# **TeNPy**

Release 0.7.0

**TeNPy Developers** 

# **USER GUIDE**

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TeNPy (short for 'Tensor Network Python') is a Python library for the simulation of strongly correlated quantum systems with tensor networks.

The philosophy of this library is to get a new balance of a good readability and usability for new-comers, and at the same time powerful algorithms and fast development of new algorithms for experts. For good readability, we include an extensive documentation next to the code, both in Python doc strings and separately as *user guides*, as well as simple example codes and even toy codes, which just demonstrate various algorithms (like TEBD and DMRG) in ~100 lines per file.

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**ONE** 

# **HOW DO I GET SET UP?**

If you have the *conda* package manager, you can install the latest released version of TeNPy with:

conda install --channel=conda-forge physics-tenpy

Further details and alternative methods can be found the file doc/INSTALL.rst. The latest version of the source code can be obtained from https://github.com/tenpy/tenpy.

**TWO** 

# HOW TO READ THE DOCUMENTATION

The **documentation is available online** at https://tenpy.readthedocs.io/. The documentation is roughly split in two parts: on one hand the full "reference" containing the documentation of all functions, classes, methods, etc., and on the other hand the "user guide" containing some introductions and additional explanations.

The documentation is based on Python's docstrings, and some additional \*.rst files located in the folder *doc/* of the repository. All documentation is formated as reStructuredText, which means it is quite readable in the source plain text, but can also be converted to other formats. If you like it simple, you can just use intective python *help()*, Python IDEs of your choice or jupyter notebooks, or just read the source. Moreover, the documentation gets converted into HTML using Sphinx, and is made available online at https://tenpy.readthedocs.io/. The big advantages of the (online) HTML documentation are a lot of cross-links between different functions, and even a search function. If you prefer yet another format, you can try to build the documentation yourself, as described in doc/contr/build\_doc.rst.

**THREE** 

# HELP - I LOOKED AT THE DOCUMENTATION, BUT I DON'T UNDERSTAND HOW ...?

We have set up a **community forum** at https://tenpy.johannes-hauschild.de/, where you can post questions and hopefully find answers. Once you got some experience with TeNPy, you might also be able to contribute to the community and answer some questions yourself;-) We also use this forum for official annoucements, for example when we release a new version.

# **FOUR**

# I FOUND A BUG

You might want to check the github issues, if someone else already reported the same problem. To report a new bug, just open a new issue on github. If you already know how to fix it, you can just create a pull request:) If you are not sure whether your problem is a bug or a feature, you can also ask for help in the TeNPy forum.

# **CITING TENPY**

When you use TeNPy for a work published in an academic journal, you can cite this paper to acknowledge the work put into the development of TeNPy. (The license of TeNPy does not force you, however.) For example, you could add the sentence "Calculations were performed using the TeNPy Library (version X.X. X)\cite{tenpy}." in the acknowledgements or in the main text.

The corresponding BibTex Entry would be the following (the  $\ullet$  requires  $\ullet$  usepackage {hyperref} in the LaTeX preamble.):

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# **ACKNOWLEDGMENT**

This work was funded by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, Materials Sciences and Engineering Division under Contract No. DE-AC02-05- CH11231 through the Scientific Discovery through Advanced Computing (SciDAC) program (KC23DAC Topological and Correlated Matter via Tensor Networks and Quantum Monte Carlo).

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### **LICENSE**

The code is licensed under GPL-v3.0 given in the file LICENSE of the repository, in the online documentation readable at https://tenpy.readthedocs.io/en/latest/install/license.html.

# 7.1 Installation instructions

With the [conda] package manager you can install python with:

```
conda install --channel=conda-forge physics-tenpy
```

More details and tricks in *Installation with conda from conda-forge*.

If you don't have conda, but you have [pip], you can:

```
pip install physics-tenpy
```

More details for this method can be found in *Installation from PyPi with pip*.

We also have a bunch of optional *Extra requirements*, which you don't have to install to use TeNPy, but you might want to.

The method with the minimal requirements is to just download the source and adjust the *PYTHONPATH*, as described in *Installation from source*. This is also the recommended way if you plan to modify parts of the source.

# 7.1.1 Installation with conda from conda-forge

We provide a package for the [conda] package manager in the *conda-forge* channel, so you can install TeNPy as:

```
conda install --channel=conda-forge physics-tenpy
```

Following the recommondation of conda-forge, you can also make conda-forge the default channel as follows:

```
conda config --add channels conda-forge conda config --set channel_priority strict
```

If you have done this, you don't need to specify the --channel=conda-forge explicitly.

Moreover, it is actually recommended to create a separate environment. To create a conda environment with the name *tenpy*, where the TeNPy package (called *physics-tenpy*) is installed:

```
conda create --name tenpy --channel=conda-forge physics-tenpy
```

In that case, you need to activate the environment each time you want to use the package with:

```
conda activate tenpy
```

The big advantage of this approach is that it allows multiple version of software to be installed in parallel, e.g., if one of your projects requires python>=3.8 and another one requires an old library which doesn't support that. Further info can be found in the conda documentation.

# 7.1.2 Installation from PyPi with pip

#### Preparation: install requirements

If you have the [conda] package manager from anaconda, you can just download the environment.yml file out of the repository and create a new environment (called tenpy, if you don't speficy another name) for TeNPy with all the required packages:

```
conda env create -f environment.yml conda activate tenpy
```

Further information on conda environments can be found in the conda documentation. Note that installing conda also installs a version of [pip].

Alternatively, if you only have [pip] (and not [conda]), install the required packages with the following command (after downloading the requirements.txt file from the repository):

```
pip install -r requirements.txt
```

Note: Make sure that the pip you call corresponds to the python version you want to use. (One way to ensure this is to use python -m pip instead of a simple pip.) Also, you might need to use the argument --user to install the packages to your home directory, if you don't have sudo rights. (Using --user with conda's pip is discouraged, though.)

**Warning:** It might just be a temporary problem, but I found that the *pip* version of numpy is incompatible with the python distribution of anaconda. If you have installed the intelpython or anaconda distribution, use the *conda* packagemanager instead of *pip* for updating the packages whenever possible!

#### Installing the latest stable TeNPy package

Now we are ready to install TeNPy. It should be as easy as (note the different package name - 'tenpy' was taken!)

```
pip install physics-tenpy
```

**Note:** If the installation fails, don't give up yet. In the minimal version, tenpy requires only pure Python with somewhat up-to-date NumPy and SciPy. See *Installation from source*.

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#### Installation of the latest version from Github

To get the latest development version from the github master branch, you can use:

```
pip install git+git://github.com/tenpy/tenpy.git
```

This should already have the lastest features described in /changelog/latest. Disclaimer: this might sometimes be broken, although we do our best to keep to keep it stable as well.

#### Installation from the downloaded source folder

Finally, if you downloaded the source and want to **modify parts of the source**, You can also install TeNPy with in development version with --editable:

```
cd $HOME/tenpy # after downloading the source, got to the repository pip install --editable .
```

#### Uninstalling a pip-installed version

In all of the above cases, you can uninstall tenpy with:

```
pip uninstall physics-tenpy
```

# 7.1.3 Updating to a new version

**Before** you update, take a look at the *Release Notes*, which lists the changes, fixes, and new stuff. Most importantly, it has a section on *backwards incompatible changes* (i.e., changes which may break your existing code) along with information how to fix it. Of course, we try to avoid introducing such incompatible changes, but sometimes, there's no way around them. If you skip some intermediate version(s) for the update, read also the release notes of those!

How to update depends a little bit on the way you installed TeNPy. Of course, you have always the option to just remove the TeNPy files (possibly with a pip uninstall physics-tenpy), and to start over with downloading and installing the newest version.

#### When installed with pip

When you installed TeNPy with [pip], you just need to do a

```
pip install --upgrade physics-tenpy
```

#### When installed from source

If you used git clone ... to download the repository, you can update to the newest version using [git]. First, briefly check that you didn't change anything you need to keep with git status. Then, do a git pull to download (and possibly merge) the newest commit from the repository.

**Note:** If some Cython file (ending in .pyx) got renamed/removed (e.g., when updating from v0.3.0 to v0.4.0), you first need to remove the corresponding binary files. You can do so with the command bash cleanup.sh.

Furthermore, whenever one of the cython files (ending in .pyx) changed, you need to re-compile it. To do that, simply call the command bash ./compile again. If you are unsure whether a cython file changed, compiling again doesn't hurt.

To summarize, you need to execute the following bash commands in the repository:

```
# 0) make a backup of the whole folder
git status # check the output whether you modified some files
git pull
bash ./cleanup.sh # (confirm with 'y')
bash ./compile.sh
```

#### 7.1.4 Installation from source

#### **Minimal Requirements**

This code works with a minimal requirement of pure Python>=3.5 and somewhat recent versions of NumPy and SciPy.

#### **Getting the source**

The following instructions are for (some kind of) Linux, and tested on Ubuntu. However, the code itself should work on other operating systems as well (in particular MacOS and Windows).

The offical repository is at https://github.com/tenpy/tenpy.git. To get the latest version of the code, you can clone it with [git] using the following commands:

```
git clone https://github.com/tenpy/tenpy.git $HOME/TeNPy
cd $HOME/TeNPy
```

**Note:** Adjust \$HOME/TeNPy to the path wherever you want to save the library.

Optionally, if you don't want to contribute, you can checkout the latest stable release:

```
git tag \# this prints the available version tags git checkout v0.3.0 \# or whatever is the lastest stable version
```

**Note:** In case you don't have [git] installed, you can download the repository as a ZIP archive. You can find it under releases, or the latest development version.

#### Minimal installation: Including tenpy into PYTHONPATH

The python source is in the directory *tenpy/* of the repository. This folder *tenpy/* should be placed in (one of the folders of) the environment variable PYTHONPATH. On Linux, you can simply do this with the following line in the terminal:

```
export PYTHONPATH=$HOME/TeNPy
```

(If you have already a path in this variable, separate the paths with a colon :.) However, if you enter this in the terminal, it will only be temporary for the terminal session where you entered it. To make it permanently, you can add

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the above line to the file \$HOME/.bashrc. You might need to restart the terminal session or need to relogin to force a reload of the ~/.bashrc.

Whenever the path is set, you should be able to use the library from within python:

tenpy.show\_config() prints the current version of the used TeNPy library as well as the versions of the used python, numpy and scipy libraries, which might be different on your computer. It is a good idea to save this data (given as string in tenpy.version.version\_summary along with your data to allow to reproduce your results exactly.

If you got a similar output as above: congratulations! You can now run the codes:)

#### Compilation of np conserved

At the heart of the TeNPy library is the module tenpy.linalg.np\_conseved, which provides an Array class to exploit the conservation of abelian charges. The data model of python is not ideal for the required book-keeping, thus we have implemented the same np\_conserved module in Cython. This allows to compile (and thereby optimize) the corresponding python module, thereby speeding up the execution of the code. While this might give a significant speed-up for code with small matrix dimensions, don't expect the same speed-up in cases where most of the CPU-time is already spent in matrix multiplications (i.e. if the bond dimension of your MPS is huge).

To compile the code, you first need to install Cython

```
conda install cython # when using anaconda, or pip install --upgrade Cython # when using pip
```

Moreover, you need a C++ compiler. For example, on Ubuntu you can install sudo apt-get install build\_essential, or on Windows you can download MS Visual Studio 2015. If you use anaconda, you can also use one conda install -c conda-forge cxx-compiler.

After that, go to the root directory of TeNPy (\$HOME/TeNPy) and simply run

```
bash ./compile.sh
```

Note that it is not required to separately download (and install) Intel MKL: the compilation just obtains the includes from numpy. In other words, if your current numpy version uses MKL (as the one provided by anaconda), the compiled TeNPy code will also use it.

After a successful compilation, the warning that TeNPy was not compiled should go away:

```
>>> import tenpy
>>> tenpy.show_config()
tenpy 0.4.0.dev0+b60bad3 (compiled from git rev._

$\infty$ b60bad3243b7e54f549f4f7c1f074dc55bb54ba3),
git revision b60bad3243b7e54f549f4f7c1f074dc55bb54ba3 using
python 3.7.3 (default, Mar 27 2019, 22:11:17)
```

```
[GCC 7.3.0]
numpy 1.16.3, scipy 1.2.1
```

**Note:** For further optimization options, look at tenpy.tools.optimization.

## 7.1.5 Extra requirements

We have some extra requirements that you don't need to install to use TeNPy, but that you might find usefull to work with. TeNPy does not import the following libraries (at least not globally), but some functions might expect arguments behaving like objects from these libraries.

Note: If you created a [conda] environment with conda env create -f environment.yml, all the extra requirements below should already be installed:) (However, a pip install -r requirements.txt does not install them.)

#### **Matplotlib**

The first extra requirement is the [matplotlib] plotting library. Some functions expect a matplotlib.axes.Axes instance as argument to plot some data for visualization.

#### Intel's Math Kernel Library (MKL)

If you want to run larger simulations, we recommend the use of Intel's MKL. It ships with a Lapack library, and uses optimization for Intel CPUs. Moreover, it uses parallelization of the LAPACK/BLAS routines, which makes execution much faster. As of now, the library itself supports no other way of parallelization.

If you don't have a python version which is built against MKL, we recommend using [conda] or directly intelpython. Conda has the advantage that it allows to use different environments for different projects. Both are available for Linux, Mac and Windows; note that you don't even need administrator rights to install it on linux. Simply follow the (straight-forward) instructions of the web page for the installation. After a successfull installation, if you run python interactively, the first output line should state the python version and contain Anaconda or Intel Corporation, respectively.

If you have a working conda package manager, you can install the numpy build against mkl with:

```
conda install mkl numpy scipy
```

**Note:** MKL uses different threads to parallelize various BLAS and LAPACK routines. If you run the code on a cluster, make sure that you specify the number of used cores/threads correctly. By default, MKL uses all the available CPUs, which might be in stark contrast than what you required from the cluster. The easiest way to set the used threads is using the environment variable *MKL\_NUM\_THREADS* (or *OMP\_NUM\_THREADS*). For a dynamic change of the used threads, you might want to look at *process*.

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#### **HDF5** file format support

We support exporting data to files in the [HDF5] format through the python interface of the h5py <https://docs.h5py.org/en/stable/> package, see Saving to disk: input/output for more information. However, that requires the installation of the HDF5 library and h5py.

#### YAML parameter files

The tenpy.tools.params.Config class supports reading and writing YAML files, which requires the package pyyaml; pip install pyyaml.

#### **Tests**

To run the tests, you need to install pytest, which you can for example do with pip install pytest. For information how to run the tests, see *Checking the installation*.

## 7.1.6 Checking the installation

The first check of whether tenpy is installed successfully, is to try to import it from within python:

```
>>> import tenpy
```

**Note:** If this raises a warning Couldn't load compiled cython code. Code will run a bit slower., something went wrong with the compilation of the Cython parts (or you didn't compile at all). While the code might run slower, the results should still be the same.

The function <code>tenpy.show\_config()</code> prints information about the used versions of tenpy, numpy and scipy, as well on the fact whether the Cython parts were compiled and could be imported.

As a further check of the installation you can try to run (one of) the python files in the *examples*/ subfolder; hopefully all of them should run without error.

You can also run the automated testsuite with pytest to make sure everything works fine. If you have pytest installed, you can go to the *tests* folder of the repository, and run the tests with:

```
cd tests
pytest
```

In case of errors or failures it gives a detailed traceback and possibly some output of the test. At least the stable releases should run these tests without any failures.

If you can run the examples but not the tests, check whether *pytest* actually uses the correct python version.

The test suite is also run automatically by github actions and with travis-ci, results can be inspected here.

### 7.1.7 TeNPy developer team

The following people are part of the TeNPy developer team. The full list of contributors can be obtained from the git repository with ``git\_  $\rightarrow$  shortlog -sn``.

Johannes Hauschild
Frank Pollmann
Michael P. Zaletel
Maximilian Schulz
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Samuel Scalet
Markus Drescher
Wilhelm Kadow
Gunnar Moeller
Jakob Unfried
Yu-Chin Tzeng

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Further, the code is based on an earlier version of the library, mainly developed by Frank Pollmann, Michael P. Zaletel and Roger S. K. Mong.

#### 7.1.8 License

The source code documented here is published under a GPL v3 license, which we include below.

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# 7.2 Release Notes

The project adheres semantic versioning.

All notable changes to the project should be documented in the changelog. The most important things should be summarized in the release notes.

The changes in /changelog/latest are implemented in the latest development version on github, but not yet released.

Changes compared to previous TeNPy highlights the most important changes compared to the other, previously developed (closed source) TeNPy version.

## 7.2.1 [0.6.1] - 2020-05-18

## **Release Notes**

This only is a follow-up release to [0.6.0] - 2020-05-16. It fixes a small bug in the examples/c\_tebd.py and some roundoff problems in the tests.

It is now possible to install TeNPy with the conda package manager:

conda install --channel=conda-forge physics-tenpy

# 7.2.2 [0.6.0] - 2020-05-16

#### **Release Notes**

This release contains a major update of the documentation, which is now hosted by "Read the Docs" at https://tenpy.readthedocs.io/. Update your bookmark:-)

Apart from that, this release introduces a format how to save and load data (in particular TeNPy classes) to HDF5 files. See *Saving to disk: input/output* for more details. To use that feature, you need to **install** the h5py package (and therefore some version of the HDF5 library). This is easy with anaconda, conda install h5py, but might be

cumbersome on your local computing cluster. (However, many university computing clusters have some version of HDF5 installed already. Check with your local sysadmin.)

Moreover, we changed how we read out parameter dictionaries - instead of the <code>get\_parameter()</code> function, we have now a <code>Config</code> class which behaves like a dictionary, you can simpy use <code>options.get(key, default)</code> for model parameters - as you would do for a python dictionary.

### Changelog

### **Backwards incompatible changes**

- Created a class Config to replace Python-native parameter dictionaries and add some useful functionality. Old code using <code>tenpy.tools.params.get\_parameter()</code> and <code>tenpy.tools.params.unused\_parameters()</code> still works as before, but raises a warning, and should be replaced. For example, if you defined your own models, you should replace calls <code>get\_parameter(model\_params, "key", "default\_value", "ModelName")</code> with <code>model\_params.get("key", "default\_value")</code>, the latter syntax being what you would use for a normal python dictionary as well.
- Renamed the following class parameter dictionaries to simply *options* for more consitency. Old code using the class attributes should still work (since we provide property aliases), but raises warnings. Note that this affects also derived classes (for example the TwoSiteDMRGEngine).
  - tenpy.algorithms.dmrg.DMRGEngine.DMRG\_params (was already renamed to engine\_params in versin 0.5.0)
  - tenpy.algorithms.mps\_common.Sweep.engine\_params
  - tenpy.algorithms.tebd.Engine.TEBD\_params
  - tenpy.algorithms.tdvp.Engine.TDVP\_params
  - tenpy.linalg.lanczos.Lanczos
- Changed the arguments of tenpy.models.model.MultiCouplingModel(): We replaced the three arguments u0, op0 and other\_op with other\_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] by single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]\*lat.dim. Note the changed order inside the tuple! Old code (which specifies opstr and category as keyword argument, if at all) still works as before, but raises a warning, and should be replaced. Since tenpy.lattice.Lattice.possible\_multi\_couplings() used similar arguments, they were changed as well.
- Don't save *H\_MPO\_graph* as model attribute anymore this also wasn't documented.
- Renamed the truncation parameter  $symmetry\_tol$  to  $degeneracy\_tol$  and make the criterion more reasonable by not checking  $log(S_i/S_j) < log(symmetry_tol)$ , but simply  $log(S_i/S_j) < degeneracy_tol$ . The latter makes more sense, as it is equivalent to  $(S_i S_j)/S_j < exp(degeneracy_tol) 1 = degeneracy_tol + \mathcal{O}(degeneracy_tol^2)$ .
- Deprecated tenpy.networks.mps.MPS.increase\_L() in favor of the newly added tenpy. networks.mps.MPS.enlarge\_mps\_unit\_cell() (taking factor instead of new\_L=factor\*L as argument).
- tenpy.networks.mps.MPS.correlation\_function() now auto-determines whether a Jordan-Wigner string is necessary. If any of the given operators is directly an npc Array, it will now raise an error; set autoJW=False in that case.
- Instead of "monkey-patching" matvec of the tenpy.algorithms.mps\_common.EffectiveH for the case that ortho\_to\_envs is not empty, we defined a proper class NpcLinearOperatorWrapper, which

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- serves as baseclass for <code>OrthogonalNpcLinearOperator</code>. The argument <code>ortho\_to\_envs</code> has been removed from <code>EffectiveH</code>.
- Switch order of the sites in the unit cell for the DualSquare, and redefine what the "default" order means.
   This is a huge optimization of DMRG, reducing the necessary MPS bond dimension for the ground state to the optimal 2<sup>L-1</sup> on each bond.
- Deprecated the Lanczos funciton/class argument *orthogonal\_to* of in LanczosGroundState. Instead, one can use the <code>OrthogonalNpcLinearOperator</code>.
- Deprecation warning for changing the default argument of *shift\_ket* for non-zero *shift\_bra* of the *TransferMatrix*.

#### Added

- tenpy.networks.mpo.MPO.variance() to calculate the variance of an MPO against a finite MPS.
- Classmethod tenpy.networks.MPS.from\_lat\_product\_state() to initialize an MPS from a product state given in lattice coordinates (independent of the *order* of the lattice).
- argument plus\_hc for tenpy.models.model.CouplingModel.add\_onsite(), tenpy.models.model.CouplingModel.add\_coupling(), and tenpy.models.model.MultiCouplingModel.add\_multi\_coupling() to simplify adding the hermitian conjugate terms.
- parameter *explicit\_plus\_hc* for *MPOMode1*, *CouplingMode1* and MPO, to reduce MPO bond dimension by not storing Hermitian conjugate terms, but computing them at runtime.
- tenpy.models.model.CouplingModel.add\_local\_term() for adding a single term to the lattice, and still handling Jordan-Wigner strings etc.
- tenpy.networks.site.Site.get\_hc\_opname() and hc\_ops to allow getting the hermitian conjugate operator (name) of the onsite operators.
- tenpy.tools.hdf5\_io with convenience functions for import and output with pickle, as well as an implementation allowing to save and load objects to HDF5 files in the format specified in Saving to disk: input/output.
- human-readable *boundary\_conditions* property in *Lattice*.
- save\_hdf5 and load\_hdf5 methods to support saving/loading to HDF5 for the following classes (and their sub-classes): ChargeInfo LegCharge LegPipe Array MPS MPO Lattice
- tenpy.networks.mps.MPSEnvironment.get\_initialization\_data() for a convenient way of saving the necessary parts of the environment after an DMRG run.
- Method enlarge\_mps\_unit\_cell for the following classes: MPS MPO Lattice Model, MPOModel, NearestNeighborModel
- tenpy.tools.misc.to\_iterable\_of\_len() for convenience of handling arguments.
- tenpy.models.lattice.Lattice.mps2lat\_values\_masked() as generalization of tenpy. models.lattice.Lattice.mps2lat\_values().
- tenpy.linalg.sparse.OrthogonalNpcLinearOperator to orthogonalize against vectors.
- tenpy.linalg.sparse.ShiftNpcLinearOperator to add a constant.
- tenpy.linalg.sparse.SumNpcLinearOperator which serves e.g. to add the h.c. during the *matvec* (in combination with the new tenpy.linalg.sparse.NpcLinearOperator.adjoint()).
- tenpy.algorithms.mps\_common.make\_eff\_H() to simplify implementations of prepare\_update().

- attribute options for the Model.
- tenpy.networks.mps.MPS.roll\_mps\_unit\_cell().

#### Changed

- DEFAULT DMRG paramter 'diag\_method' from 'lanczos' to 'default', which is the same for large bond dimensions, but performs a full exact diagonalization if the effective Hamiltonian has small dimensions. The threshold introduced is the new DMRG parameter 'max\_N\_for\_ED'.
- DEFAULT parameter charge\_sector=None instead of charge\_sector=0 in tenpy.networks. mps.MPS.overlap() to look for eigenvalues of the transfer matrix in *all* charge sectors, and not assume that it's the 0 sector.
- Derive the following classes (and their subclasses) from the new Hdf5Exportable to support saving to HDF5: Site Terms OnsiteTerms CouplingTerms Model, i.e., all model classes.
- Instead of just defining to\_matrix and adjoint for EffectiveH, define the interface directly for NpcLinearOperator.
- Try to keep the charge block structure as far as possible for add\_charge () and drop\_charge ()

#### **Fixed**

- Adjust the default DMRG parameter min\_sweeps if chi\_list is set.
- Avoid some unnecessary transpositions in MPO environments for MPS sweeps (e.g. in DMRG).
- sort (bunch=True) could return un-bunched Array, but still set the bunched flag.
- LegPipe did not initialize self. bunched correctly.
- issue #98: Error of calling *psi.canonical\_form()* directly after disabling the DMRG mixer.
- svd() with full\_matrices=True gave wrong charges.
- tenpy.linalg.np\_conserved.Array.drop\_charge() and tenpy.lina.np\_conserved. Array.drop\_charge() did not copy over labels.
- wrong pairs for the *fifth\_nearest\_neighbors* of the *Honeycomb*.
- Continue in tenpy.algorithms.dmrg.full\_diag\_effH() with a warning instaed of raising an Error, if the effective Hamltonian is zero.
- correlation\_length(): check for hermitian Flag might have raised and Error with new numpy warnings
- correlation\_function() did not respect argument str\_on\_first=False.
- tenpy.networks.mps.MPS.get\_op() worked unexpected for infinite bc with incomensurate self.L and len(op\_list).
- tenpy.networks.mps.MPS.permute\_sites() did modify the given perm.
- issue #105 Unintended side-effects using lanczos\_params.verbose in combination with orthogonal\_to
- issue #108 tenpy.linalg.sparse.FlatLinearOperator.\_matvec() changes self.\_charge\_sector

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# 7.2.3 [0.5.0] - 2019-12-18

## **Backwards incompatible changes**

- Major rewriting of the DMRG Engines, see issue #39 and issue #85 for details. The <code>EngineCombine</code> and <code>EngineFracture</code> have been combined into a single <code>TwoSiteDMRGEngine</code> with an The run function works as before. In case you have directly used the <code>EngineCombine</code> or <code>EngineFracture</code>, you should update your code and use the <code>TwoSiteEngine</code> instead.
- Moved init LP and init RP method from MPS into MPSEnvironment and MPOEnvironment.

## Changed

- Addition/subtraction of Array: check whether the both arrays have the same labels in differnt order, and in that case raise a warning that we will transpose in the future.
- Made tenpy.linalg.np\_conserved.Array.get\_block() public (previously tenpy.linalg.np\_conserved.Array.\_get\_block).
- groundstate() now returns a tuple (E0, psi0) instead of just psi0. Moreover, the argument charge\_sector was added.
- Simplification in the Lattice: Instead of having separate arguments/attributes/functions for 'nearest\_neighbors', 'next\_nearest\_neighbors', 'next\_next\_nearest\_neighbors' (Honeycomb) and possibly 'fourth\_nearest\_neighbors', 'fifth\_nearest\_neighbors', collect them in a dictionary called *pairs*. Old call structures still allowed, but deprecated.
- issue #94: Array addition and *inner()* should reflect the order of the labels, if they coincided. Will change the default behaviour in the future, raising *FutureWarning* for now.
- Default parameter for DMRG params: increased precision by setting *P\_tol\_min* down to the maximum of 1.e-30, lanczos\_params['svd\_min']\*\*2 \* P\_tol\_to\_trunc, lanczos\_params['trunc\_cut']\*\*2 \* P\_tol\_to\_trunc by default.

#### **Added**

- tenpy.algorithms.mps\_common with the Sweep class and EffectiveH to be a OneSiteH or TwoSiteH.
- Single-Site DMRG with the SingleSiteDMRG.
- Example function in examples/c\_tebd.py how to run TEBD with a model originally having next-nearest neighbors.
- increase\_L() to allow increasing the unit cell of an MPS.
- Additional option order='folded' for the Chain.
- tenpy.algorithms.exact\_diag.ExactDiag.from\_H\_mpo() wrapper as replacement for tenpy.networks.mpo.MPO.get\_full\_hamiltonian() and tenpy.networks.mpo.MPO.get\_grouped\_mpo(). The latter are now deprecated.
- Argument *max\_size* to limit the matrix dimension in *ExactDiag*.
- tenpy.linalg.sparse.FlatLinearOperator.from\_guess\_with\_pipe() to allow quickly converting matvec functions acting on multi-dimensional arrays to a FlatLinearOperator by combining the legs into a LegPipe.

- tenpy.tools.math.speigsh() for hermitian variant of speigs()
- Allow for arguments 'LA', 'SA' in argsort ().
- tenpy.linalg.lanczos.lanczos\_arpack() as possiple replacement of the self-implemented lanc-zos function.
- tenpy.algorithms.dmrg.full\_diag\_effH() as another replacement of lanczos().
- The new DMRG parameter 'diag\_method' allows to select a method for the diagonalization of the effective Hamiltonian. See tenpy.algorithms.dmrg.DMRGEngine.diag() for details.
- dtype attribute in *EffectiveH*.
- tenpy.linalg.charges.LegCharge.get\_qindex\_of\_charges() to allow selecting a block of an Array from the charges.
- tenpy.algorithms.mps\_common.EffectiveH.to\_matrix to allow contracting an EffectiveH to a matrix, as well as metadata tenpy.linalg.sparse.NpcLinearOperator.acts\_on and tenpy.algorithms.mps\_common.EffectiveH.N.
- argument only\_physical\_legs in tenpy.networks.mps.MPS.get\_total\_charge()

#### **Fixed**

- MPO expectation\_value() did not work for finite systems.
- Calling compute\_K() repeatedly with default parameters but on states with different *chi* would use the *chi* of the very first call for the truncation parameters.
- allow MPSEnvironment and MPOEnvironment to have MPS/MPO with different length
- group\_sites() didn't work correctly in some situations.
- matvec to array() returned the transposed of A.
- tenpy.networks.mps.MPS.from\_full() messed up the form of the first array.
- issue #95: blowup of errors in DMRG with *update\_env > 0*. Turns out to be a problem in the precision of the truncation error: *TruncationError.eps* was set to 0 if it would be smaller than machine precision. To fix it, I added from\_S().

## 7.2.4 [0.4.1] - 2019-08-14

# **Backwards incompatible changes**

- Switch the sign of the BoseHubbardModel and FermiHubbardModel to hopping and chemical potential having negative prefactors. Of course, the same adjustment happens in the BoseHubbardChain and FermiHubbardChain.
- moved BoseHubbardModel and BoseHubbardChain as well as FermiHubbardModel and FermiHubbardChain into the new module tenpy.models.hubbard.
- Change arguments of coupling\_term\_handle\_JW() and multi\_coupling\_term\_handle\_JW() to use strength and sites instead of op\_needs\_JW.
- Only accept valid identifiers as operator names in add\_op().

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## Changed

- grid\_concat () allows for None entries (representing zero blocks).
- from\_full() allows for 'segment' boundary conditions.
- apply\_local\_op() allows for n-site operators.

#### Added

- *max\_range* attribute in MPO and *MPOGraph*.
- is\_hermitian()
- Nearest-neighbor interaction in BoseHubbardModel
- multiply\_op\_names() to replace ' '.join(op\_names) and allow explicit compression/multiplication.
- order\_combine\_term() to group operators together.
- dagger () of MPO's (and to implement that also flip\_charges\_qconj()).
- has\_label() to check if a label exists
- qr\_li() and rq\_li()
- · Addition of MPOs
- 3 additional examples for chern insulators in examples/chern\_insulators/.
- FermionicHaldaneModel and BosonicHaldaneModel.
- from\_MPOModel() for initializing nearest-neighbor models after grouping sites.

### **Fixed**

- issue #36: long-range couplings could give IndexError.
- issue #42: Onsite-terms in Fermi Hubbard Model were wrong for lattices with non-trivial unit cell.
- Missing a factor 0.5 in GUE ().
- Allow TermList to have terms with multiple operators acting on the same site.
- Allow MPS indices outside unit cell in mps2lat\_idx() and lat2mps\_idx().
- expectation\_value() did not work for n-site operators.

# 7.2.5 [0.4.0] - 2019-04-28

## **Backwards incompatible changes**

- The argument order of tenpy.models.lattice.Lattice could be a tuple (priority, snake\_winding) before. This is no longer valid and needs to be replaced by ("standard", snake\_winding, priority).
- Moved the boundary conditions  $bc\_coupling$  from the tenpy.models.model.CouplingModel into the tenpy.models.lattice.Lattice (as bc). Using the parameter  $bc\_coupling$  will raise a FutureWarning, one should set the boundary conditions directly in the lattice.

- Added parameter permute (True by default) in tenpy.networks.mps.MPS.from\_product\_state() and tenpy.networks.mps.MPS.from\_Bflat(). The resulting state will therefore be independent of the "conserve" parameter of the Sites unlike before, where the meaning of the p\_state argument might have changed.
- Generalize and rename tenpy.networks.site.DoubleSite to tenpy.networks.site. GroupedSite, to allow for an arbitrary number of sites to be grouped. Arguments site0, site1, label0, label1 of the \_\_init\_\_ can be replaced with [site0, site1], [label0, label1] and op0, op1 of the kronecker\_product with [op0, op1]; this will recover the functionality of the DoubleSite.
- Restructured callstructure of Mixer in DMRG, allowing an implementation of other mixers. To enable the mixer, set the DMRG parameter "mixer" to True or 'DensityMatrixMixer' instead of just 'Mixer'.
- The interaction parameter in the tenpy.models.bose\_hubbbard\_chain.BoseHubbardModel (and tenpy.models.bose\_hubbbard\_chain.BoseHubbardChain) did not correspond to U/2N(N-1) as claimed in the Hamiltonian, but to  $UN^2$ . The correcting factor 1/2 and change in the chemical potential have been fixed.
- Major restructuring of tenpy.linalg.np\_conserved and tenpy.linalg.charges. This should not break backwards-compatibility, but if you compiled the cython files, you **need** to remove the old binaries in the source directory. Using bash cleanup.sh might be helpful to do that, but also remove other files within the repository, so be careful and make a backup beforehand to be on the save side. Afterwards recompile with bash compile.sh.
- Changed structure of tenpy.models.model.CouplingModel.onsite\_terms and tenpy. models.model.CouplingModel.coupling\_terms: Each of them is now a dictionary with category strings as keys and the newly introduced tenpy.networks.terms.OnsiteTerms and tenpy.networks.terms.CouplingTerms as values.
- tenpy.models.model.CouplingModel.calc\_H\_onsite() is deprecated in favor of new methods.
- Argument raise\_op2\_left of tenpy.models.model.CouplingModel.add\_coupling() is deprecated.

## Added

- tenpy.networks.mps.MPS.canonical\_form\_infinite().
- tenpy.networks.mps.MPS.expectation\_value\_term(), tenpy.networks.mps.MPS.expectation\_value\_terms\_sum() and tenpy.networks.mps.MPS.expectation\_value\_multi\_sites() for expectation values of terms.
- tenpy.networks.mpo.MPO.expectation\_value() for an MPO.
- tenpy.linalg.np\_conserved.Array.extend() and tenpy.linalg.charges.LegCharge. extend(), allowing to extend an Array with zeros.
- DMRG parameter 'orthogonal\_to' allows to calculate excited states for finite systems.
- possibility to change the number of charges after creating LegCharges/Arrays.
- more general way to specify the order of sites in a tenpy.models.lattice.Lattice.
- new tenpy.models.lattice.Triangular, tenpy.models.lattice.Honeycomb and tenpy. models.lattice.Kagome lattice
- a way to specify nearest neighbor couplings in a Lattice, along with methods to count the number of nearest neighbors for sites in the bulk, and a way to plot them (plot\_coupling() and friends)
- tenpy.networks.mpo.MPO.from\_grids() to generate the MPO from a grid.
- tenpy.models.model.MultiCouplingModel for couplings involving more than 2 sites.

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- request #8: Allow shift in boundary conditions of CouplingModel.
- Allow to use state labels in tenpy.networks.mps.MPS.from\_product\_state().
- tenpy.models.model.CouplingMPOModel structuring the default initialization of most models.
- Allow to force periodic boundary conditions for finite MPS in the CouplingMPOModel. This is not recommended, though.
- tenpy.models.model.NearestNeighborModel.calc\_H\_MPO\_from\_bond() and tenpy. models.model.MPOModel.calc\_H\_bond\_from\_MPO() for conversion of H\_bond into H\_MPO and vice versa.
- tenpy.algorithms.tebd.RandomUnitaryEvolution for random unitary circuits
- Allow documentation links to github issues, arXiv, papers by doi and the forum with e.g. :issue:`5`, :arxiv:`1805.00055`, :doi:`10.21468/SciPostPhysLectNotes.5`, :forum:`3`
- tenpy.models.model.CouplingModel.coupling\_strength\_add\_ext\_flux() for adding hoppings with external flux.
- tenpy.models.model.CouplingModel.plot\_coupling\_terms() to visualize the added coupling terms.
- tenpy.networks.terms.OnsiteTerms, tenpy.networks.terms.CouplingTerms, tenpy.networks.terms.MultiCouplingTerm containing the of terms for the CouplingModel and MultiCouplingModel. This allowed to add the category argument to add\_onsite, add\_coupling and add\_multi\_coupling.
- tenpy.networks.terms.TermList as another (more human readable) representation of terms with conversion from and to the other \*Term classes.
- tenpy.networks.mps.MPS.init\_LP() and tenpy.networks.mps.MPS.init\_RP() to initialize left and right parts of an Environment.
- tenpy.networks.mpo.MPOGraph.from\_terms() and tenpy.networks.mpo.MPOGraph.from\_term\_list().
- argument charge\_sector in tenpy.networks.mps.MPS.correlation\_length().

## Changed

- moved toycodes from the folder examples / to a new folder toycodes / to separate them clearly.
- major remodelling of the internals of tenpy.linalq.np\_conserved and tenpy.linalq.charges.
  - Introduced the new module tenpy/linalg/\_npc\_helper.pyx which contains all the Cython code, and gets imported by
  - Array now rejects addition/subtraction with other types
  - Array now rejects multiplication/division with non-scalar types
  - By default, make deep copies of npc Arrays.
- Restructured lanczos into a class, added time evolution calculating exp (A\*dt) |psi0>
- Warning for poorly conditioned Lanczos; to overcome this enable the new parameter reortho.
- Simplified call strucutre of extend(), and extend().
- Restructured tenpy.algorithms.dmrg:

- run() is now just a wrapper around the new run(), run(psi, model, pars) is roughly equivalent to eng = EngineCombine(psi, model, pars); eng.run().
- Added init\_env() and reset\_stats() to allow a simple restart of DMRG with slightly different parameters, e.g. for tuning Hamiltonian parameters.
- Call canonical\_form() for infinite systems if the final state is not in canonical form.
- Changed **default values** for some parameters:
  - set trunc\_params['chi\_max'] = 100. Not setting a *chi\_max* at all will lead to memory problems. Disable DMRG\_params['chi\_list'] = None by default to avoid conflicting settings.
  - reduce to mixer\_params['amplitude'] = 1.e-5. A too strong mixer screws DMRG up pretty
    bad.
  - increase Lanczos\_params['N\_cache'] = N\_max (i.e., keep all states)
  - set DMRG\_params['P\_tol\_to\_trunc'] = 0.05 and provide reasonable ...\_min and ...\_max
    values.
  - increased (default) DMRG accuracy by setting DMRG\_params['max\_E\_err'] = 1.e-8 and
    DMRG\_params['max\_S\_err'] = 1.e-5.
  - don't check the (absolute) energy for convergence in Lanczos.
  - set DMRG\_params ['norm\_tol'] = 1.e-5 to check whether the final state is in canonical form.
- Verbosity of get\_parameter() reduced: Print parameters only for verbosity >= 1. and default values only for verbosity >= 2.
- Don't print the energy during real-time TEBD evolution it's preserved up to truncation errors.
- Renamed the SquareLattice class to tenpy.models.lattice.Square for better consistency.
- auto-determine whether Jordan-Wigner strings are necessary in add\_coupling().
- The way the labels of npc Arrays are stored internally changed to a simple list with None entries. There is a deprecated properly setter yielding a dictionary with the labels.
- renamed first\_LP and last\_RP arguments of MPSEnvironment and MPOEnvironment to init\_LP and init\_RP.
- Testing: insetad of the (outdated) nose, we now use pytest <a href="https://pytest.org">https://pytest.org</a> for testing.

## **Fixed**

- issue #22: **Serious bug** in tenpy.linalg.np\_conserved.inner(): if do\_conj=True is used with non-zero qtotal, it returned 0. instead of non-zero values.
- avoid error in tenpy.networks.mps.MPS.apply\_local\_op()
- Don't carry around total charge when using DMRG with a mixer
- Corrected couplings of the FermionicHubbardChain
- issue #2: memory leak in cython parts when using intelpython/anaconda
- issue #4: incompatible data types.
- issue #6: the CouplingModel generated wrong Couplings in some cases
- issue #19: Convergence of energy was slow for infinite systems with N\_sweeps\_check=1
- · more reasonable traceback in case of wrong labels

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- wrong dtype of npc.Array when adding/subtracting/... arrays of different data types
- could get wrong H\_bond for completely decoupled chains.
- · SVD could return outer indices with different axes
- tenpy.networks.mps.MPS.overlap() works now for MPS with different total charge (e.g. after psi.apply\_local\_op(i, 'Sp')).
- skip existing graph edges in MPOGraph.add() when building up terms without the strength part.

#### Removed

• Attribute *chinfo* of *Lattice*.

# 7.2.6 [0.3.0] - 2018-02-19

This is the first version published on github.

## **Added**

- Cython modules for np\_conserved and charges, which can optionally be compiled for speed-ups
- tools.optimization for dynamical optimization
- · Various models.
- More predefined lattice sites.
- Example toy-codes.
- · Network contractor for general networks

#### Changed

• Switch to python3

### Removed

• Python 2 support.

# 7.2.7 [0.2.0] - 2017-02-24

- Compatible with python2 and python3 (using the 2to3 tool).
- Development version.
- · Includes TEBD and DMRG.

# 7.2.8 Changes compared to previous TeNPy

This library is based on a previous (closed source) version developed mainly by Frank Pollmann, Michael P. Zaletel and Roger S. K. Mong. While allmost all files are completely rewritten and not backwards compatible, the overall structure is similar. In the following, we list only the most important changes.

## **Global Changes**

- syntax style based on PEP8. Use \$>yapf -r -i ./ to ensure consitent formatting over the whole project. Special comments # yapf: disable and # yapf: enable can be used for manual formatting of some regions in code.
- Following PEP8, we distinguish between 'private' functions, indicated by names starting with an underscore and to be used only within the library, and the public API. The puplic API should be backwards-compatible with different releases, while private functions might change at any time.
- all modules are in the folder tenpy to avoid name conflicts with other libraries.
- withing the library, relative imports are used, e.g., from ..tools.math import (toiterable, tonparray) Exception: the files in tests/ and examples/ run as \_\_main\_\_ and can't use relative imports
  - Files outside of the library (and in *tests*/, *examples*/) should use absolute imports, e.g. import tenpy. algorithms.tebd
- renamed tenpy/mps/ to tenpy/networks, since it containes various tensor networks.
- added Site describing the local physical sites by providing the physical LegCharge and onsite operators.

#### np\_conserved

- pure python, no need to compile!
- in module tenpy.linalg instead of algorithms/linalg.
- moved functionality for charges to charges
- Introduced the classes Charge Info (basically the old q\_number, and mod\_q) and LegCharge (the old qind, qconj).
- Introduced the class <code>LegPipe</code> to replace the old <code>leg\_pipe</code>. It is derived from <code>LegCharge</code> and used as a leg in the <code>array</code> class. Thus any inherited array (after <code>tensordot</code> etc still has all the necessary information to split the legs. (The legs are shared between different arrays, so it's saved only once in memory)
- Enhanced indexing of the array class to support slices and 1D index arrays along certain axes
- more functions, e.g. grid\_outer()

## **TEBD**

- Introduced TruncationError for easy handling of total truncation error.
- some truncation parameters are renamed and may have a different meaning, e.g. svd\_max -> svd\_min has no 'log' in the definition.

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#### **DMRG**

- separate Lanczos module in *tenpy/linalg/*. Strangely, the old version orthoganalized against the complex conjugates of *orthogonal\_to* (contrary to it's doc string!) (and thus calculated 'theta\_o' as bra, not ket).
- cleaned up, provide prototypes for DMRG engine and mixer.

#### **Tools**

- added tenpy.tools.misc, which contains 'random stuff' from old tools.math like to\_iterable and to\_array (renamed to follow PEP8, documented)
- moved stuff for fitting to tenpy.tools.fit
- enhanced tenpy.tools.string.vert\_join() for nice formatting
- moved (parts of) old *cluster/omp.py* to tenpy.tools.process
- added tenpy.tools.params for a simplified handling of parameter/arguments for models and/or algorithms. Similar as the old *models.model.set\_var*, but use it also for algorithms. Also, it may modify the given dictionary.

# 7.3 Introductions

The following documents are meant as introductions to various topics relevant to TeNPy.

If you are new to TeNPy, read the *Overview*.

#### 7.3.1 Overview

#### Repository

The root directory of this git repository contains the following folders:

- tenpy The actual source code of the library. Every subfolder contains an \_\_init\_\_.py file with a summary what the modules in it are good for. (This file is also necessary to mark the folder as part of the python package. Consequently, other subfolders of the git repo should not include a \_\_init\_\_.py file.)
- **toycodes** Simple toy codes completely independet of the remaining library (i.e., codes in tenpy/). These codes should be quite readable and intend to give a flavor of how (some of) the algorithms work.

