	<pre>project() (tenpy.linalg.charges.LegCharge method),</pre>
(tenpy.models.toric_code.DualSquare method),	314
606	<pre>project() (tenpy.linalg.charges.LegPipe method), 319</pre>
possible_multi_couplings()	PurificationApplyMPO (class in
(tenpy.models.lattice.Chain method), 362	tenpy.algorithms.purification), 251
possible_multi_couplings()	PurificationTEBD2 (class in
(tenpy.models.lattice.HelicalLattice method),	tenpy.algorithms.purification), 255
369	PurificationTwoSiteU (class in
possible_multi_couplings() (tampy models lattice Horacomb mathed)	tenpy.algorithms.purification), 260
(tenpy.models.lattice.Honeycomb method), 384	Python Enhancement Proposals PEP 257,181
possible_multi_couplings()	PEP 8, 181
(tenpy.models.lattice.IrregularLattice method),	1 11 0, 101
395	Q
possible_multi_couplings()	q_map (tenpy.linalg.charges.LegPipe attribute), 318
(tenpy.models.lattice.Kagome method), 405	q_map_slices (tenpy.linalg.charges.LegPipe at-
possible_multi_couplings()	tribute), 318
(tenpy.models.lattice.Ladder method), 417	qconj (tenpy.linalg.charges.LegCharge attribute), 309
possible_multi_couplings()	QCUTOFF (in module tenpy.linalg.np_conserved), 302
(tenpy.models.lattice.Lattice method), 429	<pre>qnumber() (tenpy.linalg.charges.ChargeInfo property),</pre>
possible_multi_couplings()	307
(tenpy.models.lattice.SimpleLattice method),	qr() (in module tenpy.linalg.np_conserved), 298
439	qr_li() (in module tenpy.tools.math), 738
possible_multi_couplings()	qtotal (tenpy.linalg.np_conserved.Array attribute),
(tenpy.models.lattice.Square method), 451	273
<pre>possible_multi_couplings() (tenpy.models.lattice.Triangular method),</pre>	qtotal (tenpy.networks.mps.TransferMatrix attribute),
(tenpy.modeis.tatice.17tanguar method), 461	656
possible_multi_couplings()	QTYPE (in module tenpy.linalg.charges), 324 QTYPE (in module tenpy.linalg.np_conserved), 302
(tenpy.models.lattice.TrivialLattice method),	Q11PE (in module lenpy.linaig.np_conserved), 502
472	R
possible_multi_couplings()	
(tenpy.models.toric_code.DualSquare method),	rank (tenpy.linalg.np_conserved.Array attribute), 273 regular_lattice (tenpy.models.lattice.IrregularLattice
606	attribute), 387
post_update_local()	released (in module tenpy.version), 755
(tenpy.algorithms.dmrg.EngineCombine	= = = = = = (

remove_op() (tenpy.networks.site.BosonSite method), 614	REPR_NONE (in module tenpy.tools.hdf5_io), 716 REPR_RANGE (in module tenpy.tools.hdf5_io), 716
remove_op() (tenpy.networks.site.FermionSite	REPR_SET (in module tenpy.tools.hdf5_io), 716
method), 619	REPR_STR (in module tenpy.tools.hdf5_io), 715
remove_op() (tenpy.networks.site.GroupedSite	REPR_TUPLE (in module tenpy.tools.hdf5_io), 716
method), 623	reset_stats() (tenpy.algorithms.dmrg.EngineCombine
remove_op() (tenpy.networks.site.Site method), 628	method), 223
	reset_stats()(tenpy.algorithms.dmrg.EngineFracture
method), 634	method), 232
remove_op() (tenpy.networks.site.SpinHalfSite method), 638	${\tt reset_stats()}~(\textit{tenpy.algorithms.purification.PurificationApplyMPO}\\ \textit{method}), 252$
remove_op() (tenpy.networks.site.SpinSite method), 642	resume_from_checkpoint() (in module tenpy.simulations.simulation), 692
remove_zeros() (tenpy.networks.terms.CouplingTerms method), 673	resume_run() (tenpy.algorithms.dmrg.EngineCombine method), 224
remove_zeros() (tenpy.networks.terms.MultiCoupling: method), 679	Terensume_run() (tenpy.algorithms.dmrg.EngineFracture method), 232
remove_zeros() (tenpy.networks.terms.OnsiteTerms method), 683	resume_run() (tenpy.algorithms.purification.PurificationApplyMPO method), 252
rename_op() (tenpy.networks.site.BosonSite method), 614	resume_run() (tenpy.algorithms.purification.PurificationTEBD2 method), 257
rename_op() (tenpy.networks.site.FermionSite method), 619	resume_run() (tenpy.algorithms.tdvp.Engine method), 243
rename_op() (tenpy.networks.site.GroupedSite method), 623	resume_run() (tenpy.algorithms.tebd.Engine method), 201
rename_op() (tenpy.networks.site.Site method), 628	RHeff (tenpy.algorithms.mps_common.TwoSiteH
$\verb"rename_op" () \textit{ (tenpy.networks.site.SpinHalfFermionSite} \\$	attribute), 212
method), 634	rmatmat() (tenpy.linalg.sparse.FlatHermitianOperator
rename_op() (tenpy.networks.site.SpinHalfSite	method), 335
method), 638	rmatmat() (tenpy.linalg.sparse.FlatLinearOperator
<pre>rename_op() (tenpy.networks.site.SpinSite method),</pre>	method), 341
642	rmatvec() (tenpy.linalg.sparse.FlatHermitianOperator
replace_label() (tenpy.linalg.np_conserved.Array	method), 335
method), 277	rmatvec() (tenpy.linalg.sparse.FlatLinearOperator
replace_labels() (tenpy.linalg.np_conserved.Array	method), 342
method), 278	Rp (tenpy.algorithms.tdvp.H0_mixed attribute), 245
REPR_ARRAY (in module tenpy.tools.hdf5_io), 715	Rp (tenpy.algorithms.tdvp.H1_mixed attribute), 246
REPR_BOOL (in module tenpy.tools.hdf5_io), 715	Rp (tenpy.algorithms.tdvp.H2_mixed attribute), 247
REPR_COMPLEX (in module tenpy.tools.hdf5_io), 715	rq_li() (in module tenpy.tools.math), 738
REPR_DICT_GENERAL (in module tenpy.tools.hdf5_io), 716	run () (tenpy.algorithms.dmrg.EngineCombine method), 224
REPR_DICT_SIMPLE (in module tenpy.tools.hdf5_io), 716	run () (tenpy.algorithms.dmrg.EngineFracture method), 232
REPR_DTYPE (in module tenpy.tools.hdf5_io), 716	run () (tenpy.algorithms.purification.PurificationApplyMPO
REPR_FLOAT (in module tenpy.tools.hdf5_io), 715	method), 253
REPR_FLOAT32 (in module tenpy.tools.hdf5_io), 715	run () (tenpy.algorithms.purification.PurificationTEBD2
REPR_FLOAT64 (in module tenpy.tools.hdf5_io), 715	method), 257
REPR_HDF5EXPORTABLE (in module	run () (tenpy.algorithms.tdvp.Engine method), 243
tenpy.tools.hdf5_io), 715	run () (tenpy.algorithms.tebd.Engine method), 201
REPR_IGNORED (in module tenpy.tools.hdf5_io), 716	<pre>run_GS() (tenpy.algorithms.purification.PurificationTEBD2</pre>
REPR_INT (in module tenpy.tools.hdf5_io), 715	method), 257