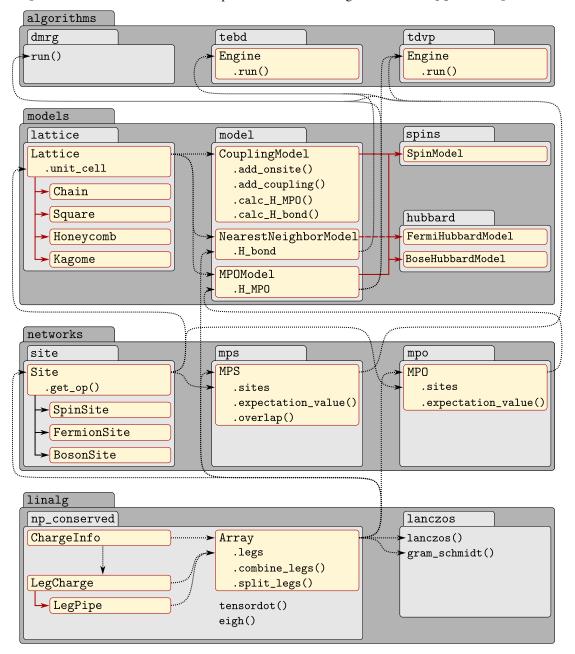
**examples** Some example files demonstrating the usage and interface of the library.

- doc A folder containing the documentation: the user guide is contained in the \*.rst files. The online documentation is autogenerated from these files and the docstrings of the library. This folder contains a make file for building the documentation, run make help for the different options. The necessary files for the reference in doc/reference can be auto-generated/updated with make src2html.
- **tests** Contains files with test routines, to be used with *pytest*. If you are set up correctly and have *pytest* installed, you can run the test suite with pytest from within the tests/ folder.
- **build** This folder is not distributed with the code, but is generated by setup.py (or compile.sh, respectively). It contains compiled versions of the Cython files, and can be ignored (and even removed without loosing functionality).

## Code structure: getting started

There are several layers of abstraction in TeNPy. While there is a certain hierarchy of how the concepts build up on each other, the user can decide to utilize only some of them. A maximal flexibility is provided by an object oriented style based on classes, which can be inherited and adjusted to individual demands.

The following figure gives an overview of the most important modules, classes and functions in TeNPy. Gray backgrounds indicate (sub)modules, yellow backgrounds indicate classes. Red arrows indicate inheritance relations, dashed black arrows indicate a direct use. (The individual models might be derived from the <code>NearestNeighborModel</code> depending on the geometry of the lattice.) There is a clear hierarchy from high-level algorithms in the <code>tenpy.algorithms</code> module down to basic operations from linear algebra in the <code>tenpy.linalg</code> module.



### Most basic level: linear algebra

**Note:** See Charge conservation with np\_conserved for more information on defining charges for arrays.

The most basic layer is given by in the <code>linalg</code> module, which provides basic features of linear algebra. In particular, the <code>np\_conserved</code> submodule implements an <code>Array</code> class which is used to represent the tensors. The basic interface of <code>np\_conserved</code> is very similar to that of the NumPy and SciPy libraries. However, the <code>Array</code> class implements abelian charge conservation. If no charges are to be used, one can use 'trivial' arrays, as shown in the following example code.

```
"""Basic use of the `Array` class with trivial arrays."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3

import tenpy.linalg.np_conserved as npc

M = npc.Array.from_ndarray_trivial([[0., 1.], [1., 0.]])
v = npc.Array.from_ndarray_trivial([2., 4. + 1.j])
v[0] = 3. # set indiviual entries like in numpy
print("|v> =", v.to_ndarray())
# |v> = [ 3.+0.j  4.+1.j]

M_v = npc.tensordot(M, v, axes=[1, 0])
print("M|v> =", M_v.to_ndarray())
# M/v> = [ 4.+1.j  3.+0.j]
print("<v|M|v> =", npc.inner(v.conj(), M_v, axes='range'))
# <v/M/v> = (24+0j)
```

The number and types of symmetries are specified in a *ChargeInfo* class. An *Array* instance represents a tensor satisfying a charge rule specifying which blocks of it are nonzero. Internally, it stores only the non-zero blocks of the tensor, along with one *LegCharge* instance for each leg, which contains the *charges* and sign *qconj* for each leg. We can combine multiple legs into a single larger *LegPipe*, which is derived from the *LegCharge* and stores all the information necessary to later split the pipe.

The following code explicitly defines the spin-1/2  $S^+, S^-, S^z$  operators and uses them to generate and diagonalize the two-site Hamiltonian  $H = \vec{S} \cdot \vec{S}$ . It prints the charge values (by default sorted ascending) and the eigenvalues of H.

```
"""Explicit definition of charges and spin-1/2 operators."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
# consider spin-1/2 with Sz-conservation
chinfo = npc.ChargeInfo([1]) # just a U(1) charge
# charges for up, down state
p_leg = npc.LegCharge.from_gflat(chinfo, [[1], [-1]])
Sz = npc.Array.from_ndarray([[0.5, 0.], [0., -0.5]], [p_leg, p_leg.conj()])
Sp = npc.Array.from_ndarray([[0., 1.], [0., 0.]], [p_leg, p_leg.conj()])
Sm = npc.Array.from_ndarray([[0., 0.], [1., 0.]], [p_leg, p_leg.conj()])
Hxy = 0.5 * (npc.outer(Sp, Sm) + npc.outer(Sm, Sp))
Hz = npc.outer(Sz, Sz)
H = Hxy + Hz
# here, H has 4 legs
H.iset_leg_labels(["s1", "t1", "s2", "t2"])
H = H.combine_legs([["s1", "s2"], ["t1", "t2"]], qconj=[+1, -1])
```

(continues on next page)

```
# here, H has 2 legs
print(H.legs[0].to_qflat().flatten())
# prints [-2 0 0 2]
E, U = npc.eigh(H) # diagonalize blocks individually
print(E)
# [ 0.25 -0.75 0.25 0.25]
```

### Sites for the local Hilbert space and tensor networks

The next basic concept is that of a local Hilbert space, which is represented by a Site in TeNPy. This class does not only label the local states and define the charges, but also provides onsite operators. For example, the SpinHalfSite provides the  $S^+, S^-, S^z$  operators under the names 'Sp', 'Sm', 'Sz', defined as Array instances similarly as in the code above. Since the most common sites like for example the SpinSite (for general spin S=0.5, 1, 1.5,...), BosonSite and FermionSite are predefined, a user of TeNPy usually does not need to define the local charges and operators explicitly. The total Hilbert space, i.e, the tensor product of the local Hilbert spaces, is then just given by a list of Site instances. If desired, different kinds of Site can be combined in that list. This list is then given to classes representing tensor networks like the MPS and MPO. The tensor network classes also use Array instances for the tensors of the represented network.

The following example illustrates the initialization of a spin-1/2 site, an MPS representing the Neel state, and an MPO representing the Heisenberg model by explicitly defining the W tensor.

```
"""Initialization of sites, MPS and MPO."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3
from tenpy.networks.site import SpinHalfSite
from tenpy.networks.mps import MPS
from tenpy.networks.mpo import MPO
spin = SpinHalfSite(conserve="Sz")
print(spin.Sz.to_ndarray())
# [[ 0.5 0. ]
# [0. -0.5]]
N = 6 # number of sites
sites = [spin] * N # repeat entry of list N times
pstate = ["up", "down"] * (N // 2) # Neel state
psi = MPS.from_product_state(sites, pstate, bc="finite")
print("<Sz> =", psi.expectation_value("Sz"))
\# \langle Sz \rangle = [0.5 - 0.5 0.5 - 0.5]
print("<Sp_i Sm_j> =", psi.correlation_function("Sp", "Sm"), sep="\n")
# <Sp_i Sm_j> =
# [[1. 0. 0. 0. 0. 0.]
  [0. 0. 0. 0. 0. 0.]
  [0. 0. 1. 0. 0. 0.]
  [0. 0. 0. 0. 0. 0.]
  [0. 0. 0. 0. 1. 0.]
  [0. 0. 0. 0. 0. 0.]]
# define an MPO
Id, Sp, Sm, Sz = spin.Id, spin.Sp, spin.Sm, spin.Sz
J, Delta, hz = 1., 1., 0.2
W_bulk = [Id, Sp, Sm, Sz, -hz * Sz], [None, None, None, None, 0.5 * J * Sm],
          [None, None, None, None, 0.5 * J * Sp], [None, None, None, J * Delta,
                                                                          (continues on next page)
```

```
[None, None, None, None, Id]]
W_first = [W_bulk[0]] # first row
W_last = [[row[-1]] for row in W_bulk] # last column
Ws = [W_first] + [W_bulk] * (N - 2) + [W_last]
H = MPO.from_grids([spin] * N, Ws, bc='finite', IdL=0, IdR=-1)
print("<psi|H|psi> =", H.expectation_value(psi))
# <psi|H|psi> = -1.25
```

#### **Models**

Note: See Models for more information on sites and how to define and extend models on your own.

Technically, the explicit definition of an MPO is already enough to call an algorithm like DMRG in <code>dmrg</code>. However, writing down the W tensors is cumbersome especially for more complicated models. Hence, TeNPy provides another layer of abstraction for the definition of models, which we discuss first. Different kinds of algorithms require different representations of the Hamiltonian. Therefore, the library offers to specify the model abstractly by the individual onsite terms and coupling terms of the Hamiltonian. The following example illustrates this, again for the Heisenberg model.

```
"""Definition of a model: the XXZ chain."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3
from tenpy.networks.site import SpinSite
from tenpy.models.lattice import Chain
from tenpy.models.model import CouplingModel, NearestNeighborModel, MPOModel
class XXZChain(CouplingModel, NearestNeighborModel, MPOModel):
   def __init__(self, L=2, S=0.5, J=1., Delta=1., hz=0.):
       spin = SpinSite(S=S, conserve="Sz")
       # the lattice defines the geometry
       lattice = Chain(L, spin, bc="open", bc_MPS="finite")
       CouplingModel.__init__(self, lattice)
       # add terms of the Hamiltonian
       self.add_coupling(J * 0.5, 0, "Sp", 0, "Sm", 1) # Sp_i Sm_{i+1}
       self.add_coupling(J * Delta, 0, "Sz", 0, "Sz", 1)
       # (for site dependent prefactors, the strength can be an array)
       self.add_onsite(-hz, 0, "Sz")
       # finish initialization
       # generate MPO for DMRG
       MPOModel.__init__(self, lat, self.calc_H_MPO())
       # generate H_bond for TEBD
       NearestNeighborModel.__init__(self, lat, self.calc_H_bond())
```

While this generates the same MPO as in the previous code, this example can easily be adjusted and generalized, for example to a higher dimensional lattice by just specifying a different lattice. Internally, the MPO is generated using a finite state machine picture. This allows not only to translate more complicated Hamiltonians into their corresponding MPOs, but also to automate the mapping from a higher dimensional lattice to the 1D chain along which the MPS winds. Note that this mapping introduces longer-range couplings, so the model can no longer be defined to be a <code>NearestNeighborModel</code> suited for TEBD if another lattice than the <code>Chain</code> is to be used. Of course, many

commonly studied models are also predefined. For example, the following code initializes the Heisenberg model on a kagome lattice; the spin liquid nature of the ground state of this model is highly debated in the current literature.

```
"""Initialization of the Heisenberg model on a kagome lattice."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3

from tenpy.models.spins import SpinModel

model_params = {
    "S": 0.5, # Spin 1/2
    "lattice": "Kagome",
    "bc_MPS": "infinite",
    "bc_y": "cylinder",
    "Ly": 2, # defines cylinder circumference
    "conserve": "Sz", # use Sz conservation
    "Jx": 1.,
    "Jy": 1.,
    "Jz": 1. # Heisenberg coupling
}
model = SpinModel (model_params)
```

## **Algorithms**

7.3. Introductions

The highest level in TeNPy is given by algorithms like DMRG and TEBD. Using the previous concepts, setting up a simulation running those algorithms is a matter of just a few lines of code. The following example runs a DMRG simulation, see <code>dmrg</code>, exemplary for the transverse field Ising model at the critical point.

```
"""Call of (finite) DMRG."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3

from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import dmrg

N = 16  # number of sites
model = TFIChain({"L": N, "J": 1., "g": 1., "bc_MPS": "finite"})
sites = model.lat.mps_sites()
psi = MPS.from_product_state(sites, ['up'] * N, "finite")
dmrg_params = {"trunc_params": {"chi_max": 100, "svd_min": 1.e-10}, "mixer": True}
info = dmrg.run(psi, model, dmrg_params)
print("E =", info['E'])
# E = -20.01638790048513
print("max. bond dimension = ", max(psi.chi))
# max. bond dimension = 27
```

The switch from DMRG to gls{iDMRG} in TeNPy is simply accomplished by a change of the parameter "bc\_MPS" from "finite" to "infinite", both for the model and the state. The returned E is then the energy density per site. Due to the translation invariance, one can also evaluate the correlation length, here slightly away from the critical point.

```
"""Call of infinite DMRG."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3

from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import dmrg
```

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```
N = 2  # number of sites in unit cell
model = TFIChain({"L": N, "J": 1., "g": 1.1, "bc_MPS": "infinite"})
sites = model.lat.mps_sites()
psi = MPS.from_product_state(sites, ['up'] * N, "infinite")
dmrg_params = {"trunc_params": {"chi_max": 100, "svd_min": 1.e-10}, "mixer": True}
info = dmrg.run(psi, model, dmrg_params)
print("E = ", info['E'])
# E = -1.342864022725017
print("max. bond dimension = ", max(psi.chi))
# max. bond dimension = 56
print("corr. length = ", psi.correlation_length())
# corr. length = 4.915809146764157
```

Running time evolution with TEBD requires an additional loop, during which the desired observables have to be measured. The following code shows this directly for the infinite version of TEBD.

```
"""Call of (infinite) TEBD."""
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3
from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import tebd
M = TFIChain({"L": 2, "J": 1., "g": 1.5, "bc_MPS": "infinite"})
psi = MPS.from_product_state(M.lat.mps_sites(), [0] * 2, "infinite")
tebd_params = {
   "order": 2,
    "delta_tau_list": [0.1, 0.001, 1.e-5],
    "max_error_E": 1.e-6,
    "trunc_params": {
       "chi_max": 30,
       "svd_min": 1.e-10
eng = tebd.Engine(psi, M, tebd_params)
eng.run_GS() # imaginary time evolution with TEBD
print("E =", sum(psi.expectation_value(M.H_bond)) / psi.L)
print("final bond dimensions: ", psi.chi)
```

# 7.3.2 Toy Codes

The following "toy codes" are included in the TeNPy repository in the folder toycodes/, we include them here in the documentation for reference. They are meant to give you a flavor of the different algorithms, while keeping the codes as readable and simple as possible. The only requirements to run them are Python 3, Numpy, and Scipy. Simply go to the folder where you downloaded them, and execute them with python.

# Toycode a\_mps.py

```
"""Toy code implementing a matrix product state."""
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
from scipy.linalg import svd
# if you get an error message "LinAlgError: SVD did not converge",
# uncomment the following line. (This requires TeNPy to be installed.)
# from tenpy.linalg.svd_robust import svd # (works like scipy.linalg.svd)
class SimpleMPS:
    """Simple class for a matrix product state.
    We index sites with `i` from 0 to L-1; bond `i` is left of site `i`.
    We *assume* that the state is in right-canonical form.
   Parameters
    Bs, Ss, bc:
       Same as attributes.
   Attributes
    Bs : list of np.Array[ndim=3]
        The 'matrices' in right-canonical form, one for each physical site
        (within the unit-cell for an infinite MPS).
       Each `B[i]` has legs (virtual left, physical, virtual right), in short ``vL i_
\hookrightarrow VR``
   Ss : list of np.Array[ndim=1]
       The Schmidt values at each of the bonds, ``Ss[i]`` is left of ``Bs[i]``.
   bc : 'infinite', 'finite'
       Boundary conditions.
        Number of sites (in the unit-cell for an infinite MPS).
   nbonds : int
       Number of (non-trivial) bonds: L-1 for 'finite' boundary conditions
    def __init__(self, Bs, Ss, bc='finite'):
        assert bc in ['finite', 'infinite']
        self.Bs = Bs
        self.Ss = Ss
        self.bc = bc
        self.L = len(Bs)
        self.nbonds = self.L - 1 if self.bc == 'finite' else self.L
    def copy(self):
        return SimpleMPS([B.copy() for B in self.Bs], [S.copy() for S in self.Ss],...
⇒self.bc)
    def get_theta1(self, i):
        """Calculate effective single-site wave function on sites i in mixed_
\hookrightarrow canonical form.
        The returned array has legs ``vL, i, vR`` (as one of the Bs).
        n n n
```

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```
return np.tensordot(np.diag(self.Ss[i]), self.Bs[i], [1, 0]) # vL [vL'],
\hookrightarrow [vL] i vR
   def get_theta2(self, i):
        """Calculate effective two-site wave function on sites i, j=(i+1) in mixed_
⇔canonical form.
        The returned array has legs ``vL, i, j, vR``.
        m m m
       j = (i + 1) % self.L
       return np.tensordot(self.get_theta1(i), self.Bs[j], [2, 0]) # vL i [vR],_
\hookrightarrow [VL] j VR
   def get_chi(self):
        """Return bond dimensions."""
       return [self.Bs[i].shape[2] for i in range(self.nbonds)]
   def site_expectation_value(self, op):
        """Calculate expectation values of a local operator at each site."""
       result = []
        for i in range(self.L):
            theta = self.get_theta1(i) # vL i vR
            op_theta = np.tensordot(op, theta, axes=[1, 1]) # i [i*], vL [i] vR
            result.append(np.tensordot(theta.conj(), op_theta, [[0, 1, 2], [1, 0, _
\hookrightarrow2]]))
            # [vL*] [i*] [vR*], [i] [vL] [vR]
       return np.real_if_close(result)
   def bond_expectation_value(self, op):
        """Calculate expectation values of a local operator at each bond."""
       result = []
       for i in range(self.nbonds):
            theta = self.get_theta2(i) # vL i j vR
            op_theta = np.tensordot(op[i], theta, axes=[[2, 3], [1, 2]])
            # i j [i*] [j*], vL [i] [j] vR
            result.append(np.tensordot(theta.conj(), op_theta, [[0, 1, 2, 3], [2, 0, ]
\rightarrow 1, 3]]))
            # [vL*] [i*] [j*] [vR*], [i] [j] [vL] [vR]
       return np.real_if_close(result)
   def entanglement entropy(self):
        """Return the (von-Neumann) entanglement entropy for a bipartition at any of
\hookrightarrowthe bonds."""
       bonds = range(1, self.L) if self.bc == 'finite' else range(0, self.L)
       result = []
        for i in bonds:
            S = self.Ss[i].copy()
            S[S < 1.e-20] = 0. # 0*log(0) should give 0; avoid warning or NaN.
            S2 = S * S
            assert abs(np.linalg.norm(S) - 1.) < 1.e-14</pre>
            result.append(-np.sum(S2 * np.log(S2)))
       return np.array(result)
   def correlation length(self):
        """Diagonalize transfer matrix to obtain the correlation length."""
       import scipy.sparse.linalg.eigen.arpack as arp
       assert self.bc == 'infinite' # works only in the infinite case
```

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```
B = self.Bs[0] # vL i vR
                       chi = B.shape[0]
                       \texttt{T} = \texttt{np.tensordot}(\texttt{B}, \, \texttt{np.conj}(\texttt{B}), \, \texttt{axes} = \texttt{[1, 1]}) \quad \# \ vL \ \texttt{[i]} \ vR, \, vL \star \ \texttt{[i*]} \ vR \star \texttt{[i
                       T = np.transpose(T, [0, 2, 1, 3]) # vL vL* vR vR*
                       for i in range(1, self.L):
                                   B = self.Bs[i]
                                   T = np.tensordot(T, B, axes=[2, 0]) # vL vL*[vR] vR*, [vL] i vR
                                   T = np.tensordot(T, np.conj(B), axes=[[2, 3], [0, 1]])
                                   # vL vL* [vR*] [i] vR, [vL*] [i*] vR*
                       T = np.reshape(T, (chi**2, chi**2))
                        # Obtain the 2nd largest eigenvalue
                       eta = arp.eigs(T, k=2, which='LM', return_eigenvectors=False, ncv=20)
                       return -self.L / np.log(np.min(np.abs(eta)))
def init_FM_MPS(L, d, bc='finite'):
            """Return a ferromagnetic MPS (= product state with all spins up)"""
           B = np.zeros([1, d, 1], np.float)
           B[0, 0, 0] = 1.
           S = np.ones([1], np.float)
           Bs = [B.copy() for i in range(L)]
           Ss = [S.copy() for i in range(L)]
           return SimpleMPS(Bs, Ss, bc)
def split_truncate_theta(theta, chi_max, eps):
            """Split and truncate a two-site wave function in mixed canonical form.
            Split a two-site wave function as follows::
                             vL --(theta) -- vR \implies vL --(A) --diag(S) -- (B) -- vR
                                             / /
                                                                                                                                 /
                                                                                                                                                                          /
           Afterwards, truncate in the new leg (labeled ``vC``).
           Parameters
           theta : np.Array[ndim=4]
                      Two-site wave function in mixed canonical form, with legs ``vL, i, j, vR``.
            chi_max : int
                     Maximum number of singular values to keep
            eps : float
                      Discard any singular values smaller than that.
           Returns
           A : np.Array[ndim=3]
                       Left-canonical matrix on site i, with legs ``vL, i, vC``
            S : np.Array[ndim=1]
                      Singular/Schmidt values.
            B : np.Array[ndim=3]
                      Right-canonical matrix on site j, with legs ``vC, j, vR``
           chivL, dL, dR, chivR = theta.shape
           theta = np.reshape(theta, [chivL * dL, dR * chivR])
           X, Y, Z = svd(theta, full_matrices=False)
            # truncate
```

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```
chivC = min(chi_max, np.sum(Y > eps))
piv = np.argsort(Y)[::-1][:chivC]  # keep the largest `chivC` singular values
X, Y, Z = X[:, piv], Y[piv], Z[piv, :]
# renormalize
S = Y / np.linalg.norm(Y)  # == Y/sqrt(sum(Y**2))
# split legs of X and Z
A = np.reshape(X, [chivL, dL, chivC])
B = np.reshape(Z, [chivC, dR, chivR])
return A, S, B
```

#### Toycode b\_model.py

```
"""Toy code implementing the transverse-field ising model."""
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
class TFIModel:
    """Simple class generating the Hamiltonian of the transverse-field Ising model.
   The Hamiltonian reads
    .. math ::
        H = -J \setminus sum_{i} \setminus sigma^x_i \setminus sigma^x_{i+1} - g \setminus sum_{i} \setminus sigma^z_i
   Parameters
    L : int
       Number of sites.
   J, g : float
       Coupling parameters of the above defined Hamiltonian.
    bc : 'infinite', 'finite'
       Boundary conditions.
   Attributes
   L : int
        Number of sites.
   bc : 'infinite', 'finite'
       Boundary conditions.
    sigmax, sigmay, sigmaz, id:
        Local operators, namely the Pauli matrices and identity.
    H_bonds : list of np.Array[ndim=4]
        The Hamiltonian written in terms of local 2-site operators, ``H = sum_i H_
⇒bonds[i]``.
       Each ``H_bonds[i]`` has (physical) legs (i out, (i+1) out, i in, (i+1) in),
        in short ``i j i* j*``.
    H_mpo : lit of np.Array[ndim=4]
        The Hamiltonian written as an MPO.
        Each ``H_mpo[i]`` has legs (virutal left, virtual right, physical out,
⇔physical in),
       in short ``wL wR i i*``.
    def __init__(self, L, J, g, bc='finite'):
        assert bc in ['finite', 'infinite']
```

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```
self.L, self.d, self.bc = L, 2, bc
       self.J, self.g = J, g
       self.sigmax = np.array([[0., 1.], [1., 0.]])
       self.sigmay = np.array([[0., -1j], [1j, 0.]])
       self.sigmaz = np.array([[1., 0.], [0., -1.]])
       self.id = np.eye(2)
       self.init_H_bonds()
       self.init_H_mpo()
   def init_H_bonds(self):
       """Initialize `H_bonds` hamiltonian.
       Called by __init__().
       sx, sz, id = self.sigmax, self.sigmaz, self.id
       d = self.d
       nbonds = self.L - 1 if self.bc == 'finite' else self.L
       H_list = []
       for i in range(nbonds):
           qL = qR = 0.5 * self.q
           if self.bc == 'finite':
               if i == 0:
                    gL = self.g
               if i + 1 == self.L - 1:
                   gR = self.g
           H_bond = -self.J * np.kron(sx, sx) - gL * np.kron(sz, id) - gR * np.
\rightarrowkron(id, sz)
            # H_bond has legs ``i, j, i*, j*``
           H_list.append(np.reshape(H_bond, [d, d, d, d]))
       self.H_bonds = H_list
   # (note: not required for TEBD)
   def init_H_mpo(self):
        """Initialize `H_mpo` Hamiltonian.
       Called by __init__().
       w_list = []
       for i in range(self.L):
           w = np.zeros((3, 3, self.d, self.d), dtype=np.float)
           w[0, 0] = w[2, 2] = self.id
           w[0, 1] = self.sigmax
           w[0, 2] = -self.g * self.sigmaz
           w[1, 2] = -self.J * self.sigmax
           w_list.append(w)
       self.H_mpo = w_list
```

### Toycode c\_tebd.py

```
"""Toy code implementing the time evolving block decimation (TEBD)."""
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
from scipy.linalg import expm
from a_mps import split_truncate_theta
def calc_U_bonds(H_bonds, dt):
    """Given the H_bonds, calculate ``U_bonds[i] = expm(-dt*H_bonds[i])``.
   Each local operator has legs (i out, (i+1) out, i in, (i+1) in), in short ``i j_

→ i * j * `

   Note that no imaginary 'i' is included, thus real `dt` means 'imaginary time'.
→evolution!
   d = H_bonds[0].shape[0]
   U_bonds = []
   for H in H_bonds:
       H = np.reshape(H, [d * d, d * d])
       U = expm(-dt * H)
       U_bonds.append(np.reshape(U, [d, d, d, d]))
   return U_bonds
def run_TEBD(psi, U_bonds, N_steps, chi_max, eps):
    """Evolve for `N_steps` time steps with TEBD."""
   Nbonds = psi.L - 1 if psi.bc == 'finite' else psi.L
   assert len(U_bonds) == Nbonds
   for n in range(N_steps):
       for k in [0, 1]: # even, odd
           for i_bond in range(k, Nbonds, 2):
               update_bond(psi, i_bond, U_bonds[i_bond], chi_max, eps)
    # done
def update_bond(psi, i, U_bond, chi_max, eps):
   """Apply `U_bond` acting on i, j=(i+1) to `psi`."""
   j = (i + 1) % psi.L
   # construct theta matrix
   theta = psi.get_theta2(i) # vL i j vR
   # apply U
   Utheta = np.tensordot(U_bond, theta, axes=([2, 3], [1, 2])) # i j [i*] [j*], vL_{\bot}
\rightarrow [i] [j] vR
   Utheta = np.transpose(Utheta, [2, 0, 1, 3]) # vL i j vR
   # split and truncate
   Ai, Sj, Bj = split_truncate_theta(Utheta, chi_max, eps)
   # put back into MPS
   psi.Bs[i] = np.tensordot(Gi, np.diag(Sj), axes=[2, 0]) # vL i [vC], [vC] vC
   psi.Ss[j] = Sj # vC
   psi.Bs[j] = Bj # vC j vR
```

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```
def example_TEBD_qs_tf_isinq_finite(L, q):
   print("finite TEBD, imaginary time evolution, transverse field Ising")
    print ("L=\{L:d\}, g=\{g:.2f\}".format (L=L, g=g))
    import a_mps
    import b_model
    M = b_model.TFIModel(L=L, J=1., g=g, bc='finite')
    psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
    for dt in [0.1, 0.01, 0.001, 1.e-4, 1.e-5]:
        U_bonds = calc_U_bonds(M.H_bonds, dt)
        run_TEBD(psi, U_bonds, N_steps=500, chi_max=30, eps=1.e-10)
        E = np.sum(psi.bond_expectation_value(M.H_bonds))
        print ("dt = \{dt:.5f\}: E = \{E:.13f\}".format (dt=dt, E=E))
   print("final bond dimensions: ", psi.get_chi())
   mag_x = np.sum(psi.site_expectation_value(M.sigmax))
   mag_z = np.sum(psi.site_expectation_value(M.sigmaz))
   print("magnetization in X = {mag_x:.5f}".format(mag_x=mag_x))
   print("magnetization in Z = \{mag_z:.5f\}".format(mag_z=mag_z))
    if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = \{E:.13f\}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_TEBD_gs_tf_ising_infinite(g):
   print("infinite TEBD, imaginary time evolution, transverse field Ising")
   print ("g = \{g:.2f\}".format (g = g))
   import a mps
    import b_model
   M = b_model.TFIModel(L=2, J=1., g=g, bc='infinite')
    psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
    for dt in [0.1, 0.01, 0.001, 1.e-4, 1.e-5]:
        U_bonds = calc_U_bonds(M.H_bonds, dt)
        run_TEBD(psi, U_bonds, N_steps=500, chi_max=30, eps=1.e-10)
        E = np.mean(psi.bond_expectation_value(M.H_bonds))
        print("dt = \{dt...5f\}: E (per site) = \{E...13f\}".format(dt=dt, E=E))
   print("final bond dimensions: ", psi.get_chi())
   mag_x = np.mean(psi.site_expectation_value(M.sigmax))
   maq_z = np.mean(psi.site_expectation_value(M.sigmaz))
   print ("<sigma_x> = \{mag_x:.5f\}".format (mag_x=mag_x))
   print("<sigma_z> = {mag_z:.5f}".format(mag_z=mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
    from tfi_exact import infinite_gs_energy
    E_{exact} = infinite_{gs_energy}(1., g)
    print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
    print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_TEBD_tf_ising_lightcone(L, g, tmax, dt):
   print("finite TEBD, real time evolution, transverse field Ising")
   print("L=\{L:d\}, g=\{g:.2f\}, tmax=\{tmax:.2f\}, dt=\{dt:.3f\}".format(L=L, g=g, _
→tmax=tmax, dt=dt))
    # find ground state with TEBD or DMRG
    # E, psi, M = example_TEBD_gs_tf_ising_finite(L, g)
```

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```
from d dmrg import example_DMRG_tf_ising_finite
   E, psi, M = example_DMRG_tf_ising_finite(L, g)
   i0 = L // 2
    # apply sigmaz on site i0
   SzB = np.tensordot(M.sigmaz, psi.Bs[i0], axes=[1, 1]) # i [i*], vL [i] vR
    psi.Bs[i0] = np.transpose(SzB, [1, 0, 2]) # vL i vR
   U_bonds = calc_U_bonds(M.H_bonds, 1.j * dt) # (imaginary dt -> realtime_
→evolution)
   S = [psi.entanglement_entropy()]
   Nsteps = int(tmax / dt + 0.5)
   for n in range(Nsteps):
       if abs((n * dt + 0.1) % 0.2 - 0.1) < 1.e-10:
            print("t = \{t:.2f\}, chi =".format(t=n * dt), psi.get_chi())
        run_TEBD(psi, U_bonds, 1, chi_max=50, eps=1.e-10)
        S.append(psi.entanglement_entropy())
   import matplotlib.pyplot as plt
   plt.figure()
   plt.imshow(S[::-1],
               aspect='auto',
               interpolation='nearest',
               extent=(0, L - 1., -0.5 * dt, (Nsteps + 0.5) * dt))
   plt.xlabel('site $i$')
   plt.ylabel('time $t/J$')
   plt.ylim(0., tmax)
   plt.colorbar().set_label('entropy $S$')
   filename = 'c_tebd_lightcone_{g:.2f}.pdf'.format(g=g)
   plt.savefig(filename)
   print("saved " + filename)
if __name__ == "__main__":
   example_TEBD_gs_tf_ising_finite(L=10, g=1.)
   print("-" * 100)
   example_TEBD_gs_tf_ising_infinite(g=1.5)
   print("-" * 100)
   example_TEBD_tf_ising_lightcone(L=20, g=1.5, tmax=3., dt=0.01)
```

## Toycode d\_dmrg.py

(continues on next page)

```
7 *
         (LP) --- (W1) -- (W2) ---- (RP)
                         /
                         j
                              vR--.
    def __init__(self, LP, RP, W1, W2):
         self.LP = LP # vL wL* vL*
         self.RP = RP # vR* wR* vR
         self.W1 = W1 # wL wC i i*
         self.W2 = W2 \# wC wR j j*
         chi1, chi2 = LP.shape[0], RP.shape[2]
         d1, d2 = W1.shape[2], W2.shape[2]
         self.theta_shape = (chi1, d1, d2, chi2) # vL i j vR
         self.shape = (chi1 * d1 * d2 * chi2, chi1 * d1 * d2 * chi2)
         self.dtype = W1.dtype
    def _matvec(self, theta):
         """Calculate |theta'> = H_eff |theta>.
         This function is used by :func:scipy.sparse.linalq.eigen.arpack.eigsh` to...
→ diagonalize
         the effective Hamiltonian with a Lanczos method, withouth generating the full,
\hookrightarrowmatrix."""
         x = np.reshape(theta, self.theta_shape) # vL i j vR
         x = \text{np.tensordot(self.LP, } x, \text{ axes=(2, 0))} \text{ # } vL \text{ } wL* \text{ } [vL*], \text{ } [vL] \text{ } i \text{ } j \text{ } vR
         x = \text{np.tensordot}(x, \text{self.W1}, \text{axes=([1, 2], [0, 3]))} \text{ # } vL \text{ [wL*] [i] } j \text{ } vR, 
\hookrightarrow [wL] \ wC \ i \ [i*]
         x = \text{np.tensordot}(x, \text{self.W2}, \text{axes=([3, 1], [0, 3])}) \text{ # vL [j] vR [wC] i, [wC]_i}
\hookrightarrow WR j [j*]
         x = \text{np.tensordot}(x, \text{self.RP}, \text{axes}=([1, 3], [0, 1])) # vL [vR] i [wR] j, 
\hookrightarrow [VR*] [WR*] VR
         x = np.reshape(x, self.shape[0])
         return x
class SimpleDMRGEngine:
    """DMRG algorithm, implemented as class holding the necessary data.
    Parameters
    psi, model, chi_max, eps:
        See attributes
    Attributes
    psi : SimpleMPS
         The current ground-state (approximation).
    model :
        The model of which the groundstate is to be calculated.
    chi_max, eps:
        Truncation parameters, see :func: `a_mps.split_truncate_theta`.
    LPs, RPs : list of np.Array[ndim=3]
         Left and right parts ("environments") of the effective Hamiltonian.
         ``LPs[i]`` is the contraction of all parts left of site `i` in the network ``
→<psi|H|psi>``,
```

(continues on next page)

```
and similar ``RPs[i]`` for all parts right of site `i`.
       Each ``LPs[i]`` has legs ``vL wL* vL*``, ``RPS[i]`` has legs ``vR* wR* vR``
   def __init__(self, psi, model, chi_max, eps):
       assert psi.L == model.L and psi.bc == model.bc # ensure compatibility
       self.H_mpo = model.H_mpo
       self.psi = psi
       self.LPs = [None] * psi.L
       self.RPs = [None] * psi.L
       self.chi_max = chi_max
       self.eps = eps
       # initialize left and right environment
       D = self.H_mpo[0].shape[0]
       chi = psi.Bs[0].shape[0]
       LP = np.zeros([chi, D, chi], dtype=np.float) # vL wL* vL*
       RP = np.zeros([chi, D, chi], dtype=np.float) # vR* wR* vR
       LP[:, 0, :] = np.eye(chi)
       RP[:, D - 1, :] = np.eye(chi)
       self.LPs[0] = LP
       self.RPs[-1] = RP
       # initialize necessary RPs
       for i in range(psi.L - 1, 1, -1):
           self.update_RP(i)
   def sweep(self):
       # sweep from left to right
       for i in range(self.psi.nbonds - 1):
           self.update_bond(i)
       # sweep from right to left
       for i in range(self.psi.nbonds - 1, 0, -1):
           self.update_bond(i)
   def update_bond(self, i):
       j = (i + 1) % self.psi.L
       # get effective Hamiltonian
       Heff = SimpleHeff(self.LPs[i], self.RPs[j], self.H_mpo[i], self.H_mpo[j])
       # Diagonalize Heff, find ground state `theta`
       theta0 = np.reshape(self.psi.get_theta2(i), [Heff.shape[0]]) # initial guess
       e, v = arp.eigsh(Heff, k=1, which='SA', return_eigenvectors=True, v0=theta0)
       theta = np.reshape(v[:, 0], Heff.theta_shape)
       # split and truncate
       Ai, Sj, Bj = split_truncate_theta(theta, self.chi_max, self.eps)
       # put back into MPS
       Gi = np.tensordot(np.diag(self.psi.Ss[i]**(-1)), Ai, axes=[1, 0]) # vL [vL*],
→ [vL] i vC
       self.psi.Bs[i] = np.tensordot(Gi, np.diag(Sj), axes=[2, 0]) # vL i [vC],...
\hookrightarrow [VC*]VC
       self.psi.Ss[j] = Sj # vC
       self.psi.Bs[j] = Bj # vC j vR
       self.update_LP(i)
       self.update_RP(j)
   def update_RP(self, i):
       """Calculate RP right of site `i-1` from RP right of site `i`."""
       j = (i - 1) % self.psi.L
       RP = self.RPs[i] # vR* wR* vR
       B = self.psi.Bs[i] # vL i vR
```

(continues on next page)

```
Bc = B.conj() # vL* i* vR*
        W = self.H_mpo[i] # wL wR i i \star
        RP = np.tensordot(B, RP, axes=[2, 0])  # vL i [vR], [vR*] wR* vR
        RP = np.tensordot(RP, W, axes=[[1, 2], [3, 1]]) # vL [i] [wR*] vR, wL [wR] i_

    [i *]

        RP = np.tensordot(RP, Bc, axes=[[1, 3], [2, 1]]) # vL [vR] wL [i], vL* [i*]
\hookrightarrow [vR *]
        self.RPs[j] = RP # vL wL vL* (== vR* wR* vR on site i-1)
    def update_LP(self, i):
        """Calculate LP left of site `i+1` from LP left of site `i`."""
        j = (i + 1) % self.psi.L
        LP = self.LPs[i] # vL wL vL*
        B = self.psi.Bs[i] # vL i vR
        G = np.tensordot(np.diag(self.psi.Ss[i]), B, axes=[1, 0]) # vL [vL*], [vL] i_
\hookrightarrow VR
        A = \text{np.tensordot}(G, \text{np.diag}(\text{self.psi.Ss}[j] * * -1), \text{axes} = [2, 0]) # vL i [vR],
\hookrightarrow [VR *] VR
        Ac = A.conj() # vL* i* vR*
        W = self.H_mpo[i] \# wL wR i i*
        LP = np.tensordot(LP, A, axes=[2, 0]) # vL wL* [vL*], [vL] i vR
        LP = np.tensordot(W, LP, axes=[[0, 3], [1, 2]]) # [wL] wR i [i*], vL [wL*]
\hookrightarrow [i] VR
        LP = np.tensordot(Ac, LP, axes=[[0, 1], [2, 1]]) # [vL*] [i*] vR*, wR [i]
\hookrightarrow [VL] VR
        self.LPs[j] = LP \# vR* wR vR (== vL wL* vL* on site i+1)
def example_DMRG_tf_ising_finite(L, g):
    print("finite DMRG, transverse field Ising")
    print("L={L:d}, g=\{g:.2f\}".format(L=L, g=g))
    import a_mps
    import b_model
    M = b_model.TFIModel(L=L, J=1., q=q, bc='finite')
    psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
    eng = SimpleDMRGEngine(psi, M, chi_max=30, eps=1.e-10)
    for i in range(10):
        eng.sweep()
        E = np.sum(psi.bond_expectation_value(M.H_bonds))
        print("sweep \{i:2d\}: E = \{E:.13f\}".format(i=i + 1, E=E))
    print("final bond dimensions: ", psi.get_chi())
    mag_x = np.sum(psi.site_expectation_value(M.sigmax))
    mag_z = np.sum(psi.site_expectation_value(M.sigmaz))
    print("magnetization in X = {mag_x:.5f}".format(mag_x=mag_x))
    print("magnetization in Z = \{mag_z: .5f\}".format(mag_z=mag_z))
    if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = {E:.13f}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_DMRG_tf_ising_infinite(g):
    print("infinite DMRG, transverse field Ising")
    print ("g=\{g:.2f\}".format (g=g))
    import a_mps
```

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```
import b model
   M = b_model.TFIModel(L=2, J=1., g=g, bc='infinite')
   psi = a_mps.init_FM_MPS(M.L, M.d, M.bc)
    eng = SimpleDMRGEngine(psi, M, chi_max=20, eps=1.e-14)
    for i in range(20):
        eng.sweep()
        E = np.mean(psi.bond_expectation_value(M.H_bonds))
        print("sweep \{i:2d\}: E (per site) = \{E:.13f\}".format(i=i + 1, E=E))
   print("final bond dimensions: ", psi.get_chi())
   mag_x = np.mean(psi.site_expectation_value(M.sigmax))
   mag_z = np.mean(psi.site_expectation_value(M.sigmaz))
   print(" < sigma_x > = \{mag_x : .5f\}".format(mag_x = mag_x))
   print("<sigma_z> = {mag_z:.5f}".format(mag_z=mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
   from tfi_exact import infinite_gs_energy
    E_exact = infinite_gs_energy(1., g)
    print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
    print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
if __name__ == "__main__":
    example_DMRG_tf_ising_finite(L=10, g=1.)
   print("-" * 100)
    example_DMRG_tf_ising_infinite(g=1.5)
```

## Toycode tfi\_exact.py

```
"""Provides exact ground state energies for the transverse field ising model for ...
→comparison.
The Hamiltonian reads
.. math ::
    H = -J \setminus sum_{i} \setminus sigma^x_i \setminus sigma^x_{i+1} - g \setminus sum_{i} \setminus sigma^z_i
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3
import numpy as np
import scipy.sparse as sparse
import scipy.sparse.linalg.eigen.arpack as arp
import warnings
import scipy.integrate
def finite_gs_energy(L, J, g):
    """For comparison: obtain ground state energy from exact diagonalization.
    Exponentially expensive in L, only works for small enough `L` <~ 20.
    if L >= 20:
        warnings.warn("Large L: Exact diagonalization might take a long time!")
    # get single site operaors
    sx = sparse.csr_matrix(np.array([[0., 1.], [1., 0.]]))
    sz = sparse.csr_matrix(np.array([[1., 0.], [0., -1.]]))
```

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```
id = sparse.csr_matrix(np.eye(2))
    sx_list = [] \# sx_list[i] = kron([id, id, ..., id, sx, id, .... id])
    sz_list = []
    for i_site in range(L):
        x_{ops} = [id] * L
        z_{ops} = [id] * L
        x_{ops[i\_site]} = sx
        z_{ops[i\_site]} = sz
        X = x_ops[0]
        Z = z_{ops}[0]
        for j in range(1, L):
            X = sparse.kron(X, x_ops[j], 'csr')
            Z = sparse.kron(Z, z_ops[j], 'csr')
        sx_list.append(X)
        sz_list.append(Z)
   H_xx = sparse.csr_matrix((2**L, 2**L))
   H_z = sparse.csr_matrix((2**L, 2**L))
    for i in range(L - 1):
        H_xx = H_xx + sx_list[i] * sx_list[(i + 1) % L]
    for i in range(L):
       H_z = H_z + sz_{list[i]}
   H = -J * H_xx - g * H_z
   E, V = arp.eigsh(H, k=1, which='SA', return_eigenvectors=True, ncv=20)
   return E[0]
def infinite_gs_energy(J, g):
    """For comparison: Calculate groundstate energy density from analytic formula.
    The analytic formula stems from mapping the model to free fermions, see P. Pfeuty,
→ The one-
   dimensional Ising model with a transverse field, Annals of Physics 57, p. 79,
\hookrightarrow (1970). Note that
   we use Pauli matrices compared this reference using spin-1/2 matrices and replace,
\hookrightarrowthe sum_k ->
   integral dk/2pi to obtain the result in the N -> infinity limit.
   def f(k, lambda_):
        return np.sqrt(1 + lambda_**2 + 2 * lambda_ * np.cos(k))
   E0_{exact} = -g / (J * 2. * np.pi) * scipy.integrate.quad(f, -np.pi, np.pi, args=(J_
\rightarrow / q, ))[0]
   return E0_exact
```

## 7.3.3 Example codes

The following "examples" are included in the TeNPy repository in the folder examples/, we include them here in the documentation for reference. Theses examples are meant to give an idea how to use the library and demonstrate parts of the interface. It might be helpful to compare some of them (e.g.,  $c_{tebd.py}$  and  $d_{teg.py}$ ) to the Toy Codes. To run these examples, you have to have TeNPy installed, see Installation instructions. You do not need to save the examples inside the tenpy folder/repository, but you can execute them from anywhere (if TeNPy is installed correctly).

#### Example code a\_np\_conserved.py