REPR_INT32 (in module tenpy.tools.hdf5_io), 715	run_GS () (tenpy.algorithms.tebd.Engine method), 201
REPR_INT64 (in module tenpy.tools.hdf5_io), 715	run_imaginary() (tenpy.algorithms.purification.PurificationTEBD2
REPR LIST (in module tenny tools hdf5 io) 716	method) 257

run_one_site() method), 243	(tenpy.algorithms.tdvp.Engi	ne save_hdf5() method	(tenpy.models.lattice.Sim I), 440	pleLattice
run_simulation()	(in module tenpy), 190		(tenpy.models.lattice.Square	method),
run_simulation()	(in modu		(- 1)	,,,
	ons.simulation), 693	save_hdf5()	(tenpy.models.lattice.	Triangular
run_two_sites()	(tenpy.algorithms.tdvp.Engi			
method), 243	(terip).attgort.tims.tavp.Eng.	save_hdf5()	(tenpy.models.lattice.Tri	vialI attice
тетои), 243		method		viaiLaiiicc
S		save_hdf5()	(tenpy.models.model.Coup	dinaModel
	· G:	method		ungmouei
S (tenpy.networks.site.Sp		save_hdf5()	(tenpy.models.model.Model	mathad)
save() (in module ten			(tenpy.modets.modet.Modet	memoa),
	f5_io.Hdf5Saver method), 709	492	(4	4DOM - 1-1
save_dataset()	(tenpy.tools.hdf5_io.Hdf5Sav		(tenpy.models.model.M	пРОмоаеі
method), 710		method		1. 16 1.1
save_dict()	(tenpy.tools.hdf5_io.Hdf5Sav		(tenpy.models.model.MultiCot	uplingModel
method), 710		method		
save_dict_conten	t()		(tenpy.models.model.NearestN	leighborModel
(tenpy.tools.ha	lf5_io.Hdf5Saver method			
710			(tenpy.models.spins.SpinChair	n method),
save_dtype()	(tenpy.tools.hdf5_io.Hdf5Sav	er 549		
method), 711	· 12	save_hdf5()	(tenpy.models.tf_ising	.TFIChain
save_global()	(tenpy.tools.hdf5_io.Hdf5Sav	er method	7), 521	
method), 711	(save_hdf5()	(tenpy.models.toric_code.D)ualSquare
	algorithms.truncation.Truncatio	nError method	7), 607	•
method), 196	ingoriminis.ir unedinom.ir unedine	save_hdf5()	(tenpy.models.xxz_chain.X	XZChain2
	(tenpy.linalg.charges.ChargeIn		= :	
method), 306	(tenpy.tinaig.enarges.enargen	, ~	(tenpy.networks.site.BosonSite	e method).
save_hdf5()	(tenpy.linalg.charges.LegCharges)	64.4	C. P.	, , , , , , , , , , , , , , , , , , , ,
method), 310	(tenpy.tinaig.charges.LegCharg	save_hdf5()	(tenpy.networks.site.Fe	ermionSite
	lingle of more at I as Discounted to		* * * * * * * * * * * * * * * * * * *	
= :	linalg.charges.LegPipe method	save_hdf5()	(tenpy.networks.site.G	rounedSite
319	(топреизне
	(tenpy.linalg.np_conserved.Arro	<i>• y</i>	(tenpy.networks.site.Site meth	ad) 620
<i>method</i>), 274				
	nodels.fermions_spinless.Ferm	ionChaine_nals()	(tenpy.networks.site.spinnatji	rermionsile
method), 564		method		· 11 100:
	nodels.hubbard.BoseHubbardC	hainsave_ndis()	(tenpy.networks.site.Sp	ппнацыие
method), 578		method		
save_hdf5()(tenpy.r	nodels.hubbard.FermiHubbard	<i>Chai</i> nave_hdf5()	(tenpy.networks.site.SpinSite	method),
method), 592		643		
save_hdf5() (tenp	y.models.lattice.Chain method), save_hdf5()	(tenpy.networks.terms.Coup	olingTerms
363		method	(I), 674	
save_hdf5() (te	enpy.models.lattice.HelicalLatti	ce save_hdf5()	(tenpy.networks.terms.Expone	ntially Decaying Term.
method), 365		method	7), 676	
	(tenpy.models.lattice.Honeycon	ab save_hdf5()	(tenpy.networks.terms.MultiCo	ouplingTerms
method), 384	(- 1) · · · · · · · · · · · · · · · · · ·	method	7), 681	
	py.models.lattice.IrregularLatti	ce save_hdf5()	(tenpy.networks.terms.O	nsiteTerms
method), 388	py.models.tatilee.frrequar	method		
	models.lattice.Kagome method		(tenpy.networks.term	s.TermList
	moueis.iuiice.Xugome meinoc	method		
406	and dela lattice I = 11		(tenpy.tools.hdf5_io.Hdf51	Exportable
	models.lattice.Ladder method	method		элрогион
417	111.00 7			Hdf5Saver
	models.lattice.Lattice method	·/•		iiujssuvei
422		method		Udf5Canar
		save_iterabl	le() (tenpy.tools.hdf5_io.	пајэзачег

method), 710	site() (tenpy.models.lattice.Honeycomb method), 384
<pre>save_iterable_content() (tenpy.tools.hdf5_io.Hdf5Saver method),</pre>	site() (tenpy.models.lattice.IrregularLattice method), 395
710	site() (tenpy.models.lattice.Kagome method), 406
save_none() (tenpy.tools.hdf5_io.Hdf5Saver	site() (tenpy.models.lattice.Ladder method), 417
method), 710	site() (tenpy.models.lattice.Lattice method), 424
save_range() (tenpy.tools.hdf5_io.Hdf5Saver	<pre>site() (tenpy.models.lattice.SimpleLattice method),</pre>
method), 711	440
save_reduce() (tenpy.tools.hdf5_io.Hdf5Saver	site() (tenpy.models.lattice.Square method), 451
method), 710	site() (tenpy.models.lattice.Triangular method), 462
save_to_hdf5() (in module tenpy.tools.hdf5_io), 714	<pre>site() (tenpy.models.lattice.TrivialLattice method),</pre>
scale_axis() (tenpy.linalg.np_conserved.Array	473
method), 285	<pre>site() (tenpy.models.toric_code.DualSquare method),</pre>
set_anonymous_svd()	607
(tenpy.algorithms.tdvp.Engine method), 243	sites (tenpy.networks.mpo.MPOGraph attribute), 665
set_B() (tenpy.algorithms.dmrg.EngineCombine	sites (tenpy.networks.site.GroupedSite attribute), 621
method), 225	size() (tenpy.linalg.np_conserved.Array property), 276
set_B() (tenpy.algorithms.dmrg.EngineFracture method), 234	Skip, 692
set_common_charges() (in module	slices (tenpy.linalg.charges.LegCharge attribute), 309
tenpy.networks.site), 645	sort () (tenpy.linalg.charges.LegCharge method), 314
set_level() (in module tenpy.tools.optimization),	sort () (tenpy.linalg.charges.LegPipe method), 319
752	sort_legcharge() (tenpy.linalg.np_conserved.Array
set_LP() (tenpy.networks.mpo.MPOEnvironment	method), 281
method), 664	sorted (tenpy.linalg.charges.LegCharge attribute), 309
set_LP() (tenpy.networks.mps.MPSEnvironment	<pre>sparse_diag() (tenpy.algorithms.exact_diag.ExactDiag</pre>
method), 653	method), 267
<pre>set_recursive() (in module tenpy.tools.misc), 733</pre>	<pre>sparse_stats() (tenpy.linalg.np_conserved.Array</pre>
set_RP() (tenpy.networks.mpo.MPOEnvironment	method), 278
method), 664	speigs() (in module tenpy.linalg.np_conserved), 299
set_RP() (tenpy.networks.mps.MPSEnvironment	speigs() (in module tenpy.tools.math), 739
method), 653	speigsh() (in module tenpy.tools.math), 739
setup_executable() (in module tenpy.tools.misc),	SpinChain (class in tenpy.models.spins), 538
733 shape (tenpy.linalg.np_conserved.Array attribute), 273	SpinHalfFermionSite (class in
shape (tenpy.linalg.sparse.FlatLinearOperator at-	tenpy.networks.site), 631 SpinHalfSite (class in tenpy.networks.site), 636
tribute), 337	SpinSite (class in tenpy.networks.site), 640
shape (tenpy.models.lattice.Lattice attribute), 420	split_legs() (tenpy.linalg.np_conserved.Array
shift () (tenpy.networks.terms.TermList method), 687	method), 282
shift_bra (tenpy.networks.mps.TransferMatrix	Square (class in tenpy.models.lattice), 442
attribute), 656	squeeze() (tenpy.linalg.np_conserved.Array method),
shift_ket (tenpy.networks.mps.TransferMatrix	283
attribute), 656	standard_normal_complex() (in module
ShiftNpcLinearOperator (class in	tenpy.linalg.random_matrix), 329
tenpy.linalg.sparse), 346	<pre>state_index() (tenpy.networks.site.BosonSite</pre>