```
"""An example code to demonstrate the usage of :class:`~tenpy.linalg.np_conserved.
→Array`.
This example includes the following steps:
1) create Arrays for an Neel MPS
2) create an MPO representing the nearest-neighbour AFM Heisenberg Hamiltonian
3) define 'environments' left and right
4) contract MPS and MPO to calculate the energy
5) extract two-site hamiltonian ``H2`` from the MPO
6) calculate ``exp(-1.j*dt*H2)`` by diagonalization of H2
7) apply ``exp(H2)`` to two sites of the MPS and truncate with svd
Note that this example uses only np_conserved, but no other modules.
Compare it to the example `b_mps.py`,
which does the same steps using a few predefined classes like MPS and MPO.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
import numpy as np
# model parameters
Jxx, Jz = 1., 1.
L = 20
dt = 0.1
cutoff = 1.e-10
print("Jxx=\{Jxx\}, Jz=\{Jz\}, L=\{L:d\}".format(Jxx=Jxx, Jz=Jz, L=L))
print("1) create Arrays for an Neel MPS")
  vL ->--B-->- vR
#
#
# create a ChargeInfo to specify the nature of the charge
chinfo = npc.ChargeInfo([1], ['2*Sz']) # the second argument is just a descriptive_
→ name
# create LegCharges on physical leg and even/odd bonds
p_leg = npc.LegCharge.from_qflat(chinfo, [[1], [-1]]) # charges for up, down
v_leg_even = npc.LegCharge.from_qflat(chinfo, [[0]])
v_leg_odd = npc.LegCharge.from_qflat(chinfo, [[1]])
B_even = npc.zeros([v_leg_even, v_leg_odd.conj(), p_leg],
```

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```
labels=['vL', 'vR', 'p']) # virtual left/right, physical
B_odd = npc.zeros([v_leg_odd, v_leg_even.conj(), p_leg], labels=['vL', 'vR', 'p'])
B_{even[0, 0, 0]} = 1. # up
B_odd[0, 0, 1] = 1. \# down
Bs = [B_even, B_odd] * (L // 2) + [B_even] * (L % 2) # (right-canonical)
Ss = [np.ones(1)] * L # Ss[i] are singular values between Bs[i-1] and Bs[i]
# Side remark:
# An MPS is expected to have non-zero entries everywhere compatible with the charges.
# In general, we recommend to use `sort_legcharge` (or `as_completely_blocked`)
# to ensure complete blocking. (But the code will also work, if you don't do it.)
# The drawback is that this might introduce permutations in the indices of single.
# which you have to keep in mind when converting dense numpy arrays to and from npc.
⊶Arrays.
print("2) create an MPO representing the AFM Heisenberg Hamiltonian")
         p*
#
         1
  WL \longrightarrow --W-->-WR
         /
#
         1
#
         p
# create physical spin-1/2 operators Sz, S+, S-
Sz = npc.Array.from_ndarray([[0.5, 0.], [0., -0.5]], [p_leg, p_leg.conj()], labels=['p
Sp = npc.Array.from_ndarray([[0., 1.], [0., 0.]], [p_leg, p_leg.conj()], labels=['p',
→ 'p*'])
Sm = npc.Array.from_ndarray([[0., 0.], [1., 0.]], [p_leg, p_leg.conj()], labels=['p',
→ 'p*'])
Id = npc.eye_like(Sz, labels=Sz.get_leg_labels()) # identity
mpo_leg = npc.LegCharge.from_qflat(chinfo, [[0], [2], [-2], [0], [0]])
W_{grid} = [Id, Sp, Sm, Sz, None]
          [None, None, None, None, 0.5 * Jxx * Sm],
          [None, None, None, None, 0.5 * Jxx * Sp],
          [None, None, None, Jz * Sz ],
          [None, None, None, Id
                                               ]] # yapf:disable
W = npc.grid_outer(W_grid, [mpo_leg, mpo_leg.conj()], grid_labels=['wL', 'wR'])
\# wL/wR = virtual left/right of the MPO
Ws = [W] * L
print("3) define 'environments' left and right")
# .--->- vR
                vL ->---.
# /
\# envL->- wR
                 wL ->-envR
#
  .--->- vR*
                 VT, *->---
```

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```
 \texttt{envL} = \texttt{npc.zeros}([\texttt{W.get\_leg}('\texttt{wL'}).\texttt{conj}(), \ \texttt{Bs}[\texttt{0}].\texttt{get\_leg}('\texttt{vL'}).\texttt{conj}(), \ \texttt{bs}[\texttt{0}].
\hookrightarrow '\veeL')],
                           labels=['wR', 'vR', 'vR*'])
envL[0, :, :] = npc.diag(1., envL.legs[1])
envR = npc.zeros([W.get_leg('wR').conj(), Bs[-1].get_leg('vR').conj(), Bs[-1].get_leg(
\hookrightarrow 'VR')],
                           labels=['wL', 'vL', 'vL*'])
envR[-1, :, :] = npc.diag(1., envR.legs[1])
print("4) contract MPS and MPO to calculate the energy <psi|H|psi>")
contr = envL
for i in range(L):
      # contr labels: wR, vR, vR*
      contr = npc.tensordot(contr, Bs[i], axes=('vR', 'vL'))
      # wR, vR*, vR, p
      contr = npc.tensordot(contr, Ws[i], axes=(['p', 'wR'], ['p*', 'wL']))
      # vR*, vR, wR, p
      contr = npc.tensordot(contr, Bs[i].conj(), axes=(['p', 'vR*'], ['p*', 'vL*']))
      # vR, wR, vR*
      # note that the order of the legs changed, but that's no problem with labels:
      # the arrays are automatically transposed as necessary
E = npc.inner(contr, envR, axes=(['vR', 'wR', 'vR*'], ['vL', 'wL', 'vL*']))
print("E =", E)
print("5) calculate two-site hamiltonian ``H2`` from the MPO")
# label left, right physical legs with p, q
W0 = W.replace_labels(['p', 'p*'], ['p0', 'p0*'])
W1 = W.replace_labels(['p', 'p*'], ['p1', 'p1*'])
H2 = npc.tensordot(W0, W1, axes=('wR', 'wL')).itranspose(['wL', 'wR', 'p0', 'p1', 'p0*
\rightarrow', 'p1*'])
H2 = H2[0, -1] # (If H has single-site terms, it's not that simple anymore)
print("H2 labels:", H2.get_leg_labels())
print ("6) calculate exp(H2) by diagonalization of H2")
# diagonalization requires to view H2 as a matrix
H2 = H2.combine_legs([('p0', 'p1'), ('p0*', 'p1*')], qconj=[+1, -1])
print("labels after combine_legs:", H2.get_leg_labels())
E2, U2 = npc.eigh(H2)
print("Eigenvalues of H2:", E2)
U_expE2 = U2.scale_axis(np.exp(-1.j * dt * E2), axis=1) # scale_axis ~= apply an_

→ diagonal matrix
exp_H2 = npc.tensordot(U_expE2, U2.conj(), axes=(1, 1))
exp_H2.iset_leg_labels(H2.get_leg_labels())
exp_H2 = exp_H2.split_legs() # by default split all legs which are `LegPipe`
# (this restores the originial labels ['p0', 'p1', 'p0*', 'p1*'] of `H2` in `exp_H2`)
print("7) apply exp(H2) to even/odd bonds of the MPS and truncate with svd")
# (this implements one time step of first order TEBD)
for even_odd in [0, 1]:
      for i in range(even_odd, L - 1, 2):
            B_L = Bs[i].scale_axis(Ss[i], 'vL').ireplace_label('p', 'p0')
             B_R = Bs[i + 1].replace_label('p', 'pl')
            theta = npc.tensordot(B_L, B_R, axes=('vR', 'vL'))
            theta = npc.tensordot(exp_H2, theta, axes=(['p0*', 'p1*'], ['p0', 'p1']))
             # view as matrix for SVD
            theta = theta.combine_legs([('vL', 'p0'), ('p1', 'vR')], new_axes=[0, 1],
                                                                                                                     (continues on next page)
 \rightarrowqconj=[+1, -1])
```

```
# now theta has labels '(vL.p0)', '(p1.vR)'
U, S, V = npc.svd(theta, inner_labels=['vR', 'vL'])
# truncate
keep = S > cutoff
S = S[keep]
invsq = np.linalg.norm(S)
Ss[i + 1] = S / invsq
U = U.iscale_axis(S / invsq, 'vR')
Bs[i] = U.split_legs('(vL.p0)').iscale_axis(Ss[i]**(-1), 'vL').ireplace_label(
-'p0', 'p')
Bs[i + 1] = V.split_legs('(p1.vR)').ireplace_label('p1', 'p')
print("finished")
```

### Example code b\_mps.py

```
"""Simplified version of `a_np_conserved.py` making use of other classes (like MPS,_
This example includes the following steps:
1) create Arrays for an Neel MPS
2) create an MPO representing the nearest-neighbour AFM Heisenberg Hamiltonian
3) define 'environments' left and right
4) contract MPS and MPO to calculate the energy
5) extract two-site hamiltonian ``H2`` from the MPO
6) calculate ``exp(-1.j*dt*H2)`` by diagonalization of H2
7) apply ``exp(H2)`` to two sites of the MPS and truncate with svd
Note that this example performs the same steps as `a_np_conserved.py`,
but makes use of other predefined classes except npc.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
import numpy as np
# some more imports
from tenpy.networks.site import SpinHalfSite
from tenpy.models.lattice import Chain
from tenpy.networks.mps import MPS
from tenpy.networks.mpo import MPO, MPOEnvironment
from tenpy.algorithms.truncation import svd_theta
# model parameters
Jxx, Jz = 1., 1.
L = 20
dt = 0.1
cutoff = 1.e-10
print("Jxx=\{Jxx\}, Jz=\{Jz\}, L=\{L:d\}".format(Jxx=Jxx, Jz=Jz, L=L))
print("1) create Arrays for an Neel MPS")
site = SpinHalfSite(conserve='Sz') # predefined charges and Sp,Sm,Sz operators
p_leg = site.leg
chinfo = p_leg.chinfo
# make lattice from unit cell and create product state MPS
lat = Chain(L, site, bc_MPS='finite')
```

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```
state = ["up", "down"] * (L // 2) + ["up"] * (L % 2) # Neel state
print("state = ", state)
psi = MPS.from_product_state(lat.mps_sites(), state, lat.bc_MPS)
print("2) create an MPO representing the AFM Heisenberg Hamiltonian")
# predefined physical spin-1/2 operators Sz, S+, S-
Sz, Sp, Sm, Id = site.Sz, site.Sp, site.Sm, site.Id
mpo_leg = npc.LegCharge.from_qflat(chinfo, [[0], [2], [-2], [0], [0]])
W_grid = [[Id,
                 Sp,
                       Sm,
                             Sz,
                                   None
          [None, None, None, None, 0.5 * Jxx * Sm],
          [None, None, None, None, 0.5 * Jxx * Sp],
          [None, None, None, Jz * Sz
                                                1,
          [None, None, None, Id
                                                ]] # yapf:disable
W = npc.grid_outer(W_grid, [mpo_leg, mpo_leg.conj()], grid_labels=['wL', 'wR'])
# wL/wR = virtual left/right of the MPO
Ws = [W] * L
Ws[0] = W[:1, :]
Ws[-1] = W[:, -1:]
H = MPO(psi.sites, Ws, psi.bc, IdL=0, IdR=-1)
print("3) define 'environments' left and right")
# this is automatically done during initialization of MPOEnvironment
env = MPOEnvironment(psi, H, psi)
envL = env.qet_LP(0)
envR = env.get_RP(L - 1)
print("4) contract MPS and MPO to calculate the energy <psi|H|psi>")
E = env.full\_contraction(L - 1)
print("E = ", E)
print("5) calculate two-site hamiltonian ``H2`` from the MPO")
# label left, right physical legs with p, q
W0 = H.get_W(0).replace_labels(['p', 'p*'], ['p0', 'p0*'])
W1 = H.qet_W(1).replace_labels(['p', 'p*'], ['p1', 'p1*'])
H2 = npc.tensordot(W0, W1, axes=('wR', 'wL')).itranspose(['wL', 'wR', 'p0', 'p1', 'p0*
→', 'p1*'])
H2 = H2[H.IdL[0], H.IdR[2]] # (If H has single-site terms, it's not that simple_
→anvmore)
print("H2 labels:", H2.get_leg_labels())
print ("6) calculate exp(H2) by diagonalization of H2")
# diagonalization requires to view H2 as a matrix
H2 = H2.combine_legs([('p0', 'p1'), ('p0*', 'p1*')], qconj=[+1, -1])
print("labels after combine_legs:", H2.get_leg_labels())
E2, U2 = npc.eigh(H2)
print("Eigenvalues of H2:", E2)
U_expE2 = U2.scale_axis(np.exp(-1.j * dt * E2), axis=1) # scale_axis ~= apply a.

→ diagonal matrix
exp_H2 = npc.tensordot(U_expE2, U2.conj(), axes=(1, 1))
exp_H2.iset_leg_labels(H2.get_leg_labels())
exp_H2 = exp_H2.split_legs() # by default split all legs which are `LegPipe`
```

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```
# (this restores the originial labels ['p0', 'p1', 'p0*', 'p1*'] of `H2` in `exp_H2`)
# alternative way: use :func:`~tenpy.linalg.np_conserved.expm`
exp_H2_alternative = npc.expm(-1.j * dt * H2).split_legs()
assert (npc.norm(exp_H2_alternative - exp_H2) < 1.e-14)</pre>
print("7) apply exp(H2) to even/odd bonds of the MPS and truncate with svd")
# (this implements one time step of first order TEBD)
trunc_par = {'svd_min': cutoff, 'trunc_cut': None, 'verbose': 0}
for even_odd in [0, 1]:
   for i in range(even_odd, L - 1, 2):
        theta = psi.get_theta(i, 2) # handles canonical form (i.e. scaling with 'S')
        theta = npc.tensordot(exp_H2, theta, axes=(['p0*', 'p1*'], ['p0', 'p1']))
        # view as matrix for SVD
        theta = theta.combine_legs([('vL', 'p0'), ('p1', 'vR')], new_axes=[0, 1],...
\rightarrowqconj=[+1, -1])
        \# now theta has labels '(vL.p0)', '(p1.vR)'
        U, S, V, err, invsq = svd_theta(theta, trunc_par, inner_labels=['vR', 'vL'])
        psi.set_SR(i, S)
        A_L = U.split_legs('(vL.p0)').ireplace_label('p0', 'p')
        B_R = V.split_legs('(p1.vR)').ireplace_label('p1', 'p')
        psi.set_B(i, A_L, form='A') # left-canonical form
        psi.set_B(i + 1, B_R, form='B') # right-canonical form
print("finished")
```

#### Example code c\_tebd.py

```
"""Example illustrating the use of TEBD in tenpy.
The example functions in this class do the same as the ones in `toycodes/c_tebd.py`, _
⇒but make use
of the classes defined in tenpy.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import tebd
def example_TEBD_gs_tf_ising_finite(L, g, verbose=True):
   print("finite TEBD, imaginary time evolution, transverse field Ising")
   print("L=\{L:d\}, g=\{g:.2f\}".format(L=L, g=g))
   model_params = dict(L=L, J=1., g=g, bc_MPS='finite', conserve=None,_
→verbose=verbose)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
   tebd_params = {
        'order': 2,
        'delta_tau_list': [0.1, 0.01, 0.001, 1.e-4, 1.e-5],
        'N_steps': 10,
        'max_error_E': 1.e-6,
```

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```
'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        'verbose': verbose,
   eng = tebd.Engine(psi, M, tebd_params)
   eng.run_GS() # the main work...
    # expectation values
    E = np.sum(M.bond_energies(psi)) # M.bond_energies() works only a for_
→ NearestNeighborModel
    # alternative: directly measure E2 = np.sum(psi.expectation_value(M.H_bond[1:]))
   print("E = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.sum(psi.expectation_value("Sigmax"))
   mag_z = np.sum(psi.expectation_value("Sigmaz"))
    print("magnetization in X = {mag_x:.5f}".format(mag_x=mag_x))
    print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
    if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = {E:.13f}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_TEBD_gs_tf_ising_infinite(g, verbose=True):
    print ("infinite TEBD, imaginary time evolution, transverse field Ising")
    print("g = \{g:.2f\}".format(g = g))
   model_params = dict(L=2, J=1., g=g, bc_MPS='infinite', conserve=None,...
→verbose=verbose)
   M = TFIChain(model_params)
    product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
   tebd_params = {
        'order': 2,
        'delta_tau_list': [0.1, 0.01, 0.001, 1.e-4, 1.e-5],
        'N_steps': 10,
        'max_error_E': 1.e-8,
        'trunc params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        'verbose': verbose,
    eng = tebd.Engine(psi, M, tebd_params)
   eng.run_GS() # the main work...
    E = np.mean(M.bond_energies(psi)) # M.bond_energies() works only a for_
\rightarrow NearestNeighborModel
    \# alternative: directly measure E2 = np.mean(psi.expectation\_value(M.H\_bond))
   print("E (per site) = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.mean(psi.expectation_value("Sigmax"))
   mag_z = np.mean(psi.expectation_value("Sigmaz"))
   print("<sigma_x> = {mag_x:.5f}".format(mag_x=mag_x))
   print("<sigma_z> = \{mag_z:.5f\}".format(mag_z=mag_z))
```

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```
print("correlation length:", psi.correlation_length())
    # compare to exact result
   from tfi_exact import infinite_gs_energy
    E_exact = infinite_gs_energy(1., g)
    print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
    print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_TEBD_tf_ising_lightcone(L, g, tmax, dt, verbose=True):
   print("finite TEBD, real time evolution")
    print("L=\{L:d\}, g=\{g:.2f\}, tmax=\{tmax:.2f\}, dt=\{dt:.3f\}".format(L=L, g=g,_
\rightarrowtmax=tmax, dt=dt))
    # find ground state with TEBD or DMRG
    # E, psi, M = example_TEBD_qs_tf_ising_finite(L, q)
   from d_dmrg import example_DMRG_tf_ising_finite
   print("(run DMRG to get the groundstate)")
   E, psi, M = example_DMRG_tf_ising_finite(L, g, verbose=False)
   print("(DMRG finished)")
    i0 = L // 2
    # apply sigmaz on site i0
   psi.apply_local_op(i0, 'Sigmaz', unitary=True)
   dt_measure = 0.05
    # tebd.Engine makes 'N_steps' steps of `dt` at once; for second order this is_
→more efficient.
   tebd_params = {
        'order': 2,
        'dt': dt,
        'N_steps': int(dt_measure / dt + 0.5),
        'trunc_params': {
            'chi_max': 50,
            'svd_min': 1.e-10,
            'trunc_cut': None
        }.
        'verbose': verbose,
    }
   eng = tebd.Engine(psi, M, tebd_params)
    S = [psi.entanglement_entropy()]
   for n in range(int(tmax / dt_measure + 0.5)):
        eng.run()
        S.append(psi.entanglement_entropy())
    import matplotlib.pyplot as plt
   plt.figure()
   plt.imshow(S[::-1],
               vmin=0.,
               aspect='auto',
               interpolation='nearest',
               extent=(0, L - 1., -0.5 * dt_measure, eng.evolved_time + 0.5 * dt_
→measure))
   plt.xlabel('site $i$')
   plt.ylabel('time $t/J$')
   plt.ylim(0., tmax)
   plt.colorbar().set_label('entropy $S$')
   filename = 'c_tebd_lightcone_{g:.2f}.pdf'.format(g=g)
   plt.savefig(filename)
   print("saved " + filename)
```

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```
def example_TEBD_gs_tf_ising_next_nearest_neighbor(L, g, Jp, verbose=True):
    from tenpy.models.spins_nnn import SpinChainNNN2
    from tenpy.models.model import NearestNeighborModel
    print("finite TEBD, imaginary time evolution, transverse field Ising next-nearest_
→neighbor")
   print ("L=\{L:d\}, q=\{q:.2f\}, Jp=\{Jp:.2f\}".format (L=L, q=q, Jp=Jp))
    model_params = dict(L=L,
                        Jx=1.
                        Jy=0.,
                        Jz=0.,
                        Jxp=Jp,
                        Jyp=0.,
                        Jzp=0.,
                        hz=q,
                        bc_MPS='finite',
                        conserve=None,
                        verbose=verbose)
    # we start with the non-grouped sites, but next-nearest neighbor interactions, ...
⇒building the MPO
   M = SpinChainNNN2(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    # now we group each to sites ...
   psi.group_sites(n=2) # ... in the state
   M.group_sites(n=2) # ... and model
    # now, M has only 'nearest-neighbor' interactions with respect to the grouped,
-sites
    # thus, we can convert the MPO into H_bond terms:
   M_nn = NearestNeighborModel.from_MPOModel(M) # hence, we can initialize H_bond,
\rightarrow from the MPO
    # now, we continue to run TEBD as before
    tebd_params = {
        'order': 2,
        'delta_tau_list': [0.1, 0.01, 0.001, 1.e-4, 1.e-5],
        'N_steps': 10,
        'max_error_E': 1.e-6,
        'trunc_params': {
            'chi max': 30,
            'svd_min': 1.e-10
        'verbose': verbose,
   eng = tebd.Engine(psi, M_nn, tebd_params) # use M_nn and grouped psi
   eng.run_GS() # the main work...
    # expectation values:
   E = np.sum(M_nn.bond_energies(psi)) # bond_energies() works only a for_
→ NearestNeighborModel
   print("E = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
    # we can split the sites of the state again for an easier evaluation of...
→expectation values
   psi.group_split()
   mag_x = 2. * np.sum(psi.expectation_value("Sx")) # factor of 2 for Sx vs Sigmax
```

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```
mag_z = 2. * np.sum(psi.expectation_value("Sz"))
print("magnetization in X = {mag_x:.5f}".format(mag_x=mag_x))
print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
return E, psi, M

if __name__ == "__main__":
    example_TEBD_gs_tf_ising_finite(L=10, g=1.)
    print("-" * 100)
    example_TEBD_gs_tf_ising_infinite(g=1.5)
    print("-" * 100)
    example_TEBD_tf_ising_lightcone(L=20, g=1.5, tmax=3., dt=0.01)
    print("-" * 100)
    example_TEBD_gs_tf_ising_next_nearest_neighbor(L=10, g=1.0, Jp=0.1)
```

### Example code d\_dmrg.py

```
"""Example illustrating the use of DMRG in tenpy.
The example functions in this class do the same as the ones in `toycodes/d_dmrg.py`,
but make use of the classes defined in tenpy.
.. todo ::
Docstrings?
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.models.spins import SpinModel
from tenpy.algorithms import dmrg
def example_DMRG_tf_ising_finite(L, g, verbose=True):
   print("finite DMRG, transverse field Ising model")
   print("L=\{L:d\}, g=\{g:.2f\}".format(L=L, g=g))
   model_params = dict(L=L, J=1., g=g, bc_MPS='finite', conserve=None,_
→verbose=verbose)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
   dmrg_params = {
        'mixer': None, # setting this to True helps to escape local minima
        'max_E_err': 1.e-10,
        'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        'verbose': verbose,
        'combine': True
   info = dmrg.run(psi, M, dmrg_params) # the main work...
   E = info['E']
```

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```
print("E = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.sum(psi.expectation_value("Sigmax"))
   mag_z = np.sum(psi.expectation_value("Sigmaz"))
    print("magnetization in X = {mag_x:.5f}".format(mag_x=mag_x))
    print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
    if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = {E:.13f}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_1site_DMRG_tf_ising_finite(L, g, verbose=True):
    print("single-site finite DMRG, transverse field Ising model")
   print ("L={L:d}, g=\{g:.2f\}".format (L=L, g=g))
   model_params = dict(L=L, J=1., g=g, bc_MPS='finite', conserve=None,_
→verbose=verbose)
   M = TFIChain(model_params)
    product_state = ["up"] * M.lat.N_sites
    psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    dmrg_params = {
        'mixer': True, # setting this to True is essential for the 1-site algorithm,
\hookrightarrowto work.
        'max_E_err': 1.e-10,
        'trunc_params': {
            'chi max': 30,
            'svd_min': 1.e-10
        },
        'verbose': verbose,
        'combine': False,
        'active_sites': 1 # specifies single-site
    info = dmrg.run(psi, M, dmrg_params)
   E = info['E']
   print ("E = \{E:.13f\}".format (E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.sum(psi.expectation_value("Sigmax"))
   mag_z = np.sum(psi.expectation_value("Sigmaz"))
   print("magnetization in X = \{mag_x : .5f\}".format(mag_x=mag_x))
   print("magnetization in Z = {mag_z:.5f}".format(mag_z=mag_z))
    if L < 20: # compare to exact result</pre>
        from tfi_exact import finite_gs_energy
        E_exact = finite_gs_energy(L, 1., g)
        print("Exact diagonalization: E = \{E:.13f\}".format(E=E_exact))
        print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_DMRG_tf_ising_infinite(q, verbose=True):
    print("infinite DMRG, transverse field Ising model")
   print ("g = \{g: .2f\}".format (g = g))
   model_params = dict(L=2, J=1., g=g, bc_MPS='infinite', conserve=None,_
→verbose=verbose)
   M = TFIChain(model_params)
    product_state = ["up"] * M.lat.N_sites
```

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```
psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    dmrg_params = {
        'mixer': True, # setting this to True helps to escape local minima
        'trunc_params': {
            'chi_max': 30,
            'svd_min': 1.e-10
        },
        'max_E_err': 1.e-10,
        'verbose': verbose,
    }
    # Sometimes, we want to call a 'DMRG engine' explicitly
    eng = dmrg.TwoSiteDMRGEngine(psi, M, dmrg_params)
    E, psi = eng.run() # equivalent to dmrg.run() up to the return parameters.
   print("E = \{E:.13f\}".format(E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.mean(psi.expectation_value("Sigmax"))
   mag_z = np.mean(psi.expectation_value("Sigmaz"))
   print(" < sigma_x > = \{mag_x : .5f\}".format(mag_x = mag_x))
    print("<sigma_z> = \{mag_z:.5f\}".format(mag_z=mag_z))
    print("correlation length:", psi.correlation_length())
    # compare to exact result
    from tfi_exact import infinite_gs_energy
   E_exact = infinite_gs_energy(1., g)
    print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
   print("relative error: ", abs((E - E_exact) / E_exact))
    return E, psi, M
def example_1site_DMRG_tf_ising_infinite(g, verbose=True):
   print("single-site infinite DMRG, transverse field Ising model")
    print ("g = \{g: .2f\}".format (g = g))
   model_params = dict(L=2, J=1., g=g, bc_MPS='infinite', conserve=None,_
→verbose=verbose)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
    psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
    dmrg_params = {
        'mixer': True, # setting this to True is essential for the 1-site algorithm.
⇔to work.
        'trunc_params': {
            'chi max': 30,
            'svd_min': 1.e-10
        'max_E_err': 1.e-10,
        'verbose': verbose,
        'combine': True
    }
   eng = dmrg.SingleSiteDMRGEngine(psi, M, dmrg_params)
   E, psi = eng.run()  # equivalent to dmrg.run() up to the return parameters.
   print ("E = \{E:.13f\}".format (E=E))
   print("final bond dimensions: ", psi.chi)
   mag_x = np.mean(psi.expectation_value("Sigmax"))
   mag_z = np.mean(psi.expectation_value("Sigmaz"))
   print ("<sigma_x> = \{mag_x:.5f\}".format (mag_x=mag_x))
   print ("<sigma_z> = \{mag_z:.5f\}".format (mag_z=mag_z))
   print("correlation length:", psi.correlation_length())
    # compare to exact result
```

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```
from tfi_exact import infinite_gs_energy
   E_exact = infinite_gs_energy(1., g)
   print("Analytic result: E (per site) = {E:.13f}".format(E=E_exact))
   print("relative error: ", abs((E - E_exact) / E_exact))
def example_DMRG_heisenberg_xxz_infinite(Jz, conserve='best', verbose=True):
   print("infinite DMRG, Heisenberg XXZ chain")
   print("Jz={Jz:.2f}, conserve={conserve!r}".format(Jz=Jz, conserve=conserve))
   model_params = dict(
       L=2,
       S=0.5,
               # spin 1/2
       Jx=1.,
        Jy=1.,
        Jz=Jz, # couplings
       bc_MPS='infinite',
       conserve=conserve,
       verbose=verbose)
   M = SpinModel(model_params)
    product_state = ["up", "down"] # initial Neel state
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
   dmrg_params = {
        'mixer': True, # setting this to True helps to escape local minima
        'trunc_params': {
            'chi_max': 100,
            'svd_min': 1.e-10,
        },
        'max_E_err': 1.e-10,
        'verbose': verbose,
   info = dmrg.run(psi, M, dmrg_params)
   E = info['E']
   print ("E = \{E:.13f\}".format (E=E))
   print("final bond dimensions: ", psi.chi)
   Sz = psi.expectation_value("Sz") # Sz instead of Sigma z: spin-1/2 operators!
   maq_z = np.mean(Sz)
   print("<S_z> = [\{Sz0:.5f\}, \{Sz1:.5f\}]; mean =\{mag_z:.5f\}".format(Sz0=Sz[0],
                                                                      Sz1=Sz[1],
                                                                      mag_z=mag_z))
    # note: it's clear that mean(<Sz>) is 0: the model has Sz conservation!
   print("correlation length:", psi.correlation_length())
   corrs = psi.correlation_function("Sz", "Sz", sites1=range(10))
   print("correlations <Sz_i Sz_j> =")
   print(corrs)
   return E, psi, M
if __name__ == "__main__":
   example_DMRG_tf_ising_finite(L=10, g=1., verbose=True)
   print("-" * 100)
   example_1site_DMRG_tf_ising_finite(L=10, g=1., verbose=True)
   print("-" * 100)
   example_DMRG_tf_ising_infinite(g=1.5, verbose=True)
   print("-" * 100)
   example_1site_DMRG_tf_ising_infinite(g=1.5, verbose=True)
   print("-" * 100)
   example_DMRG_heisenberg_xxz_infinite(Jz=1.5)
```

# Example code e\_tdvp.py

```
"""Example illustrating the use of TDVP in tenpy.
As of now, we have TDVP only for finite systems. The call structure is quite similar,
\rightarrowto TEBD. A
difference is that we can run one-site TDVP or two-site TDVP. In the former, the bond,
→dimension can
not grow; the latter allows to grow the bond dimension and hence requires a,
⇔truncation.
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3
import numpy as np
import tenpy.linalg.np_conserved as npc
import tenpy.models.spins
import tenpy.networks.mps as mps
import tenpy.networks.site as site
from tenpy.algorithms import tdvp
from tenpy.networks.mps import MPS
import copy
def run_out_of_equilibrium():
   L = 10
   chi = 5
   delta_t = 0.1
   model_params = {
        'L': L,
        'S': 0.5,
        'conserve': 'Sz',
        'Jz': 1.0,
        'Jy': 1.0,
        'Jx': 1.0,
        'hx': 0.0,
        'hy': 0.0,
        'hz': 0.0,
        'muJ': 0.0,
        'bc_MPS': 'finite',
   heisenberg = tenpy.models.spins.SpinChain(model_params)
    product_state = ["up"] * (L // 2) + ["down"] * (L - L // 2)
    # starting from a domain-wall product state which is not an eigenstate of the
→ Heisenberg model
    psi = MPS.from_product_state(heisenberg.lat.mps_sites(),
                                 product_state,
                                 bc=heisenberg.lat.bc_MPS,
                                 form='B')
    tdvp_params = {
        'start_time': 0,
        'dt': delta_t,
        'trunc_params': {
            'chi_max': chi,
            'svd_min': 1.e-10,
            'trunc_cut': None
        }
```

(continues on next page)

```
tdvp_engine = tdvp.Engine(psi, heisenberg, tdvp_params)
   times = []
   S_mid = []
    for i in range(30):
       tdvp_engine.run_two_sites(N_steps=1)
        times.append(tdvp_engine.evolved_time)
       S_mid.append(psi.entanglement_entropy(bonds=[L // 2])[0])
    for i in range(30):
       tdvp_engine.run_one_site(N_steps=1)
        #psi_2=copy.deepcopy(psi)
        #psi_2.canonical_form()
       times.append(tdvp_engine.evolved_time)
        S_mid.append(psi.entanglement_entropy(bonds=[L // 2])[0])
   import matplotlib.pyplot as plt
   plt.figure()
   plt.plot(times, S_mid)
   plt.xlabel('t')
   plt.ylabel('S')
   plt.axvline(x=3.1, color='red')
   plt.text(0.0, 0.0000015, "Two sites update")
   plt.text(3.1, 0.0000015, "One site update")
   plt.show()
if __name__ == "__main__":
   run_out_of_equilibrium()
```

#### Example code tfi\_exact.py

```
"""Provides exact ground state energies for the transverse field ising model for ...
⇔comparison.
The Hamiltonian reads
.. math ::
   H = -J \setminus sum_{i} \setminus sigma^x_i \setminus sigma^x_{i+1} - g \setminus sum_{i} \setminus sigma^z_i
# Copyright 2019-2020 TeNPy Developers, GNU GPLv3
import numpy as np
import scipy.sparse as sparse
import scipy.sparse.linalg.eigen.arpack as arp
import warnings
import scipy.integrate
def finite_gs_energy(L, J, g):
    """For comparison: obtain ground state energy from exact diagonalization.
    Exponentially expensive in L, only works for small enough `L` <~ 20.
    m m m
    if L >= 20:
        warnings.warn("Large L: Exact diagonalization might take a long time!")
    # get single site operaors
    sx = sparse.csr_matrix(np.array([[0., 1.], [1., 0.]]))
```

(continues on next page)

```
sz = sparse.csr_matrix(np.array([[1., 0.], [0., -1.]]))
    id = sparse.csr_matrix(np.eye(2))
    sx_list = [] # sx_list[i] = kron([id, id, ..., id, sx, id, .... id])
    sz_list = []
    for i_site in range(L):
        x_{ops} = [id] * L
        z\_ops = [id] * L
        x_{ops[i\_site]} = sx
        z_{ops[i\_site]} = sz
        X = x_ops[0]
        Z = z_{ops}[0]
        for j in range(1, L):
            X = sparse.kron(X, x_ops[j], 'csr')
            Z = sparse.kron(Z, z_ops[j], 'csr')
        sx_list.append(X)
        sz_list.append(Z)
    H_xx = sparse.csr_matrix((2**L, 2**L))
    H_z = sparse.csr_matrix((2**L, 2**L))
    for i in range(L - 1):
        H_xx = H_xx + sx_list[i] * sx_list[(i + 1) % L]
    for i in range(L):
       H_z = H_z + sz_{list[i]}
    H = -J * H_xx - g * H_z
   E, V = arp.eigsh(H, k=1, which='SA', return_eigenvectors=True, ncv=20)
    return E[0]
def infinite_gs_energy(J, g):
    """For comparison: Calculate groundstate energy density from analytic formula.
   The analytic formula stems from mapping the model to free fermions, see P. Pfeuty,
→ The one-
    dimensional Ising model with a transverse field, Annals of Physics 57, p. 79,
\hookrightarrow (1970). Note that
   we use Pauli matrices compared this reference using spin-1/2 matrices and replace.
\hookrightarrowthe sum_k ->
   integral dk/2pi to obtain the result in the N -> infinity limit.
   def f(k, lambda_):
        return np.sqrt(1 + lambda_**2 + 2 * lambda_ * np.cos(k))
   E0_{exact} = -g / (J * 2. * np.pi) * scipy.integrate.quad(f, -np.pi, np.pi, args=(J_i)
\rightarrow / g, ))[0]
   return E0_exact
```

### Example code purication.py

### Example code z\_exact\_diag.py

```
"""A simple example comparing DMRG output with full diagonalization (ED).

Sorry that this is not well documented! ED is meant to be used for debugging only;)

"""

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```

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```
import tenpy.linalg.np_conserved as npc
from tenpy.models.xxz_chain import XXZChain
from tenpy.networks.mps import MPS
from tenpy.algorithms.exact_diag import ExactDiag
from tenpy.algorithms import dmrg
def example_exact_diagonalization(L, Jz):
   xxz_pars = dict(L=L, Jxx=1., Jz=Jz, hz=0.0, bc_MPS='finite')
   M = XXZChain(xxz_pars)
   product_state = ["up", "down"] * (xxz_pars['L'] // 2) # this selects a charge...
⇔sector!
   psi_DMRG = MPS.from_product_state(M.lat.mps_sites(), product_state)
   charge_sector = psi_DMRG.get_total_charge(True) # ED charge sector should match
   ED = ExactDiag(M, charge_sector=charge_sector, max_size=2.e6)
   ED.build_full_H_from_mpo()
    # ED.build_full_H_from_bonds() # whatever you prefer
   print("start diagonalization")
   ED.full_diagonalization() # the expensive part for large L
   E0_ED, psi_ED = ED.groundstate() # return the ground state
   print("psi_ED =", psi_ED)
   print("run DMRG")
   dmrg.run(psi_DMRG, M, {'verbose': 0}) # modifies psi_DMRG in place!
    # first way to compare ED with DMRG: convert MPS to ED vector
   psi_DMRG_full = ED.mps_to_full(psi_DMRG)
   print("psi_DMRG_full =", psi_DMRG_full)
   ov = npc.inner(psi_ED, psi_DMRG_full, axes='range', do_conj=True)
   print("<psi_ED|psi_DMRG_full> =", ov)
   assert (abs(abs(ov) - 1.) < 1.e-13)
    # second way: convert ED vector to MPS
   psi_ED_mps = ED.full_to_mps(psi_ED)
   ov2 = psi_ED_mps.overlap(psi_DMRG)
   print("<psi_ED_mps|psi_DMRG> =", ov2)
   assert (abs(abs(ov2) - 1.) < 1.e-13)
   assert (abs (ov - ov2) < 1.e-13)
    # -> advantage: expectation value etc. of MPS are available!
   print("<Sz> =", psi_ED_mps.expectation_value('Sz'))
if __name__ == "__main__":
    example_exact_diagonalization(10, 1.)
```

# **Advanced examples**

The following "advanced examples" are included in the TeNPy repository in the folder examples/advanced, we include them here in the documentation for reference. Theses "advanced examples" go beyond the basic usage of the algorithm, but can give you an idea for certain tasks. It's a somewhat random collection, feel free to suggest further examples.

# Advanced example xxz\_corr\_length.py

```
"""Calculate the correleation legnth of the transferse field Ising model for various_
\hookrightarrow h_zz.
This example uses DMRG to find the ground state of the transverse field Ising model.
→when tuning
through the phase transition by changing the field `hz`. It uses
:meth:`~tenpy.networks.mps.MPS.correlation_length` to extract the correlation length_
→of the ground
state, and plots it vs. hz in the end.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
from tenpy.models.spins import SpinChain
from tenpy.networks.mps import MPS
from tenpy.algorithms import dmrq
import matplotlib.pyplot as plt
def run(Jzs):
   L = 2
   model_params = dict(L=L, Jx=1., Jy=1., Jz=Jzs[0], bc_MPS='infinite', conserve='Sz
\hookrightarrow', verbose=0)
   chi = 300
    dmrg_params = {
        'trunc_params': {
            'chi_max': chi,
            'svd_min': 1.e-10,
            'trunc_cut': None
        },
        'update_env': 20,
        'start_env': 20,
        'max_E_err': 0.0001,
        'max_S_err': 0.0001,
        'verbose': 1,
        'mixer': False
    }
   M = SpinChain(model_params)
   psi = MPS.from_product_state(M.lat.mps_sites(), (["up", "down"] * L)[:L], M.lat.
→bc_MPS)
   engine = dmrg.TwoSiteDMRGEngine(psi, M, dmrg_params)
   np.set_printoptions(linewidth=120)
    corr_length = []
    for Jz in Jzs:
```

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```
print("-" * 80)
        print("Jz = ", Jz)
        print("-" * 80)
        model_params['Jz'] = Jz
        M = SpinChain(model_params)
        engine.init_env(model=M) # (re)initialize DMRG environment with new model
        # this uses the result from the previous DMRG as first initial quess
        engine.run()
        # psi is modified by engine.run() and now represents the ground state for the
\hookrightarrow current `Jz`.
        corr_length.append(psi.correlation_length(tol_ev0=1.e-3))
        print("corr. length", corr_length[-1])
        print("<Sz>", psi.expectation_value('Sz'))
        dmrg_params['start_env'] = 0 # (some of) the parameters are read out again
    corr_length = np.array(corr_length)
    results = {
        'model_params': model_params,
        'dmrg_params': dmrg_params,
        'Jzs': Jzs,
        'corr_length': corr_length,
        'eval_transfermatrix': np.exp(-1. / corr_length)
    return results
def plot(results, filename):
   corr_length = results['corr_length']
   Jzs = results['Jzs']
   plt.plot(Jzs, np.exp(-1. / corr_length))
   plt.xlabel(r'$J_z/J_x$')
   plt.ylabel(r'$t = \exp(-\frac{1}{xi})$')
   plt.savefig(filename)
    print("saved to " + filename)
if __name__ == "__main__":
   filename = 'xxz_corrlength.pkl'
    import pickle
    import os.path
   if not os.path.exists(filename):
        results = run(list(np.arange(4.0, 1.5, -0.25)) + list(np.arange(1.5, 0.8, -0.8))
\hookrightarrow05)))
        with open(filename, 'wb') as f:
            pickle.dump(results, f)
    else:
        print("just load the data")
        with open(filename, 'rb') as f:
            results = pickle.load(f)
    plot(results, filename[:-4] + '.pdf')
```

# Advanced example central\_charge\_ising.py

```
"""Example to extract the central charge from the entranglement scaling.
This example code evaluate the central charge of the transverse field Ising model,
→using IDMRG.
The expected value for the central charge c = 1/2. The code always recycle the
\rightarrowenvironment from
the previous simulation, which can be seen at the "age".
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
import tenpy
import time
from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import dmrg
def example_DMRG_tf_ising_infinite_S_xi_scaling(g):
   model_params = dict(L=2, J=1., g=g, bc_MPS='infinite', conserve='best', verbose=0)
   M = TFIChain(model_params)
   product_state = ["up"] * M.lat.N_sites
   psi = MPS.from_product_state(M.lat.mps_sites(), product_state, bc=M.lat.bc_MPS)
   dmrg_params = {
        'start_env': 10,
        'mixer': False,
        # 'mixer_params': {'amplitude': 1.e-3, 'decay': 5., 'disable_after': 50},
        'trunc_params': {
            'chi_max': 5,
            'svd_min': 1.e-10
        'max_E_err': 1.e-9,
        'max_S_err': 1.e-6,
        'update_env': 0,
        'verbose': 0
   }
   chi_list = np.arange(7, 31, 2)
   s list = []
   xi_list = []
   eng = dmrg.TwoSiteDMRGEngine(psi, M, dmrg_params)
   for chi in chi_list:
        t0 = time.time()
       eng.reset_stats(
       ) # necessary if you for example have a fixed numer of sweeps, if you don't,
→set this you option your simulation stops after initial number of sweeps!
       eng.trunc_params['chi_max'] = chi
        ## DMRG Calculation
       print("Start IDMRG CALCULATION")
       eng.run()
        eng.options['mixer'] = None
       psi.canonical_form()
```

(continues on next page)

```
Calculating bond entropy and correlation length ##
        s_list.append(np.mean(psi.entanglement_entropy()))
        xi_list.append(psi.correlation_length())
        print (chi,
              time.time() - t0,
              np.mean(psi.expectation_value(M.H_bond)),
              s_list[-1],
              xi_list[-1],
              flush=True)
        tenpy.tools.optimization.optimize(3) # quite some speedup for small chi
        print("SETTING NEW BOND DIMENSION")
   return s_list, xi_list
def fit_plot_central_charge(s_list, xi_list, filename):
    """Plot routine in order to determine the cental charge."""
    import matplotlib.pyplot as plt
    from scipy.optimize import curve_fit
   def fitFunc(Xi, c, a):
       return (c / 6) * np.log(Xi) + a
   Xi = np.array(xi_list)
   S = np.array(s_list)
   LXi = np.log(Xi) # Logarithm of the correlation length xi
   fitParams, fitCovariances = curve_fit(fitFunc, Xi, S)
    # Plot fitting parameter and covariances
   print('c =', fitParams[0], 'a =', fitParams[1])
   print('Covariance Matrix', fitCovariances)
    # plot the data as blue circles
   plt.errorbar(LXi,
                 fmt='o',
                 c='blue',
                 ms=5.5
                 markerfacecolor='white',
                 markeredgecolor='blue',
                 markeredgewidth=1.4)
    # plot the fitted line
   plt.plot(LXi,
             fitFunc(Xi, fitParams[0], fitParams[1]),
             linewidth=1.5,
             c='black',
             label='fit c={c:.2f}'.format(c=fitParams[0]))
   plt.xlabel(r'$\log{\, }xi_{\cosh}$', fontsize=16)
   plt.ylabel(r'$S$', fontsize=16)
   plt.legend(loc='lower right', borderaxespad=0., fancybox=True, shadow=True,...
→fontsize=16)
   plt.savefig(filename)
```

(continues on next page)

```
if __name__ == "__main__":
    s_list, xi_list = example_DMRG_tf_ising_infinite_S_xi_scaling(g=1)
    fit_plot_central_charge(s_list, xi_list, "central_charge_ising.pdf")
```

# Advanced example mpo\_exponential\_decay.py

```
"""Demonstration of the mpo.MPO.from_grids method.
We construct a MPO model for a spin 1/2 Heisenberg chain with an infinite number of
2-sites interactions, with strength that decays exponentially with the distance
\rightarrowbetween the sites.
Because of the infinite number of couplings it is not possible to construct the MPO_
coupling model. However the tensors that form the MPO have a surprisingly simple
form (see the grid below).
We run the iDMRG algorithm to find the ground state and energy density of the
system in the thermodynamic limit.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
import tenpy.linalg.np_conserved as npc
from tenpy.networks.mpo import MPO
from tenpy.networks.mps import MPS
from tenpy.networks.site import SpinHalfSite
from tenpy.models.model import MPOModel
from tenpy.models.lattice import Chain
from tenpy.algorithms import dmrg
from tenpy.tools.params import asConfig
class ExponentiallyDecayingHeisenberg(MPOModel):
   r"""Spin-1/2 Heisenberg Chain with exponentially decaying interactions.
    The Hamiltonian reads:
    .. math ::
        H = \sum_{j>i} \exp(-\frac{j-i-1}{\mathrm{xi}}) (
                  \mathcal{J}_{XX}/2 (S^{+}_i S^{-}_j + S^{-}_i S^{+}_j)
                + \mathtt{Jz} S^z_i S^z_j
            - \sum_i \mathtt{hz} S^z_i
   All parameters are collected in a single dictionary `model_params`.
    Parameters
    L : int
       Length of the chain.
   Jxx, Jz, hz, xi: float
```

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```
Coupling parameters as defined for the Hamiltonian above.
   bc_MPS : {'finite' | 'infinte'}
      MPS boundary conditions.
   conserve : 'Sz' | 'parity' | None
      What should be conserved. See :class:`~tenpy.networks.Site.SpinHalfSite`.
   def __init__(self, model_params):
       # model parameters
       model_params = asConfig(model_params, "ExponentiallyDecayingHeisenberg")
       L = model_params.get('L', 2)
       xi = model_params.get('xi', 0.5)
       Jxx = model_params.get('Jxx', 1.)
       Jz = model_params.get('Jz', 1.5)
       hz = model_params.get('hz', 0.)
       conserve = model_params.get('conserve', 'Sz')
       if xi == 0.:
          g = 0.
       elif xi == np.inf:
          g = 1.
       else:
           q = np.exp(-1 / (xi))
       # Define the sites and the lattice, which in this case is a simple uniform,
⇔ chain
       # of spin 1/2 sites
       site = SpinHalfSite(conserve=conserve)
       lat = Chain(L, site, bc_MPS='infinite', bc='periodic')
       # The operators that appear in the Hamiltonian. Standard spin operators are
       # already defined for the spin 1/2 site, but it is also possible to add new
       # operators using the add_op method
       Sz, Sp, Sm, Id = site.Sz, site.Sp, site.Sm, site.Id
       # yapf:disable
       # The grid (list of lists) that defines the MPO. It is possible to define the
       # operators in the grid in the following ways:
       # 1) NPC arrays, defined above:
                                       -hz*Sz
       grid = [[Id, Sp, Sm, Sz,
               [None, g*Id, None, None, 0.5*Jxx*Sm],
               [None, None, q*Id, None, 0.5*Jxx*Sp],
               [None, None, None, g*Id, Jz*Sz
                                                1.
               [None, None, None, Id
                                                 ]]
       # 2) In the form [("OpName", strength)], where "OpName" is the name of the
       # operator (e.g. "Sm" for Sm) and "strength" is a number that multiplies it.
       grid = [[[("Id", 1)], [("Sp",1)], [("Sm",1)], [("Sz",1)], [("Sz", -hz)] ],
                                                          , [("Sm", 0.5*Jxx)]],
                          , [("Id",g)], None , None
               None
                                   , [("Id",g)], None
                                                             , [("Sp", 0.5*Jxx)]],
               None
                           , None
                                              , [("Id",g)], [("Sz",Jz)]
                          , None
                                      , None
               None
                                                                               1,
                           , None
                                   , None
                                                  , None
                                                           , [("Id",1)]
               None
                                                                                11
       # 3) It is also possible to write a single "OpName", equivalent to
       # [("OpName", 1)].
                                                  , "Sz"
                          , "Sp"
       grid = [["Id"
                                     , "Sm"
                                                            , [("Sz", -hz)]
                                                             , [("Sm", 0.5*Jxx)]],
                          , [("Id",g)], None
               None
                                                  , None
                                   , [("Id",g)], None
                          , None
                                                              , [("Sp", 0.5*Jxx)]],
               None
                                      , None
                          , None
                                               , [("Id",g)], [("Sz",Jz)]
               None
                                                                                ],
                                                            , "Id"
                                      , None
                                                  , None
               None
                           , None
                                                                                ]]
       # yapf:enable
```

(continues on next page)

```
grids = [grid] * L
        # Generate the MPO from the grid. Note that it is not necessary to specify
        # the physical legs and their charges, since the from_grids method can extract
        # this information from the position of the operators inside the grid.
        H = MPO.from_grids(lat.mps_sites(), grids, bc='infinite', IdL=0, IdR=-1)
        MPOModel.__init__(self, lat, H)
def example_run_dmrg():
    """Use iDMRG to extract information about the ground state of the system."""
   model_params = dict(L=2, Jxx=1, Jz=1.5, xi=0.8, verbose=1)
   model = ExponentiallyDecayingHeisenberg(model_params)
   psi = MPS.from_product_state(model.lat.mps_sites(), ["up", "down"], bc='infinite')
   dmrq params = {
        'mixer': True,
        'chi_list': {
           0: 100
        'trunc_params': {
            'svd_min': 1.e-10
        },
        'verbose': 1
    }
   results = dmrg.run(psi, model, dmrg_params)
   print("Energy per site: ", results['E'])
   print("<Sz>: ", psi.expectation_value('Sz'))
if __name__ == "__main__":
   example_run_dmrg()
```

# 7.3.4 Charge conservation with np\_conserved

The basic idea is quickly summarized: By inspecting the Hamiltonian, you can identify symmetries, which correspond to conserved quantities, called **charges**. These charges divide the tensors into different sectors. This can be used to infer for example a block-diagonal structure of certain matrices, which in turn speeds up SVD or diagonalization a lot. Even for more general (non-square-matrix) tensors, charge conservation imposes restrictions which blocks of a tensor can be non-zero. Only those blocks need to be saved, which ultimately (= for large enough arrays) leads to a speedup of many routines, e.g., tensordot.