short_version (in module tenpy.version), 755	method), 615
show_config() (in module tenpy), 192	state_index() (tenpy.networks.site.FermionSite
SimpleLattice (class in tenpy.models.lattice), 432	method), 619
SingleSiteMixer (class in tenpy.algorithms.dmrg),	state_index() (tenpy.networks.site.GroupedSite
235	method), 624
Site (class in tenpy.networks.site), 625	state_index() (tenpy.networks.site.Site method),
site() (tenpy.models.lattice.Chain method), 363	628
site() (tenpy.models.lattice.HelicalLattice method),	state_index() (tenpy.networks.site.SpinHalfFermionSite

state_index() (tenpy.networks.si	te.SpinHalfSite	method), 225
method), 639		ep() (tenpy.algorithms.dmrg.EngineFracture
	ks.site.SpinSite	method), 234
method), 643		ep() (tenpy.algorithms.purification.PurificationApplyMPO
state_indices() (tenpy.networks		method), 253
method), 615		ep_left_right()
state_indices() (tenpy.networks.s		(tenpy.algorithms.tdvp.Engine method), 243
method), 619		ep_left_right_two()
state_indices() (tenpy.networks.si	-	(tenpy.algorithms.tdvp.Engine method), 244
method), 624		ep_right_left()
state_indices() (tenpy.networks.sit		(tenpy.algorithms.tdvp.Engine method), 244
		ep_right_left_two()
state_indices() (tenpy.networks.site method), 634	эринацгеншоные	(tenpy.algorithms.tdvp.Engine method), 244
state_indices() (tenpy.networks.si	te.SpinHalfSite T	
method), 639	J	
	T()	(tenpy.linalg.sparse.FlatHermitianOperator prop-
method), 643	ks.site.SpinSite	erty), 332
· · · · · · · · · · · · · · · · · · ·		(tenpy.linalg.sparse.FlatLinearOperator property),
state_labels(tenpy.networks.site.Site		340
states (tenpy.networks.mpo.MPOGraph		e_slice() (tenpy.linalg.np_conserved.Array
stored_blocks() (tenpy.linalg.np_co		method), 278
property), 276		porary_level (class in tenpy.tools.optimization),
strength (tenpy.networks.terms.Terml		750
686		porary_level (<i>tenpy.tools.optimization.temporary_level</i>
subqshape (tenpy.linalg.charges.LegP	pe attribute),	attribute), 750
318	ten	
subshape (tenpy.linalg.charges.LegPipe		module, 189
subspace_expand()	ten	py.algorithms
(tenpy.algorithms.dmrg.SingleSi		module, 193
method), 236	ten	py.algorithms.algorithm
subspace_expand()	M	module, 194
(tenpy.algorithms.dmrg.TwoSite)	Aixer method), ten	py.algorithms.dmrg
238	. 742	module, 240
sum_of_exp() (in module tenpy.tools.fi		py.algorithms.exact_diag
1 1	ass in	module, 267
tenpy.linalg.sparse), 347		py.algorithms.mpo_evolution
suzuki_trotter_decomposition	()	module, 261
statio method) 257	<i>injication1EBD2</i> ten	py.algorithms.mps_common
static method), 257	/ \	module, 213
suzuki_trotter_decomposition (tenpy.algorithms.tebd.Engine s		py.algorithms.network_contractor
202	* *	module, 263
	ten	py.algorithms.purification
suzuki_trotter_time_steps()	wife ation TEDD 2	module, 261
(tenpy.algorithms.purification.P	<i>injication1EBD2</i> ten	
static method), 257		module, 248
suzuki_trotter_time_steps()	ten	py.algorithms.tebd
(tenpy.algorithms.tebd.Engine s	**	module, 204
202	ten	py.algorithms.truncation
svd() (in module tenpy.linalg.np_conser		module, 198
svd() (in module tenpy.linalg.svd_robust		py.linalg
svd_gesvd() (in module tenpy.linalg.sv		module, 269
svd_theta() (in module tenpy.algorith	ns.iruncanon), ten	py.linalg.charges
197		module, 324
sweep() (tenpy.algorithms.dmrg.E	ngineCombine ten	py.linalg.lanczos

module, 351	module, 699
tenpy.linalg.np_conserved	
	tenpy.tools
module, 302	module, 701
tenpy.linalg.random_matrix	tenpy.tools.events
module, 329	module, 725
tenpy.linalg.sparse	tenpy.tools.fit
module, 348	module,744
tenpy.linalg.svd_robust	tenpy.tools.hdf5_io
module, 326	module,714
tenpy.models	tenpy.tools.math
module, 353	module,740
tenpy.models.fermions_spinless	tenpy.tools.misc
module, 565	module,736
tenpy.models.haldane	tenpy.tools.optimization
module, 596	module, 753
tenpy.models.hofstadter	tenpy.tools.params
module, 595	module,720
tenpy.models.hubbard	tenpy.tools.process
module, 593	module, 748
tenpy.models.lattice	tenpy.tools.string
module, 476	module, 745
tenpy.models.model	tenpy.version
module, 506	module,755
tenpy.models.spins	tensordot() (in module tenpy.linalg.np_conserved),
module, 549	300
tenpy.models.spins_nnn	TermList (class in tenpy.networks.terms), 685
module, 550	terms (tenpy.networks.terms.TermList attribute), 685
tenpy.models.tf_ising	test_contractible()
module, 521	(tenpy.linalg.charges.LegCharge method),
tenpy.models.toric_code	312
module, 607	test_contractible()
tenpy.models.xxz_chain	(tenpy.linalg.charges.LegPipe method), 323
module, 535	test_equal() (tenpy.linalg.charges.LegCharge
tenpy.networks	method), 313
module, 609	test_equal() (tenpy.linalg.charges.LegPipe
tenpy.networks.mpo	method), 323
module, 669	test_sanity() (tenpy.linalg.charges.ChargeInfo
tenpy.networks.mps	method), 307
module, 658	test_sanity() (tenpy.linalg.charges.LegCharge
tenpy.networks.purification_mps	method), 312
module, 689	test_sanity() (tenpy.linalg.charges.LegPipe
tenpy.networks.site	method), 319
module, 648	test_sanity() (tenpy.linalg.np_conserved.Array
	method), 273
tenpy.networks.terms	
module, 688	test_sanity() (tenpy.models.fermions_spinless.FermionChain
tenpy.simulations	method), 564
module, 691	test_sanity() (tenpy.models.hubbard.BoseHubbardChain
tenpy.simulations.ground_state_search	method), 579
module, 698	test_sanity() (tenpy.models.hubbard.FermiHubbardChain
tenpy.simulations.measurement	method), 592
module, 697	test_sanity() (tenpy.models.lattice.Chain method),
tenpy.simulations.simulation	363
module, 693	test_sanity() (tenpy.models.lattice.HelicalLattice
tenpy.simulations.time evolution	method), 373

test_sanity() (tenpy.models.lattice.Honeycomb	(tenpy.algorithms.tdvp.Engine method), 244
method), 384	to_array() (in module tenpy.tools.misc), 734
<pre>test_sanity() (tenpy.models.lattice.IrregularLattice</pre>	to_Arrays() (tenpy.networks.terms.OnsiteTerms
method), 395	method), 683
test_sanity() (tenpy.models.lattice.Kagome	to_iterable() (in module tenpy.tools.misc), 734
method), 406	to_iterable_arrays() (in module
test_sanity() (tenpy.models.lattice.Ladder	tenpy.linalg.np_conserved), 301
method), 417	to_iterable_of_len() (in module
test_sanity() (tenpy.models.lattice.Lattice	tenpy.tools.misc), 734
method), 422	to_LegCharge() (tenpy.linalg.charges.LegPipe
test_sanity() (tenpy.models.lattice.SimpleLattice	method), 319
method), 440	to_mathematica_lists() (in module
test_sanity() (tenpy.models.lattice.Square	tenpy.tools.string), 744
- · · · · · · · · · · · · · · · · · · ·	
method), 451	to_matrix() (tenpy.algorithms.mps_common.EffectiveH
test_sanity() (tenpy.models.lattice.Triangular	method), 208
method), 462	to_matrix() (tenpy.algorithms.mps_common.OneSiteH
test_sanity() (tenpy.models.lattice.TrivialLattice	method), 210
method), 473	to_matrix() (tenpy.algorithms.mps_common.TwoSiteH
test_sanity() (tenpy.models.model.CouplingModel	method), 213
method), 480	$\verb"to_matrix"() (tenpy.algorithms.purification.PurificationTwoSiteUp) and the property of t$
test_sanity()(tenpy.models.model.MultiCouplingMo	del method), 260
method), 502	to_matrix() (tenpy.linalg.sparse.NpcLinearOperator
test_sanity() (tenpy.models.spins.SpinChain	method), 343
method), 549	<pre>to_matrix() (tenpy.linalg.sparse.NpcLinearOperatorWrapper</pre>
test_sanity() (tenpy.models.tf_ising.TFIChain	method), 344
method), 521	to_matrix() (tenpy.linalg.sparse.OrthogonalNpcLinearOperator
test_sanity()(tenpy.models.toric_code.DualSquare	method), 346
method), 607	to_matrix() (tenpy.linalg.sparse.ShiftNpcLinearOperator
test_sanity() (tenpy.models.xxz_chain.XXZChain2	method), 346
method), 535	to_matrix() (tenpy.linalg.sparse.SumNpcLinearOperator
test_sanity()(tenpy.networks.mpo.MPOEnvironment	
method), 662	to_matrix() (tenpy.networks.mps.TransferMatrix
test_sanity() (tenpy.networks.mpo.MPOGraph	method), 657