This introduction covers our implementation of charges; explaining mathematical details of the underlying symmetry is beyond its scope. We refer you to the corresponding chapter in our [TeNPyNotes] for a more general introduction of the idea (also stating the "charge rule" introduced below). Ref. [Singh2009] explains why it works form a mathematical point of view, [Singh2010] has the focus on a U(1) symmetry and might be easier to read.

#### **Notations**

Lets fix the notation of certain terms for this introduction and the doc-strings in np\_conserved. This might be helpful if you know the basics from a different context. If you're new to the subject, keep reading even if you don't understand each detail, and come back to this section when you encounter the corresponding terms again.

A Array is a multi-dimensional array representing a **tensor** with the entries:

$$T_{a_0,a_1,...a_{rank-1}}$$
 with  $a_i \in \{0,...,n_i-1\}$ 

Each  $leg a_i$  corresponds the a vector space of dimension  $n_i$ .

An **index** of a leg is a particular value  $a_i \in \{0, ..., n_i - 1\}$ .

The **rank** is the number of legs, the **shape** is  $(n_0, ..., n_{rank-1})$ .

We restrict ourselfes to abelian charges with entries in  $\mathbb{Z}$  or in  $\mathbb{Z}_m$ . The nature of a charge is specified by m; we set m=1 for charges corresponding to  $\mathbb{Z}$ . The number of charges is referred to as **qnumber** as a short hand, and the collection of m for each charge is called **qmod**. The qnumber, qmod and possibly descriptive names of the charges are saved in an instance of Charge Info.

To each index of each leg, a value of the charge(s) is associated. A **charge block** is a contiguous slice corresponding to the same charge(s) of the leg. A **qindex** is an index in the list of charge blocks for a certain leg. A **charge sector** is for given charge(s) is the set of all qindices of that charge(s). A leg is **blocked** if all charge sectors map one-to-one to qindices. Finally, a leg is **sorted**, if the charges are sorted lexiographically. Note that a *sorted* leg is always *blocked*. We can also speak of the complete array to be **blocked by charges** or **legcharge-sorted**, which means that all of its legs are blocked or sorted, respectively. The charge data for a single leg is collected in the class <code>LegCharge</code>. A <code>LegCharge</code> has also a flag **qconj**, which tells whether the charges point <code>inward</code> (+1) or <code>outward</code> (-1). What that means, is explained later in <code>Which entries of the npc Array can be non-zero?</code>.

For completeness, let us also summarize also the internal structure of an <code>Array</code> here: The array saves only non-zero **blocks**, collected as a list of <code>np.array</code> in <code>self.\_data</code>. The qindices necessary to map these blocks to the original leg indices are collected in <code>self.\_qdata</code> An array is said to be **qdata-sorted** if its <code>self.\_qdata</code> is lexiographically sorted. More details on this follow <code>later</code>. However, note that you usually shouldn't access <code>\_qdata</code> and <code>\_data</code> directly this is only necessary from within <code>tensordot</code>, <code>svd</code>, etc. Also, an array has a **total charge**, defining which entries can be non-zero - details in <code>Which entries of the npc Array can be non-zero?</code>.

Finally, a **leg pipe** (implemented in LegPipe) is used to formally combine multiple legs into one leg. Again, more details follow *later*.

### **Physical Example**

For concreteness, you can think of the Hamiltonian  $H=-t\sum_{< i,j>}(c_i^{\dagger}c_j+H.c.)+Un_in_j$  with  $n_i=c_i^{\dagger}c_i$ . This Hamiltonian has the global U(1) gauge symmetry  $c_i\to c_ie^{i\phi}$ . The corresponding charge is the total number of particles  $N=\sum_i n_i$ . You would then introduce one charge with m=1.

Note that the total charge is a sum of local terms, living on single sites. Thus, you can infer the charge of a single physical site: it's just the value  $q_i = n_i \in \mathbb{N}$  for each of the states.

Note that you can only assign integer charges. Consider for example the spin 1/2 Heisenberg chain. Here, you can naturally identify the magnetization  $S^z = \sum_i S_i^z$  as the conserved quantity, with values  $S_i^z = \pm \frac{1}{2}$ . Obviously, if  $S^z$  is conserved, then so is  $2S^z$ , so you can use the charges  $q_i = 2S_i^z \in \{-1, +1\}$  for the *down* and *up* states, respectively. Alternatively, you can also use a shift and define  $q_i = S_i^z + \frac{1}{2} \in \{0, 1\}$ .

As another example, consider BCS like terms  $\sum_k (c_k^{\dagger} c_{-k}^{\dagger} + H.c.)$ . These terms break the total particle conservation, but they preserve the total parity, i.e., N

In the above examples, we had only a single charge conserved at a time, but you might be lucky and have multiple conserved quantities, e.g. if you have two chains coupled only by interactions. TeNPy is designed to handle the general

case of multiple charges. When giving examples, we will restrict to one charge, but everything generalizes to multiple charges.

### The different formats for LegCharge

As mentioned above, we assign charges to each index of each leg of a tensor. This can be done in three formats: **qflat**, as **qind** and as **qdict**. Let me explain them with examples, for simplicity considering only a single charge (the most inner array has one entry for each charge).

**qflat form: simply a list of charges for each index.** An example:

```
qflat = [[-2], [-1], [-1], [0], [0], [0], [3], [3]]
```

This tells you that the leg has size 9, the charges for are [-2], [-1], [-1], ..., [3] for the indices 0, 1, 2, 3,..., 8. You can identify four *charge blocks* slice (0, 1), slice (1, 3), slice (3, 7), slice (7, 9) in this example, which have charges [-2], [-1], [0], [3]. In other words, the indices 1, 2 (which are in slice (1, 3)) have the same charge value [-1]. A *qindex* would just enumerate these blocks as 0, 1, 2, 3.

**qind form: a 1D array** *slices* **and a 2D array** *charges*. This is a more compact version than the *qflat* form: the *slices* give a partition of the indices and the *charges* give the charge values. The same example as above would simply be:

```
slices = [0, 1, 3, 7, 9]
charges = [[-2], [-1], [0], [3]]
```

Note that *slices* includes 0 as first entry and the number of indices (here 9) as last entries. Thus it has len block\_number + 1, where block\_number (given by block\_number) is the number of charge blocks in the leg, i.e. a *qindex* runs from 0 to block\_number-1. On the other hand, the 2D array *charges* has shape (block\_number, qnumber), where qnumber is the number of charges (given by *qnumber*).

In that way, the *qind* form maps an *qindex*, say qi, to the indices slice (slices [qi], slices [qi+1]) and the charge(s) charges [qi].

**qdict form: a dictionary in the other direction than qind, taking charge tuples to slices.** Again for the same example:

```
{(-2,): slice(0, 1),
(-1,): slice(1, 3),
(0,): slice(3, 7),
(3,): slice(7, 9)}
```

Since the keys of a dictionary are unique, this form is only possible if the leg is *completely blocked*.

The LegCharge saves the charge data of a leg internally in *qind* form, directly in the attribute *slices* and *charges*. However, it also provides convenient functions for conversion between from and to the *qflat* and *qdict* form.

The above example was nice since all charges were sorted and the charge blocks were 'as large as possible'. This is however not required.

The following example is also a valid *qind* form:

```
slices = [0, 1, 3, 5, 7, 9]
charges = [[-2], [-1], [0], [3]]
```

This leads to the *same qflat* form as the above examples, thus representing the same charges on the leg indices. However, regarding our Arrays, this is quite different, since it diveds the leg into 5 (instead of previously 4) charge blocks. We say the latter example is *not bunched*, while the former one is *bunched*.

To make the different notions of *sorted* and *bunched* clearer, consider the following (valid) examples:

charges					bunched	sorted	blocked
[[-2],	[-1],	[0],	[1],	[3]]	True	True	True
[[-2],	[-1],	[0],	[0],	[3]]	False	True	False
[[-2],	[0],	[-1],	[1],	[3]]	True	False	True
[[-2],	[0],	[-1],	[0],	[3]]	True	False	False

If a leg is *bunched* and *sorted*, it is automatically *blocked* (but not vice versa). See also *below* for further comments on that.

# Which entries of the npc Array can be non-zero?

The reason for the speedup with np\_conserved lies in the fact that it saves only the blocks 'compatible' with the charges. But how is this 'compatible' defined?

Assume you have a tensor, call it T, and the LegCharge for all of its legs, say a, b, c, ...

Remeber that the LegCharge associates to each index of the leg a charge value (for each of the charges, if *qnumber* > 1). Let a.to\_qflat() [ia] denote the charge(s) of index ia for leg a, and similar for other legs.

In addition, the LegCharge has a flag *qconj*. This flag **qconj** is only a sign, saved as +1 or -1, specifying whether the charges point 'inward' (+1, default) or 'outward' (-1) of the tensor.

Then, the **total charge of an entry** T[ia, ib, ic, ...] of the tensor is defined as:

```
qtotal[ia, ib, ic, ...] = a.to_qflat()[ia] * a.qconj + b.to_qflat()[ib] * b.qconj + c.
    →to_qflat()[ic] * c.qconj + ... modulo qmod
```

The rule which entries of the a Array can be non-zero (i.e., are 'compatible' with the charges), is then very simple:

#### Rule for non-zero entries

An entry ia, ib, ic, ... of a *Array* can only be non-zero, if qtotal[ia, ib, ic, ...] matches the *unique* qtotal attribute of the class.

In other words, there is a *single* **total charge** .qtotal attribute of a *Array*. All indices ia, ib, ic, ... for which the above defined qtotal[ia, ib, ic, ...] matches this *total charge*, are said to be **compatible with the charges** and can be non-zero. All other indices are **incompatible with the charges** and must be zero.

In case of multiple charges, qnumber > 1, is a straigth-forward generalization: an entry can only be non-zero if it is compatible with each of the defined charges.

### The pesky qconj - contraction as an example

Why did we introduce the qconj flag? Remember it's just a sign telling whether the charge points inward or outward. So whats the reasoning?

The short answer is, that LegCharges actually live on bonds (i.e., legs which are to be contracted) rather than individual tensors. Thus, it is convenient to share the LegCharges between different legs and even tensors, and just adjust the sign of the charges with *qconj*.

As an example, consider the contraction of two tensors,  $C_{ia,ic} = \sum_{ib} A_{ia,ib} B_{ib,ic}$ . For simplicity, say that the total charge of all three tensors is zero. What are the implications of the above rule for non-zero entries? Or rather, how can

we ensure that C complies with the above rule? An entry C[ia,ic] will only be non-zero, if there is an ib such that both A[ia,ib] and B[ib,ic] are non-zero, i.e., both of the following equations are fullfilled:

(A.legs [0] is the LegCharge saving the charges of the first leg (with index ia) of A.)

For the uncontracted legs, we just keep the charges as they are:

```
C.legs = [A.legs[0], B.legs[1]]
```

It is then straight-forward to check, that the rule is fullfilled for C, if the following condition is met:

```
A.qtotal + B.qtotal - C.qtotal == A.legs[1].to_qflat()[ib] A.b.qconj + B.legs[0].to_

qflat()[ib] B.b.qconj modulo qmod
```

The easiest way to meet this condition is (1) to require that A.b and B.b share the *same* charges b.to\_qflat(), but have opposite *qconj*, and (2) to define C.qtotal = A.qtotal + B.qtotal. This justifies the introduction of *qconj*: when you define the tensors, you have to define the *LegCharge* for the *b* only once, say for A.legs[1]. For B.legs[0] you simply use A.legs[1].conj() which creates a copy of the LegCharge with shared *slices* and *charges*, but opposite *qconj*. As a more impressive example, all 'physical' legs of an MPS can usually share the same *LegCharge* (up to different qconj if the local Hilbert space is the same). This leads to the following convention:

#### Convention

When an npc algorithm makes tensors which share a bond (either with the input tensors, as for tensordot, or amongst the output tensors, as for SVD), the algorithm is free, but not required, to use the **same** LegCharge for the tensors sharing the bond, *without* making a copy. Thus, if you want to modify a LegCharge, you **must** make a copy first (e.g. by using methods of LegCharge for what you want to acchive).

### Assigning charges to non-physical legs

From the above physical examples, it should be clear how you assign charges to physical legs. But what about other legs, e.g, the virtual bond of an MPS (or an MPO)?

The charge of these bonds must be derived by using the 'rule for non-zero entries', as far as they are not arbitrary. As a concrete example, consider an MPS on just two spin 1/2 sites:

The two legs p are the physical legs and share the same charge, as they both describe the same local Hilbert space. For better distincition, let me label the indices of them by  $\uparrow = 0$  and  $\downarrow = 1$ . As noted above, we can associate the charges 1  $(p = \uparrow)$  and -1  $(p = \downarrow)$ , respectively, so we define:

```
chinfo = npc.ChargeInfo([1], ['2*Sz'])
p = npc.LegCharge.from_qflat(chinfo, [1, -1], qconj=+1)
```

For the qconj signs, we stick to the convention used in our MPS code and indicated by the arrows in above 'picture': physical legs are incoming (qconj=+1), and from left to right on the virtual bonds. This is achieved by using [p, x, y.conj()] as *legs* for A, and [p, y, z.conj()] for B, with the default qconj=+1 for all p, x, y, z: y.conj() has the same charges as y, but opposite qconj=-1.

The legs x and z of an L=2 MPS, are 'dummy' legs with just one index 0. The charge on one of them, as well as the total charge of both A and B is arbitrary (i.e., a gauge freedom), so we make a simple choice: total charge 0 on both arrays, as well as for x = 0,  $x = npc.LegCharge.from_qflat(chinfo, [0], qconj=+1)$ .

The charges on the bonds y and z then depend on the state the MPS represents. Here, we consider a singlet  $\psi = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$  as a simple example. A possible MPS representation is given by:

```
A[up, :, :] = [[1/2.**0.5, 0]] B[up, :, :] = [[0], [-1]] A[down, :, :] = [[0, 1/2.**0.5]] B[down, :, :] = [[1], [0]]
```

There are two non-zero entries in A, for the indices  $(a, x, y) = (\uparrow, 0, 0)$  and  $(\downarrow, 0, 1)$ . For  $(a, x, y) = (\uparrow, 0, 0)$ , we want:

This fixes the charge of y=0 to 1. A similar calculation for  $(a, x, y) = (\downarrow, 0, 1)$  yields the charge -1 for y=1. We have thus all the charges of the leg y and can define y = npc.LegCharge.from\_qflat(chinfo, [1, -1], qconj=+1).

Now take a look at the entries of B. For the non-zero entry  $(b, y, z) = (\uparrow, 1, 0)$ , we want:

This implies the charge 0 for z = 0, thus z = npc.LegCharge.form\_qflat (chinfo, [0], qconj=+1). Finally, note that the rule for  $(b, y, z) = (\downarrow, 0, 0)$  is automatically fullfilled! This is an implication of the fact that the singlet has a well defined value for  $S_a^z + S_b^z$ . For other states without fixed magnetization (e.g.,  $|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$ ) this would not be the case, and we could not use charge conservation.

As an exercise, you can calculate the charge of z in the case that A.qtotal=5, B.qtotal = -1 and charge 2 for x=0. The result is -2.

**Note:** This section is meant be an pedagogical introduction. In you program, you can use the functions detect\_legcharge() (which does exactly what's described above) or detect\_qtotal() (if you know all LegCharges, but not qtotal).

# **Array creation**

Making an new Array requires both the tensor entries (data) and charge data.

The default initialization a = Array(...) creates an empty Array, where all entries are zero (equivalent to zeros()). (Non-zero) data can be provided either as a dense np.array to from\_ndarray(), or by providing a numpy function such as np.random, np.ones etc. to from\_func().

In both cases, the charge data is provided by one Charge Info, and a LegCharge instance for each of the legs.

**Note:** The charge data instances are not copied, in order to allow it to be shared between different Arrays. Consequently, you *must* make copies of the charge data, if you manipulate it directly. (However, methods like *sort* () do that for you.)

Of course, a new Array can also created using the charge data from existing Arrays, for examples with zeros\_like() or creating a (deep or shallow) copy(). Further, there are the higher level functions like tensordot() or svd(), which also return new Arrays.

Further, new Arrays are created by the various functions like *tensordot* or *svd* in *np\_conserved*.

### **Complete blocking of Charges**

While the code was designed in such a way that each charge sector has a different charge, the code should still run correctly if multiple charge sectors (for different qindex) correspond to the same charge. In this sense <code>Array</code> can act like a sparse array class to selectively store subblocks. Algorithms which need a full blocking should state that explicitly in their doc-strings. (Some functions (like <code>svd</code> and <code>eigh</code>) require complete blocking internally, but if necessary they just work on a temporary copy returned by <code>as\_completely\_blocked()</code>).

If you expect the tensor to be dense subject to charge constraints (as for MPS), it will be most efficient to fully block by charge, so that work is done on large chunks.

However, if you expect the tensor to be sparser than required by charge (as for an MPO), it may be convenient not to completely block, which forces smaller matrices to be stored, and hence many zeroes to be dropped. Nevertheless, the algorithms were not designed with this in mind, so it is not recommended in general. (If you want to use it, run a benchmark to check whether it is really faster!)

If you haven't created the array yet, you can call <code>sort()</code> (with bunch=True) on each <code>LegCharge</code> which you want to block. This sorts by charges and thus induces a permution of the indices, which is also returned as an 1D array perm. For consistency, you have to apply this permutation to your flat data as well.

Alternatively, you can simply call <code>sort\_legcharge()</code> on an existing <code>Array</code>. It calls <code>sort()</code> internally on the specified legs and performs the necessary permutations directly to (a copy of) <code>self</code>. Yet, you should keep in mind, that the axes are permuted afterwards.

#### Internal Storage schema of npc Arrays

The actual data of the tensor is stored in \_data. Rather than keeping a single np.array (which would have many zeros in it), we store only the non-zero sub blocks. So \_data is a python list of *np.array*'s. The order in which they are stored in the list is not physically meaningful, and so not guaranteed (more on this later). So to figure out where the sub block sits in the tensor, we need the \_qdata structure (on top of the LegCharges in legs).

Consider a rank 3 tensor T, with the first leg like:

```
legs[0].slices = np.array([0, 1, 4, ...])
legs[0].charges = np.array([[-2], [1], ...])
```

Each row of *charges* gives the charges for a *charge block* of the leg, with the actual indices of the total tensor determined by the *slices*. The *qindex* simply enumerates the charge blocks of a lex. Picking a qindex (and thus a *charge block*) from each leg, we have a subblock of the tensor.

For each (non-zero) subblock of the tensor, we put a (numpy) ndarray entry in the \_data list. Since each subblock of the tensor is specified by *rank* qindices, we put a corresponding entry in \_qdata, which is a 2D array of shape (#stored\_blocks, rank). Each row corresponds to a non-zero subblock, and there are rank columns giving the corresponding qindex for each leg.

Example: for a rank 3 tensor we might have:

The third subblock has an ndarray t3, and qindices [4 2 2] for the three legs.

• To find the position of t3 in the actual tensor you can use get\_slice():

```
T.legs[0].get_slice(4), T.legs[1].get_slice(2), T.legs[2].get_slice(2)
```

The function leg.get\_charges(qi) simply returns slice(leg.slices[qi], leg.slices[qi+1])

• To find the charges of t3, we an use get\_charge():

```
T.legs[0].get_charge(2), T.legs[1].get_charge(2), T.legs[2].get_charge(2)
```

The function leg.get\_charge(qi) simply returns leg.charges[qi] \*leg.qconj.

**Note:** Outside of  $np\_conserved$ , you should use the API to access the entries. If you really need to iterate over all blocks of an Array T, try for (block, blockslices, charges, qindices) in T: do\_something().

The order in which the blocks stored in \_data/\_qdata is arbitrary (although of course \_data and \_qdata must be in correspondence). However, for many purposes it is useful to sort them according to some convention. So we include a flag .\_qdata\_sorted to the array. So, if sorted (with <code>isort\_qdata()</code>, the \_qdata example above goes to

Note that *np.lexsort* chooses the right-most column to be the dominant key, a convention we follow throughout.

If \_qdata\_sorted == True, \_qdata and \_data are guaranteed to be lexsorted. If \_qdata\_sorted == False, there is no gaurantee. If an algorithm modifies \_qdata, it must set \_qdata\_sorted = False (unless it gaurantees it is still sorted). The routine sort\_qdata() brings the data to sorted form.

# **Indexing of an Array**

Although it is usually not necessary to access single entries of an Array, you can of course do that. In the simplest case, this is something like A[0, 2, 1] for a rank-3 Array A. However, accessing single entries is quite slow and usually not recommended. For small Arrays, it may be convenient to convert them back to flat numpy arrays with to\_ndarray().

On top of that very basic indexing, *Array* supports slicing and some kind of advanced indexing, which is however different from the one of numpy arrarys (described here). Unlike numpy arrays, our Array class does not broadcast existing index arrays – this would be terribly slow. Also, *np.newaxis* is not supported, since inserting new axes requires additional information for the charges.

Instead, we allow just indexing of the legs independent of each other, of the form A[i0, i1, ...]. If all indices i0, i1, ... are integers, the single corresponding entry (of type *dtype*) is returned.

However, the individual 'indices' i0 for the individual legs can also be one of what is described in the following list. In that case, a new Array with less data (specified by the indices) is returned.

The 'indices' can be:

- an *int*: fix the index of that axis, return array with one less dimension. See also take slice().
- a slice (None) or :: keep the complete axis
- an Ellipsis or . . .: shorthand for slice (None) for missing axes to fix the len
- an 1D bool *ndarray* mask: apply a mask to that axis, see *iproject* ().
- a slice (start, stop, step) or start:stop:step: keep only the indices specified by the slice. This is also implemented with *iproject*.
- an 1D int *ndarray* mask: keep only the indices specified by the array. This is also implemented with *iproject*.

For slices and 1D arrays, additional permuations may be perfored with the help of permute().

If the number of indices is less than rank, the remaining axes remain free, so for a rank 4 Array A, A[i0, i1] == A[i0, i1, ...] == A[i0, i1, :, :].

Note that indexing always **copies** the data – even if *int* contains just slices, in which case numpy would return a view. However, assigning with A[:, [3, 5], 3] = B should work as you would expect.

```
Warning: Due to numpy's advanced indexing, for 1D integer arrays a0 and a1 the following holds
```

```
A[a0, a1].to_ndarray() == A.to_ndarray()[np.ix_(a0, a1)] != A.to_ndarray()[a0, a1]
```

For a combination of slices and arrays, things get more complicated with numpys advanced indexing. In that case, a simple  $np.ix_(...)$  doesn't help any more to emulate our version of indexing.

### Introduction to combine\_legs, split\_legs and LegPipes

Often, it is necessary to "combine" multiple legs into one: for example to perform a SVD, a tensor needs to be viewed as a matrix. For a flat array, this can be done with np.reshape, e.g., if A has shape (10, 3, 7) then B = np. reshape (A, (30, 7)) will result in a (view of the) array with one less dimension, but a "larger" first leg. By default (order='C'), this results in

```
B[i*3 + j, k] == A[i, j, k] for i in range(10) for j in range(3) for k in range(7)
```

While for a np.array, also a reshaping  $(10, 3, 7) \rightarrow (2, 21, 5)$  would be allowed, it does not make sense physically. The only sensible "reshape" operation on an Array are

- 1) to combine multiple legs into one leg pipe (LeqPipe) with combine\_legs(), or
- 2) to **split** a pipe of previously combined legs with *split* legs().

Each leg has a Hilbert space, and a representation of the symmetry on that Hilbert space. Combining legs corresponds to the tensor product operation, and for abelian groups, the corresponding "fusion" of the representation is the simple addition of charge.

Fusion is not a lossless process, so if we ever want to split the combined leg, we need some additional data to tell us how to reverse the tensor product. This data is saved in the class <code>LegPipe</code>, derived from the <code>LegCharge</code> and used as new <code>leg</code>. Details of the information contained in a LegPipe are given in the class doc string.

The rough usage idea is as follows:

- 1) You can call <code>combine\_legs</code> () without supplying any LegPipes, <code>combine\_legs</code> will then make them for you. Nevertheless, if you plan to perform the combination over and over again on sets of legs you know to be identical [with same charges etc, up to an overall -1 in <code>qconj</code> on all incoming and outgoing Legs] you might make a LegPipe anyway to save on the overhead of computing it each time.
- 2) In any way, the resulting Array will have a LegPipe as a LegCharge on the combined leg. Thus, it and all tensors inheriting the leg (e.g. the results of svd, tensordot etc.) will have the information how to split the LegPipe back to the original legs.
- 3) Once you performed the necessary operations, you can call split\_legs(). This uses the information saved in the *LegPipe* to split the legs, recovering the original legs.

For a LegPipe, <code>conj()</code> changes <code>qconj</code> for the outgoing pipe and the incoming legs. If you need a LegPipe with the same incoming <code>qconj</code>, use <code>outer\_conj()</code>.

### Leg labeling

It's convenient to name the legs of a tensor: for instance, we can name legs 0, 1, 2 to be 'a', 'b', 'c':  $T_{i_a,i_b,i_c}$ . That way we don't have to remember the ordering! Under tensordot, we can then call

```
U = npc.tensordot(S, T, axes = [ [...], ['b'] ] )
```

without having to remember where exactly 'b' is. Obviously U should then inherit the name of its legs from the uncontracted legs of S and T. So here is how it works:

- Labels can *only* be strings. The labels should not include the characters . or ?. Internally, the labels are stored as dict a.labels = {label: leg position, ...}. Not all legs need a label.
- To set the labels, call

```
A.set_labels(['a', 'b', None, 'c', ... ])
```

```
which will set up the labeling \{'a': 0, 'b': 1, 'c': 3 \ldots \}.
```

- (Where implemented) the specification of axes can use either the labels **or** the index positions. For instance, the call tensordot (A, B, [['a', 2, 'c'], [...]]) will interpret 'a' and 'c' as labels (calling get\_leg\_indices() to find their positions using the dict) and 2 as 'the 2nd leg'. That's why we require labels to be strings!
- Labels will be intelligently inherited through the various operations of np\_conserved.
  - Under *transpose*, labels are permuted.
  - Under *tensordot*, labels are inherited from uncontracted legs. If there is a collision, both labels are dropped.

- Under combine\_legs, labels get concatenated with a . delimiter and sourrounded by brackets. Example: let a.labels = {'a': 1, 'b': 2, 'c': 3}. Then if b = a. combine\_legs([[0, 1], [2]]), it will have b.labels = {'(a.b)': 0, '(c)': 1}. If some sub-leg of a combined leg isn't named, then a '?#' label is inserted (with # the leg index), e.g., 'a.?0.c'.
- Under *split\_legs*, the labels are split using the delimiters (and the '?#' are dropped).
- Under conj, iconj: take 'a' -> 'a\*', 'a\*' -> 'a', and '(a.(b\*.c))' -> '(a\*.(b.c\*))'
- Under svd, the outer labels are inherited, and inner labels can be optionally passed.
- Under *pinv*, the labels are transposed.

#### See also

- The module tenpy.linalg.np\_conserved should contain all the API needed from the point of view of the algorithms. It contians the fundamental Array class and functions for working with them (creating and manipulating).
- The module tenpy.linalg.charges contains implementations for the charge structure, for example the classes ChargeInfo, LegCharge, and LegPipe. As noted above, the 'public' API is imported to (and accessible from) np\_conserved.

### A full example code for spin-1/2

Below follows a full example demonstrating the creation and contraction of Arrays. (It's the file  $a\_np\_conserved.py$  in the examples folder of the tenpy source.)

```
"""An example code to demonstrate the usage of :class:`~tenpy.linalg.np_conserved.
→Array`.
This example includes the following steps:
1) create Arrays for an Neel MPS
2) create an MPO representing the nearest-neighbour AFM Heisenberg Hamiltonian
3) define 'environments' left and right
4) contract MPS and MPO to calculate the energy
5) extract two-site hamiltonian ``H2`` from the MPO
6) calculate ``exp(-1.j*dt*H2)`` by diagonalization of H2
7) apply ``exp(H2)`` to two sites of the MPS and truncate with svd
Note that this example uses only np_conserved, but no other modules.
Compare it to the example `b_mps.py`,
which does the same steps using a few predefined classes like MPS and MPO.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import tenpy.linalg.np_conserved as npc
import numpy as np
# model parameters
Jxx, Jz = 1., 1.
L = 20
dt = 0.1
cutoff = 1.e-10
print("Jxx=\{Jxx\}, Jz=\{Jz\}, L=\{L:d\}".format(Jxx=Jxx, Jz=Jz, L=L))
```

(continues on next page)

```
print("1) create Arrays for an Neel MPS")
    vL ->--B-->- vR
#
                    1
                    p
# create a ChargeInfo to specify the nature of the charge
chinfo = npc.ChargeInfo([1], ['2*Sz']) # the second argument is just a descriptive_
⇔name
# create LegCharges on physical leg and even/odd bonds
p_leg = npc.LegCharge.from_qflat(chinfo, [[1], [-1]]) # charges for up, down
v_leg_even = npc.LegCharge.from_qflat(chinfo, [[0]])
v_leg_odd = npc.LegCharge.from_qflat(chinfo, [[1]])
B_even = npc.zeros([v_leg_even, v_leg_odd.conj(), p_leg],
                                       labels=['vL', 'vR', 'p']) # virtual left/right, physical
B_odd = npc.zeros([v_leg_odd, v_leg_even.conj(), p_leg], labels=['vL', 'vR', 'p'])
B_even[0, 0, 0] = 1. # up
B_odd[0, 0, 1] = 1. \# down
Bs = [B\_even, B\_odd] * (L // 2) + [B\_even] * (L % 2) # (right-canonical)
Ss = [np.ones(1)] * L # Ss[i] are singular values between Bs[i-1] and Bs[i]
# Side remark:
# An MPS is expected to have non-zero entries everywhere compatible with the charges.
# In general, we recommend to use `sort_legcharge` (or `as_completely_blocked`)
# to ensure complete blocking. (But the code will also work, if you don't do it.)
# The drawback is that this might introduce permutations in the indices of single.
→ leas.
# which you have to keep in mind when converting dense numpy arrays to and from npc.
⊶Arrays.
print("2) create an MPO representing the AFM Heisenberg Hamiltonian")
#
                    p*
#
                     1
#
     WT, ->--W-->-WR
#
                    1
#
                    p
# create physical spin-1/2 operators Sz, S+, S-
Sz = npc.Array.from_ndarray([[0.5, 0.], [0., -0.5]], [p_leg, p_leg.conj()], labels=['p_leg.conj()], 
→', 'p*'])
Sp = npc.Array.from_ndarray([[0., 1.], [0., 0.]], [p_leg, p_leg.conj()], labels=['p',
Sm = npc.Array.from_ndarray([[0., 0.], [1., 0.]], [p_leg, p_leg.conj()], labels=['p',

  'p*'])

Id = npc.eye_like(Sz, labels=Sz.get_leg_labels()) # identity
```

(continues on next page)

```
mpo_leq = npc.LegCharge.from_qflat(chinfo, [[0], [2], [-2], [0], [0]])
W_{grid} = [[Id, Sp,
                      Sm,
                            Sz, None
          [None, None, None, O.5 * Jxx * Sm],
          [None, None, None, None, 0.5 * Jxx * Sp],
          [None, None, None, Jz * Sz ],
          [None, None, None, Id
                                                 ]]
                                                     # yapf:disable
W = npc.grid_outer(W_grid, [mpo_leg, mpo_leg.conj()], grid_labels=['wL', 'wR'])
\# wL/wR = virtual left/right of the MPO
Ws = [W] * L
print("3) define 'environments' left and right")
  .--->- vR
                vL ->---.
# /
\# envL \rightarrow WR
                 wI_i \rightarrow -envR
  .--->- vR*
                 VT. *->---
envL = npc.zeros([W.qet_leg('wL').conj(), Bs[0].qet_leg('vL').conj(), Bs[0].qet_leg(
\hookrightarrow '\veeL')],
                 labels=['wR', 'vR', 'vR*'])
envL[0, :, :] = npc.diag(1., envL.legs[1])
envR = npc.zeros([W.get_leg('wR').conj(), Bs[-1].get_leg('vR').conj(), Bs[-1].get_leg(
\hookrightarrow 'VR')],
                 labels=['wL', 'vL', 'vL*'])
envR[-1, :, :] = npc.diag(1., envR.legs[1])
print("4) contract MPS and MPO to calculate the energy <psi|H|psi>")
contr = envL
for i in range(L):
    # contr labels: wR, vR, vR*
   contr = npc.tensordot(contr, Bs[i], axes=('vR', 'vL'))
    # wR, vR*, vR, p
   contr = npc.tensordot(contr, Ws[i], axes=(['p', 'wR'], ['p*', 'wL']))
    # vR*, vR, wR, p
   contr = npc.tensordot(contr, Bs[i].conj(), axes=(['p', 'vR*'], ['p*', 'vL*']))
    # vR, wR, vR*
    # note that the order of the legs changed, but that's no problem with labels:
    # the arrays are automatically transposed as necessary
E = npc.inner(contr, envR, axes=(['vR', 'wR', 'vR*'], ['vL', 'wL', 'vL*']))
print("E =", E)
print("5) calculate two-site hamiltonian ``H2`` from the MPO")
# label left, right physical legs with p, q
W0 = W.replace_labels(['p', 'p*'], ['p0', 'p0*'])
W1 = W.replace_labels(['p', 'p*'], ['p1', 'p1*'])
H2 = npc.tensordot(W0, W1, axes=('wR', 'wL')).itranspose(['wL', 'wR', 'p0', 'p1', 'p0*])
→', 'p1*'])
H2 = H2[0, -1] # (If H has single-site terms, it's not that simple anymore)
print("H2 labels:", H2.get_leg_labels())
print ("6) calculate exp(H2) by diagonalization of H2")
# diagonalization requires to view H2 as a matrix
H2 = H2.combine_legs([('p0', 'p1'), ('p0*', 'p1*')], qconj=[+1, -1])
print("labels after combine_legs:", H2.get_leg_labels())
                                                                          (continues on next page)
```

```
E2, U2 = npc.eigh(H2)
print("Eigenvalues of H2:", E2)
U_expE2 = U2.scale_axis(np.exp(-1.j * dt * E2), axis=1) # scale_axis ~= apply an_
 \rightarrow diagonal matrix
exp_H2 = npc.tensordot(U_expE2, U2.conj(), axes=(1, 1))
exp_H2.iset_leg_labels(H2.get_leg_labels())
exp_H2 = exp_H2.split_legs() # by default split all legs which are `LegPipe`
# (this restores the originial labels ['p0', 'p1', 'p0*', 'p1*'] of `H2` in `exp_H2`)
print("7) apply exp(H2) to even/odd bonds of the MPS and truncate with svd")
# (this implements one time step of first order TEBD)
for even_odd in [0, 1]:
         for i in range(even_odd, L - 1, 2):
                   B_L = Bs[i].scale_axis(Ss[i], 'vL').ireplace_label('p', 'p0')
                    B_R = Bs[i + 1].replace_label('p', 'p1')
                   theta = npc.tensordot(B_L, B_R, axes=('vR', 'vL'))
                    theta = npc.tensordot(exp_H2, theta, axes=(['p0*', 'p1*'], ['p0', 'p1']))
                    # view as matrix for SVD
                    theta = theta.combine_legs([('vL', 'p0'), ('p1', 'vR')], new_axes=[0, 1],_
 \rightarrowqconj=[+1, -1])
                    # now theta has labels '(vL.p0)', '(p1.vR)'
                    U, S, V = npc.svd(theta, inner_labels=['vR', 'vL'])
                    # truncate
                   keep = S > cutoff
                    S = S[keep]
                   invsq = np.linalg.norm(S)
                    Ss[i + 1] = S / invsq
                    U = U.iscale_axis(S / invsq, 'vR')
                   Bs[i] = U.split_legs('(vL.p0)').iscale_axis(Ss[i] **(-1), 'vL').ireplace_label('.orange, -1) = U.split_legs('(vL.p0)').ireplace_label('.orange, -1) = U.split_legs('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').ireplace_label('(vL.p0)').i
 \rightarrow'p0', 'p')
                   Bs[i + 1] = V.split_legs('(p1.vR)').ireplace_label('p1', 'p')
print("finished")
```

### **7.3.5 Models**

#### What is a model?

Abstractly, a **model** stands for some physical (quantum) system to be described. For tensor networks algorithms, the model is usually specified as a Hamiltonian written in terms of second quantization. For example, let us consider a spin-1/2 Heisenberg model described by the Hamiltonian

$$H = J \sum_{i} S_{i}^{x} S_{i+1}^{x} + S_{i}^{y} S_{i+1}^{y} + S_{i}^{z} S_{i+1}^{z}$$

Note that a few things are defined more or less implicitly.

- The local Hilbert space: it consists of Spin-1/2 degrees of freedom with the usual spin-1/2 operators  $S^x$ ,  $S^y$ ,  $S^z$ .
- The geometric (lattice) strucuture: above, we spoke of a 1D "chain".
- The boundary conditions: do we have open or periodic boundary conditions? The "chain" suggests open boundaries, which are in most cases preferable for MPS-based methods.
- The range of i: How many sites do we consider (for a 2D system: in each direction)?

Obviously, these things need to be specified in TeNPy in one way or another, if we want to define a model.

Ultimately, our goal is to run some algorithm. However, different algorithm requires the model and Hamiltonian to be specified in different forms. We have one class for each such required form. For example <code>dmrg</code> requires an <code>MPOModel</code>, which contains the Hamiltonian written as an MPO. So a new model class suitable for DMRG should have this general structure:

```
class MyNewModel(MPOModel):
    def __init__(self, model_params):
        lattice = somehow_generate_lattice(model_params)
        H_MPO = somehow_generate_MPO(lattice, model_params)
        # initialize MPOModel
        MPOModel.__init__(self, lattice, H_MPO)
```

On the other hand, if we want to evolve a state with tebd we need a NearestNeighborModel, in which the Hamiltonian is written in terms of two-site bond-terms to allow a Suzuki-Trotter decomposition of the time-evolution operator:

```
class MyNewMode12 (NearestNeighborModel):
    """General strucutre for a model suitable for TEBD."""
    def __init__(self, model_params):
        lattice = somehow_generate_lattice(model_params)
        H_bond = somehow_generate_H_bond(lattice, model_params)
        # initialize MPOModel
        NearestNeighborModel.__init__(self, lattice, H_bond)
```

Of course, the difficult part in these examples is to generate the H\_MPO and H\_bond in the required form. If you want to write it down by hand, you can of course do that. But it can be quite tedious to write every model multiple times, just because we need different representations of the same Hamiltonian. Luckily, there is a way out in TeNPy: the CouplingModel. Before we describe this class, let's discuss the background of the Site and Site class.

# The Hilbert space

The **local Hilbert** space is represented by a Site (read its doc-string!). In particular, the Site contains the local LegCharge and hence the meaning of each basis state needs to be defined. Beside that, the site contains the local operators - those give the real meaning to the local basis. Having the local operators in the site is very convenient, because it makes them available by name for example when you want to calculate expectation values. The most common sites (e.g. for spins, spin-less or spin-full fermions, or bosons) are predefined in the module tenpy.networks.site, but if necessary you can easily extend them by adding further local operators or completely write your own subclasses of Site.

The full Hilbert space is a tensor product of the local Hilbert space on each site.

Note: The LegCharge of all involved sites need to have a common ChargeInfo in order to allow the contraction of tensors acting on the various sites. This can be ensured with the function  $multi\_sites\_combine\_charges()$ .

An example where  $multi_sites_combine_charges()$  is needed would be a coupling of different types of sites, e.g., when a tight binding chain of fermions is coupled to some local spin degrees of freedom. Another use case of this function would be a model with a U(1) symmetry involving only half the sites, say  $\sum_{i=0}^{L/2} n_{2i}$ .

**Note:** If you don't know about the charges and  $np\_conserved$  yet, but want to get started with models right away, you can set conserve=None in the existing sites or use  $leg = tenpy.linalg.np\_conserved.LegCharge.$  from\_trivial(d) for an implementation of your custom site, where d is the dimension of the local Hilbert space. Alternatively, you can find some introduction to the charges in the *Charge conservation with np\\_conserved*.

### The geometry: lattice class

The geometry is usually given by some kind of **lattice** structure how the sites are arranged, e.g. implicitly with the sum over nearest neighbours  $\sum_{\langle i,j \rangle}$ . In TeNPy, this is specified by a Lattice class, which contains a unit cell of a few Site which are shifted periodically by its basis vectors to form a regular lattice. Again, we have pre-defined some basic lattices like a Chain, two chains coupled as a Ladder or 2D lattices like the Square, Honeycomb and Kagome lattices; but you are also free to define your own generalizations.

MPS based algorithms like DMRG always work on purely 1D systems. Even if our model "lives" on a 2D lattice, these algorithms require to map it onto a 1D chain (probably at the cost of longer-range interactions). This mapping is also done by the lattice by defining the **order** (order) of the sites.

**Note:** Further details on the lattice geometry can be found in *Details on the lattice geometry*.

### The CouplingModel: general structure

The CouplingModel provides a general, quite abstract way to specify a Hamiltonian of two-site couplings on a given lattice. Once initialized, its methods add\_onsite() and  $add_coupling()$  allow to add onsite and coupling terms repeated over the different unit cells of the lattice. In that way, it basically allows a straight-forward translation of the Hamiltonian given as a math forumla  $H = \sum_i A_i B_{i+dx} + ...$  with onsite operators A, B,... into a model class.

The general structure for a new model based on the CouplingModel is then:

```
class MyNewModel3(CouplingModel, MPOModel, NearestNeighborModel):
    def __init__(self, ...):
        ... # follow the basic steps explained below
```

In the initialization method \_\_init\_\_ (self, ...) of this class you can then follow these basic steps:

- 0. Read out the parameters.
- 1. Given the parameters, determine the charges to be conserved. Initialize the *LegCharge* of the local sites accordingly.
- 2. Define (additional) local operators needed.
- 3. Initialize the needed Site.