method), 666	
**	to_ndarray() (tenpy.linalg.np_conserved.Array
test_sanity() (tenpy.networks.mps.MPSEnvironment	method), 278
method), 651	to_nn_bond_Arrays()
test_sanity() (tenpy.networks.site.BosonSite	(tenpy.networks.terms.CouplingTerms method),
method), 615	673
test_sanity() (tenpy.networks.site.FermionSite	
method), 619	(tenpy. networks. terms. Multi Coupling Terms
test_sanity() (tenpy.networks.site.GroupedSite	method), 681
method), 624	to_OnsiteTerms_CouplingTerms()
<pre>test_sanity() (tenpy.networks.site.Site method),</pre>	(tenpy.networks.terms.TermList method),
627	686
<pre>test_sanity() (tenpy.networks.site.SpinHalfFermionS</pre>	iteo_OptimizationFlag() (in module
method), 634	tenpy.tools.optimization), 752
test_sanity() (tenpy.networks.site.SpinHalfSite	to_qdict() (tenpy.linalg.charges.LegCharge method),
method), 639	312
test_sanity() (tenpy.networks.site.SpinSite	to_qdict() (tenpy.linalg.charges.LegPipe method),
method), 643	324
TFIChain (class in tenpy.models.tf_ising), 510	to_qflat() (tenpy.linalg.charges.LegCharge method),
theta_svd_left_right()	312
(tenpy.algorithms.tdvp.Engine method), 244	to_qflat() (tenpy.linalg.charges.LegPipe method),
theta_svd_right_left()	324

<pre>to_TermList() (tenpy.networks.terms.CouplingTerms</pre>	tenpy.tools.hdf5_io), 716
to_TermList() (tenpy.networks.terms.ExponentiallyDemethod), 676	u_close_1() (in module tenpy.linalg.random_matrix),
to_TermList() (tenpy.networks.terms.MultiCouplingTemethod), 680	erms 329
to_TermList() (tenpy.networks.terms.OnsiteTerms method), 684	unary_blockwise() (tenpy.linalg.np_conserved.Array method), 286
trace() (in module tenpy.linalg.np_conserved), 301 TransferMatrix (class in tenpy.networks.mps), 655	unit_cell_(tenpy.models.lattice.Lattice attribute), 421 unit_cell_positions (tenpy.models.lattice.Lattice
transpose (tenpy.networks.mps.TransferMatrix attribute), 656	attribute), 421 unused_parameters() (in module
transpose() (tenpy.linalg.np_conserved.Array method), 285	tenpy.tools.params), 719 unwrapped () (tenpy.linalg.sparse.NpcLinearOperatorWrapper
transpose() (tenpy.linalg.sparse.FlatHermitianOperat method), 335	or method), 344 unwrapped () (tenpy.linalg.sparse.OrthogonalNpcLinearOperator
transpose() (tenpy.linalg.sparse.FlatLinearOperator method), 342	method), 346 unwrapped() (tenpy.linalg.sparse.ShiftNpcLinearOperator
<pre>transpose_list_list() (in module</pre>	method), 347 unwrapped() (tenpy.linalg.sparse.SumNpcLinearOperator
Triangular (class in tenpy.models.lattice), 453 trivial_like_NNModel()	method), 348
(tenpy.models.fermions_spinless.FermionChain method), 564	update() (tenpy.algorithms.purification.PurificationTEBD2 method), 255
trivial_like_NNModel() (tenpy.models.hubbard.BoseHubbardChain	update() (tenpy.algorithms.tebd.Engine method), 202 update_amplitude()
<pre>method), 579 trivial_like_NNModel()</pre>	(tenpy.algorithms.dmrg.DensityMatrixMixer method), 217
(tenpy.models.hubbard.FermiHubbardChain method), 592	<pre>update_amplitude() (tenpy.algorithms.dmrg.SingleSiteMixer method), 236</pre>
<pre>trivial_like_NNModel() (tenpy.models.model.NearestNeighborModel method), 505</pre>	update_amplitude() (tenpy.algorithms.dmrg.TwoSiteMixer method), 238
trivial_like_NNModel() (tenpy.models.spins.SpinChain method), 549	update_bond() (tenpy.algorithms.purification.PurificationTEBD2 method), 258
trivial_like_NNModel() (tenpy.models.tf_ising.TFIChain method),	<pre>update_bond() (tenpy.algorithms.tebd.Engine</pre>
521 trivial_like_NNModel()	(tenpy.algorithms.purification.PurificationTEBD2 method), 258
(tenpy.models.xxz_chain.XXZChain2 method), 535	update_bond_imag() (tenpy.algorithms.tebd.Engine method), 203
<pre>TrivialLattice (class in tenpy.models.lattice), 464 trunc_err_bonds()</pre>	update_imag() (tenpy.algorithms.purification.PurificationTEBD2 method). 258