**Note:** Using pre-defined sites like the SpinHalfSite is recommended and can replace steps 1-3.

- 4. Initialize the lattice (or if you got the lattice as a parameter, set the sites in the unit cell).
- 5. Initialize the CouplingModel with CouplingModel.\_\_init\_\_(self, lat).
- 6. Use add\_onsite() and add\_coupling() to add all terms of the Hamiltonian. Here, the pairs of the lattice can come in handy, for example:

```
self.add_onsite(-np.asarray(h), 0, 'Sz')
for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
    self.add_coupling(0.5*J, u1, 'Sp', u2, 'Sm', dx, plus_hc=True)
    self.add_coupling( J, u1, 'Sz', u2, 'Sz', dx)
```

**Note:** The method  $add\_coupling()$  adds the coupling only in one direction, i.e. not switching i and j in a  $\sum_{\langle i,j\rangle}$ . If you have terms like  $c_i^{\dagger}c_j$  or  $S_i^+S_j^-$  in your Hamiltonian, you *need* to add it in both directions to

get a Hermitian Hamiltonian! The easiest way to do that is to use the  $plus\_hc$  option of  $add\_onsite()$  and  $add\_coupling()$ , as we did for the  $J/2(S_i^+S_j^-+h.c.)$  terms of the Heisenberg model above. Alternatively, you can add the hermitian conjugate terms explicitly, see the examples in  $add\_coupling()$  for more details.

Note that the *strength* arguments of these functions can be (numpy) arrays for site-dependent couplings. If you need to add or multipliy some parameters of the model for the *strength* of certain terms, it is recommended use np.asarray beforehand – in that way lists will also work fine.

- 7. Finally, if you derived from the <code>MPOModel</code>, you can call <code>calc\_H\_MPO()</code> to build the MPO and use it for the initialization as <code>MPOModel.\_\_init\_\_(self, lat, self.calc\_H\_MPO())</code>.
- 8. Similarly, if you derived from the <code>NearestNeighborModel</code>, you can call <code>calc\_H\_bond()</code> to initialze it as <code>NearestNeighborModel.\_\_init\_\_(self, lat, self.calc\_H\_bond())</code>. Calling <code>self.calc\_H\_bond()</code> will fail for models which are not nearest-neighbors (with respect to the MPS ordering), so you should only subclass the <code>NearestNeighborModel</code> if the lattice is a simple <code>Chain</code>.

The <code>CouplingModel</code> works for Hamiltonians which are a sum of terms involving at most two sites. The generalization <code>MultiCouplingModel</code> can be used for Hamiltonians with coupling terms acting on more than 2 sites at once. Follow the exact same steps in the initialization, and just use the <code>add\_multi\_coupling()</code> instead or in addition to the <code>add\_coupling()</code>. A prototypical example is the exactly solvable <code>ToricCode</code>.

The code of the module <code>tenpy.models.xxz\_chain</code> is included below as an illustrative example how to implement a Model. The implementation of the <code>XXZChain</code> directly follows the steps outline above. The <code>XXZChain2</code> implements the very same model, but based on the <code>CouplingMPOModel</code> explained in the next section.

```
"""Prototypical example of a 1D quantum model: the spin-1/2 XXZ chain.
The XXZ chain is contained in the more general :class: `~tenpy.models.spins.SpinChain`;
→ the idea of
this module is more to serve as a pedagogical example for a model.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
from .lattice import Site, Chain
from .model import CouplingModel, NearestNeighborModel, MPOModel, CouplingMPOModel
from ..linalg import np_conserved as npc
from ..tools.params import asConfig
from ..networks.site import SpinHalfSite # if you want to use the predefined site
__all__ = ['XXZChain', 'XXZChain2']
class XXZChain(CouplingModel, NearestNeighborModel, MPOModel):
   r"""Spin-1/2 XXZ chain with Sz conservation.
    The Hamiltonian reads:
    .. math ::
       H = \sum_{i=1}^{n} \frac{1}{2} (S^{+}_{i} S^{-}_{i+1} + S^{-}_{i+1})
               + \mathtt{Jz} S^z_i S^z_{i+1} \\
            - \sum_i \mathtt{hz} S^z_i
   All parameters are collected in a single dictionary `model_params`, which
   is turned into a :class: `~tenpy.tools.params.Config` object.
    Parameters
```

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```
model_params : :class:`~tenpy.tools.params.Config`
       Parameters for the model. See :cfg:config:`XXZChain` below.
   Options
   .. cfg:config :: XXZChain
       :include: CouplingMPOModel
       L : int
           Length of the chain.
       Jxx, Jz, hz : float | array
           Coupling as defined for the Hamiltonian above.
       bc_MPS : {'finite' | 'infinte'}
           MPS boundary conditions. Coupling boundary conditions are chosen,
→appropriately.
   n n n
   def __init__(self, model_params):
       # 0) read out/set default parameters
       model_params = asConfig(model_params, "XXZChain")
       L = model_params.get('L', 2)
       Jxx = model_params.get('Jxx', 1.)
       Jz = model_params.get('Jz', 1.)
       hz = model_params.get('hz', 0.)
       bc_MPS = model_params.get('bc_MPS', 'finite')
       # 1-3):
       USE_PREDEFINED_SITE = False
       if not USE_PREDEFINED_SITE:
            # 1) charges of the physical leg. The only time that we actually define.
⇒charges!
           leg = npc.LegCharge.from_qflat(npc.ChargeInfo([1], ['2*Sz']), [1, -1])
            # 2) onsite operators
           Sp = [[0., 1.], [0., 0.]]
           Sm = [[0., 0.], [1., 0.]]
           Sz = [[0.5, 0.], [0., -0.5]]
            # (Can't define Sx and Sy as onsite operators: they are incompatible with,
\hookrightarrow Sz charges.)
            # 3) local physical site
           site = Site(leg, ['up', 'down'], Sp=Sp, Sm=Sm, Sz=Sz)
       else:
            # there is a site for spin-1/2 defined in TeNPy, so just we can just use.
\hookrightarrow it.
            # replacing steps 1-3)
           site = SpinHalfSite(conserve='Sz')
        # 4) lattice
       bc = 'periodic' if bc_MPS == 'infinite' else 'open'
       lat = Chain(L, site, bc=bc, bc_MPS=bc_MPS)
       # 5) initialize CouplingModel
       CouplingModel.__init__(self, lat)
       # 6) add terms of the Hamiltonian
        # (u is always 0 as we have only one site in the unit cell)
       self.add_onsite(-hz, 0, 'Sz')
       self.add_coupling(Jxx * 0.5, 0, 'Sp', 0, 'Sm', 1, plus_hc=True)
       # instead of plus_hc=True, we could explicitly add the h.c. term with:
       self.add_coupling(Jz, 0, 'Sz', 0, 'Sz', 1)
        # 7) initialize H_MPO
```

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```
MPOModel.__init__(self, lat, self.calc_H_MPO())
        # 8) initialize H_bond (the order of 7/8 doesn't matter)
        NearestNeighborModel.__init__(self, lat, self.calc_H_bond())
class XXZChain2(CouplingMPOModel, NearestNeighborModel):
    """Another implementation of the Spin-1/2 XXZ chain with Sz conservation.
    This implementation takes the same parameters as the :class:`XXZChain`, but is,
→ implemented
   based on the :class: `~tenpy.models.model.CouplingMPOModel`.
   Parameters
   model_params : dict | :class:`~tenpy.tools.params.Config`
       See :cfg:config:`XXZChain`
   def __init__(self, model_params):
       model_params = asConfig(model_params, "XXZChain2")
       model_params.setdefault('lattice', "Chain")
        CouplingMPOModel.__init__(self, model_params)
   def init_sites(self, model_params):
        return SpinHalfSite(conserve='Sz') # use predefined Site
   def init_terms(self, model_params):
        # read out parameters
        Jxx = model_params.get('Jxx', 1.)
        Jz = model_params.get('Jz', 1.)
       hz = model_params.get('hz', 0.)
        # add terms
        for u in range(len(self.lat.unit_cell)):
           self.add_onsite(-hz, u, 'Sz')
        for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
            self.add_coupling(Jxx * 0.5, u1, 'Sp', u2, 'Sm', dx, plus_hc=True)
            self.add_coupling(Jz, u1, 'Sz', u2, 'Sz', dx)
```

#### The easy easy way: the CouplingMPOModel

Since many of the basic steps above are always the same, we don't need to repeat them all the time. So we have yet another class helping to structure the initialization of models: the <code>CouplingMPOModel</code>. The general structure of this class is like this:

```
class CouplingMPOModel(CouplingModel, MPOModel):
    def __init__(self, model_param):
        # ... follows the basic steps 1-8 using the methods
        lat = self.init_lattice(self, model_param) # for step 4
        # ...
        self.init_terms(self, model_param) # for step 6
        # ...

def init_sites(self, model_param):
        # You should overwrite this

def init_lattice(self, model_param):
```

(continues on next page)

```
sites = self.init_sites(self, model_param) # for steps 1-3
# initialize an arbitrary pre-defined lattice
# using model_params['lattice']

def init_terms(self, model_param):
# does nothing.
# You should overwrite this
```

The XXZChain2 included above illustrates, how it can be used. You need to implement steps 1-3) by overwriting the method init\_sites() Step 4) is performed in the method init\_lattice(), which initializes arbitrary 1D or 2D lattices; by default a simple 1D chain. If your model only works for specific lattices, you can overwrite this method in your own class. Step 6) should be done by overwriting the method init\_terms(). Steps 5,7,8 and calls to the init\_... methods for the other steps are done automatically if you just call the CouplingMPOModel. \_\_init\_\_(self, model\_param).

The XXZChain and XXZChain2 work only with the Chain as lattice, since they are derived from the NearestNeighborModel. This allows to use them for TEBD in 1D (yeah!), but we can't get the MPO for DMRG on a e.g. a Square lattice cylinder - although it's intuitively clear, what the Hamiltonian there should be: just put the nearest-neighbor coupling on each bond of the 2D lattice.

It's not possible to generalize a <code>NearestNeighborModel</code> to an arbitrary lattice where it's no longer nearest Neigbors in the MPS sense, but we can go the other way around: first write the model on an arbitrary 2D lattice and then restrict it to a 1D chain to make it a <code>NearestNeighborModel</code>.

Let me illustrate this with another standard example model: the transverse field Ising model, implemented in the module <code>tenpy.models.tf\_ising</code> included below. The <code>TFIModel</code> works for arbitrary 1D or 2D lattices. The <code>TFIChain</code> is then taking the exact same model making a <code>NearestNeighborModel</code>, which only works for the 1D chain.

```
"""Prototypical example of a quantum model: the transverse field Ising model.
Like the :class:`~tenpy.models.xxz_chain.XXZChain`, the transverse field ising chain
:class:`TFIChain` is contained in the more general :class:`~tenpy.models.spins.
→ SpinChain`;
the idea is more to serve as a pedagogical example for a 'model'.
We choose the field along z to allow to conserve the parity, if desired.
# Copyright 2018-2020 TeNPy Developers, GNU GPLv3
import numpy as np
from .model import CouplingMPOModel, NearestNeighborModel
from ..tools.params import asConfig
from ..networks.site import SpinHalfSite
__all__ = ['TFIModel', 'TFIChain']
class TFIModel(CouplingMPOModel):
   r"""Transverse field Ising model on a general lattice.
    The Hamiltonian reads:
    .. math ::
       H = - \sum_{i=1}^{n} \frac{1}{j} \operatorname{dist}(J) \operatorname{dist}(J)
```

(continues on next page)

```
- \sum_{i} \mathtt{q} \sigma^z_i
   Here, :math: \ \ i \ j \ angle, i < j \ denotes nearest neighbor pairs, each pair 
→appearing
   exactly once.
   All parameters are collected in a single dictionary `model_params`, which
    is turned into a :class: `~tenpy.tools.params.Config` object.
   Parameters
   model_params : :class:`~tenpy.tools.params.Config`
       Parameters for the model. See :cfg:config:`TFIModel` below.
   Options
    .. cfg:config :: TFIModel
        :include: CouplingMPOModel
        conserve : None | 'parity'
           What should be conserved. See :class:`~tenpy.networks.Site.SpinHalfSite`.
        J, q : float | array
            Coupling as defined for the Hamiltonian above.
    n n n
    def init_sites(self, model_params):
       conserve = model_params.get('conserve', 'parity')
        assert conserve != 'Sz'
        if conserve == 'best':
            conserve = 'parity'
           if self.verbose >= 1.:
                print(self.name + ": set conserve to", conserve)
        site = SpinHalfSite(conserve=conserve)
        return site
   def init_terms(self, model_params):
       J = np.asarray(model_params.get('J', 1.))
        g = np.asarray(model_params.get('g', 1.))
        for u in range(len(self.lat.unit_cell)):
           self.add_onsite(-g, u, 'Sigmaz')
        for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
            self.add_coupling(-J, u1, 'Sigmax', u2, 'Sigmax', dx)
        # done
class TFIChain(TFIModel, NearestNeighborModel):
    """The :class: `TFIModel` on a Chain, suitable for TEBD.
    See the :class:`TFIModel` for the documentation of parameters.
    def __init__(self, model_params):
       model_params = asConfig(model_params, self.__class__.__name__)
       model_params.setdefault('lattice', "Chain")
        CouplingMPOModel.__init__(self, model_params)
```

## **Automation of Hermitian conjugation**

As most physical Hamiltonians are Hermitian, these Hamiltonians are fully determined when only half of the mutually conjugate terms is defined. For example, a simple Hamiltonian:

$$H = \sum_{\langle i,j \rangle, i < j} - J(c_i^{\dagger} c_j + c_j^{\dagger} c_i)$$

is fully determined by the term  $c_i^{\dagger}c_j$  if we demand that Hermitian conjugates are included automatically. In TeNPy, whenever you add a coupling using  $add\_onsite()$ ,  $add\_coupling()$ , or  $add\_multi\_coupling()$ , you can use the optional argument  $plus\_hc$  to automatically create and add the Hermitian conjugate of that coupling term - as shown above.

Additionally, in an MPO, explicitly adding both a non-Hermitian term and its conjugate increases the bond dimension of the MPO, which increases the memory requirements of the MPOEnvironment. Instead of adding the conjugate terms explicitly, you can set a flag *explicit\_plus\_hc* in the MPOCouplingModel parameters, which will ensure two things:

- 1. The model and the MPO will only store half the terms of each Hermitian conjugate pair added, but the flag  $explicit\_plus\_hc$  indicates that they  $represent\ self+h.c.$ . In the example above, only the term  $c_i^{\dagger}c_j$  would be saved.
- 2. At runtime during DMRG, the Hermitian conjugate of the (now non-Hermitian) MPO will be computed and applied along with the MPO, so that the effective Hamiltonian is still Hermitian.

**Note:** The model flag *explicit\_plus\_hc* should be used in conjunction with the flag *plus\_hc* in *add\_coupling()* or *add\_multi\_coupling()*. If *plus\_hc* is *False* while *explicit\_plus\_hc* is *True* the MPO bond dimension will not be reduced, but you will still pay the additional computational cost of computing the Hermitian conjugate at runtime.

Thus, we end up with several use cases, depending on your preferences. Consider the FermionModel. If you do not care about the MPO bond dimension, and want to add Hermitian conjugate terms manually, you would set  $model\_par[\'explicit\_plus\_hc'] = False$  and write:

```
self.add_coupling(-J, u1, 'Cd', u2, 'C', dx)
self.add_coupling(np.conj(-J), u2, 'Cd', u1, 'C', -dx)
```

If you wanted to save the trouble of the extra line of code (but still did not care about MPO bond dimension), you would keep the *model\_par*, but instead write:

```
self.add_coupling(-J, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Finally, if you wanted a reduction in MPO bond dimension, you would need to set *model\_par['explicit\_plus\_hc'] = True*, and write:

```
self.add_coupling(-J, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

#### Some random remarks on models

- Needless to say that we have also various predefined models under tenpy.models.
- Of course, an MPO is all you need to initialize a MPOModel to be used for DMRG; you don't have to use the CouplingModel or CouplingMPOModel. For example an exponentially decaying long-range interactions are not supported by the coupling model but straight-forward to include to an MPO, as demonstrated in the example examples/mpo\_exponentially\_decaying.py.
- If the model of your interest contains Fermions, you should read the *Fermions and the Jordan-Wigner transformation*.
- We suggest writing the model to take a single parameter dictionary for the initialization, as the CouplingMPOModel does. The CouplingMPOModel converts the dictionary to a dict-like Config with some additional features before passing it on to the *init\_lattice*, *init\_site*, ... methods. It is recommended to read out providing default values with model\_params.get("key", default\_value), see get().
- When you write a model and want to include a test that it can be at least constructed, take a look at tests/test\_model.py.

# 7.3.6 Details on the lattice geometry

The Lattice class defines the geometry of the system. In the basic form, it represents a unit cell of a few sites repeated in one or multiple directions. Moreover, it maps this higher-dimensional geometry to a one-dimensional chain for MPS-based algorithms.

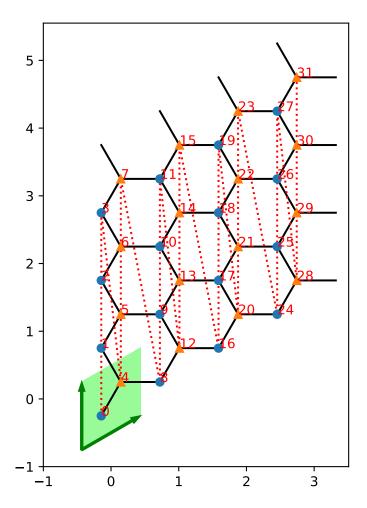
#### Visualization

A plot of the lattice can greatly help to understand which sites are connected by what couplings. The methods plot\_\* of the Lattice can do a good job for a quick illustration. Let's look at the Honeycomb lattice as an example.

```
import matplotlib.pyplot as plt
from tenpy.models import lattice

plt.figure(figsize=(5, 6))
ax = plt.gca()
lat = lattice.Honeycomb(Lx=4, Ly=4, sites=None, bc='periodic')
lat.plot_coupling(ax)
lat.plot_order(ax, linestyle=':')
lat.plot_sites(ax)
lat.plot_basis(ax, origin=-0.5*(lat.basis[0] + lat.basis[1]))
ax.set_aspect('equal')
ax.set_xlim(-1)
ax.set_ylim(-1)
plt.show()
```

In this case, the unit cell (shaded green) consists of two sites, which for the purpose of plotting we just set to sites=None; in general you should specify instances of Site for that. The unit cell gets repeated in the directions given by the lattice basis (green arrows at the unit cell boundary). Hence, we can label each site by a **lattice** index (x, y, u) in this case, where x in range (Lx), y in range (Ly) specify the translation of the unit cell and u in range  $(len(unit_cell))$ , here u in [0, 1], specifies the index within the unit cell.



#### How an MPS winds through the lattice: the order

For MPS-based algorithms, we need to map a 2D lattice like the one above to a 1D chain. The red, dashed line in the plot indicates how an MPS winds through the 2D lattice. The **MPS index** i is a simple enumeration of the sites along this line, shown as numbers next to the sites in the plot. The methods  $mps2lat_idx()$  and  $lat2mps_idx()$  map indices of the MPS to and from indices of the lattice.

The MPS class itself is (mostly) agnostic of the underlying geometry. For example, expectation\_value() will return a 1D array of the expectation value on each site indexed by the MPS index *i*. If you have a two-dimensional lattice, you can use mps2lat\_values() to map this result to a 2D array index by the lattice indices.

A suitable order is critical for the efficiency of MPS-based algorithms. On one hand, different orderings can lead to different MPO bond-dimensions, with direct impact on the complexity scaling. On the other hand, it influences how much entanglement needs to go through each bonds of the underlying MPS, e.g., the ground strate to be found in DMRG, and therefore influences the required MPS bond dimensions. For the latter reason, the "optimal" ordering can not be known a priori and might even depend on your coupling parameters (and the phase you are in). In the end, you can just try different orderings and see which one works best.

The simplets way to *change* the order is to use a non-default value for the initialization parameter *order* of the *Lattice* class. This gets passed on to *ordering()*, which you an override in a custom lattice class to define new possible orderings. Alternatively, you can go the most general way and simply set the attribute *order* to be a 2D numpy array with lattice indices as rows, in the order you want.

```
import matplotlib.pyplot as plt
from tenpy.models import lattice
Lx, Ly = 3, 3
fig, axes = plt.subplots(2, 2, figsize=(7, 8))
lat1 = lattice.Honeycomb(Lx, Ly, sites=None, bc='periodic') # default order
lat2 = lattice.Honeycomb(Lx, Ly, sites=None, bc='periodic',
                       order="Cstyle") # first method to change order
# alternative: directly set "Cstyle" order
lat3 = lattice.Honeycomb(Lx, Ly, sites=None, bc='periodic')
lat3.order = lat2.ordering("Cstyle") # now equivalent to lat2
# general: can apply arbitrary permutation to the order
lat4 = lattice. Honeycomb (Lx, Ly, sites=None, bc='periodic',
                        order="Cstyle")
old_order = lat4.order
permutation = []
for i in range(0, len(old_order), 2):
   permutation.append(i+1)
   permutation.append(i)
lat4.order = old_order[permutation, :]
for lat, label, ax in zip([lat1, lat2, lat3, lat4],
                          ["order='default'",
                           "order='Cstyle'",
                           "order='Cstyle'",
                           "custom permutation"],
                          axes.flatten()):
   lat.plot_coupling(ax)
   lat.plot_sites(ax)
   lat.plot_order(ax, linestyle=':', linewidth=2.)
   ax.set_aspect('equal')
   ax.set_title('order = ' + repr(label))
```

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```
plt.show()
```

#### **Boundary conditions**

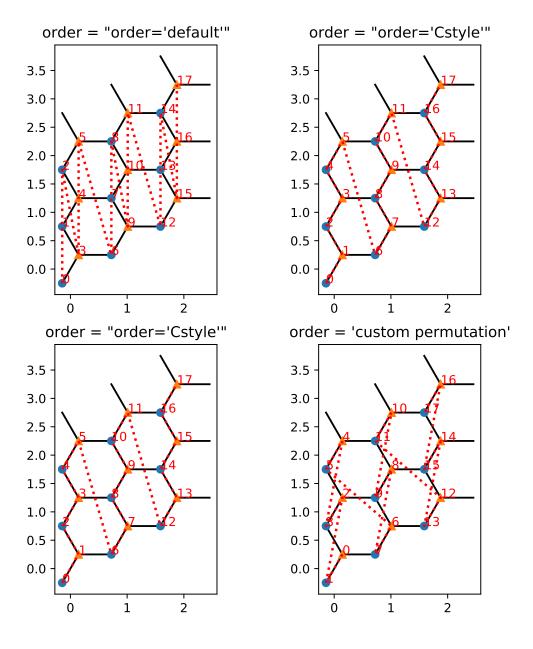
The Lattice defines the **boundary conditions** bc in each direction. It can be one of the usual 'open' or 'periodic' in each direction.

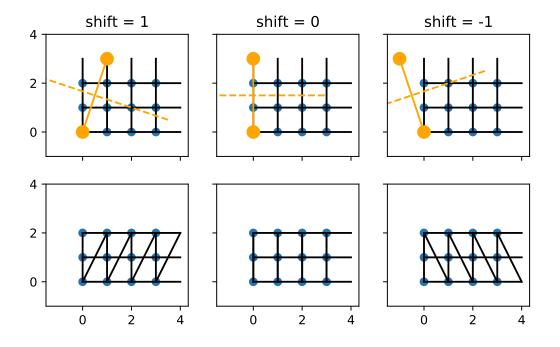
On top of that, there is the *bc\_MPS* boundary condition of the MPS, one of 'finite', 'segment', 'infinite'. For an 'infinite' MPS, the whole lattice is repeated in the direction of the *first* basis vector of the lattice. For bc\_MPS='infinite', the first direction should always be 'periodic', but you *can* also define a lattice with bc\_MPS='finite', bc=['periodic', 'perioid'] for a finite system on the torus. This is discouraged, though, because the ground state MPS will require the *squared* bond dimension for the *same* precision in this case!

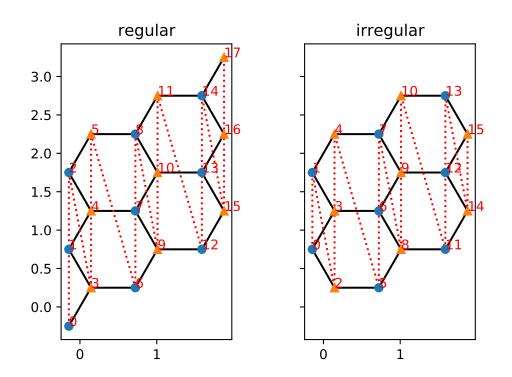
For two (or higher) dimensional lattices, e.g for DMRG on an infinite cylinder, you can also specify an integer *shift* instead of just saying 'periodic': Rolling the 2D lattice up into a cylinder, you have a degree of freedom which sites to connect. This is illustrated in the figure below for a *Square* lattice with bc=['periodic', shift] for shift in [-1, 0, 1] (different columns). In the first row, the orange markers indicate a pair of identified sites (see plot\_bc\_shift()). The dashed orange line indicates the direction of the cylinder axis. The line where the cylinder is "cut open" therefore winds around the the cylinder for a non-zero *shift*. (A similar thing happens even for shift=0 for more complicated lattices with non-orthogonal basis.) In the second row, we directly draw lines between all sites connected by nearest-neighbor couplings, as they appear in the MPO.

## **Irregular Lattices**

The IrregularLattice allows to add or remove sites from/to an existing regular lattice. The doc-string of IrregularLattice contains several examples, let us consider another one here, where we use the IrregularLattice to "fix" the boundary of the Honeycomb lattice: when we use "open" boundary conditions for a finite system, there are two sites (on the lower left, and upper right), wich are not included into any hexagonal. The following example shows how to remove them from the system:







# 7.3.7 Fermions and the Jordan-Wigner transformation

The Jordan-Wigner tranformation maps fermionic creation- and annihilation operators to (bosonic) spin-operators.

#### Spinless fermions in 1D

Let's start by explicitly writing down the transformation. With the Pauli matrices  $\sigma_j^{x,y,z}$  and  $\sigma_j^{\pm} = (\sigma_j^x \pm i\sigma_j^y)/2$  on each site, we can map

$$\begin{split} n_j &\leftrightarrow (\sigma_j^z + 1)/2 \\ c_j &\leftrightarrow (-1)^{\sum_{l < j} n_l} \sigma_j^- \\ c_j^\dagger &\leftrightarrow (-1)^{\sum_{l < j} n_l} \sigma_j^+ \end{split}$$

The  $n_l$  in the second and third row are defined in terms of Pauli matrices according to the first row. We do not interpret the Pauli matrices as spin-1/2; they have nothing to do with the spin in the spin-full case. If you really want to interpret them physically, you might better think of them as hard-core bosons  $(b_j = \sigma_j^-, b_j^\dagger = \sigma_j^+)$ , with a spin of the fermions mapping to a spin of the hard-core bosons.

Note that this transformation maps the fermionic operators  $c_j$  and  $c_j^{\dagger}$  to global operators; although they carry an index j indicating a site, they actually act on all sites 1 <= j! Thus, clearly the operators C and Cd defined in the FermionSite do not directly correspond to  $c_j$  and  $c_j^{\dagger}$ . The part  $(-1)^{\sum_{l < j} n_l}$  is called Jordan-Wigner string and in the FermionSite is given by the local operator  $JW := (-1)^{n_l}$  acting all sites 1 < j. Since this important, let me stress it again:

**Warning:** The fermionic operator  $c_j$  (and similar  $c_j^{\dagger}$ ) maps to a *global* operator consisting of the Jordan-Wigner string built by the local operator JW on sites 1 < j and the local operator C (or Cd, respectively) on site j.

On the sites itself, the onsite operators C and Cd in the FermionSite fulfill the correct anti-commutation relation, without the need to include JW strings. The JW string is necessary to ensure the anti-commutation for operators acting on different sites.

Written in terms of *onsite* operators defined in the FermionSite, with the *i*-th entry entry in the list acting on site *i*, the relations are thus:

```
["JW", ..., "JW", "C", "Id", ..., "Id"] # for the annihilation operator
["JW", ..., "JW", "Cd", "Id", ..., "Id"] # for the creation operator
```

Note that "JW" squares to the identity, "JW JW" == "Id", which is the reason that the Jordan-wigner string completely cancels in  $n_j = c_j^{\dagger} c_j$ . In the above notation, this can be written as:

```
["JW", ..., "JW", "Cd", "Id", ..., "Id"] * ["JW", ..., "JW", "C", "Id", ..., "Id"]

== ["JW JW", ..., "JW JW", "Cd C", "Id Id", ..., "Id Id"] # by definition of the tensorproduct

== ["Id", ..., "Id", "N", "Id", ..., "Id"] # by definition of the local operators

# ("X Y" stands for the local operators X and Y applied on the same site. We assume that the "Cd" and "C" on the first line act on the same site.)
```

For a pair of operators acting on different sites, JW strings have to be included for every site between the operators. For example, taking i < j,  $c_i^{\dagger}c_j \leftrightarrow \sigma_i^+(-1)^{\sum_{i < = l < j} n_l}\sigma_j^-$ . More explicitly, for j = i+2 we get:

```
["JW", ..., "JW", "Cd", "Id", "Id", "Id", ..., "Id"] * ["JW", ..., "JW", "JW", "JW", "JW", "C", "Id"] * ["JW JW", ..., "Id"] * ["Id", ..., "Id"] * ["Id", ..., "Id"] * ["Id", ..., "Id"]
```

In other words, the Jordan-Wigner string appears only in the range i <= 1 < j, i.e. between the two sites *and* on the smaller/left one of them. (You can easily generalize this rule to cases with more than two c or  $c^{\dagger}$ .)

This last line (as well as the last line of the previous example) can be rewritten by changing the order of the operators  $Cd \ JW \ to \ "JW \ Cd" == - \ "Cd"$ . (This is valid because either site i is occupied, yielding a minus sign from the JW, or it is empty, yielding a 0 from the Cd.)

This is also the case for j < i, say j = i-2:  $c_i^{\dagger}c_j \leftrightarrow (-1)^{\sum_{j < = l < i} n_l} \sigma_i^+ \sigma_j^-$ . As shown in the following, the JW again appears on the left site, but this time acting *after* C:

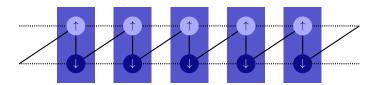
```
["JW", ..., "JW", "JW", "Cd", "Id", ..., "Id"] * ["JW", ..., "JW", "C", "Id", 

→"Id", "Id", ..., "Id"]
== ["JW JW", ..., "JW JW", "JW C", "JW", "Cd Id", ..., "Id"]
== ["Id", ..., "Id", "JW C", "JW", "Cd", ..., "Id"]
```

#### **Higher dimensions**

For an MPO or MPS, you always have to define an ordering of all your sites. This ordering effectifely maps the higher-dimensional lattice to a 1D chain, usually at the expence of long-range hopping/interactions. With this mapping, the Jordan-Wigner transformation generalizes to higher dimensions in a straight-forward way.

#### Spinful fermions



As illustrated in the above picture, you can think of spin-1/2 fermions on a chain as spinless fermions living on a ladder (and analogous mappings for higher dimensional lattices). Each rung (a blue box in the picture) forms a SpinHalfFermionSite which is composed of two FermionSite (the circles in the picture) for spin-up and spin-down. The mapping of the spin-1/2 fermions onto the ladder induces an ordering of the spins, as the final result must again be a one-dimensional chain, now containing both spin species. The solid line indicates the convention for the ordering, the dashed lines indicate spin-preserving hopping  $c_{s,i}^{\dagger}c_{s,i+1} + h.c.$  and visualize the ladder structure. More generally, each species of fermions appearing in your model gets a separate label, and its Jordan-Wigner string includes the signs  $(-1)^{n_l}$  of all species of fermions to the 'left' of it (in the sense of the ordering indicated by the solid line in the picture).

In the case of spin-1/2 fermions labeled by  $\uparrow$  and  $\downarrow$  on each *site*, the complete mapping is given (where j and l are indices of the FermionSite):

$$n_{\uparrow,j} \leftrightarrow (\sigma_{\uparrow,j}^z + 1)/2$$

$$n_{\downarrow,j} \leftrightarrow (\sigma_{\downarrow,j}^z + 1)/2$$

$$c_{\uparrow,j} \leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} \sigma_{\uparrow,j}^-$$

$$c_{\uparrow,j}^{\dagger} \leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} \sigma_{\uparrow,j}^+$$

$$c_{\downarrow,j} \leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} (-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^-$$

$$c_{\downarrow,j}^{\dagger} \leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} (-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^+$$

$$c_{\downarrow,j}^{\dagger} \leftrightarrow (-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} (-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^+$$

In each of the above mappings the operators on the right hand sides commute; we can rewrite  $(-1)^{\sum_{l < j} n_{\uparrow,l} + n_{\downarrow,l}} = \prod_{l < j} (-1)^{n_{\uparrow,l}} (-1)^{n_{\downarrow,l}}$ , which resembles the actual structure in the code more closely. The parts of the operator acting in the same box of the picture, i.e. which have the same index j or l, are the 'onsite' operators in the SpinHalfFermionSite: for example JW on site j is given by  $(-1)^{n_{\uparrow,j}} (-1)^{n_{\downarrow,j}}$ , Cu is just the  $\sigma_{\uparrow,j}^-$ , Cdu is  $\sigma_{\uparrow,j}^+$ , Cd is  $(-1)^{n_{\uparrow,j}} \sigma_{\downarrow,j}^-$ . Note the asymmetry regarding the spin in the definition of the onsite operators: the spin-down operators include Jordan-Wigner signs for the spin-up fermions on the same site. This asymetry stems from the ordering convention introduced by the solid line in the picture, according to which the spin-up site is "left" of the spin-down site. With the above definition, the operators within the same SpinHalfFermionSite fulfill the expected commutation relations, for example "Cu Cdd" == - "Cdd Cu", but again the JW on sites left of the operator pair is crucial to get the correct commutation relations globally.

**Warning:** Again, the fermionic operators  $c_{\downarrow,j}, c_{\downarrow,j}^{\dagger}, c_{\downarrow,j}, c_{\downarrow,j}^{\dagger}$  correspond to *global* operators consisting of the Jordan-Wigner string built by the local operator JW on sites 1 < j and the local operators 'Cu', 'Cdu', 'Cd', 'Cdd' on site j.

Written explicitly in terms of onsite operators defined in the FermionSite, with the *j*-th entry entry in the list acting on site *j*, the relations are:

```
["JW", ..., "JW", "Cu", "Id", ..., "Id"] # for the annihilation operator spin-up
["JW", ..., "JW", "Cd", "Id", ..., "Id"] # for the annihilation operator spin-down
["JW", ..., "JW", "Cdu", "Id", ..., "Id"] # for the creation operator spin-up
["JW", ..., "JW", "Cdd", "Id", ..., "Id"] # for the creation operator spin-down
```

As you can see, the asymmetry regaring the spins in the definition of the local onsite operators "Cu", "Cdu", "Cd

#### How to handle Jordan-Wigner strings in practice

There are only a few pitfalls where you have to keep the mapping in mind: When **building a model**, you map the physical fermionic operators to the usual spin/bosonic operators. The algorithms don't care about the mapping, they just use the given Hamiltonian, be it given as MPO for DMRG or as nearest neighbor couplings for TEBD. Only when you do a **measurement** (e.g. by calculating an expectation value or a correlation function), you have to reverse this mapping. Be aware that in certain cases, e.g. when calculating the entanglement entropy on a certain bond, you cannot reverse this mapping (in a straightforward way), and thus your results might depend on how you defined the Jordan-Wigner string.

Whatever you do, you should first think about if (and how much of) the Jordan-Wigner string cancels. For example for many of the onsite operators (like the particle number operator N or the spin operators in the SpinHalfFermionSite) the Jordan-Wigner string cancels completely and you can just ignore it both in onsite-terms and couplings. In case of two operators acting on different sites, you typically have a Jordan-Wigner string inbetween (e.g. for the  $c_i^{\dagger}c_j$  examples described above and below) or no Jordan-Wigner strings at all (e.g. for density-density interactions  $n_i n_j$ ). In fact, the case that the Jordan Wigner string on the left of the first non-trivial operator does not cancel is currently not supported for models and expectation values, as it usually doesn't appear in practice. For terms involving more operators, things tend to get more complicated, e.g.  $c_i^{\dagger}c_j^{\dagger}c_kc_l$  with i < j < k < l requires a Jordan-Wigner string on sites m with  $i \le m < j$  or  $k \le m < l$ , but not for j < m < k.

**Note:** TeNPy keeps track of which onsite operators need a Jordan-Wigner string in the Site class, specifically in  $need\_JW\_string$  and  $op\_needs\_JW()$ . Hence, when you define custom sites or add extra operators to the sites, make sure that  $op\_needs\_JW()$  returns the expected results.

When **building a model** the Jordan-Wigner strings need to be taken into account. If you just specify the *H\_MPO* or *H\_bond*, it is *your* responsibility to use the correct mapping. However, if you use the <code>add\_coupling()</code> method of the <code>CouplingModel</code>, (or the generalization <code>add\_multi\_coupling()</code> for more than 2 operators), TeNPy can use the information from the <code>Site</code> class to <code>automatically add Jordan-Wigner</code> strings as needed. Indeed, with the default argument <code>op\_string=None</code>, <code>add\_coupling</code> will automatically check whether the operators need Jordan-Wigner strings and correspondlingly set <code>op\_string='JW'</code>, <code>str\_on\_first=True</code>, if necessary. For <code>add\_multi\_coupling</code>, you cann't even explicitly specify the correct Jordan-Wigner strings, but you <code>must use op\_string=None</code>, from which it will automatically determine where Jordan-Wigner strings are needed.

Obviously, you should be careful about the convention which of the operators is applied first (in a physical sense as an operator acting on a state), as this corresponds to a sign of the prefactor. Read the doc-strings of add\_coupling() add\_multi\_coupling() for details.

As a concrete example, let us specify a hopping  $\sum_i (c_i^{\dagger} c_{i+1} + h.c.) = \sum_i (c_i^{\dagger} c_{i+1} + c_i^{\dagger} c_{i-1})$  in a 1D chain of FermionSite with add\_coupling(). The recommended way is just:

```
add_coupling(strength, 0, 'Cd', 0, 'C', 1, plus_hc=True)
```

If you want to specify both the Jordan-Wigner string and the h.c. term explicitly, you can use:

```
add_coupling(strength, 0, 'Cd', 0, 'C', 1, op_string='JW', str_on_first=True) add_coupling(strength, 0, 'Cd', 0, 'C', -1, op_string='JW', str_on_first=True)
```

Slightly more complicated, to specify the hopping  $\sum_{\langle i,j\rangle,s}(c_{s,i}^{\dagger}c_{s,j}+h.c.)$  in the Fermi-Hubbard model on a 2D square lattice, we could use:

```
for (dx, dy) in [(1, 0), (0, 1)]:
    add_coupling(strength, 0, 'Cdu', 0, 'Cu', (dx, dy), plus_hc=True) # spin up
    add_coupling(strength, 0, 'Cdd', 0, 'Cd', (dx, dy), plus_hc=True) # spin down

# or without `plus_hc`
for (dx, dy) in [(1, 0), (-1, 0), (0, 1), (0, -1)]: # include -dx !
    add_coupling(strength, 0, 'Cdu', 0, 'Cu', (dx, dy)) # spin up
    add_coupling(strength, 0, 'Cdd', 0, 'Cd', (dx, dy)) # spin down

# or specifying the 'JW' string explicitly
for (dx, dy) in [(1, 0), (-1, 0), (0, 1), (0, -1)]:
    add_coupling(strength, 0, 'Cdu', 0, 'Cu', (dx, dy), 'JW', True) # spin up
    add_coupling(strength, 0, 'Cdd', 0, 'Cd', (dx, dy), 'JW', True) # spin down
```

The most important functions for doing **measurements** are probably expectation\_value() and correlation\_function(). Again, if all the Jordan-Wigner strings cancel, you don't have to worry about them at all, e.g. for many onsite operators or correlation functions involving only number operators. If you build multi-site operators to be measured by *expectation\_value*, take care to include the Jordan-Wigner string correctly.

Some MPS methods like <code>correlation\_function()</code>, <code>expectation\_value\_term()</code> and <code>expectation\_value\_terms\_sum()</code> automatically add Jordan-Wignder strings (at least with default arguments). Other more low-level functions like <code>expectation\_value\_multi\_sites()</code> don't do it. Hence, you should always watch out during measurements, if the function used needs special treatment for Jordan-Wigner strings.

# 7.3.8 Saving to disk: input/output

## Using pickle

A simple and pythonic way to store data of TeNPy arrays is to use pickle from the Python standard library. Pickle allows to store (almost) arbitrary python objects, and the Array is no exception (and neither are other TeNPy classes).

Say that you have run DMRG to get a ground state *psi* as an MPS. With pickle, you can save it to disk as follows:

```
import pickle
with open('my_psi_file.pkl', 'wb') as f:
    pickle.dump(psi, f)
```

Here, the with ...: structure ensures that the file gets closed after the pickle dump, and the 'wb' indicates the file opening mode "write binary". Reading the data from disk is as easy as ('rb' for reading binary):

```
with open('my_psi_file.pkl', 'rb') as f:
    psi = pickle.load(f)
```

**Note:** It is a good (scientific) practice to include meta-data to the file, like the parameters you used to generate that state. Instead of just the *psi*, you can simply store a dictionary containing *psi* and other data, e.g., data = {'psi': psi, 'dmrg\_params': dmrg\_params, 'model\_params': model\_params}. This can *save you a lot of pain*, when you come back looking at the files a few month later and forgot what you've done to generate them!

In some cases, compression can significantly reduce the space needed to save the data. This can for example be done with gzip (as well in the Python standard library). However, be warned that it might cause longer loading and saving times, i.e. it comes at the penalty of more CPU usage for the input/output. In Python, this requires only small adjustments:

```
import pickle
import gzip

# to save:
with gzip.open('my_data_file.pkl', 'wb') as f:
    pickle.dump(data, f)
# and to load:
with gzip.open('my_data_file.pkl', 'rb') as f:
    data = pickle.load(data, f)
```

## Using HDF5 with h5py

While pickle is great for simple input/output of python objects, it also has disadvantages. The probably most dramatic one is the limited portability: saving data on one PC and loading it on another one might fail! Even exporting data from Python 2 to load them in Python 3 on the same machine can give quite some troubles. Moreover, pickle requires to load the whole file at once, which might be unnecessary if you only need part of the data, or even lead to memory problems if you have more data on disk than fits into RAM.

Hence, we support saving to HDF5 files as an alternative. The h5py package provides a dictionary-like interface for the file/group objects with numpy-like data sets, and is quite easy to use. If you don't know about HDF5, read the quickstart of the h5py documentation (and this guide).

The implementation can be found in the tenpy.tools.hdf5\_io module with the Hdf5Saver and Hdf5Loader classes and the wrapper functions save\_to\_hdf5(), load\_from\_hdf5().

The usage is very similar to pickle:

**Note:** The hickle package imitates the pickle functionality while saving the data to HDF5 files. However, since it aims to be close to pickle, it results in a more complicated data structure than we want here.

**Note:** To use the export/import features to HDF5, you need to install the h5py python package (and hence some version of the HDF5 library).

### Data format specification for saving to HDF5

This section motivates and defines the format how we save data of TeNPy-defined classes. The goal is to have the <code>save\_to\_hdf5()</code> function for saving sufficiently simple enough python objects (supported by the format) to disk in an HDF5 file, such that they can be reconstructed with the <code>load\_from\_hdf5()</code> function, as outlined in the example code above.

Guidelines of the format:

- 0. Store enough data such that <code>load\_from\_hdf5()</code> can reconstruct a copy of the object (provided that the save did not fail with an error).
- 1. Objects of a type supported by the HDF5 datasets (with the h5py interface) should be directly stored as h5py Dataset. Such objects are for example numpy arrays (of non-object *dtype*), scalars and strings.
- 2. Allow to save (nested) python lists, tuples and dictionaries with values (and keys) which can be saved.
- 3. Allow user-defined classes to implement a well-defined interface which allows to save instances of that class, hence extending what data can be saved. An instance of a class supporting the interface gets saved as an HDF5 Group. Class attributes are stored as entries of the group, metadata like the type should be stored in HDF5 attributes, see attributes.
- 4. Simple and intuitive, human-readable structure for the HDF5 paths. For example, saving a simple dictionary {'a': np.arange(10), 'b': 123.45} should result in an HDF5 file with just the two data sets /a and /b.
- 5. Allow loading only a subset of the data by specifying the *path* of the HDF5 group to be loaded. For the above example, specifying the path /b should result in loading the float 123.45, not the array.
- 6. Avoid unnecessary copies if the same python object is referenced by different names, e.g, for the data { 'c': large\_obj, 'd': large\_obj} with to references to the same large\_obj, save it only once and use HDF5 hard-links such that /c and /d are the same HDF5 dataset/group. Also avoid the copies during the

- loading, i.e., the loaded dictionary should again have two references to a single object *large\_obj*. This is also necessary to allow saving and loading of objects with cyclic references.
- 7. Loading a dataset should be (fairly) secure and not execute arbitrary python code (even if the dataset was manipulated), as it is the case for pickle.
  - *Disclaimer*: I'm not an security expert, so I can't guarantee that... Also, loading a HDF5 file can import other python modules, so importing a manipulated file is not secure if you downloaded a malicious python file as well.

The full format specification is given by the what the code in hdf5\_io does... Since this is not trivial to understand, let me summarize it here:

- Following 1), simple scalars, strings and numpy arrays are saved as Dataset. Other objects are saved as a HDF5 Group, with the actual data being saved as group members (as sub-groups and sub-datasets) or as attributes (for metadata or simple data).
- The type of the object is stored in the HDF5 attribute 'type', which is one of the global REPR\_\* variables in tenpy.tools.hdf5\_io. The type determines the format for saving/loading of builtin types (list, ...)
- Userdefined classes which should be possible to export/import need to implement the methods <code>save\_hdf5</code> and <code>from\_hdf5</code> as specified in <code>Hdf5Exportable</code>. When saving such a class, the attribute 'type' is automatically set to 'instance', and the class name and module are saved under the attributes 'module' and 'class'. During loading, this information is used to automatically import the module, get the class and call the classmethod <code>from\_hdf5</code> for reconstruction. This can only work if the class definition already exists, i.e., you can only save class instances, not classes itself.
- For most (python) classes, simply subclassing <code>Hdf5Exportable</code> should work to make the class exportable. The latter saves the contents of <code>\_\_dict\_\_</code>, with the extra attribute 'format' specifying whether the dictionary is "simple" (see below.).
- The None object is saved as a group with the attribute 'type' being 'None' and no subgroups.
- For iterables (list, tuple and set), we simple enumerate the entries and save entries as group members under the names '0', '1', '2', ..., and a maximum 'len' attribute.
- The format for dictionaries depends on whether all keys are "simple", which we define as being strings which are valid path names in HDF5, see <a href="valid\_hdf5\_path\_component">valid\_hdf5\_path\_component</a> (). Following 4), the keys of a simple dictionary are directly used as names for group members, and the values being whatever object the group member represents.
- Partial loading along 5) is possible by directly specifying the subgroup or the path to <code>load\_from\_hdf5()</code>.
- Guideline 6) is ensured as much as possible. However, there is a bug/exception: tuples with cyclic references are not re-constructed correctly; the inner objects will be lists instead of tuples (but with the same object entries).

Finally, we have to mention that many TeNPy classes are *Hdf5Exportable*. In particular, the *Array* supports this. To see what the exact format for those classes is, look at the *save hdf5* and *from hdf5* methods of those classes.

**Note:** There can be multiple possible output formats for the same object. The dictionary – with the format for simple keys or general keys – is such an example, but userdefined classes can use the same technique in their *from\_hdf5* method. The user might also explicitly choose a "lossy" output format (e.g. "flat" for np\_conserved Arrays and LegCharges).

**Tip:** The above format specification is quite general and not bound to TeNPy. Feel free to use it in your own projects ;-) To separate the development, versions and issues of the format clearly from TeNPy, we maintain the code for it in a separate git repository, https://github.com/tenpy/hdf5\_io

## 7.4 Literature

This is a (by far non-exhaustive) list of some references for the various ideas behind the code. They can be cited from the python doc-strings using the format [Author####]\_. Within each category, we sort the references by year and author.

# 7.4.1 TeNPy related sources

[TeNPyNotes] are lecture notes, meant as an introduction to tensor networks (focusing on MPS), and introduced TeNPy to the scientific community by giving examples how to call the algorithms in TeNPy. [TeNPySource] is the location of the source code, and the place where you can report bugs. [TeNPyDoc] is where the location is hosted online. [TeNPyForum] is the place where you can ask questions and look for help, when you are stuck with implementing something.

# 7.4.2 General reading

[Schollwoeck2011] is an extensive introduction to MPS, DMRG and TEBD with lots of details on the implementations, and a classic read, although a bit lengthy. Our [TeNPyNotes] are a shorter summary of the important concepts, similar as [Orus2014]. [Hubig2019] is a very good, recent review focusing on time evolution with MPS. The lecture notes of [Eisert2013] explain the area law as motivation for tensor networks very well. PEPS are for example reviewed in [Verstraete2009], [Eisert2013] and [Orus2014]. [Stoudenmire2011] reviews the use of DMRG for 2D systems. [Cirac2009] discusses the different groups of tensor network states.

# 7.4.3 Algorithm developments

[White1992] is the invention of DMRG, which started everything. [Vidal2004] introduced TEBD. [White2005] and [Hubig2015] solved problems for single-site DMRG. [McCulloch2008] was a huge step forward to solve convergence problems for infinite DMRG. [Singh2009], [Singh2010] explain how to incorporate Symmetries. [Haegeman2011] introduced TDVP, again explained more accessible in [Haegeman2016]. [Zaletel2015] is another standard method for time-evolution with long-range Hamiltonians. [Karrasch2013] gives some tricks to do finite-temperature simulations (DMRG), which is a bit extended in [Hauschild2018]. [Vidal2007] introduced MERA.

# 7.4.4 Related theory

The following are referenced from somewhere in the algorithms.

#### 7.4.5 Software-related

The following are not physics-related, but are good to know if you want to work with TeNPy (or more generally Python).

# 7.5 Contributing

There are lots of things where you can help, even if you don't wont to dig deep into the source code. You are welcome to do any of the following things, all of them are very helpful!

- Report bugs and problems, such that they can be fixed.
- Implement new models.
- Update and extend the documentation.
- Give feedback on how you like TeNPy and what you would like to see improved.
- · Help fixing bugs.
- Help fixing minor issues.
- Extend the functionality by implementing new functions, methods, and algorithms.

The code is maintained in a git repository, the official repository is on github. Even if you're not yet on the developer team, you can still submit pull requests on github. If you're unsure how or what to do, you can ask for help in the [TeNPyForum]. If you want to become a member of the developer team, just ask;-)

Thank You!

# 7.5.1 Coding Guidelines

To keep consistency, we ask you to comply with the following guidelines for contributions. However, these are just guidelines - it still helps if you contribute something, even if doesn't follow these rules ;-)

• Use a code style based on **PEP 8**. The git repo includes a config file .style.yapf for the python package yapf. yapf is a tool to auto-format code, e.g., by the command yapf -i some/file (-i for "in place"). We run yapf on a regular basis on the github master branch. If your branch diverged, it might help to run yapf before merging.

**Note:** Since no tool is perfect, you can format some regions of code manually and enclose them with the special comments # yapf: disable and # yapf: enable.

• Every function/class/module should be documented by its doc-string, see **PEP 257**. We auto-format the doc-strings with docformatter on a regular basis.

Additional documentation for the user guide is in the folder doc/.

The documentation uses *reStructuredText*. If you are new to *reStructuredText*, read this introduction. We use the *numpy* style for doc-strings (with the napoleon extension to sphinx). You can read abouth them in these Instructions for the doc strings. In addition, you can take a look at the following example file. Helpful hints on top of that:

```
r"""<- this r makes me a raw string, thus '\' has no special meaning.
Otherwise you would need to escape backslashes, e.g. in math formulas.

You can include cross references to classes, methods, functions, modules like :class:`~tenpy.linalg.np_conserved.Array`, :meth:`~tenpy.linalg.np_conserved.

Array.to_ndarray`,
:func:`tenpy.tools.math.toiterable`, :mod:`tenpy.linalg.np_conserved`.
The ~ in the beginning makes only the last part of the name appear in the_

generated documentation.

Documents of the userguide can be referenced with :doc:`/intro_npc` even from_

inside the doc-strings.

(continues on next page)
```

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```
You can also cross-link to other documentations, e.g. :class:`numpy.ndarray`, ... :func`scipy.linalg.svd` and :mod: will work.

Moreover, you can link to github issues, arXiv papers, dois, and topics in the ... community forum with e.g. :issue:`5`, :arxiv:`1805.00055`, :doi:`10.1000/1` and :forum:`3`.

Write inline formulas as :math:`H |\Psi\rangle = E |\Psi\rangle` or displayed. ... equations as ... math ::
    e^{{i\pi} + 1 = 0}

In doc-strings, math can only be used in the Notes section. To refer to variables within math, use `\mathtt{varname}`.

.. todo ::
    This block can describe things which need to be done and is automatically. ... included in a section of :doc:`todo`. """
```

• Use relative imports within TeNPy. Example:

```
from ..linalg import np_conserved as npc
```

• Use the python package pytest for testing. Run it simply with pytest in *tests/*. You should make sure that all tests run through, before you git push back into the public repo. Long-running tests are marked with the attribute *slow*; for a quick check you can also run pytest -m "not slow".

We have set up github actions to automatically run the tests.

- Reversely, if you write new functions, please also include suitable tests!
- During development, you might introduce # TODO comments. But also try to remove them again later! If you're not 100% sure that you will remove it soon, please add a doc-string with a .. todo :: block, such that we can keep track of it.

Unfinished functions should raise NotImplementedError().

- Summarize the changes you have made in the Changelog under /changelog/latest.
- If you want to try out new things in temporary files: any folder named playground is ignored by git.
- If you add a new toycode or example: add a reference to include it in the documentation.
- We've created a sphinx extensions for documenting config-option dictionaries. If a class takes a dictionary of options, we usually call it *options*, convert it to a Config at the very beginning of the \_\_init\_\_ with asConfig(), save it as self.options, and document it in the class doc-string with a . . cfg:config :: directive. The name of the config should usually be the class-name (if that is sufficiently unique), or for algorithms directly the common name of the algorithm, e.g. "DMRG"; use the same name for the use the same name for the documentation of the .. cfg:config :: directive as for the Config class instance. Attributes which are simply read-out options should be documented by just referencing the options with the :cfg:option:`configname.optionname` role.

# 7.5.2 Bulding the documentation

You can use Sphinx to generate the full documentation in various formats (including HTML or PDF) yourself, as described in the following. First, install the extra requirements, i.e., Sphinx, with:

pip install -r doc/requirements.txt

**Note:** Plotting the inheritance graphs also requires Graphviz. If you have *conda*, installing it requires just conda install graphviz.

Afterwards, simply go to the folder doc/ and run the following command:

make html

This should generate the html documentation in the folder *doc/sphinx\_build/html*. Open this folder (or to be precise: the file *index.html* in it) in your webbroser and enjoy this and other documentation beautifully rendered, with cross links, math formulas and even a search function. Other output formats are available as other make targets, e.g., make latexpdf.

**Note:** Building the documentation with sphinx requires loading the modules. The *conf.py* adjusts the python path to include the *tenpy* from root directory of the repository.

## 7.5.3 To-Do list

You can check https://github.com/tenpy/tenpy/issues for things to be done.

The following list is auto-generated by sphinx, extracting . . todo :: blocks from doc-strings of the code.

Todo: Write UserGuide!!!

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/algorithms/dmrg.py:do of tenpy.algorithms.dmrg, line 30.)

**Todo:** Rebuild TDVP engine as subclasses of sweep.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/algorithms/mps\_comm of tenpy.algorithms.mps\_common, line 21.)

#### Todo:

- implement or wrap netcon.m, a function to find optimal contractionn sequences (arXiv:1304.6112)
- · improve helpfulness of Warnings
- \_do\_trace: trace over all pairs of legs at once. need the corresponding npc function first.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/algorithms/network\_co of tenpy.algorithms.network\_contractor, line 10.)

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**Todo:** This is still a beta version, use with care. The interface might still change.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/algorithms/tdvp.py:docoftenpy.algorithms.tdvp, line 12.)

**Todo:** long-term: Much of the code is similar as in DMRG. To avoid too much duplicated code, we should have a general way to sweep through an MPS and updated one or two sites, used in both cases.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/algorithms/tdvp.py:docoftenpy.algorithms.tdvp, line 16.)

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/fermions\_spin

**Todo:** add further terms (e.g. c^dagger c^dagger + h.c.) to the Hamiltonian.

of tenpy.models.fermions\_spinless, line 3.)

**Todo:** WARNING: These models are still under development and not yet tested for correctness. Use at your own risk! Replicate known results to confirm models work correctly. Long term: implement different lattices. Long term: implement variable hopping strengths Jx, Jy.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/hofstadter.py:coof tenpy.models.hofstadter, line 3.)

**Todo:** make sure this function is used for expectation values...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/lattice.py:docs of tenpy.models.lattice.Honeycomb.mps2lat\_values, line 53.)

**Todo:** make sure this function is used for expectation values...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/lattice.py:docs of tenpy.models.lattice.IrregularLattice.mps2lat\_values, line 53.)

**Todo:** make sure this function is used for expectation values...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/lattice.py:docs of tenpy.models.lattice.Kagome.mps2lat\_values, line 53.)

**Todo:** make sure this function is used for expectation values...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/lattice.py:docs of tenpy.models.lattice.Ladder.mps2lat\_values, line 53.)

**Todo:** make sure this function is used for expectation values...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/lattice.py:docs of tenpy.models.lattice.Lattice.mps2lat\_values, line 53.)

**Todo:** make sure this function is used for expectation values...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/lattice.py:docs of tenpy.models.lattice.TrivialLattice.mps2lat\_values, line 53.)

**Todo:** implement MPO for time evolution...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/model.py:docs of tenpy.models.model.MPOModel, line 7.)

**Todo:** make sure this function is used for expectation values...

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/models/toric\_code.py: of tenpy.models.toric\_code.DualSquare.mps2lat\_values, line 53.)

**Todo:** This is a naive, expensive implementation contracting the full network. Try to follow arXiv:1711.01104 for a better estimate; would that even work in the infinite limit?

(The *original entry* is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/networks/mpo.py:docs of tenpy.networks.mpo.MPO.variance, line 5.)

**Todo:** might be useful to add a "cleanup" function which removes operators cancelling each other and/or unused states. Or better use a 'compress' of the MPO?

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/networks/mpo.py:docs of tenpy.networks.mpo.MPOGraph, line 17.)

**Todo:** Make more general: it should be possible to specify states as strings.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/networks/mps.py:docs of tenpy.networks.mps.build\_initial\_state, line 14.)

**Todo:** One can also look at the canonical ensembles by defining the conserved quantities differently, see Barthel (2016), arXiv:1607.01696 for details. Idea: usual charges on p, trivial charges on q; fix total charge to desired value. I think it should suffice to implement another  $from\_infiniteT$ .

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/networks/purification\_of tenpy.networks.purification\_mps, line 106.)

**Todo:** Check if Jordan-Wigner strings for 4x4 operators are correct.

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/networks/site.py:docst of tenpy.networks.site.SpinHalfFermionSite, line 61.)

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**Todo:** For memory caching with big MPO environments, we need a Hdf5Cacher clearing the memo's every now and then (triggered by what?).

(The original entry is located in /home/docs/checkouts/readthedocs.org/user\_builds/tenpy/checkouts/v0.7.0/tenpy/tools/hdf5\_io.py:docst of tenpy.tools.hdf5\_io, line 60.)

# 7.6 Tenpy main module

• full name: tenpy

• parent module: tenpy

• type: module

#### **Submodules**

algorithms	A collection of algorithms such as TEBD and DMRG.
linalg	Linear-algebra tools for tensor networks.
models	Definition of the various models.
networks	Definitions of tensor networks like MPS and MPO.
tools	A collection of tools: mostly short yet quite useful func-
	tions.
version	Access to version of this library.

## **Module description**

TeNPy - a Python library for Tensor Network Algorithms

TeNPy is a library for algorithms working with tensor networks, e.g., matrix product states and -operators, designed to study the physics of strongly correlated quantum systems. The code is intended to be accessible for newcommers and yet powerful enough for day-to-day research.

```
tenpy.__version__ = '0.7.0'
   hard-coded version string

tenpy.__full_version__ = '0.7.0'
   full version from git description, and numpy/scipy/python versions
tenpy.show_config()
   Print information about the version of tenpy and used libraries.
```

The information printed is tenpy.version.version\_summary.

## **Submodules**

algorithms	A collection of algorithms such as TEBD and DMRG.
linalg	Linear-algebra tools for tensor networks.
models	Definition of the various models.
networks	Definitions of tensor networks like MPS and MPO.
tools	A collection of tools: mostly short yet quite useful func-
	tions.
version	Access to version of this library.

# 7.7 algorithms

• full name: tenpy.algorithms

• parent module: tenpy

• type: module

# **Module description**

A collection of algorithms such as TEBD and DMRG.

## **Submodules**

truncation	Truncation of Schmidt values.
tebd	Time evolving block decimation (TEBD).
mps_common	'Sweep' algorithm and effective Hamiltonians.
dmrg	Density Matrix Renormalization Group (DMRG).
tdvp	Time Dependant Variational Principle (TDVP) with
	MPS (finite version only).
purification	Algorithms for using Purification.
mpo_evolution	Time evolution using the WI or WII approximation of
	the time evolution operator.
network_contractor	Network Contractor.
exact_diag	Full diagonalization (ED) of the Hamiltonian.

# 7.7.1 truncation

• full name: tenpy.algorithms.truncation

• parent module: tenpy.algorithms

• type: module

7.7. algorithms

## Classes

 ${\bf Truncation Error}$ 

TruncationError([eps, ov])	Class representing a truncation error.
iraneacronerror([cps, ov])	Class representing a truncation error.

## **TruncationError**

• full name: tenpy.algorithms.truncation.TruncationError

• parent module: tenpy.algorithms.truncation

• type: class

# **Inheritance Diagram**

TruncationError

# **Methods**

TruncationErrorinit([eps, ov])	Initialize self.
TruncationError.copy()	Return a copy of self.
TruncationError.from_S(S_discarded[,	Construct TruncationError from discarded singular val-
norm_old])	ues.
<pre>norm_old]) TruncationError.from_norm(norm_new[,</pre>	ues.  Construct TruncationError from norm after and before

#### **Class Attributes and Properties**

TruncationError.ov\_err

Error 1. -ov of the overlap with the correct state.

class tenpy.algorithms.truncation.TruncationError (eps=0.0, ov=1.0)

Bases: object

Class representing a truncation error.

The default initialization represents "no truncation".

**Warning:** For imaginary time evolution, this is *not* the error you are interested in!

Parameters ov (eps,) - See below.

eps

The total sum of all discared Schmidt values squared. Note that if you keep singular values up to 1.e-14 (= a bit more than machine precision for 64bit floats), *eps* is on the order of 1.e-28 (due to the square)!

Type float

ov

A lower bound for the overlap  $|\langle \psi_{trunc} | \psi_{correct} \rangle|^2$  (assuming normalization of both states). This is probably the quantity you are actually interested in. Takes into account the factor 2 explained in the section on Errors in the *TEBD Wikipedia article <a href="https://en.wikipedia.org/wiki/Time-evolving\_block\_decimation">https://en.wikipedia.org/wiki/Time-evolving\_block\_decimation</a>*.

Type float

#### **Examples**

```
>>> TE = TruncationError()
>>> TE += tebd.time_evolution(...) # add `eps`, multiply `ov`
```

copy()

Return a copy of self.

 $\verb|classmethod from_norm| (norm_new, norm_old=1.0)|$ 

Construct TruncationError from norm after and before the truncation.

#### **Parameters**

- norm\_new (float) Norm of Schmidt values kept,  $\sqrt{\sum_{akept} \lambda_a^2}$  (before renormalization).
- norm\_old (float) Norm of all Schmidt values before truncation,  $\sqrt{\sum_a \lambda_a^2}$ .

classmethod from\_S (S\_discarded, norm\_old=None)

Construct TruncationError from discarded singular values.

#### **Parameters**

- **S\_discarded** (1D numpy array) The singular values discarded.
- norm\_old (float) Norm of all Schmidt values before truncation,  $\sqrt{\sum_a \lambda_a^2}$ . Default (None) is 1.

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#### property ov\_err

Error 1. -ov of the overlap with the correct state.

#### **Functions**

svd_theta(theta, trunc_par[, qtotal_LR,])	Performs SVD of a matrix <i>theta</i> (= the wavefunction)
	and truncates it.
truncate(S, options)	Given a Schmidt spectrum S, determine which values to
	keep.

## svd\_theta

- full name: tenpy.algorithms.truncation.svd\_theta
- parent module: tenpy.algorithms.truncation
- · type: function

tenpy.algorithms.truncation.svd\_theta(theta,  $trunc\_par$ ,  $qtotal\_LR=[None, None]$ ,  $in-ner\_labels=['vR', 'vL']$ )

Performs SVD of a matrix theta (= the wavefunction) and truncates it.

Perform a singular value decomposition (SVD) with svd() and truncates with truncate(). The result is an approximation theta  $\sim=$  tensordot(U.scale\_axis(S\*renormalization, 1), VH, axes=1)

#### **Parameters**

- theta (Array, shape (M, N)) The matrix, on which the singular value decomposition (SVD) is performed. Usually, *theta* represents the wavefunction, such that the SVD is a Schmidt decomposition.
- trunc\_par (dict) truncation parameters as described in truncate ().
- qtotalLR((charges, charges)) The total charges for the returned *U* and *VH*.
- inner\_labels ((string, string)) Labels for the *U* and *VH* on the newlycreated bond.

#### Returns

- U (Array) Matrix with left singular vectors as columns. Shape (M, M) or (M, K) depending on *full\_matrices*.
- **S**(*1D ndarray*) The singluar values of the array. If no *cutoff* is given, it has length min (M, N). Normalized to np.linalg.norm(S) == 1.
- VH (Array) Matrix with right singular vectors as rows. Shape (N, N) or (K, N) depending on *full\_matrices*.
- **err** (*TruncationError*) The truncation error introduced.
- renormalization (*float*) Factor, by which S was renormalized.

## **Module description**

Truncation of Schmidt values.

Often, it is necessary to truncate the number of states on a virtual bond of an MPS, keeping only the state with the largest Schmidt values. The function truncate () picks exactly those from a given Schmidt spectrum  $\lambda_a$ , depending on some parameters explained in the doc-string of the function.

Further, we provide TruncationError for a simple way to keep track of the total truncation error.

The SVD on a virtual bond of an MPS actually gives a Schmidt decomposition  $|\psi\rangle = \sum_a \lambda_a |L_a\rangle |R_a\rangle$  where  $|L_a\rangle$  and  $|R_a\rangle$  form orthonormal bases of the parts left and right of the virtual bond. Let us assume that the state is properly normalized,  $\langle \psi | \psi \rangle = \sum_a \lambda_a^2 = 1$ . Assume that the singular values are ordered descending, and that we keep the first  $\chi_c$  of the initially  $\chi$  Schmidt values.

Then we decompose the untruncated state as  $|\psi\rangle = \sqrt{1-\epsilon}|\psi_{tr}\rangle + \sqrt{\epsilon}|\psi_{tr}^{\perp}\rangle$  where  $|\psi_{tr}\rangle = \frac{1}{\sqrt{1-\epsilon}}\sum_{a<\chi_c}\lambda_a|L_a\rangle|R_a\rangle$  is the truncated state kept (normalized to 1),  $|\psi_{tr}^{\perp}\rangle = \frac{1}{\sqrt{\epsilon}}\sum_{a>=\chi_c}\lambda_a|L_a\rangle|R_a\rangle$  is the discarded part (orthogonal to the kept part) and the *truncation error of a single truncation* is defined as  $\epsilon = 1 - |\langle\psi|\psi_{tr}\rangle|^2 = \sum_{a>=\chi_c}\lambda_a^2$ .

**Warning:** For imaginary time evolution (e.g. with TEBD), you try to project out the ground state. Then, looking at the truncation error defined in this module does *not* give you any information how good the found state coincides with the actual ground state! (Instead, the returned truncation error depends on the overlap with the initial state, which is arbitrary > 0)

**Warning:** This module takes only track of the errors coming from the truncation of Schmidt values. There might be other sources of error as well, for example TEBD has also an discretisation error depending on the chosen time step.

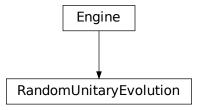
#### 7.7.2 tebd

full name: tenpy.algorithms.tebd

• parent module: tenpy.algorithms

• type: module

#### **Classes**



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Engine(psi, model, options)	Time Evolving Block Decimation (TEBD) algorithm.
RandomUnitaryEvolution(psi, options)	Evolution of an MPS with random two-site unitaries in
	a TEBD-like fashion.

#### **Module description**

Time evolving block decimation (TEBD).

The TEBD algorithm (proposed in [Vidal2004]) uses a trotter decomposition of the Hamiltonian to perform a time evoltion of an MPS. It works only for nearest-neighbor hamiltonians (in tenpy given by a NearestNeighborModel), which can be written as  $H = H^{even} + H^{odd}$ , such that  $H^{even}$  contains the the terms on even bonds (and similar  $H^{odd}$  the terms on odd bonds). In the simplest case, we apply first  $U = \exp(-i*dt*H^{even})$ , then  $U = \exp(-i*dt*H^{odd})$  for each time step dt. This is correct up to errors of  $O(dt^2)$ , but to evolve until a time T, we need T/dt steps, so in total it is only correct up to error of O(T\*dt). Similarly, there are higher order schemata (in dt) (for more details see Engine.update()).

Remember, that bond i is between sites (i-1, i), so for a finite MPS it looks like:

After each application of a *Ui*, the MPS needs to be truncated - otherwise the bond dimension *chi* would grow indefinitely. A bound for the error introduced by the truncation is returned.

If one chooses imaginary dt, the exponential projects (for sufficiently long 'time' evolution) onto the ground state of the Hamiltonian.

**Note:** The application of DMRG is typically much more efficient than imaginary TEBD! Yet, imaginary TEBD might be usefull for cross-checks and testing.

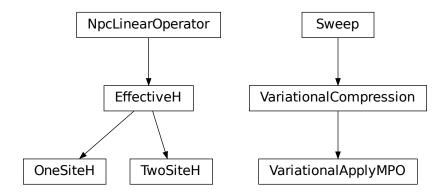
# 7.7.3 mps\_common

• full name: tenpy.algorithms.mps\_common

• parent module: tenpy.algorithms

• type: module

## Classes



EffectiveH(env, i0[, combine, move_right])	Prototype class for local effective Hamiltonians used in sweep algorithms.
OneSiteH(env, i0[, combine, move_right])	Class defining the one-site effective Hamiltonian for
	Lanczos.
Sweep(psi, model, options)	Prototype class for a 'sweeping' algorithm.
TwoSiteH(env, i0[, combine, move_right])	Class defining the two-site effective Hamiltonian for
	Lanczos.
VariationalApplyMPO(psi, U_MPO, options)	Variational compression for applying an MPO to an
	MPS (in place).
VariationalCompression(psi, options)	Variational compression of an MPS (in place).

## **EffectiveH**

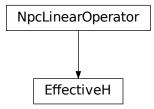
• full name: tenpy.algorithms.mps\_common.EffectiveH

• parent module: tenpy.algorithms.mps\_common

• type: class

7.7. algorithms

## **Inheritance Diagram**



#### Methods

EffectiveHinit(env, i0[, combine,])	Initialize self.
EffectiveH.adjoint()	Return the hermitian conjugate of <i>self</i>
EffectiveH.combine_theta(theta)	Combine the legs of theta, such that it fits to how we
	combined the legs of self.
EffectiveH.matvec(vec)	Calculate the action of the operator on a vector <i>vec</i> .
EffectiveH.to_matrix()	Contract self to a matrix.

## **Class Attributes and Properties**

```
EffectiveH.acts_on
EffectiveH.length
```

**class** tenpy.algorithms.mps\_common.**EffectiveH** (env, i0, combine=False, move\_right=True)
Bases: tenpy.linalg.sparse.NpcLinearOperator

Prototype class for local effective Hamiltonians used in sweep algorithms.

As an example, the local effective Hamiltonian for a two-site (DMRG) algorithm looks like:

```
| .-- --.
| | | | | |
| LP---H0-H1--RP
| | | | |
```

where HO and H1 are MPO tensors.

## **Parameters**

- **env** (MPOEnvironment) Environment for contraction <psi|H|psi>.
- i0 (int) Index of the active site if length=1, or of the left-most active site if length>1.
- **combine** (bool, optional) Whether to combine legs into pipes as far as possible. This reduces the overhead of calculating charge combinations in the contractions.

• move\_right (bool, optional) - Whether the sweeping algorithm that calls for an *EffectiveH* is moving to the right.

#### length

Number of (MPS) sites the effective hamiltonian covers. NB: Class attribute.

Type int

#### dtype

The data type of the involved arrays.

Type np.dtype

N

Contracting *self* with as\_matrix() will result in an *N*'*x*'*N* matrix.

Type int

#### acts\_on

Labels of the state on which self acts. NB: class attribute. Overwritten by normal attribute, if combine.

Type list of str

#### combine

Whether to combine legs into pipes as far as possible. This reduces the overhead of calculating charge combinations in the contractions.

Type bool

#### move\_right

Whether the sweeping algorithm that calls for an *EffectiveH* is moving to the right.

Type bool

## combine\_theta(theta)

Combine the legs of *theta*, such that it fits to how we combined the legs of *self*.

**Parameters theta** (Array) – Wave function to apply the effective Hamiltonian to, with uncombined legs.

Returns theta – Wave function with labels as given by self.acts\_on.

Return type Array

#### adjoint()

Return the hermitian conjugate of self

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

#### matvec(vec)

Calculate the action of the operator on a vector vec.

Note that we don't require *vec* to be one-dimensional. However, for square operators we require that the result of *matvec* has the same legs (in the same order) as *vec* such that they can be added. Note that this excludes a non-trivial *qtotal* for square operators.

#### to\_matrix()

Contract self to a matrix.

If *self* represents an operator with very small shape, e.g. because the MPS bond dimension is very small, an algorithm might choose to contract *self* to a single tensor.

**Returns matrix** – Contraction of the represented operator.

Return type Array

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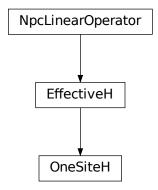
## **OneSiteH**

• full name: tenpy.algorithms.mps\_common.OneSiteH

• parent module: tenpy.algorithms.mps\_common

• type: class

## **Inheritance Diagram**



#### Methods

OneSiteHinit(env, i0[,	combine,	Initialize self.
move_right])		
OneSiteH.adjoint()		Return the hermitian conjugate of <i>self</i> .
OneSiteH.combine_Heff()		Combine LP and RP with W to form LHeff and RHeff,
		depending on the direction.
OneSiteH.combine_theta(theta)		Combine the legs of <i>theta</i> , such that it fits to how we
		combined the legs of <i>self</i> .
OneSiteH.matvec(theta)		Apply the effective Hamiltonian to <i>theta</i> .
OneSiteH.to_matrix()		Contract <i>self</i> to a matrix.

## **Class Attributes and Properties**

```
OneSiteH.acts_on
OneSiteH.length
```

**class** tenpy.algorithms.mps\_common.**OneSiteH**(*env*, *i0*, *combine=False*, *move\_right=True*)

Bases: tenpy.algorithms.mps\_common.EffectiveH

Class defining the one-site effective Hamiltonian for Lanczos.

The effective one-site Hamiltonian looks like this:

```
| .-- --.
| | | | |
| LP---W0---RP
| | | | |
```

If *combine* is True, we define either *LHeff* as contraction of *LP* with *W* (in the case *move\_right* is True) or *RHeff* as contraction of *RP* and *W*.

#### **Parameters**

- **env** (MPOEnvironment) Environment for contraction <psi|H|psi>.
- i0 (int) Index of the active site if length=1, or of the left-most active site if length>1.
- **combine** (bool) Whether to combine legs into pipes. This combines the virtual and physical leg for the left site (when moving right) or right side (when moving left) into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive,  $O(2d^3\chi^3D)$ . Is originally from the wo-site method; unclear if it works well for 1 site.
- move\_right (bool) Whether the the sweep is moving right or left for the next update.

### length

Number of (MPS) sites the effective hamiltonian covers.

```
Type int
```

#### acts on

Labels of the state on which self acts. NB: class attribute. Overwritten by normal attribute, if combine.

Type list of str

### combine, move\_right

See above.

Type bool

### LHeff, RHeff

Only set if combine, and only one of them depending on move\_right. If  $move\_right$  was True, LHeff is set with labels '(vR\*.p0)', 'wR', '(vR.p0\*)' for bra, MPO, ket; otherwise RHeff is set with labels '(p0\*.vL)', 'wL', '(p0, vL\*)'

```
Type Array
```

#### LP, WO, RP

Tensors making up the network of self.

```
Type Array
```

#### matvec(theta)

Apply the effective Hamiltonian to theta.

**Parameters** theta (Array) – Labels: vL, p0, vR if combine=False, (vL.p0), vR or vL, (p0.vR) if True (depending on the direction of movement)

Returns Product of theta and the effective Hamiltonian.

Return type theta Array

#### combine Heff()

Combine LP and RP with W to form LHeff and RHeff, depending on the direction.

In a move to the right, we need LHeff. In a move to the left, we need RHeff. Both contain the same W.

```
combine_theta (theta)
   Combine the legs of theta, such that it fits to how we combined the legs of self.

Parameters theta (Array) - Wave function with labels 'vL', 'p0', 'p1', 'vR'

Returns theta - Wave function with labels 'vL', 'p0', 'p1', 'vR'

Return type Array

to_matrix()
   Contract self to a matrix.

adjoint()
```

### **TwoSiteH**

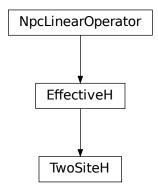
• full name: tenpy.algorithms.mps\_common.TwoSiteH

Return the hermitian conjugate of self.

• parent module: tenpy.algorithms.mps\_common

• type: class

### **Inheritance Diagram**



### **Methods**

TwoSiteHinit(env, i0[,	combine,	Initialize self.
move_right])		
TwoSiteH.adjoint()		Return the hermitian conjugate of <i>self</i> .
TwoSiteH.combine_Heff()		Combine LP and RP with W to form LHeff and RHeff.
TwoSiteH.combine_theta(theta)		Combine the legs of theta, such that it fits to how we
		combined the legs of <i>self</i> .
TwoSiteH.matvec(theta)		Apply the effective Hamiltonian to <i>theta</i> .
TwoSiteH.to_matrix()		Contract self to a matrix.

### **Class Attributes and Properties**

```
TwoSiteH.acts_on
TwoSiteH.length
```

```
class tenpy.algorithms.mps_common.TwoSiteH (env, i0, combine=False, move_right=True)
Bases: tenpy.algorithms.mps_common.EffectiveH
```

Class defining the two-site effective Hamiltonian for Lanczos.

The effective two-site Hamiltonian looks like this:

```
| .-- --.
| | | | | |
| LP---W0--W1---RP
| | | | |
```

If *combine* is True, we define *LHeff* and *RHeff*, which are the contractions of *LP* with *W0*, and *RP* with *W1*, respectively.

### **Parameters**

- **env** (MPOEnvironment) Environment for contraction <psi|H|psi>.
- **i0** (*int*) Index of the active site if length=1, or of the left-most active site if length>1.
- **combine** (bool) Whether to combine legs into pipes. This combines the virtual and physical leg for the left site (when moving right) or right side (when moving left) into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive,  $O(2d^3\chi^3D)$ .
- move\_right (bool) Whether the the sweep is moving right or left for the next update.

#### combine

Whether to combine legs into pipes. This combines the virtual and physical leg for the left site and right site into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive,  $O(2d^3\chi^3D)$ .

```
Type bool
```

### length

Number of (MPS) sites the effective hamiltonian covers.

```
Type int
```

#### acts\_on

Labels of the state on which *self* acts. NB: class attribute. Overwritten by normal attribute, if *combine*.

```
Type list of str
```

#### LHeff

Left part of the effective Hamiltonian. Labels '(vR\*.p0)', 'wR', '(vR.p0\*)' for bra, MPO, ket.

Type Array

### RHeff

Right part of the effective Hamiltonian. Labels '(p1\*.vL)', 'wL', '(p1.vL\*)' for ket, MPO, bra.

Type Array

```
LP, W0, W1, RP
     Tensors making up the network of self.
         Type Array
matvec(theta)
     Apply the effective Hamiltonian to theta.
         Parameters theta (Array) - Labels: vL, p0, p1, vR if combine=False, (vL.p0),
             (p1.vR) if True
         Returns Product of theta and the effective Hamiltonian.
         Return type theta Array
combine Heff()
     Combine LP and RP with W to form LHeff and RHeff.
     Combine LP with W0 and RP with W1 to get the effective parts of the Hamiltonian with piped legs.
combine_theta(theta)
     Combine the legs of theta, such that it fits to how we combined the legs of self.
         Parameters theta (Array) - Wave function with labels 'vL', 'p0', 'p1', 'vR'
         Returns theta – Wave function with labels 'vL', 'p0', 'p1', 'vR'
         Return type Array
to matrix()
     Contract self to a matrix.
adjoint()
     Return the hermitian conjugate of self.
```

#### Module description

'Sweep' algorithm and effective Hamiltonians.

Many MPS-based algorithms use a 'sweep' structure, wherein local updates are performed on the MPS tensors sequentially, first from left to right, then from right to left. This procedure is common to DMRG, TDVP, sequential time evolution, etc.

Another common feature of these algorithms is the use of an effective local Hamiltonian to perform the local updates. The most prominent example of this is probably DMRG, where the local MPS object is optimized with respect to the rest of the MPS-MPO-MPS network, the latter forming the effective Hamiltonian.

The Sweep class attempts to generalize as many aspects of 'sweeping' algorithms as possible. *EffectiveH* and its subclasses implement the effective Hamiltonians mentioned above. Currently, effective Hamiltonians for 1-site and 2-site optimization are implemented.

The VariationalCompression and VariationalApplyMPO implemented here also directly use the Sweep class.

Todo: Rebuild TDVP engine as subclasses of sweep.

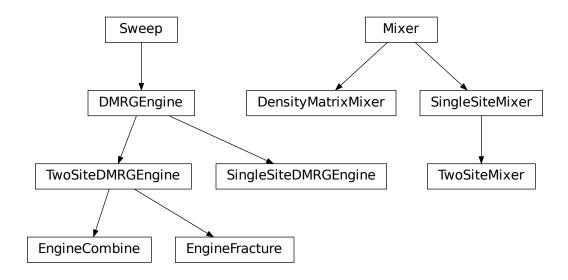
# 7.7.4 dmrg

• full name: tenpy.algorithms.dmrg

• parent module: tenpy.algorithms

• type: module

### **Classes**



DMRGEngine(psi, model, options)	DMRG base class.'Engine' for the DMRG algorithm.	
DensityMatrixMixer(options)	Mixer based on density matrices.	
EngineCombine(psi, model, DMRG_params)	Engine which combines legs into pipes as far as possi-	
	ble.	
EngineFracture(psi, model, DMRG_params)	Engine which keeps the legs separate.	
Mixer(options)	Base class of a general Mixer.	
SingleSiteDMRGEngine(psi, model, options)	'Engine' for the single-site DMRG algorithm.	
SingleSiteMixer(options)	Mixer for single-site DMRG.	
TwoSiteDMRGEngine(psi, model, options)	'Engine' for the two-site DMRG algorithm.	
TwoSiteMixer(options)	Mixer for two-site DMRG.	

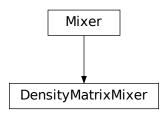
### **DensityMatrixMixer**

• full name: tenpy.algorithms.dmrg.DensityMatrixMixer

• parent module: tenpy.algorithms.dmrg

• type: class

### **Inheritance Diagram**



### **Methods**

DensityMatrixMixerinit(options)	Initialize self.	
DensityMatrixMixer.get_xL(wL_leg, Id_L,	Generate the coupling of the MPO legs for the reduced	
Id_R)	density matrix.	
DensityMatrixMixer.get_xR(wR_leg, Id_L,	Generate the coupling of the MPO legs for the reduced	
Id_R)	density matrix.	
DensityMatrixMixer.mix_rho_L(engine,	Calculated mixed reduced density matrix for left site.	
theta,)		
DensityMatrixMixer.mix_rho_R(engine,	Calculated mixed reduced density matrix for left site.	
theta,)		
DensityMatrixMixer.perturb_svd(engine,	Mix extra terms to theta and perform an SVD.	
)		
DensityMatrixMixer.	Update the amplitude, possibly disable the mixer.	
update_amplitude(sweeps)		

class tenpy.algorithms.dmrg.DensityMatrixMixer(options)

Bases: tenpy.algorithms.dmrg.Mixer

Mixer based on density matrices.

This mixer constructs density matrices as described in the original paper [White2005].

perturb\_svd (engine, theta, i0, update\_LP, update\_RP)

Mix extra terms to theta and perform an SVD.

We calculate the left and right reduced density using the mixer (which might include applications of H). These density matrices are diagonalized and truncated such that we effectively perform a svd for the case mixer.amplitude=0.

### **Parameters**

- engine (SingleSiteDMRGEngine|TwoSiteDMRGEngine) The DMRG engine calling the mixer.
- theta (Array) The optimized wave function, prepared for svd.
- **i0** (int) Site index; *theta* lives on i0, i0+1.
- update\_LP (bool) Whether to calculate the next env.LP[i0+1].
- update\_RP (bool) Whether to calculate the next env.RP[i0].

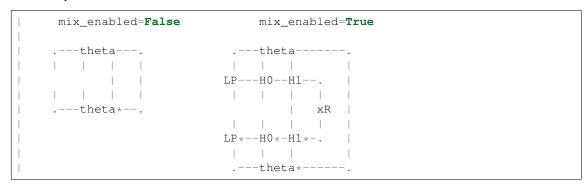
#### **Returns**

- U (Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- **S** (1D ndarray | 2D *Array*) Without mixer just the singluar values of the array; with mixer it might be a general matrix; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(p1.vR)'.
- **err** (*TruncationError*) The truncation error introduced.

### mix\_rho\_L (engine, theta, i0, mix\_enabled)

Calculated mixed reduced density matrix for left site.

### Pictorially:



#### **Parameters**

- engine (Engine) The DMRG engine calling the mixer.
- **theta** (*Array*) Ground state of the effective Hamiltonian, prepared for svd.
- **i0** (*int*) Site index; theta lives on i0, i0+1.
- mix\_enabled (bool) Whether we should perturb the density matrix.

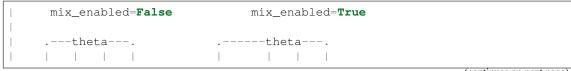
**Returns rho\_L** – A (hermitian) square array with labels '(vL.p0)', '(vL\*.p0\*)', Mainly the reduced density matrix of the left part, but with some additional mixing.

### Return type Array

### mix\_rho\_R (engine, theta, i0, mix\_enabled)

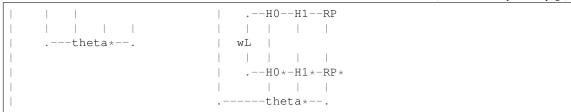
Calculated mixed reduced density matrix for left site.

### Pictorially:



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#### **Parameters**

- engine (Engine) The DMRG engine calling the mixer.
- theta (Array) Ground state of the effective Hamiltonian, prepared for svd.
- **i0** (*int*) Site index; *theta* lives on i0, i0+1.
- mix\_enabled (bool) Whether we should perturb the density matrix.

**Returns rho\_R** – A (hermitian) square array with labels '(p1.vR)', '(p1\*.vR\*)'. Mainly the reduced density matrix of the right part, but with some additional mixing.

### Return type Array

### $get_xR(wR\_leg, Id\_L, Id\_R)$

Generate the coupling of the MPO legs for the reduced density matrix.

#### **Parameters**

- wR\_leg (LegCharge) LegCharge to be connected to.
- IdL (int | None) Index within the leg for which the MPO has only identities to the left.
- IdR (int | None) Index within the leg for which the MPO has only identities to the right.

#### **Returns**

- **mixed\_xR** (*Array*) Connection of the MPOs on the right for the reduced density matrix *rhoL*. Labels ('wL', 'wL\*').
- add\_separate\_Id (bool) If Id\_L is None, we can't include the identity into mixed\_xR, so it has to be added directly in mix rho L().

### get\_xL (wL\_leg, Id\_L, Id\_R)

Generate the coupling of the MPO legs for the reduced density matrix.

### **Parameters**

- wL\_leg (LegCharge) LegCharge to be connected to.
- Id\_L (int | None) Index within the leg for which the MPO has only identities to the left.
- Id\_R (int | None) Index within the leg for which the MPO has only identities to the right.

### Returns

- mixed\_xL (Array) Connection of the MPOs on the left for the reduced density matrix *rhoR*. Labels ('wR', 'wR\*').
- add\_separate\_Id (bool) If Id\_R is None, we can't include the identity into mixed\_xL, so it has to be added directly in mix\_rho\_R().

### update\_amplitude (sweeps)

Update the amplitude, possibly disable the mixer.

**Parameters** sweeps (int) – The number of performed sweeps, to check if we need to disable the mixer.

**Returns mixer** – Returns *self* if we should continue mixing, or None, if the mixer should be disabled.

Return type Mixer | None

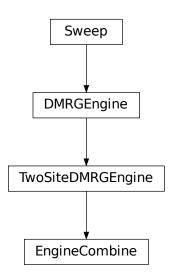
# **EngineCombine**

• full name: tenpy.algorithms.dmrg.EngineCombine

• parent module: tenpy.algorithms.dmrg

• type: class

## **Inheritance Diagram**



### **Methods**

EngineCombineinit(psi, model,	Initialize self.
DMRG_params)	
EngineCombine.diag(theta_guess)	Diagonalize the effective Hamiltonian represented by
	self.
EngineCombine.environment_sweeps( $N_sweeps$	epsDerform N_sweeps sweeps without optimization to up-
	date the environment.
EngineCombine.get_sweep_schedule()	Define the schedule of the sweep.
EngineCombine.init_env([model])	(Re-)initialize the environment.

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Table	18 – continued from pre	vious page
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EngineCombine.make_eff_H()	Create new instance of self.EffectiveH at self.i0 and set
	it to self.eff_H.
EngineCombine.mixed_svd(theta)	Get (truncated) B from the new theta (as returned by
	diag).
EngineCombine.mixer_activate()	Set <i>self.mixer</i> to the class specified by <i>options['mixer']</i> .
EngineCombine.mixer_cleanup()	Cleanup the effects of a mixer.
EngineCombine.plot_sweep_stats([axes,	Plot sweep_stats to display the convergence with
])	the sweeps.
EngineCombine.plot_update_stats(axes[,	Plot update_stats to display the convergence dur-
])	ing the sweeps.
EngineCombine.post_update_local(update_o	lataPerform post-update actions.
EngineCombine.prepare_svd(theta)	Transform theta into matrix for svd.
EngineCombine.prepare_update()	Prepare <i>self</i> for calling update_local() on sites
	(i0, i0+1).
EngineCombine.reset_stats()	Reset the statistics, useful if you want to start a new
	sweep run.
EngineCombine.run()	Run the DMRG simulation to find the ground state.
EngineCombine.set_B(U, S, VH)	Update the MPS with the U, S, VH returned by
	self.mixed_svd.
EngineCombine.sweep([optimize,	One 'sweep' of a the algorithm.
meas_E_trunc])	
EngineCombine.update_LP $(U)$	Update left part of the environment.
EngineCombine.update_RP(VH)	Update right part of the environment.
EngineCombine.update_local(theta[,])	Perform bond-update on the sites (i0, i0+1).