(tenpy.algorithms.purification.PurificationTEBD property), 258	2update_imag() (tenpy.algorithms.tebd.Engine method), 203
trunc_err_bonds() (tenpy.algorithms.tebd.Engine property), 202	update_local() (tenpy.algorithms.dmrg.EngineCombine method), 226
TruncationError (class in tenpy.algorithms.truncation), 195	update_local()(tenpy.algorithms.dmrg.EngineFracture method), 234
TwoSiteH (class in tenpy.algorithms.mps_common), 211	update_local()(tenpy.algorithms.purification.PurificationApplyMF method), 251
TwoSiteMixer (class in tenpy.algorithms.dmrg), 237 TYPES_FOR_HDF5_DATASETS (in module	update_LP() (tenpy.algorithms.dmrg.EngineCombine method), 225

```
\verb"update_LP"()" (\textit{tenpy.algorithms.dmrg.EngineFracture} \quad Z
                  method), 234
                                                                                                                  zero_if_close() (in module tenpy.tools.misc), 735
update_new_psi() (tenpy.algorithms.purification.Purification_App_vmPOdule tenpy.linalg.np_conserved), 301
                  method), 251
                                                                                                                  zeros like()
                                                                                                                                                               (tenpy.linalg.np_conserved.Array
update_recursive() (in module tenpy.tools.misc),
                                                                                                                                    method), 276
                   735
update RP() (tenpy.algorithms.dmrg.EngineCombine
                  method), 226
update_RP() (tenpy.algorithms.dmrg.EngineFracture
                  method), 234
update_s_h0()
                                                   (tenpy.algorithms.tdvp.Engine
                  method), 244
\verb"update_step"()" (\textit{tenpy.algorithms.purification.PurificationTEBD2") and the property of t
                  method), 255
                                                   (tenpy.algorithms.tebd.Engine
update_step()
                   method), 203
update_theta_h1() (tenpy.algorithms.tdvp.Engine
                  method), 244
update_theta_h2() (tenpy.algorithms.tdvp.Engine
                   method), 244
use_cython() (in module tenpy.tools.optimization),
                   752
V
∨ (tenpy.algorithms.exact_diag.ExactDiag attribute), 265
valid_hdf5_path_component()
                                                                                   (in
                                                                                              module
                   tenpy.tools.hdf5_io), 714
valid_opname()
                                                   (tenpy.networks.site.BosonSite
                  method), 615
valid_opname()
                                               (tenpy.networks.site.FermionSite
                   method), 619
valid_opname()
                                              (tenpy.networks.site.GroupedSite
                  method), 624
valid_opname() (tenpy.networks.site.Site method),
valid_opname() (tenpy.networks.site.SpinHalfFermionSite
                  method), 635
valid_opname()
                                              (tenpy.networks.site.SpinHalfSite
                  method), 639
valid opname()
                                                      (tenpy.networks.site.SpinSite
                  method), 643
                                (tenpy.linalg.sparse.FlatLinearOperator
vec_label
                  attribute), 337
version (in module tenpy.version), 755
version_summary (in module tenpy.version), 755
vert_join() (in module tenpy.tools.string), 745
W
W (tenpy.algorithms.tdvp.H1_mixed attribute), 246
W0 (tenpy.algorithms.tdvp.H2_mixed attribute), 247
W1 (tenpy.algorithms.tdvp.H2_mixed attribute), 247
X
XXZChain2 (class in tenpy.models.xxz_chain), 524
```