### **Class Attributes and Properties**

EngineCombine.DMRG_params	
EngineCombine.engine_params	

### class tenpy.algorithms.dmrg.EngineCombine(psi, model, DMRG\_params)

Bases: tenpy.algorithms.dmrg.TwoSiteDMRGEngine

Engine which combines legs into pipes as far as possible.

This engine combines the virtual and physical leg for the left site and right site into pipes. This reduces the overhead of calculating charge combinations in the contractions, but one matvec() is formally more expensive,  $O(2d^3\chi^3D)$ .

Deprecated since version 0.5.0: Directly use the <code>TwoSiteDMRGEngine</code> with the DMRG parameter <code>combine=True</code>.

### **EffectiveH**

alias of tenpy.algorithms.mps\_common.TwoSiteH

### diag (theta\_guess)

Diagonalize the effective Hamiltonian represented by self.

### option DMRGEngine.max\_N\_for\_ED: int

Maximum matrix dimension of the effective hamiltonian up to which the 'default' diag\_method uses ED instead of Lanczos.

# option DMRGEngine.diag\_method: str One of the folloing strings:

- 'default' Same as 'lanczos' for large bond dimensions, but if the total dimension of the effective Hamiltonian does not exceed the DMRG parameter 'max\_N\_for\_ED' it uses 'ED\_block'.
- 'lanczos' lanczos () Default, the Lanczos implementation in TeNPy.
- 'arpack' lanczos\_arpack() Based on scipy.linalg.sparse.eigsh(). Slower than 'lanczos', since it needs to convert the npc arrays to numpy arrays during *each* matvec, and possibly does many more iterations.
- **'ED\_block'** full\_diag\_effH() Contract the effective Hamiltonian to a (large!) matrix and diagonalize the block in the charge sector of the initial state. Preserves the charge sector of the explicitly conserved charges. However, if you don't preserve a charge explicitly, it can break it. For example if you use a SpinChain({'conserve': 'parity'}), it could change the total "Sz", but not the parity of 'Sz'.
- **'ED\_all'** full\_diag\_effH() Contract the effective Hamiltonian to a (large!) matrix and diagonalize it completely. Allows to change the charge sector even for explicitly conserved charges. For example if you use a SpinChain({'conserve': 'Sz'}), it can change the total "Sz".

**Parameters theta\_guess** (Array) - Initial guess for the ground state of the effective Hamiltonian.

### Returns

- **E0** (*float*) Energy of the found ground state.
- theta (Array) Ground state of the effective Hamiltonian.
- N (*int*) Number of Lanczos iterations used. –1 if unknown.
- **ov\_change** (*float*) Change in the wave function 1. abs(<theta\_guess|theta\_diag>)

### environment\_sweeps (N\_sweeps)

Perform *N\_sweeps* sweeps without optimization to update the environment.

**Parameters N\_sweeps** (int) – Number of sweeps to run without optimization

#### get\_sweep\_schedule()

Define the schedule of the sweep.

One 'sweep' is a full sequence from the leftmost site to the right and back. Only those *LP* and *RP* that can be used later should be updated.

**Returns schedule** — Schedule for the sweep. Each entry is (i0, move\_right, (update\_LP, update\_RP)), where i0 is the leftmost of the self.EffectiveH. length sites to be updated in update\_local(), move\_right indicates whether the next i0 in the schedule is rigth (True) of the current one, and update\_LP, update\_RP indicate whether it is necessary to update the LP and RP. The latter are chosen such that the environment is growing for infinite systems, but we only keep the minimal number of environment tensors in memory.

Return type iterable of (int, bool, (bool, bool))

### $init\_env (model=None)$

(Re-)initialize the environment.

This function is useful to (re-)start a Sweep with a slightly different model or different (engine) parameters. Note that we assume that we still have the same *psi*. Calls <code>reset\_stats()</code>.

**Parameters model** (MPOModel) – The model representing the Hamiltonian for which we want to find the ground state. If None, keep the model used before.

### **Options**

Deprecated since version 0.6.0: Options LP, LP\_age, RP and RP\_age are now collected in a dictionary init\_env\_data with different keys init\_LP, init\_RP, age\_LP, age\_RP

```
option Sweep.init_env_data: dict
   Dictionary as returned by self.env.get_initialization_data() from
   get_initialization_data().
```

```
option Sweep.orthogonal_to: list of MPSEnvironment
```

List of other matrix product states to orthogonalize against. Works only for finite systems. This parameter can be used to find (a few) excited states as follows. First, run DMRG to find the ground state and then run DMRG again while orthogonalizing against the ground state, which yields the first excited state (in the same symmetry sector), and so on.

```
option Sweep.start_env: int
```

Number of sweeps to be performed without optimization to update the environment.

**Raises ValueError** – If the engine is re-initialized with a new model, which legs are incompatible with those of hte old model.

```
make_eff_H()
```

Create new instance of *self.EffectiveH* at *self.i0* and set it to *self.eff\_H*.

#### mixed\_svd(theta)

Get (truncated) B from the new theta (as returned by diag).

The goal is to split theta and truncate it:

```
| -- theta -- ==> -- U -- S -- VH -
```

Without a mixer, this is done by a simple svd and truncation of Schmidt values.

With a mixer, the state is perturbed before the SVD. The details of the perturbation are defined by the Mixer class.

Note that the returned S is a general (not diagonal) matrix, with labels 'vL', 'vR'.

Parameters theta (Array) – The optimized wave function, prepared for svd.

#### Returns

- U (Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- **S** (1D ndarray | 2D *Array*) Without mixer just the singluar values of the array; with mixer it might be a general matrix with labels 'vL', 'vR'; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(p1.vR)'.
- $\bullet$   $\,$  err ( TruncationError ) The truncation error introduced.

#### mixer\_activate()

Set *self.mixer* to the class specified by *options['mixer']*.

```
option TwoSiteDMRGEngine.mixer: str | class | bool
```

Chooses the Mixer to be used. A string stands for one of the mixers defined in this module, a class is used as custom mixer. Default (None) uses no mixer, True uses <code>DensityMatrixMixer</code> for the 2-site case and <code>SingleSiteMixer</code> for the 1-site case.

```
option TwoSiteDMRGEngine.mixer_params: dict
```

Mixer parameters as described in Mixer.

#### mixer cleanup()

Cleanup the effects of a mixer.

A sweep () with an enabled Mixer leaves the MPS psi with 2D arrays in S. To recover the originial form, this function simply performs one sweep with disabled mixer.

```
plot_sweep_stats (axes=None, xaxis='time', yaxis='E', y_exact=None, **kwargs)

Plot sweep stats to display the convergence with the sweeps.
```

#### **Parameters**

- axes (matplotlib.axes.Axes) The axes to plot into. Defaults to matplotlib. pyplot.gca()
- yaxis (xaxis,) Key of sweep\_stats to be used for the x-axis and y-axis of the plots.
- **y\_exact** (float) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y\_exact) /y\_exact) on a log-scale yaxis.
- \*\*kwargs Further keyword arguments given to axes.plot(...).

```
plot_update_stats (axes, xaxis='time', yaxis='E', y_exact=None, **kwargs)
```

Plot update\_stats to display the convergence during the sweeps.

#### **Parameters**

- axes (matplotlib.axes.Axes) The axes to plot into. Defaults to matplotlib. pyplot.gca()
- xaxis ('N\_updates' | 'sweep' | keys of update\_stats) Key of update\_stats to be used for the x-axis of the plots. 'N\_updates' is just enumerating the number of bond updates, and 'sweep' corresponds to the sweep number (including environment sweeps).
- yaxis ('E' | keys of update\_stats) Key of update\_stats to be used for the y-axis of the plots. For 'E', use the energy (per site for infinite systems).
- **y\_exact** (float) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y\_exact) /y\_exact) on a log-scale yaxis.
- \*\*kwargs Further keyword arguments given to axes.plot(...).

```
post_update_local (update_data)
```

Perform post-update actions.

Compute truncation energy, remove LP/RP that are no longer needed and collect statistics.

```
Parameters update data (dict) – What was returned by update local().
```

```
prepare_svd(theta)
```

Transform theta into matrix for svd.

```
prepare_update()
```

Prepare self for calling update\_local() on sites (i0, i0+1).

Returns theta – Current best guess for the ground state, which is to be optimized. Labels 'vL', 'p0', 'vR', 'p1'.

Return type Array

### reset\_stats()

Reset the statistics, useful if you want to start a new sweep run.

#### option DMRGEngine.chi list: dict | None

A dictionary to gradually increase the *chi\_max* parameter of *trunc\_params*. The key defines starting from which sweep *chi\_max* is set to the value, e.g. {0: 50, 20: 100} uses chi\_max=50 for the first 20 sweeps and chi\_max=100 afterwards. Overwrites *trunc\_params['chi\_list']'*. By default (None) this feature is disabled.

#### option DMRGEngine.sweep\_0: int

The number of sweeps already performed. (Useful for re-start).

#### run()

Run the DMRG simulation to find the ground state.

#### Returns

- E (*float*) The energy of the resulting ground state MPS.
- **psi** (MPS) The MPS representing the ground state after the simluation, i.e. just a reference to psi.

#### **Options**

### option DMRGEngine.diag\_method: str

Method to be used for diagonalization, default 'default'. For possible arguments see DMRGEngine.diag().

#### option DMRGEngine.E\_tol\_to\_trunc: float

It's reasonable to choose the Lanczos convergence criteria 'E\_tol' not many magnitudes lower than the current truncation error. Therefore, if  $E\_tol\_to\_trunc$  is not None, we update  $E\_tol$  of  $lanczos\_params$  to max\_E\_trunc\*E\_tol\_to\_trunc, restricted to the interval  $[E\_tol\_min, E\_tol\_max]$ , where max\_E\_trunc is the maximal energy difference due to truncation right after each Lanczos optimization during the sweeps.

# option DMRGEngine.E\_tol\_max: float

See *E\_tol\_to\_trunc* 

#### option DMRGEngine.E\_tol\_min: float

See *E\_tol\_to\_trunc* 

### option DMRGEngine.max\_E\_err: float

Convergence if the change of the energy in each step satisfies  $-Delta E / max(|E|, 1) < max_E_err.$  Note that this is also satisfied if Delta E > 0, i.e., if the energy increases (due to truncation).

#### option DMRGEngine.max\_hours: float

If the DMRG took longer (measured in wall-clock time), 'shelve' the simulation, i.e. stop and return with the flag shelve=True.

### option DMRGEngine.max\_S\_err: float

Convergence if the relative change of the entropy in each step satisfies  $|Delta S|/S < max_S_err$ 

#### option DMRGEngine.max sweeps: int

Maximum number of sweeps to be performed.

### option DMRGEngine.min\_sweeps: int

Minimum number of sweeps to be performed. Defaults to 1.5\*N\_sweeps\_check.

### option DMRGEngine.N\_sweeps\_check: int

Number of sweeps to perform between checking convergence criteria and giving a status update.

#### option DMRGEngine.norm tol: float

After the DMRG run, update the environment with at most *norm\_tol\_iter* sweeps until np.linalg.norm(psi.norm\_err()) < norm\_tol.

#### option DMRGEngine.norm\_tol\_iter: float

Perform at most *norm\_tol\_iter*`\*`*update\_env* sweeps to converge the norm error below *norm\_tol*. If the state is not converged after that, call canonical\_form() instead.

### option DMRGEngine.P\_tol\_to\_trunc: float

It's reasonable to choose the Lanczos convergence criteria 'P\_tol' not many magnitudes lower than the current truncation error. Therefore, if  $P\_tol\_to\_trunc$  is not None, we update  $P\_tol$  of  $lanczos\_params$  to max\_trunc\_err\*P\_tol\_to\_trunc, restricted to the interval  $[P\_tol\_min, P\_tol\_max]$ , where max\_trunc\_err is the maximal truncation error (discarded weight of the Schmidt values) due to truncation right after each Lanczos optimization during the sweeps.

```
option DMRGEngine.P_tol_max: float
```

See *P\_tol\_to\_trunc* 

option DMRGEngine.P\_tol\_min: float

See *P\_tol\_to\_trunc* 

```
option DMRGEngine.update_env: int
```

Number of sweeps without bond optimizaiton to update the environment for infinite boundary conditions, performed every *N\_sweeps\_check* sweeps.

### $set_B(U, S, VH)$

Update the MPS with the U, S, VH returned by *self.mixed\_svd*.

#### **Parameters**

- VH  $(U_{r})$  Left and Right-canonical matrices as returned by the SVD.
- **S** (1D array | 2D Array) The middle part returned by the SVD, theta = U S VH. Without a mixer just the singular values, with enabled *mixer* a 2D array.

```
sweep (optimize=True, meas_E_trunc=False)
```

One 'sweep' of a the algorithm.

Iteratate over the bond which is optimized, to the right and then back to the left to the starting point.

#### **Parameters**

- **optimize** (bool, optional) Whether we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).
- meas\_E\_trunc (bool, optional) Whether to measure truncation energies.

#### **Returns**

- max\_trunc\_err (float) Maximal truncation error introduced.
- max\_E\_trunc (None | float) None if meas\_E\_trunc is False, else the maximal change of the energy due to the truncation.

### $update_LP(U)$

Update left part of the environment.

We always update the environment at site i0 + 1: this environment then contains the site where we just performed a local update (when sweeping right).

Parameters U (Array) - The U as returned by the SVD, with combined legs, labels 'vL. p0', 'vR'.

```
update RP (VH)
```

Update right part of the environment.

We always update the environment at site i0: this environment then contains the site where we just performed a local update (when sweeping left).

```
Parameters VH (Array) – The VH as returned by SVD, with combined legs, labels 'vL', '(vR.p1)'.
```

```
update_local (theta, optimize=True, meas_E_trunc=False)
```

Perform bond-update on the sites (i0, i0+1).

#### **Parameters**

- **theta** (*Array*) Initial guess for the ground state of the effective Hamiltonian.
- **optimize** (bool) Wheter we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).
- $meas_E_trunc(bool)$  Wheter to measure the energy after truncation.

#### Returns

**update\_data** – Data computed during the local update, as described in the following:

- **E0** [float] Total energy, obtained *before* truncation (if optimize=True), or *after* truncation (if optimize=False) (but never None).
- **N** [int] Dimension of the Krylov space used for optimization in the lanczos algorithm. 0 if optimize=False.
- **age** [int] Current size of the DMRG simulation: number of physical sites involved into the contraction.

```
U, VH: Array U and VH returned by mixed\_svd().
```

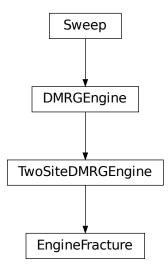
ov\_change: float Change in the wave function 1. - abs(<theta\_guess|theta>)
induced by diag(), not including the truncation!

Return type dict

### **EngineFracture**

- full name: tenpy.algorithms.dmrg.EngineFracture
- parent module: tenpy.algorithms.dmrg
- · type: class

# **Inheritance Diagram**



### **Methods**

EngineFractureinit(psi, model,	Initialize self.
DMRG_params)	
EngineFracture.diag(theta_guess)	Diagonalize the effective Hamiltonian represented by
	self.
EngineFracture.environment_sweeps(N_sweeps	ee Pse) form N_sweeps sweeps without optimization to up-
	date the environment.
EngineFracture.get_sweep_schedule()	Define the schedule of the sweep.
EngineFracture.init_env([model])	(Re-)initialize the environment.
EngineFracture.make_eff_H()	Create new instance of self.EffectiveH at self.i0 and set
	it to self.eff_H.
EngineFracture.mixed_svd(theta)	Get (truncated) $B$ from the new theta (as returned by
	diag).
EngineFracture.mixer_activate()	Set <i>self.mixer</i> to the class specified by <i>options['mixer']</i> .
EngineFracture.mixer_cleanup()	Cleanup the effects of a mixer.
EngineFracture.plot_sweep_stats([axes,	Plot sweep_stats to display the convergence with
])	the sweeps.
EngineFracture.plot_update_stats(axes[,	Plot update_stats to display the convergence dur-
])	ing the sweeps.
EngineFracture.post_update_local(update_	date) form post-update actions.
EngineFracture.prepare_svd(theta)	Transform theta into matrix for svd.
EngineFracture.prepare_update()	Prepare self for calling update_local() on sites
	(i0, i0+1).
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EngineFracture.reset_stats()	Reset the statistics, useful if you want to start a new
	sweep run.
EngineFracture.run()	Run the DMRG simulation to find the ground state.
EngineFracture.set_B(U, S, VH)	Update the MPS with the U, S, VH returned by
	self.mixed_svd.
EngineFracture.sweep([optimize,	One 'sweep' of a the algorithm.
meas_E_trunc])	
EngineFracture.update_LP(U)	Update left part of the environment.
EngineFracture.update_RP(VH)	Update right part of the environment.
EngineFracture.update_local(theta[,])	Perform bond-update on the sites (i0, i0+1).

### **Class Attributes and Properties**

EngineFracture.DMRG_params	
EngineFracture.engine_params	

### class tenpy.algorithms.dmrg.EngineFracture(psi, model, DMRG\_params)

 $Bases: {\tt tenpy.algorithms.dmrg.TwoSiteDMRGEngine}$ 

Engine which keeps the legs separate.

Due to a different contraction order in matvec (), this engine might be faster than EngineCombine, at least for large physical dimensions and if the MPO is sparse. One matvec () is  $O(2\chi^3 d^2W + 2\chi^2 d^3W^2)$ .

Deprecated since version 0.5.0: Directly use the TwoSiteDMRGEngine with the DMRG parameter combine=False.

### **EffectiveH**

 $a lias \ of \ tenpy. algorithms. mps\_common. Two Site H$ 

#### diag(theta\_guess)

Diagonalize the effective Hamiltonian represented by self.

#### option DMRGEngine.max\_N\_for\_ED: int

Maximum matrix dimension of the effective hamiltonian up to which the 'default' diag\_method uses ED instead of Lanczos.

### option DMRGEngine.diag method: str

One of the folloing strings:

'default' Same as 'lanczos' for large bond dimensions, but if the total dimension of the effective Hamiltonian does not exceed the DMRG parameter 'max\_N\_for\_ED' it uses 'ED\_block'.

'lanczos' lanczos () Default, the Lanczos implementation in TeNPy.

- 'arpack' lanczos\_arpack() Based on scipy.linalg.sparse.eigsh(). Slower than 'lanczos', since it needs to convert the npc arrays to numpy arrays during *each* matvec, and possibly does many more iterations.
- **'ED\_block'** <code>full\_diag\_effH()</code> Contract the effective Hamiltonian to a (large!) matrix and diagonalize the block in the charge sector of the initial state. Preserves the charge sector of the explicitly conserved charges. However, if you don't preserve a charge explicitly, it can break it. For example if you use a <code>SpinChain({'conserve': 'parity'})</code>, it could change the total "Sz", but not the parity of 'Sz'.
- **'ED\_all'** full\_diag\_effH() Contract the effective Hamiltonian to a (large!) matrix and diagonalize it completely. Allows to change the charge sector even for explicitly conserved charges. For example if you use a SpinChain({'conserve': 'Sz'}), it can change the total "Sz".

Parameters theta\_guess (Array) - Initial guess for the ground state of the effective Hamiltonian.

#### Returns

- **E0** (*float*) Energy of the found ground state.
- theta (Array) Ground state of the effective Hamiltonian.
- N (*int*) Number of Lanczos iterations used. –1 if unknown.
- **ov\_change** (*float*) Change in the wave function 1. abs(<theta\_quess|theta\_diaq>)

#### environment\_sweeps (N\_sweeps)

Perform *N\_sweeps* sweeps without optimization to update the environment.

**Parameters N\_sweeps** (int) – Number of sweeps to run without optimization

```
get_sweep_schedule()
```

Define the schedule of the sweep.

One 'sweep' is a full sequence from the leftmost site to the right and back. Only those *LP* and *RP* that can be used later should be updated.

Returns schedule — Schedule for the sweep. Each entry is (i0, move\_right, (update\_LP, update\_RP)), where i0 is the leftmost of the self.EffectiveH. length sites to be updated in update\_local(), move\_right indicates whether the next i0 in the schedule is rigth (True) of the current one, and update\_LP, update\_RP indicate whether it is necessary to update the LP and RP. The latter are chosen such that the environment is growing for infinite systems, but we only keep the minimal number of environment tensors in memory.

**Return type** iterable of (int, bool, (bool, bool))

```
init_env (model=None)
```

(Re-)initialize the environment.

This function is useful to (re-)start a Sweep with a slightly different model or different (engine) parameters. Note that we assume that we still have the same *psi*. Calls reset\_stats().

**Parameters model** (MPOModel) – The model representing the Hamiltonian for which we want to find the ground state. If None, keep the model used before.

### **Options**

Deprecated since version 0.6.0: Options LP, LP\_age, RP and RP\_age are now collected in a dictionary init\_env\_data with different keys init\_LP, init\_RP, age\_LP, age\_RP

```
option Sweep.init_env_data: dict
   Dictionary as returned by self.env.get_initialization_data() from
   get_initialization_data().
```

### option Sweep.orthogonal\_to: list of MPSEnvironment

List of other matrix product states to orthogonalize against. Works only for finite systems. This parameter can be used to find (a few) excited states as follows. First, run DMRG to find the ground state and then run DMRG again while orthogonalizing against the ground state, which yields the first excited state (in the same symmetry sector), and so on.

```
option Sweep.start_env: int
```

Number of sweeps to be performed without optimization to update the environment.

**Raises** ValueError – If the engine is re-initialized with a new model, which legs are incompatible with those of hte old model.

#### make\_eff\_H()

Create new instance of *self.EffectiveH* at *self.i0* and set it to *self.eff\_H*.

#### mixed svd(theta)

Get (truncated) B from the new theta (as returned by diag).

The goal is to split theta and truncate it:

```
| -- theta -- ==> -- U -- S -- VH -
```

Without a mixer, this is done by a simple svd and truncation of Schmidt values.

With a mixer, the state is perturbed before the SVD. The details of the perturbation are defined by the Mixer class.

Note that the returned S is a general (not diagonal) matrix, with labels 'vL', 'vR'.

**Parameters** theta (Array) – The optimized wave function, prepared for svd.

#### **Returns**

- U (Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- S (1D ndarray | 2D Array) Without mixer just the singluar values of the array; with mixer it might be a general matrix with labels 'vL', 'vR'; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(p1.vR)'.
- **err** (*TruncationError*) The truncation error introduced.

#### mixer activate()

Set *self.mixer* to the class specified by *options['mixer']*.

```
option TwoSiteDMRGEngine.mixer: str | class | bool
```

Chooses the Mixer to be used. A string stands for one of the mixers defined in this module, a class is used as custom mixer. Default (None) uses no mixer, True uses <code>DensityMatrixMixer</code> for the 2-site case and <code>SingleSiteMixer</code> for the 1-site case.

```
option TwoSiteDMRGEngine.mixer_params: dict
```

Mixer parameters as described in Mixer.

#### mixer\_cleanup()

Cleanup the effects of a mixer.

A sweep () with an enabled Mixer leaves the MPS psi with 2D arrays in S. To recover the originial form, this function simply performs one sweep with disabled mixer.

```
plot_sweep_stats (axes=None, xaxis='time', yaxis='E', y_exact=None, **kwargs)
```

Plot sweep stats to display the convergence with the sweeps.

#### **Parameters**

- axes (matplotlib.axes.Axes) The axes to plot into. Defaults to matplotlib. pyplot.gca()
- yaxis (xaxis,) Key of sweep\_stats to be used for the x-axis and y-axis of the plots.
- **y\_exact** (float) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y\_exact) /y\_exact) on a log-scale yaxis.

```
• **kwargs - Further keyword arguments given to axes.plot(...).
```

```
plot_update_stats (axes, xaxis='time', yaxis='E', y_exact=None, **kwargs)
```

Plot update\_stats to display the convergence during the sweeps.

#### **Parameters**

- axes (matplotlib.axes.Axes) The axes to plot into. Defaults to matplotlib. pyplot.gca()
- **xaxis** ('N\_updates' | 'sweep' | keys of update\_stats) Key of update\_stats to be used for the x-axis of the plots. 'N\_updates' is just enumerating the number of bond updates, and 'sweep' corresponds to the sweep number (including environment sweeps).
- yaxis ('E' | keys of update\_stats) Key of update\_stats to be used for the y-axis of the plots. For 'E', use the energy (per site for infinite systems).
- **y\_exact** (*float*) Exact value for the quantity on the y-axis for comparison. If given, plot abs ((y-y\_exact) /y\_exact) on a log-scale yaxis.
- \*\*kwargs Further keyword arguments given to axes.plot(...).

#### post\_update\_local (update\_data)

Perform post-update actions.

Compute truncation energy, remove *LP/RP* that are no longer needed and collect statistics.

```
Parameters update_data (dict) - What was returned by update_local().
```

```
prepare svd(theta)
```

Transform theta into matrix for svd.

```
prepare_update()
```

Prepare *self* for calling *update\_local()* on sites (i0, i0+1).

**Returns theta** – Current best guess for the ground state, which is to be optimized. Labels 'vL', 'p0', 'vR', 'p1'.

```
Return type Array
```

#### reset\_stats()

Reset the statistics, useful if you want to start a new sweep run.

```
option DMRGEngine.chi_list: dict | None
```

A dictionary to gradually increase the *chi\_max* parameter of *trunc\_params*. The key defines starting from which sweep *chi\_max* is set to the value, e.g. {0: 50, 20: 100} uses chi\_max=50 for the first 20 sweeps and chi\_max=100 afterwards. Overwrites *trunc\_params['chi\_list']*'. By default (None) this feature is disabled.

```
option DMRGEngine.sweep_0: int
```

The number of sweeps already performed. (Useful for re-start).

### run()

Run the DMRG simulation to find the ground state.

#### **Returns**

- E (*float*) The energy of the resulting ground state MPS.
- **psi** (MPS) The MPS representing the ground state after the simluation, i.e. just a reference to psi.

#### **Options**

### option DMRGEngine.diag\_method: str

Method to be used for diagonalization, default 'default'. For possible arguments see DMRGEngine.diag().

#### option DMRGEngine.E\_tol\_to\_trunc: float

It's reasonable to choose the Lanczos convergence criteria 'E\_tol' not many magnitudes lower than the current truncation error. Therefore, if  $E\_tol\_to\_trunc$  is not None, we update  $E\_tol$  of  $lanczos\_params$  to max\_E\_trunc\*E\_tol\_to\_trunc, restricted to the interval  $[E\_tol\_min, E\_tol\_max]$ , where max\_E\_trunc is the maximal energy difference due to truncation right after each Lanczos optimization during the sweeps.

### option DMRGEngine.E\_tol\_max: float

See *E\_tol\_to\_trunc* 

### option DMRGEngine.E\_tol\_min: float

See *E\_tol\_to\_trunc* 

#### option DMRGEngine.max\_E\_err: float

Convergence if the change of the energy in each step satisfies  $-Delta E / max(|E|, 1) < max_E_err$ . Note that this is also satisfied if Delta E > 0, i.e., if the energy increases (due to truncation).

#### option DMRGEngine.max\_hours: float

If the DMRG took longer (measured in wall-clock time), 'shelve' the simulation, i.e. stop and return with the flag shelve=True.

### option DMRGEngine.max\_S\_err: float

Convergence if the relative change of the entropy in each step satisfies  $|Delta S|/S < max_S_err$ 

#### option DMRGEngine.max\_sweeps: int

Maximum number of sweeps to be performed.

### option DMRGEngine.min\_sweeps: int

Minimum number of sweeps to be performed. Defaults to 1.5\*N\_sweeps\_check.

#### option DMRGEngine.N sweeps check: int

Number of sweeps to perform between checking convergence criteria and giving a status update.

### option DMRGEngine.norm\_tol: float

After the DMRG run, update the environment with at most *norm\_tol\_iter* sweeps until np.linalg.norm(psi.norm\_err()) < norm\_tol.

#### option DMRGEngine.norm\_tol\_iter: float

Perform at most *norm\_tol\_iter*`\*`*update\_env* sweeps to converge the norm error below *norm\_tol*. If the state is not converged after that, call canonical\_form() instead.

#### option DMRGEngine.P\_tol\_to\_trunc: float

It's reasonable to choose the Lanczos convergence criteria 'P\_tol' not many magnitudes lower than the current truncation error. Therefore, if  $P\_tol\_to\_trunc$  is not None, we update  $P\_tol$  of  $lanczos\_params$  to max\_trunc\_err\*P\_tol\_to\_trunc, restricted to the interval  $[P\_tol\_min, P\_tol\_max]$ , where max\_trunc\_err is the maximal truncation error (discarded weight of the Schmidt values) due to truncation right after each Lanczos optimization during the sweeps.

### option DMRGEngine.P\_tol\_max: float

See *P\_tol\_to\_trunc* 

### option DMRGEngine.P\_tol\_min: float

See *P\_tol\_to\_trunc* 

```
option DMRGEngine.update env: int
```

Number of sweeps without bond optimizaiton to update the environment for infinite boundary conditions, performed every *N\_sweeps\_check* sweeps.

#### set B(U, S, VH)

Update the MPS with the U, S, VH returned by self.mixed\_svd.

#### **Parameters**

- VH  $(U_{\star})$  Left and Right-canonical matrices as returned by the SVD.
- **S** (1D array | 2D Array) The middle part returned by the SVD, theta = U S VH. Without a mixer just the singular values, with enabled *mixer* a 2D array.

```
sweep (optimize=True, meas_E_trunc=False)
```

One 'sweep' of a the algorithm.

Iteratate over the bond which is optimized, to the right and then back to the left to the starting point.

#### **Parameters**

- **optimize** (bool, optional) Whether we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).
- meas\_E\_trunc (bool, optional) Whether to measure truncation energies.

#### Returns

- max\_trunc\_err (float) Maximal truncation error introduced.
- max\_E\_trunc (None | float) None if meas\_E\_trunc is False, else the maximal change of the energy due to the truncation.

#### $\mathtt{update\_LP}\left(U\right)$

Update left part of the environment.

We always update the environment at site i0 + 1: this environment then contains the site where we just performed a local update (when sweeping right).

```
Parameters U (Array) – The U as returned by the SVD, with combined legs, labels 'vL. p0', 'vR'.
```

#### update\_RP(VH)

Update right part of the environment.

We always update the environment at site i0: this environment then contains the site where we just performed a local update (when sweeping left).

```
Parameters VH (Array) - The VH as returned by SVD, with combined legs, labels 'vL', '(vR.p1)'.
```

```
update_local (theta, optimize=True, meas_E_trunc=False)
```

Perform bond-update on the sites (i0, i0+1).

#### **Parameters**

- theta (Array) Initial guess for the ground state of the effective Hamiltonian.
- **optimize** (bool) Wheter we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).
- **meas\_E\_trunc** (bool) Wheter to measure the energy after truncation.

### Returns

**update\_data** – Data computed during the local update, as described in the following:

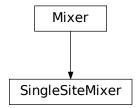
- **E0** [float] Total energy, obtained *before* truncation (if optimize=True), or *after* truncation (if optimize=False) (but never None).
- **N** [int] Dimension of the Krylov space used for optimization in the lanczos algorithm. 0 if optimize=False.
- **age** [int] Current size of the DMRG simulation: number of physical sites involved into the contraction.
- **U, VH:** Array U and VH returned by  $mixed\_svd()$ .
- ov\_change: float Change in the wave function 1. abs(<theta\_guess|theta>)
  induced by diag(), not including the truncation!

Return type dict

### **SingleSiteMixer**

- full name: tenpy.algorithms.dmrg.SingleSiteMixer
- parent module: tenpy.algorithms.dmrg
- type: class

### **Inheritance Diagram**



### **Methods**

SingleSiteMixerinit(options)	Initialize self.	
SingleSiteMixer.perturb_svd(engine, theta,	Mix extra terms to theta and perform an SVD.	
)		
SingleSiteMixer.subspace_expand(engine,	Expand the MPS subspace, to allow the bond dimension	
)	to increase.	
SingleSiteMixer.	Update the amplitude, possibly disable the mixer.	
update_amplitude(sweeps)		

class tenpy.algorithms.dmrg.SingleSiteMixer(options)

Bases: tenpy.algorithms.dmrg.Mixer

Mixer for single-site DMRG.

Performs a subspace expansion following [Hubig2015].

```
perturb_svd (engine, theta, i0, move_right, next_B)
```

Mix extra terms to theta and perform an SVD.

We calculate the left and right reduced density matrix using the mixer (which might include applications of H). These density matrices are diagonalized and truncated such that we effectively perform a svd for the case mixer.amplitude=0.

#### **Parameters**

- engine (Engine) The DMRG engine calling the mixer.
- theta (Array) The optimized wave function, prepared for svd.
- i0 (int) The site index where theta lives.
- move\_right (bool) Whether we move to the right (True) or left (False).
- next\_B (Array) The subspace expansion requires to change the tensor on the next site as well. If move\_right, it should correspond to engine.psi.get\_B(i0+1, form='B'). If not move\_right, it should correspond to engine.psi.get\_B(i0-1, form='A').

#### Returns

- U (Array) Left-canonical part of tensordot(theta, next\_B). Labels '(vL.p0)', 'vR'.
- S (1D ndarray) (Perturbed) singular values on the new bond (between theta and next B).
- VH (Array) Right-canonical part of tensordot(theta, next\_B). Labels 'vL', '(p1. vR)'.
- **err** (*TruncationError*) The truncation error introduced.

### subspace\_expand (engine, theta, i0, move\_right, next\_B)

Expand the MPS subspace, to allow the bond dimension to increase.

This is the subspace expansion following [Hubig2015].

### **Parameters**

- engine (SingleSiteDMRGEngine | TwoSiteDMRGEngine) 'Engine' for the DMRG algorithm
- **theta** (*Array*) Optimized guess for the ground state of the effective local Hamiltonian.
- i0 (int) Site index at which the local update has taken place.
- **move\_right** (bool) Whether the next *i0* of the sweep will be right or left of the current one.
- next\_B (Array) The subspace expansion requires to change the tensor on the next site as well. If move\_right, it should correspond to engine.psi.get\_B(i0+1, form='B'). If not move\_right, it should correspond to engine.psi.get\_B(i0-1, form='A').

#### Returns

- *theta* Local MPS tensor at site *i0* after subspace expansion.
- next\_B MPS tensor at site i0+1 or i0-1 (depending on sweep direction) after subspace expansion.

### update\_amplitude(sweeps)

Update the amplitude, possibly disable the mixer.

**Parameters** sweeps (int) – The number of performed sweeps, to check if we need to disable the mixer.

**Returns mixer** – Returns *self* if we should continue mixing, or None, if the mixer should be disabled.

Return type Mixer | None

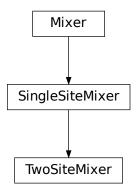
### **TwoSiteMixer**

• full name: tenpy.algorithms.dmrg.TwoSiteMixer

• parent module: tenpy.algorithms.dmrg

• type: class

# **Inheritance Diagram**



### **Methods**

<pre>TwoSiteMixerinit(options)</pre>	Initialize self.
TwoSiteMixer.perturb_svd(engine, theta, i0,	Mix extra terms to theta and perform an SVD.
)	
TwoSiteMixer.subspace_expand(engine,	Expand the MPS subspace, to allow the bond dimension
theta,)	to increase.
TwoSiteMixer.update_amplitude(sweeps)	Update the amplitude, possibly disable the mixer.

class tenpy.algorithms.dmrg.TwoSiteMixer(options)

Bases: tenpy.algorithms.dmrg.SingleSiteMixer

Mixer for two-site DMRG.

This is the two-site version of the mixer described in [Hubig2015]. Equivalent to the <code>DensityMatrixMixer</code>, but never construct the full density matrix.

### perturb\_svd (engine, theta, i0, move\_right)

Mix extra terms to theta and perform an SVD.

#### **Parameters**

- engine (Engine) The DMRG engine calling the mixer.
- theta (Array) The optimized wave function, prepared for svd.
- i0 (int) Site index; theta lives on i0, i0+1.
- update\_LP (bool) Whether to calculate the next env. LP [i0+1].
- update\_RP (bool) Whether to calculate the next env.RP[i0].

#### Returns

- U(Array) Left-canonical part of theta. Labels '(vL.p0)', 'vR'.
- **S** (1D ndarray | 2D Array) Without mixer just the singluar values of the array; with mixer it might be a general matrix; see comment above.
- VH (Array) Right-canonical part of theta. Labels 'vL', '(vR.p1)'.
- **err** (*TruncationError*) The truncation error introduced.

### subspace\_expand (engine, theta, i0, move\_right, next\_B)

Expand the MPS subspace, to allow the bond dimension to increase.

This is the subspace expansion following [Hubig2015].

#### **Parameters**

- engine (SingleSiteDMRGEngine | TwoSiteDMRGEngine) 'Engine' for the DMRG algorithm
- theta (Array) Optimized guess for the ground state of the effective local Hamiltonian.
- i0 (int) Site index at which the local update has taken place.
- move\_right (bool) Whether the next i0 of the sweep will be right or left of the current one.
- next\_B (Array) The subspace expansion requires to change the tensor on the next site as well. If move\_right, it should correspond to engine.psi.get\_B(i0+1, form='B'). If not move\_right, it should correspond to engine.psi.get\_B(i0-1, form='A').

#### Returns

- *theta* Local MPS tensor at site *i0* after subspace expansion.
- next\_B MPS tensor at site i0+1 or i0-1 (depending on sweep direction) after subspace expansion.

#### update\_amplitude (sweeps)

Update the amplitude, possibly disable the mixer.

**Parameters** sweeps (int) – The number of performed sweeps, to check if we need to disable the mixer.

**Returns mixer** – Returns *self* if we should continue mixing, or None, if the mixer should be disabled.

### Return type Mixer | None

### **Functions**

<pre>chi_list(chi_max[, dchi, nsweeps])</pre>	Compute a 'ramping-up' chi_list.
<pre>full_diag_effH(effH, theta_guess[, keep_sector])</pre>	Perform an exact diagonalization of <i>effH</i> .
run(psi, model, options)	Run the DMRG algorithm to find the ground state of the
	given model.

### chi\_list

- full name: tenpy.algorithms.dmrg.chi\_list
- parent module: tenpy.algorithms.dmrg
- type: function

```
tenpy.algorithms.dmrg.chi_list(chi_max, dchi=20, nsweeps=20)
Compute a 'ramping-up' chi_list.
```

The resulting chi\_list allows to increases *chi* by *dchi* every *nsweeps* sweeps up to a given maximal *chi\_max*.

### **Parameters**

- **chi\_max** (*int*) Final value for the bond dimension.
- dchi (int) Step size how to increase chi
- nsweeps (int) Step size for sweeps

**Returns chi\_list** – To be used as *chi\_list* parameter for DMRG, see run(). Keys increase by *nsweeps*, values by *dchi*, until a maximum of *chi\_max* is reached.

Return type dict

### full\_diag\_effH

- full name: tenpy.algorithms.dmrg.full\_diag\_effH
- parent module: tenpy.algorithms.dmrg
- type: function

```
tenpy.algorithms.dmrg.full_diag_effH(effH, theta_guess, keep_sector=True)
Perform an exact diagonalization of effH.
```

This function offers an alternative to lanczos().

#### **Parameters**

- **effH** (EffectiveH) The effective Hamiltonian.
- **theta\_guess** (*Array*) Current guess to select the charge sector. Labels as specified by effH.acts\_on.

### **Module description**

Density Matrix Renormalization Group (DMRG).

Although it was originally not formulated with tensor networks, the DMRG algorithm (invented by Steven White in 1992 [White1992]) opened the whole field with its enormous success in finding ground states in 1D.

We implement DMRG in the modern formulation of matrix product states [Schollwoeck2011], both for finite systems ('finite' or 'segment' boundary conditions) and in the thermodynamic limit ('infinite' b.c.).

The function run () - well - runs one DMRG simulation. Internally, it generates an instance of an Sweep. This class implements the common functionality like defining a *sweep*, but leaves the details of the contractions to be performed to the derived classes.

Currently, there are two derived classes implementing the contractions: SingleSiteDMRGEngine and TwoSiteDMRGEngine. They differ (as their name implies) in the number of sites which are optimized simultaneously. They should both give the same results (up to rounding errors). However, if started from a product state, SingleSiteDMRGEngine depends critically on the use of a Mixer, while TwoSiteDMRGEngine is in principle more computationally expensive to run and has occasionally displayed some convergence issues.. Which one is preffered in the end is not obvious a priori and might depend on the used model. Just try both of them.

A Mixer should be used initially to avoid that the algorithm gets stuck in local energy minima, and then slowly turned off in the end. For SingleSiteDMRGEngine, using a mixer is crucial, as the one-site algorithm cannot increase the MPS bond dimension by itself.

Todo: Write UserGuide!!!

# 7.7.5 tdvp

• full name: tenpy.algorithms.tdvp

• parent module: tenpy.algorithms

• type: module

### **Classes**

Engine H0\_mixed H1\_mixed H2\_mixed

<pre>Engine(psi, model, options[, environment])</pre>	Time dependent variational principle 'Engine'.
HO_mixed(Lp, Rp)	Class defining the zero site Hamiltonian for Lanczos.
H1_mixed(Lp, Rp, W)	Class defining the one site Hamiltonian for Lanczos.
H2_mixed(Lp, Rp, W0, W1)	Class defining the two sites Hamiltonian for Lanczos.

### H0\_mixed

- full name: tenpy.algorithms.tdvp.H0\_mixed
- parent module: tenpy.algorithms.tdvp
- type: class

### **Inheritance Diagram**

H0 mixed

### Methods

HO_mixedinit(Lp, Rp)	Initialize self.	
$ t H0_{mixed.matvec(x)}$		

```
class tenpy.algorithms.tdvp.{\tt H0\_mixed}\,(Lp,Rp) Bases: object
```

Class defining the zero site Hamiltonian for Lanczos.

### **Parameters**

- Lp (tenpy.linalg.np\_conserved.Array) left part of the environment
- $\mathbf{Rp}$  (tenpy.linalg.np\_conserved.Array)  $\mathbf{right}$  part of the environment

Lр

left part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

Rр

right part of the environment

Type tenpy.linalg.np\_conserved.Array

### H1\_mixed

- full name: tenpy.algorithms.tdvp.H1\_mixed
- parent module: tenpy.algorithms.tdvp
- type: class

### **Inheritance Diagram**

H1\_mixed

### Methods

H1_mixedinit(Lp, Rp, W)	Initialize self.	
H1_mixed.matvec(theta)		_

```
class tenpy.algorithms.tdvp.{\tt H1\_mixed}\,(Lp,Rp,W) Bases: object
```

Class defining the one site Hamiltonian for Lanczos.

### **Parameters**

- Lp (tenpy.linalg.np\_conserved.Array) left part of the environment
- $\mathbf{Rp}$  (tenpy.linalg.np\_conserved.Array) right part of the environment
- M (tenpy.linalg.np\_conserved.Array) MPO which is applied to the 'p' leg of theta

Lр

left part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

Rр

right part of the environment

Type tenpy.linalg.np\_conserved.Array

W

MPO which is applied to the 'p0' leg of theta

Type tenpy.linalg.np\_conserved.Array

### H2\_mixed

- $\bullet \ \, full \ name: tenpy.algorithms.tdvp.H2\_mixed$
- $\bullet \ \ parent \ module: \ \textit{tenpy.algorithms.tdvp}$
- type: class

### **Inheritance Diagram**

H2\_mixed

### Methods

H2_mixedinit(Lp, Rp, W0, W1)	Initialize self.	
H2_mixed.matvec(theta)		

```
class tenpy.algorithms.tdvp.{\tt H2\_mixed}\,(Lp,Rp,W0,W1) Bases: object
```

Class defining the two sites Hamiltonian for Lanczos.

### **Parameters**

- Lp  $(tenpy.linalg.np\_conserved.Array)$  left part of the environment
- $\mathbf{Rp}$  (tenpy.linalg.np\_conserved.Array) right part of the environment
- W (tenpy.linalg.np\_conserved.Array) MPO which is applied to the 'p0' leg of theta

Lp

left part of the environment

```
\textbf{Type} \ \textit{tenpy.linalg.np\_conserved.} \\ \textit{Array}
```

Rр

right part of the environment

```
Type tenpy.linalg.np_conserved.Array
```

WΟ

MPO which is applied to the 'p0' leg of theta

```
Type tenpy.linalg.np_conserved.Array
```

W1

MPO which is applied to the 'p1' leg of theta

Type tenpy.linalg.np\_conserved.Array

### **Module description**

Time Dependant Variational Principle (TDVP) with MPS (finite version only).

The TDVP MPS algorithm was first proposed by [Haegeman2011]. However the stability of the algorithm was later improved in [Haegeman2016], that we are following in this implementation. The general idea of the algorithm is to project the quantum time evolution in the manyfold of MPS with a given bond dimension. Compared to e.g. TEBD, the algorithm has several advantages: e.g. it conserves the unitarity of the time evolution and the energy (for the single-site version), and it is suitable for time evolution of Hamiltonian with arbitrary long range in the form of MPOs. We have implemented the one-site formulation which **does not** allow for growth of the bond dimension, and the two-site algorithm which does allow the bond dimension to grow - but requires truncation as in the TEBD case.

**Todo:** This is still a beta version, use with care. The interface might still change.

**Todo:** long-term: Much of the code is similar as in DMRG. To avoid too much duplicated code, we should have a general way to sweep through an MPS and updated one or two sites, used in both cases.

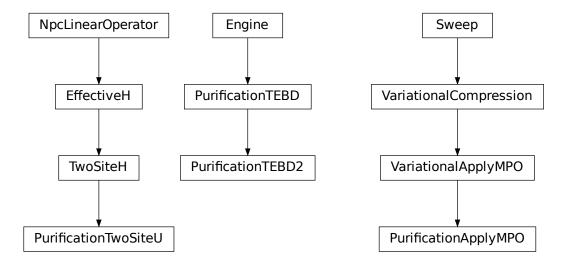
# 7.7.6 purification

· full name: tenpy.algorithms.purification

• parent module: tenpy.algorithms

• type: module

### Classes



PurificationApplyMPO(psi, U_MPO, options)	Variant of VariationalApplyMPO suitable for purifica-
	tion.
PurificationTEBD(psi, model, options)	Time evolving block decimation (TEBD) for purifica-
	tion MPS.
PurificationTEBD2(psi, model, options)	Similar as PurificationTEBD, but perform sweeps in-
	stead of brickwall.
PurificationTwoSiteU(env, i0[, combine,])	Variant of <i>TwoSiteH</i> suitable for purification.

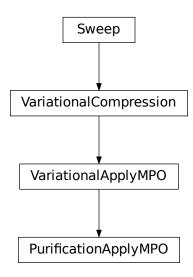
# **PurificationApplyMPO**

• full name: tenpy.algorithms.purification.PurificationApplyMPO

• parent module: tenpy.algorithms.purification

• type: class

# **Inheritance Diagram**



### **Methods**

PurificationApplyMPOinit(psi,	Initialize self.
U_MPO,)	
PurificationApplyMPO.	Perform <i>N_sweeps</i> sweeps without optimization to up-
$environment\_sweeps(N\_sweeps)$	date the environment.
PurificationApplyMPO.	Define the schedule of the sweep.
get_sweep_schedule()	

continues on next page

Table 30 – continued from previous page

	1 1 5
PurificationApplyMPO.init_env(U_MPO)	Initialize the environment.
PurificationApplyMPO.make_eff_H()	Create new instance of self.EffectiveH at self.i0 and set
	it to self.eff_H.
PurificationApplyMPO.	Algorithm-specific actions to be taken after local up-
<pre>post_update_local()</pre>	date.
PurificationApplyMPO.prepare_update()	Prepare everything algorithm-specific to perform a local
	update.
PurificationApplyMPO.reset_stats()	Reset the statistics.
PurificationApplyMPO.run()	Run the compression.
PurificationApplyMPO.sweep([optimize])	One 'sweep' of a sweeper algorithm.
PurificationApplyMPO.update_LP(_)	
PurificationApplyMPO.update_RP(_)	
PurificationApplyMPO.update_local(_[,	Perform local update.
optimize])	
PurificationApplyMPO.	
_update_new_psi(theta)	

### **Class Attributes and Properties**

PurificationApplyMPO.engine\_params

**class** tenpy.algorithms.purification.**PurificationApplyMPO**(psi, U\_MPO, options)

Bases: tenpy.algorithms.mps\_common.VariationalApplyMPO

Variant of Variational ApplyMPO suitable for purification.

### EffectiveH

alias of PurificationTwoSiteU

### update\_local (\_, optimize=True)

Perform local update.

This simply contracts the environments and *theta* from the *ket* to get an updated *theta* for the bra *self.psi* (to be changed in place).

#### environment\_sweeps (N\_sweeps)

Perform *N\_sweeps* sweeps without optimization to update the environment.

**Parameters** N\_sweeps (int) – Number of sweeps to run without optimization

### get\_sweep\_schedule()

Define the schedule of the sweep.

One 'sweep' is a full sequence from the leftmost site to the right and back. Only those *LP* and *RP* that can be used later should be updated.

Returns schedule - Schedule for the sweep. Each entry is (i0, move\_right, (update\_LP, update\_RP)), where i0 is the leftmost of the self.EffectiveH. length sites to be updated in update\_local(), move\_right indicates whether the next i0 in the schedule is rigth (True) of the current one, and update\_LP, update\_RP indicate whether it is necessary to update the LP and RP. The latter are chosen such that the environment is growing for infinite systems, but we only keep the minimal number of environment tensors in memory.

**Return type** iterable of (int, bool, (bool, bool))

```
init env(U MPO)
```

Initialize the environment.

**Parameters U\_MPO** (MPO) – The MPO to be applied to the sate.

```
make_eff_H()
```

Create new instance of self. EffectiveH at self. i0 and set it to self. eff\_H.

```
post_update_local (update_data)
```

Algorithm-specific actions to be taken after local update.

An example would be to collect statistics.

```
prepare_update()
```

Prepare everything algorithm-specific to perform a local update.

```
reset_stats()
```

Reset the statistics. Useful if you want to start a new Sweep run.

This method is expected to be overwritten by subclass, and should then define self.update\_stats and self.sweep\_stats dicts consistent with the statistics generated by the algorithm particular to that subclass.

```
option Sweep.sweep_0: int
```

Number of sweeps that have already been performed.

run()

Run the compression.

The state psi is compressed in place.

**Returns** max\_trunc\_err – The maximal truncation error of a two-site wave function.

Return type TruncationError

```
sweep (optimize=True)
```

One 'sweep' of a sweeper algorithm.

Iteratate over the bond which is optimized, to the right and then back to the left to the starting point. If optimize=False, don't actually diagonalize the effective hamiltonian, but only update the environment.

**Parameters optimize** (bool, optional) – Whether we actually optimize to find the ground state of the effective Hamiltonian. (If False, just update the environments).

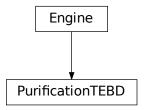
**Returns** max\_trunc\_err - Maximal truncation error introduced.

Return type float

#### **PurificationTEBD**

- full name: tenpy.algorithms.purification.PurificationTEBD
- $\bullet \ parent \ module: \ tenpy.algorithms.purification$
- type: class

# Inheritance Diagram



# Methods

PurificationTEBDinit(psi, model, op-	Initialize self.
tions)	
PurificationTEBD.calc_U(order, delta_t[,])	see calc_U()
PurificationTEBD.disentangle(theta)	Disentangle <i>theta</i> before splitting with svd.
PurificationTEBD.	Try global disentangling by determining the maximally
disentangle_global([pair])	entangled pairs of sites.
PurificationTEBD.	Perform a sweep through the system and disentangle
disentangle_global_nsite([n])	<pre>with disentangle_n_site().</pre>
PurificationTEBD.disentangle_n_site(i,	Generalization of disentangle () to n sites.
n, theta)	
PurificationTEBD.run()	(Real-)time evolution with TEBD (time evolving block
	decimation).
PurificationTEBD.run_GS()	TEBD algorithm in imaginary time to find the ground
	state.
PurificationTEBD.run_imaginary(beta)	Run imaginary time evolution to cool down to the given
	beta.
PurificationTEBD.	Returns list of necessary steps for the suzuki trotter de-
suzuki_trotter_decomposition()	composition.
PurificationTEBD.	Return time steps of U for the Suzuki Trotter decompo-
suzuki_trotter_time_steps(order)	sition of desired order.
PurificationTEBD.update(N_steps)	Evolve by N_steps * U_param['dt'].
PurificationTEBD.update_bond(i, U_bond)	Updates the B matrices on a given bond.
PurificationTEBD.update_bond_imag $(i,$	Update a bond with a (possibly non-unitary) <i>U_bond</i> .
U_bond)	
PurificationTEBD.update_imag(N_steps)	Perform an update suitable for imaginary time evolu-
	tion.
PurificationTEBD.update_step(U_idx_dt,	Updates either even <i>or</i> odd bonds in unit cell.
odd)	

## **Class Attributes and Properties**

PurificationTEBD.TEBD_params	
PurificationTEBD.disent_iterations	For each bond the total number of iterations performed
	in any Disentangler.
PurificationTEBD.trunc_err_bonds	truncation error introduced on each non-trivial bond.

## class tenpy.algorithms.purification.PurificationTEBD (psi, model, options)

Bases: tenpy.algorithms.tebd.Engine

Time evolving block decimation (TEBD) for purification MPS.

Deprecated since version 0.6.0: Renamed parameter/attribute *TEBD\_params* to options.

#### **Parameters**

- **psi** (PurificationMPS) Initial state to be time evolved. Modified in place.
- model (NearestNeighborModel) The model representing the Hamiltonian for which we want to find the ground state.
- options (dict) Further optional parameters as described in the following table. Use verbose=1 to print the used parameters during runtime. See run() and run\_GS() for more details.

#### used\_disentangler

The disentangler to be used on the auxiliar indices. Chosen by <code>get\_disentangler()</code>, called with the TEBD parameter 'disentangle'. Defaults to the trivial disentangler for <code>options['disentangle']=None</code>.

Type Disentangler

## \_disent\_iterations

Number of iterations performed on all bonds, including trivial bonds; lenght L.

Type 1D ndarray

## \_guess\_U\_disent

Same index strucuture as *self.\_U*: for each two-site U of the physical time evolution the disentangler from the last application. Initialized to identities.

Type list of list of npc.Array

#### run imaginary (beta)

Run imaginary time evolution to cool down to the given beta.

Note that we don't change the *norm* attribute of the MPS, i.e. normalization is preserved.

**Parameters** beta (float) - The inverse temperature beta = 1/T, by which we should cool down. We evolve to the closest multiple of options ['dt'], see also evolved\_time.

#### property disent\_iterations

For each bond the total number of iterations performed in any Disentangler.

calc\_U (order, delta\_t, type\_evo='real', E\_offset=None)
see calc\_U()

## update\_bond(i, U\_bond)

Updates the B matrices on a given bond.

Function that updates the B matrices, the bond matrix s between and the bond dimension chi for bond i. This would look something like:

#### **Parameters**

- i(int) Bond index; we update the matrices at sites i-1, i.
- **U\_bond** (Array) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0\*', 'p1\*' for  $U_bond$ .

**Returns trunc\_err** – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

## update\_bond\_imag(i, U\_bond)

Update a bond with a (possibly non-unitary) *U\_bond*.

Similar as update\_bond(); but after the SVD just keep the A, S, B canonical form. In that way, one can sweep left or right without using old singular values, thus preserving the canonical form during imaginary time evolution.

#### **Parameters**

- i(int) Bond index; we update the matrices at sites i-1, i.
- **U\_bond** (Array) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0\*', 'p1\*'.

**Returns trunc\_err** – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

#### disentangle (theta)

Disentangle theta before splitting with svd.

For the purification we write  $\rho_P = Tr_Q |\psi_{P,Q}> <\psi_{P,Q}|$ . Thus, we can actually apply any unitary to the auxiliar Q space of  $|\psi>$  without changing the result.

**Note:** We have to apply the *same* unitary to the 'bra' and 'ket' used for expectation values / correlation functions!

The behaviour of this function is set by <code>used\_disentangler</code>, which in turn is obtained from <code>get\_disentangler(options['disentangle'])</code>, see <code>get\_disentangler()</code> for details on the syntax.

Parameters theta (Array) - Wave function to disentangle, with legs 'vL', 'vR', 'p0', 'p1', 'q0', 'q1'.

## Returns

• theta\_disentangled (Array) - Disentangled theta; npc.tensordot(U, theta, axes=[['q0\*', 'q1\*'], ['q0', 'q1']]).

• U(Array) – The unitary used to disentangle *theta*, with labels 'q0', 'q1', 'q0\*', 'q1\*'. If no unitary was found/applied, it might also be None.

## disentangle\_global (pair=None)

Try global disentangling by determining the maximally entangled pairs of sites.

Caclulate the mutual information (in the auxiliar space) between two sites and determine where it is maximal. Disentangle these two sites with <code>disentangle()</code>

#### disentangle global nsite(n=2)

Perform a sweep through the system and disentangle with disentangle\_n\_site().

**Parameters n** (*int*) – maximal number of sites to disentangle at once.

### disentangle\_n\_site(i, n, theta)

Generalization of disentangle () to n sites.

Simply group left and right n/2 physical legs, adjust labels, and apply disentangle() to disentangle the central bond. Recursively proceed to disentangle left and right parts afterwards. Scales (for even n) as  $O(\chi^3 d^n d^{n/2})$ .

#### run()

(Real-)time evolution with TEBD (time evolving block decimation).

```
option TEBD.dt: float
    Time step.
```

#### option TEBD.N\_steps: int

Number of time steps dt to evolve. The Trotter decompositions of order > 1 are slightly more efficient if more than one step is performed at once.

```
option TEBD.order: int
```

Order of the algorithm. The total error scales as O (t\*dt^order).

## run\_GS()

TEBD algorithm in imaginary time to find the ground state.

**Note:** It is almost always more efficient (and hence advisable) to use DMRG. This algorithms can nonetheless be used quite well as a benchmark and for comparison.

```
option TEBD.delta_tau_list: list
```

A list of floats: the timesteps to be used. Choosing a large timestep  $delta\_tau$  introduces large (Trotter) errors, but a too small time step requires a lot of steps to reach  $exp(-tau \ H) \ --> |psi0><psi0|$ . Therefore, we start with fairly large time steps for a quick time evolution until convergence, and the gradually decrease the time step.

```
option TEBD.order: int
```

Order of the Suzuki-Trotter decomposition.

```
option TEBD.N_steps: int
```

Number of steps before measurement can be performed

#### static suzuki\_trotter\_decomposition(order, N\_steps)

Returns list of necessary steps for the suzuki trotter decomposition.

We split the Hamiltonian as  $H = H_{even} + H_{odd} = H[0] + H[1]$ . The Suzuki-Trotter decomposition is an approximation  $\exp(tH) \approx prod_{(j,k) \in ST} \exp(d[j]tH[k]) + O(t^{order+1})$ .

**Parameters order** (1, 2, 4, '4\_opt') – The desired order of the Suzuki-Trotter decomposition. Order 1 approximation is simply  $e^A a^B$ . Order 2 is the "leapfrog"  $e^A A^2 e^B e^A A^2$ . Order 4 is the fourth-order from [Suzuki1991] (also referenced

in [Schollwoeck2011]), and '4\_opt' gives the optmized version of Equ. (30a) in [Barthel2020].

**Returns ST\_decomposition** - Indices j, k of the time-steps d = suzuki\_trotter\_time\_step(order) and the decomposition of H. They are chosen such that a subsequent application of exp(d[j] t H[k]) to a given state |psi> yields (exp(N\_steps t H[k]) + O(N\_steps t^{order+1})) |psi>.

**Return type** list of (int, int)

## static suzuki\_trotter\_time\_steps(order)

Return time steps of U for the Suzuki Trotter decomposition of desired order.

See suzuki\_trotter\_decomposition() for details.

**Parameters order** (*int*) – The desired order of the Suzuki-Trotter decomposition.

**Returns time\_steps** – We need U = exp(-i H\_{even/odd} delta\_t \* dt) for the *dt* returned in this list.

Return type list of float

## property trunc\_err\_bonds

truncation error introduced on each non-trivial bond.

# update (N\_steps)

Evolve by N\_steps \* U\_param['dt'].

**Parameters** N\_steps (int) - The number of steps for which the whole lattice should be updated.

**Returns trunc\_err** – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

## update\_imag(N\_steps)

Perform an update suitable for imaginary time evolution.

Instead of the even/odd brick structure used for ordinary TEBD, we 'sweep' from left to right and right to left, similar as DMRG. Thanks to that, we are actually able to preserve the canonical form.

**Parameters** N\_steps (int) - The number of steps for which the whole lattice should be updated.

**Returns trunc\_err** – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

#### update\_step(U\_idx\_dt, odd)

Updates either even *or* odd bonds in unit cell.

Depending on the choice of p, this function updates all even  $(\mathbb{E}, odd=False, 0)$  or odd  $(\bigcirc)$  (odd=True, 1) bonds:

Note that finite boundary conditions are taken care of by having Us [0] = None.

## **Parameters**

- U\_idx\_dt (int) Time step index in self.\_U, evolve with Us[i] = self. U[U\_idx\_dt][i] at bond (i-1,i).
- **odd** (bool/int) Indication of whether to update even (odd=False, 0) or even (odd=True, 1) sites

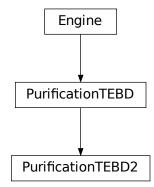
**Returns trunc\_err** – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

#### PurificationTEBD2

- full name: tenpy.algorithms.purification.PurificationTEBD2
- parent module: tenpy.algorithms.purification
- type: class

# **Inheritance Diagram**



#### **Methods**

PurificationTEBD2init(psi, model, op-	Initialize self.
tions)	
PurificationTEBD2.calc_U(order, delta_t[,	see calc_U()
])	
PurificationTEBD2.disentangle(theta)	Disentangle <i>theta</i> before splitting with svd.
PurificationTEBD2.	Try global disentangling by determining the maximally
disentangle_global([pair])	entangled pairs of sites.
	continues on next page

	Table	34 –	continued	from	previous	page
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	· · · · · · · · · · · · · · · · · · ·
PurificationTEBD2.	Perform a sweep through the system and disentangle
disentangle_global_nsite([n])	<pre>with disentangle_n_site().</pre>
PurificationTEBD2.	Generalization of disentangle () to n sites.
disentangle_n_site(i, n, theta)	
PurificationTEBD2.run()	(Real-)time evolution with TEBD (time evolving block
	decimation).
PurificationTEBD2.run_GS()	TEBD algorithm in imaginary time to find the ground
	state.
PurificationTEBD2.run_imaginary(beta)	Run imaginary time evolution to cool down to the given
	beta.
PurificationTEBD2.	Returns list of necessary steps for the suzuki trotter de-
$suzuki\_trotter\_decomposition()$	composition.
PurificationTEBD2.	Return time steps of U for the Suzuki Trotter decompo-
suzuki_trotter_time_steps(order)	sition of desired order.
PurificationTEBD2.update(N_steps)	Evolve by N_steps * U_param['dt'].
PurificationTEBD2.update_bond(i, U_bond)	Updates the B matrices on a given bond.
PurificationTEBD2.update_bond_imag(i,	Update a bond with a (possibly non-unitary) <i>U_bond</i> .
U_bond)	
Purification TEBD $2$ . update $\_$ imag $(N_{\_}$ steps)	Perform an update suitable for imaginary time evolu-
	tion.
PurificationTEBD2.update_step(U_idx_dt,	Updates bonds in unit cell.
odd)	

# **Class Attributes and Properties**

PurificationTEBD2.TEBD_params	
PurificationTEBD2.disent_iterations	For each bond the total number of iterations performed
	in any Disentangler.
PurificationTEBD2.trunc_err_bonds	truncation error introduced on each non-trivial bond.

**class** tenpy.algorithms.purification.**PurificationTEBD2** (psi, model, options)
Bases: tenpy.algorithms.purification.PurificationTEBD

Similar as PurificationTEBD, but perform sweeps instead of brickwall.

Instead of the A-B pattern of even/odd bonds used in TEBD, perform sweeps similar as in DMRG for real-time evolution (similar as update\_imag() does for imaginary time evolution).

```
update (N_steps)
     Evolve by N_steps * U_param['dt'].
```

**Parameters N\_steps** (int) - The number of steps for which the whole lattice should be updated.

**Returns trunc\_err** – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

# $update\_step(U\_idx\_dt, odd)$

Updates bonds in unit cell.

Depending on the choice of *odd*, perform a sweep to the left or right, updating once per site with a time step given by U\_idx\_dt.

#### **Parameters**

- **U\_idx\_dt** (*int*) Time step index in self.\_U, evolve with Us[i] = self. U[U\_idx\_dt][i] at bond (i-1,i).
- **odd** (bool/int) Indication of whether to update even (odd=False, 0) or even (odd=True, 1) sites

**Returns trunc\_err** – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

```
Return type TruncationError
```

```
calc_U (order, delta_t, type_evo='real', E_offset=None)
see calc_U()
```

#### property disent\_iterations

For each bond the total number of iterations performed in any Disentangler.

## disentangle (theta)

Disentangle *theta* before splitting with svd.

For the purification we write  $\rho_P = Tr_Q |\psi_{P,Q}> <\psi_{P,Q}|$ . Thus, we can actually apply any unitary to the auxiliar Q space of  $|\psi>$  without changing the result.

**Note:** We have to apply the *same* unitary to the 'bra' and 'ket' used for expectation values / correlation functions!

The behaviour of this function is set by used\_disentangler, which in turn is obtained from get\_disentangler(options['disentangle']), see get\_disentangler() for details on the syntax.

```
Parameters theta (Array) - Wave function to disentangle, with legs 'vL', 'vR', 'p0', 'p1', 'q0', 'q1'.
```

#### Returns

- theta\_disentangled (Array) Disentangled theta; npc.tensordot(U, theta, axes=[['q0\*', 'q1\*'], ['q0', 'q1']]).
- U(Array) The unitary used to disentangle *theta*, with labels 'q0', 'q1', 'q0\*', 'q1\*'. If no unitary was found/applied, it might also be None.

## disentangle\_global(pair=None)

Try global disentangling by determining the maximally entangled pairs of sites.

Caclulate the mutual information (in the auxiliar space) between two sites and determine where it is maximal. Disentangle these two sites with <code>disentangle()</code>

## disentangle\_global\_nsite(n=2)

Perform a sweep through the system and disentangle with disentangle\_n\_site().

**Parameters n** (int) – maximal number of sites to disentangle at once.

#### disentangle\_n\_site(i, n, theta)

Generalization of disentangle () to n sites.

Simply group left and right n/2 physical legs, adjust labels, and apply disentangle() to disentangle the central bond. Recursively proceed to disentangle left and right parts afterwards. Scales (for even n) as  $O(\chi^3 d^n d^{n/2})$ .

#### run()

(Real-)time evolution with TEBD (time evolving block decimation).

# option TEBD.dt: float

Time step.

#### option TEBD.N\_steps: int

Number of time steps dt to evolve. The Trotter decompositions of order > 1 are slightly more efficient if more than one step is performed at once.

## option TEBD.order: int

Order of the algorithm. The total error scales as O(t\*dt^order).

#### run GS()

TEBD algorithm in imaginary time to find the ground state.

**Note:** It is almost always more efficient (and hence advisable) to use DMRG. This algorithms can nonetheless be used quite well as a benchmark and for comparison.

#### option TEBD.delta\_tau\_list: list

A list of floats: the timesteps to be used. Choosing a large timestep *delta\_tau* introduces large (Trotter) errors, but a too small time step requires a lot of steps to reach exp(-tau H) --> |psi0><psi0|. Therefore, we start with fairly large time steps for a quick time evolution until convergence, and the gradually decrease the time step.

#### option TEBD.order: int

Order of the Suzuki-Trotter decomposition.

#### option TEBD.N\_steps: int

Number of steps before measurement can be performed

## run\_imaginary(beta)

Run imaginary time evolution to cool down to the given beta.

Note that we don't change the norm attribute of the MPS, i.e. normalization is preserved.

**Parameters** beta (float) - The inverse temperature beta = 1/T, by which we should cool down. We evolve to the closest multiple of options ['dt'], see also evolved\_time.

## static suzuki\_trotter\_decomposition(order, N\_steps)

Returns list of necessary steps for the suzuki trotter decomposition.

We split the Hamiltonian as  $H = H_{even} + H_{odd} = H[0] + H[1]$ . The Suzuki-Trotter decomposition is an approximation  $\exp(tH) \approx prod_{(j,k) \in ST} \exp(d[j]tH[k]) + O(t^{order+1})$ .

**Parameters order** (1, 2, 4, '4\_opt') – The desired order of the Suzuki-Trotter decomposition. Order 1 approximation is simply  $e^A a^B$ . Order 2 is the "leapfrog"  $e^A A^2 e^B e^A A^2$ . Order 4 is the fourth-order from [Suzuki1991] (also referenced in [Schollwoeck2011]), and '4\_opt' gives the optmized version of Equ. (30a) in [Barthel2020].

**Returns ST\_decomposition** - Indices j, k of the time-steps d = suzuki\_trotter\_time\_step(order) and the decomposition of H. They are chosen such that a subsequent application of exp(d[j] t H[k]) to a given state |psi> yields (exp(N\_steps t H[k]) + O(N\_steps t^{order+1})) |psi>.

**Return type** list of (int, int)

## static suzuki\_trotter\_time\_steps(order)

Return time steps of U for the Suzuki Trotter decomposition of desired order.

See suzuki\_trotter\_decomposition() for details.

**Parameters order** (int) – The desired order of the Suzuki-Trotter decomposition.

**Returns time\_steps** – We need U = exp(-i H\_{even/odd} delta\_t \* dt) for the *dt* returned in this list.

Return type list of float

#### property trunc\_err\_bonds

truncation error introduced on each non-trivial bond.

#### update\_bond(i, U\_bond)

Updates the B matrices on a given bond.

Function that updates the B matrices, the bond matrix s between and the bond dimension chi for bond i. This would look something like:

#### **Parameters**

- i(int) Bond index; we update the matrices at sites i-1, i.
- **U\_bond** (Array) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0\*', 'p1\*' for  $U_bond$ .

**Returns trunc\_err** – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

## update\_bond\_imag(i, U\_bond)

Update a bond with a (possibly non-unitary) *U\_bond*.

Similar as <code>update\_bond()</code>; but after the SVD just keep the A, S, B canonical form. In that way, one can sweep left or right without using old singular values, thus preserving the canonical form during imaginary time evolution.

#### **Parameters**

- i(int) Bond index; we update the matrices at sites i-1, i.
- **U\_bond** (Array) The bond operator which we apply to the wave function. We expect labels 'p0', 'p1', 'p0\*', 'p1\*'.

**Returns trunc\_err** – The error of the represented state which is introduced by the truncation during this update step.

Return type TruncationError

## update\_imag(N\_steps)

Perform an update suitable for imaginary time evolution.

Instead of the even/odd brick structure used for ordinary TEBD, we 'sweep' from left to right and right to left, similar as DMRG. Thanks to that, we are actually able to preserve the canonical form.

**Parameters**  $N_{steps}(int)$  – The number of steps for which the whole lattice should be updated.

**Returns trunc\_err** – The error of the represented state which is introduced due to the truncation during this sequence of update steps.

Return type TruncationError

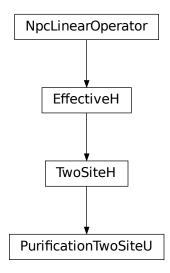
# **PurificationTwoSiteU**

• full name: tenpy.algorithms.purification.PurificationTwoSiteU

• parent module: tenpy.algorithms.purification

• type: class

# **Inheritance Diagram**



# **Methods**

PurificationTwoSiteUinit(env, i0[,])	Initialize self.
PurificationTwoSiteU.adjoint()	Return the hermitian conjugate of <i>self</i> .
PurificationTwoSiteU.combine_Heff()	Combine LP and RP with W to form LHeff and RHeff.
PurificationTwoSiteU.	Combine the legs of <i>theta</i> , such that it fits to how we
combine_theta(theta)	combined the legs of <i>self</i> .
PurificationTwoSiteU.matvec(theta)	Apply the effective Hamiltonian to theta.
PurificationTwoSiteU.to_matrix()	Contract <i>self</i> to a matrix.

# **Class Attributes and Properties**

```
PurificationTwoSiteU.acts_on
 PurificationTwoSiteU.length
class tenpy.algorithms.purification.PurificationTwoSiteU(env, i0, combine=False,
                                                                          move_right=True)
     Bases: tenpy.algorithms.mps_common.TwoSiteH
     Variant of TwoSiteH suitable for purification.
     The MPO gets only applied to the physical legs p0, p1, the ancialla legs q0, q1 of theta are ignored.
     combine Heff()
          Combine LP and RP with W to form LHeff and RHeff.
          Combine LP with W0 and RP with W1 to get the effective parts of the Hamiltonian with piped legs.
     adjoint()
          Return the hermitian conjugate of self.
     combine_theta(theta)
          Combine the legs of theta, such that it fits to how we combined the legs of self.
              Parameters theta (Array) - Wave function with labels 'vL', 'p0', 'p1', 'vR'
              Returns theta – Wave function with labels 'vL', 'p0', 'p1', 'vR'
              Return type Array
     matvec(theta)
          Apply the effective Hamiltonian to theta.
              Parameters theta (Array) - Labels: vL, p0, p1, vR if combine=False, (vL.p0),
                  (p1.vR) if True
              Returns Product of theta and the effective Hamiltonian.
              Return type theta Array
     to_matrix()
          Contract self to a matrix.
```

# **Module description**

Algorithms for using Purification.

An example is shown in *Example code purication.py*.

# 7.7.7 mpo evolution

• full name: tenpy.algorithms.mpo\_evolution

• parent module: tenpy.algorithms

• type: module

# Classes

# ExpMPOEvolution

ExpMPOEvolution(psi, model, options)	Time evolution of an MPS using the W_I or W_II ap-
	proximation for exp(H dt).

# **Module description**

Time evolution using the WI or WII approximation of the time evolution operator.

# 7.7.8 network contractor

• full name: tenpy.algorithms.network\_contractor

• parent module: tenpy.algorithms

· type: module

## **Functions**

contract(tensor_list[, tensor_names,])	Contract a network of tensors.
ncon(tensor_list, leg_links, sequence)	Implementation of ncon.m for TeNPy Arrays.

## contract

- full name: tenpy.algorithms.network\_contractor.contract
- parent module: tenpy.algorithms.network\_contractor
- type: function

tenpy.algorithms.network\_contractor.contract (tensor\_list, tensor\_names=None, leg\_contractions=None, open\_legs=None, sequence=None)

Contract a network of tensors.

Based on the MatLab function ncon.m as described in arXiv:1402.0939.

# **Parameters**

- tensor\_list (list of Array) The tensors to be contracted.
- leg\_contractions (list of [n1, 11, n2, 12]) A list of contraction instructions. An entry of leg\_contractions has the form [n1, 11, n2, 12], where n1, n2 are entries of tensor\_names and each identify an Array in tensor\_list. 11, 12 are leg labels of

the corresponding Array. The instruction implies to contract leg 11 of tensor n1 with leg 12 of tensor n2.

- open\_legs (list of [n1, 11, 1]) A list of instructions for "open" (uncontracted) legs. [n1, 11, 1] implies that leg 11 of tensor n1 is not contracted and is labelled 1 in the result.
- tensor\_names (list of str) A list of names for each tensor, to be used in leg\_contractions and open\_legs. The default value is list(range(len(tensor\_list))), so that the tensor "names" are 0, 1, 2, ....
- **sequence** (*list of int*) The order in which the leg\_contractions are to be performed. An entry of network\_contractor.outer\_product indicates performing an outer product. This corresponds to the zero-in-sequence convention of arXiv:1304.6112

**Returns** result – The number or tensor resulting from the contraction.

Return type Array | complex

#### ncon

- full name: tenpy.algorithms.network\_contractor.ncon
- parent module: tenpy.algorithms.network\_contractor
- type: function

tenpy.algorithms.network\_contractor.ncon(tensor\_list, leg\_links, sequence) Implementation of ncon.m for TeNPy Arrays.

This function is a python implementation of ncon.m (arXiv:1304.6112) for tenpy Array. contract () is a wrapper that translates from a more python/tenpy input style

#### **Parameters**

- tensor\_list (list of :class:'Array') Tensors to be contracted.
- **leg\_links** (*list of list of int*) Each entry of leg\_links describes the connectivity of the corresponding tensor in *tensor\_list*. Each entry is a list that has an entry for each leg of the corresponding tensor. Values 0, 1, 2, ... are labels of contracted legs and should appear exactly twice in *leg\_links*. Values -1, -2, -3, ... are labels of uncontracted legs and indicate the final ordering (-1 is first axis).
- **sequence** (list of int) The order in which the contractions are to be performed. An entry of network\_contractor.outer\_product indicates performing an outer product. This corresponds to the zero-in-sequence convention of arXiv:1304.6112

**Returns** result – The number or tensor resulting from the contraction.

Return type Array | complex

# **Module description**

Network Contractor.

A tool to contract a network of multiple tensors.

This is an implementation of 'NCON: A tensor network contractor for MATLAB' by Robert N. C. Pfeifer, Glen Evenbly, Sukhwinder Singh, Guifre Vidal, see arXiv:1402.0939

```
tenpy.algorithms.network_contractor.outer_product = -66666666
a constant that represents an outer product in the sequence of ncon
```

## **Todo:**

- implement or wrap netcon.m, a function to find optimal contractionn sequences (arXiv:1304.6112)
- improve helpfulness of Warnings
- \_do\_trace: trace over all pairs of legs at once. need the corresponding npc function first.

# 7.7.9 exact\_diag

• full name: tenpy.algorithms.exact\_diag

• parent module: tenpy.algorithms

• type: module

# Classes

ExactDiag

ExactDiag(model[, charge\_sector, sparse, ...])

(Full) exact diagonalization of the Hamiltonian.

# **ExactDiag**

• full name: tenpy.algorithms.exact\_diag.ExactDiag

• parent module: tenpy.algorithms.exact\_diag

· type: class

# **Inheritance Diagram**

ExactDiag

## Methods

<pre>ExactDiaginit(model[, charge_sector,</pre>	Initialize self.
])	
ExactDiag.build_full_H_from_bonds()	Calculate self.full_H from self.mpo.
ExactDiag.build_full_H_from_mpo()	Calculate self.full_H from self.mpo.
ExactDiag.exp_H(dt)	Return U(dt) := exp(-i H dt).
ExactDiag.from_H_mpo(H_MPO, *args,	Wrapper taking directly an MPO instead of a Model.
**kwargs)	
ExactDiag.full_diagonalization(*args,	Full diagonalization to obtain all eigenvalues and eigen-
**kwargs)	vectors.
<pre>ExactDiag.full_to_mps(psi[, canonical_form])</pre>	Convert a full state (with a single leg) to an MPS.
<pre>ExactDiag.groundstate([charge_sector])</pre>	Pick the ground state energy and ground state from
	self.V.
ExactDiag.matvec(psi)	Allow to use <i>self</i> as LinearOperator for lanczos.
ExactDiag.mps_to_full(mps)	Contract an MPS along the virtual bonds and combine
	its legs.
ExactDiag.sparse_diag(k, *args, **kwargs)	Call speigs().

Bases: object

(Full) exact diagonalization of the Hamiltonian.

## **Parameters**

- model (MPOmodel | CouplingModel) The model which is to be diagonalized.
- charge\_sector (None | charges) If not None, project onto the given charge sector.
- **sparse** (bool) If True, don't sort/bunch the LegPipe used to combine the physical legs. This results in array *blocks* with just one entry, requires much more charge data, and is not what *np\_conserved* was designed for, so it's not recommended.
- max\_size (int) The build\_H\_\* functions will do nothing (but emit a warning) if the total size of the Hamiltonian would be larger than this.

# model

The model which is to be diagonalized.

Type MPOmodel|CouplingModel

## chinfo

The nature of the charge (which is the same for all sites).

```
Type ChargeInfo
charge_sector
     If not None, we project onto the given charge sector.
         Type None I charges
max size
     The build_H_* functions will do nothing (but emit a warning) if the total size of the Hamiltonian would
     be larger than this.
         Type int
full_H
     The full Hamiltonian to be diagonalized with legs '(p0.p1...)', '(p0*,p1*...)' (in that or-
     der). None if the build_H_* functions haven't been called yet, or if max_size would have been ex-
     ceeded.
         Type Array | None
E
     1D array of eigenvalues.
         Type ndarray | None
v
     Eigenvectors. First leg 'ps' are physical legs, the second leg 'ps*' corresponds to the eigenvalues.
         Type Array | None
sites
     The sites in the given order.
         Type list of Site
labels p
     The labels use for the physical legs; just ['p0', 'p1', ..., 'p{L-1}'].
         Type list or str
_labels_pconj
     Just each of \_labels\_p with an *.
         Type list or str
_pipe
     The pipe from the single physical legs to the full combined leg.
         Type LegPipe
_pipe_conj
     Just _pipe.conj().
         Type LegPipe
_{\mathtt{mask}}
     Bool mask, which of the indices of the pipe are in the desired charge_sector.
         Type 1D bool ndarray | None
classmethod from_H_mpo (H_MPO, *args, **kwargs)
     Wrapper taking directly an MPO instead of a Model.
         Parameters
             • H_MPO (MPO) – The MPO representing the Hamiltonian.
```

```
• *args – Further keyword arguments as for the ___init___ of the class.
             • **kwargs – Further keyword arguments as for the ___init___ of the class.
build_full_H_from_mpo()
     Calculate self.full_H from self.mpo.
build full H from bonds()
     Calculate self.full_H from self.mpo.
full_diagonalization(*args, **kwargs)
     Full diagonalization to obtain all eigenvalues and eigenvectors.
     Arguments are given to eigh.
groundstate (charge_sector=None)
     Pick the ground state energy and ground state from self. V.
         Parameters charge_sector (None | 1D ndarray) - By default (None), consider all
             charge sectors. Alternatively, give the qtotal which the returned state should have.
         Returns
             • E0 (float) – Ground state energy (possibly in the given sector).
             • psi0 (Array) – Ground state (possibly in the given sector).
exp_H(dt)
     Return U(dt) := \exp(-i H dt).
mps_to_full (mps)
     Contract an MPS along the virtual bonds and combine its legs.
         Parameters mps (MPS) – The MPS to be contracted.
         Returns psi – The MPO contracted along the virtual bonds.
         Return type Array
full_to_mps (psi, canonical_form='B')
     Convert a full state (with a single leg) to an MPS.
         Parameters
             • psi (Array) – The state (with a single leg) which should be splitted into an MPS.
             • canonical_from (Array) - The form in which the MPS will be afterwards.
         Returns mps – An normalized MPS representation in canonical form.
         Return type MPS
matvec (psi)
     Allow to use self as LinearOperator for lanczos.
     Just applies full_H to (the first axis of) the given psi.
sparse_diag(k, *args, **kwargs)
     Call speigs ().
```

# **Module description**

Full diagonalization (ED) of the Hamiltonian.

The full diagonalization of a small system is a simple approach to test other algorithms. In case you need the full spectrum, a full diagonalization is often the only way. This module provides functionality to quickly diagonalize the Hamiltonian of a given model. This might be used to obtain the spectrum, the ground state or highly excited states.

**Note:** Good use of symmetries is crucial to increase the treatable system size. While we can simply use the defined *LegCharge* of a model, we don't make use of any other symmetries like translation symmetry, SU(2) symmetry or inversion symmetries. In other words, this code does not aim to provide state-of-the-art exact diagonalization, but just the ability to diagonalize the defined models for small system sizes without addional extra work.

# 7.8 linalg

full name: tenpy.linalgparent module: tenpy

• type: module

# **Module description**

Linear-algebra tools for tensor networks.

Most notably is the module  $np\_conserved$ , which contains everything needed to make use of charge conservervation in the context of tensor networks.

Relevant contents of *charges* are imported to *np\_conserved*, so you propably won't need to import *charges* directly.

## **Submodules**

works.  charges Basic definitions of a charge.  svd_robust (More) robust version of singular value  random_matrix Provide some random matrix ensembles  sparse Providing support for sparse algorithms vector products only).	n in tensor net-
svd_robust(More) robust version of singular valuerandom_matrixProvide some random matrix ensemblessparseProviding support for sparse algorithms	
random_matrixProvide some random matrix ensemblessparseProviding support for sparse algorithms	
sparse Providing support for sparse algorithms	decomposition.
-	for numpy.
vector products only).	(using matrix-
r	
1anczos Lanczos algorithm for np_conserved ara	ays.

# 7.8.1 np\_conserved

full name: tenpy.linalg.np\_conservedparent module: tenpy.linalg

• type: module

# **Classes**

Array

Array(legcharges[, dtype, qtotal, labels])	A multidimensional array (=tensor) for using charge
	conservation.

# **Array**

• full name: tenpy.linalg.np\_conserved.Array

• parent module: tenpy.linalg.np\_conserved

• type: class

# **Inheritance Diagram**

Array

# Methods

Arrayinit(legcharges[, dtype, qtotal,])	see help(self)
Array.add_charge(add_legs[, chinfo, qtotal])	Add charges.
Array.add_leg(leg, i[, axis, label])	Add a leg to <i>self</i> , setting the current array as slice for a
	given index.
Array.add_trivial_leg([axis, label, qconj])	Add a trivial leg (with just one entry) to self.
	continues on next page

Table 44 – continued from previous page

Table 44 – continued from previous page	
Array.as_completely_blocked()	Gives a version of self which is completely blocked by
	charges.
Array.astype(dtype[, copy])	Return copy with new dtype, upcasting all blocks in
	_data.
Array.binary_blockwise(func, other, *args,	Roughly return func(self, other), block-
)	wise.
Array.change_charge(charge, new_qmod[,])	Change the <i>qmod</i> of one charge in <i>chinfo</i> .
Array.combine_legs(combine_legs[, new_axes,	Reshape: combine multiple legs into multiple pipes.
])	
Array.complex_conj()	Return copy which is complex conjugated without con-
mray . compren_comp	jugating the charge data.
Array.conj([complex_conj, inplace])	Conjugate: complex conjugate data, conjugate charge
Array: conj([complex_conj, inplace])	data.
A many gany ([doon])	
Array.copy([deep])	Return a (deep or shallow) copy of self.
Array.drop_charge([charge, chinfo])	Drop (one of) the charges.
Array.extend(axis, extra)	Increase the dimension of a given axis, filling the values
	with zeros.
Array.from_func(func, legcharges[, dtype,])	Create an Array from a numpy func.
Array.from_func_square(func, leg[, dtype,	Create an Array from a (numpy) function.
])	
Array.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Array.from_ndarray(data_flat, legcharges[,])	convert a flat (numpy) ndarray to an Array.
Array.from_ndarray_trivial(data_flat[,])	convert a flat numpy ndarray to an Array with trivial
	charge conservation.
Array.gauge_total_charge(axis[, newqtotal,	Changes the total charge by adjusting the charge on a
])	certain leg.
Array.get_block(qindices[, insert])	Return the ndarray in _data representing the block
	corresponding to <i>qindices</i> .
Array.get_leg(label)	Return self.legs[self.
-1-5( )	<pre>get_leg_index(label)].</pre>
Array.get_leg_index(label)	translate a leg-index or leg-label to a leg-index.
Array.get_leg_indices(labels)	Translate a list of leg-indices or leg-labels to leg indices.
Array.get_leg_labels()	Return list of the leg labels, with <i>None</i> for anonymous
nriay.get_reg_rabers()	legs.
Array.has_label(label)	Check whether a given label exists.
Array.iadd_prefactor_other(prefactor,	self += prefactor * other for scalar prefac-
other)	tor and Array other.
· · · · · · · · · · · · · · · · · · ·	
Array.ibinary_blockwise(func, other, *args,	Roughly self = func(self, other), block-
)	wise; in place.
Array.iconj([complex_conj])	Wraper around self.conj() with
7 7 7 7 7 7 11 1 1 1 1 1 1	inplace=True.
Array.idrop_labels([old_labels])	Remove leg labels from self; in place.
Array.iproject(mask, axes)	Applying masks to one or multiple axes; in place.
Array.ipurge_zeros([cutoff, norm_order])	Removes selfdata blocks with <i>norm</i> less than
	cutoff; in place.
Array.ireplace_label(old_label, new_label)	Replace the leg label <i>old_label</i> with <i>new_label</i> ; in place.
Array.ireplace_labels(old_labels,	Replace leg label old_labels[i] with
new_labels)	new_labels[i]; in place.
Array.is_completely_blocked()	Return bool whether all legs are blocked by charge.
	Scale with varying values along an axis; in place.
Array.iscale_axis(\$ , axis )	Scale with varying values along all axis, in place.
Array.iscale_axis(s[, axis])  Array.iscale prefactor(prefactor)	
Array.iscale_axis(s[, axis]) Array.iscale_prefactor(prefactor)	self *= prefactor for scalar <i>prefactor</i> .  continues on next page

Table 44 – continued from previous page

Array.isort_qdata()(Lexiographically) sort selfqdata; in place.Array.iswapaxes(axis1, axis2)Similar as np.swapaxes; in place.Array.itranspose([axes])Transpose axes like np.transpose; in place.Array.iunary_blockwise(func, *args, Roughly self = f(self), block-wise; in place.**kwargs)Array.make_pipe(axes, **kwargs)Generates a LegPipe for specified axes.
Array.itranspose([axes])  Array.iunary_blockwise(func, *args, Roughly self = f(self), block-wise; in place.  **kwargs)
Array.iunary_blockwise(func, *args, Roughly self = f(self), block-wise; in place. **kwargs)
**kwargs)
Array.make pipe(axes. **kwargs) Generates a LegPipe for specified axes.
Array.matvec(other)  This function is used by the Lanczos algorithm needed for DMRG.
Array.norm([ord, convert_to_float]) Norm of flattened data.
Array.permute(perm, axis) Apply a permutation in the indices of an axis.
Array.replace_label(old_label, new_label) Return a shallow copy with the leg label old_label re-
placed by new_label.
Array.replace_labels(old_labels, new_labels) Return a shallow copy with old_labels[i] re-
placed by new_labels[i].
Array.save_hdf5(hdf5_saver, h5gr, subpath) Export self into a HDF5 file.
Array.scale_axis(s[, axis]) Same as iscale_axis(), but return a (deep) copy.
Array.sort_legcharge([sort, bunch]) Return a copy with one or all legs sorted by charges.
Array.sparse_stats() Returns a string detailing the sparse statistics.
Array.split_legs([axes, cutoff]) Reshape: opposite of combine_legs: split (some) legs
which are LegPipes.
Array.squeeze([axes]) Like np.squeeze.
Array.take_slice(indices, axes) Return a copy of self fixing indices along one or multi-
ple axes.
Array.test_sanity() Sanity check.
Array.to_ndarray() Convert self to a dense numpy ndarray.
Array.transpose([axes]) Like itranspose(), but on a deep copy.
Array.unary_blockwise(func, *args, **kwargs) Roughly return func(self), block-wise.
Array.zeros_like() Return a copy of self with only zeros as entries, contain-
ing no _data.

## **Class Attributes and Properties**

Array.labels	
Array.ndim	Alias for rank or len(self.shape).
Array.size	The number of dtype-objects stored.
Array.stored_blocks	The number of (non-zero) blocks stored in _data.

class tenpy.linalg.np\_conserved.Array(legcharges, dtype=<class 'numpy.float64'>, qtotal=None, labels=None)

Bases: object

A multidimensional array (=tensor) for using charge conservation.

An *Array* represents a multi-dimensional tensor, together with the charge structure of its legs (for abelian charges). Further information can be found in *Charge conservation with np\_conserved*.

The default \_\_init\_\_() (i.e. Array(...)) does not insert any data, and thus yields an Array 'full' of zeros, equivalent to zeros(). Further, new arrays can be created with one of from\_ndarray\_trivial(), from\_ndarray(), or from\_func(), and of course by copying/tensordot/svd etc.

In-place methods are indicated by a name starting with i. (But is\_completely\_blocked is not inplace...)

#### **Parameters**

- **legcharges** (list of *LegCharge*) The leg charges for each of the legs. The ChargeInfo is read out from it.
- **dtype** (type or string) The data type of the array entries. Defaults to np.float64.
- **qtotal** (1D array of QTYPE) The total charge of the array. Defaults to 0.
- labels (list of {str | None}) Labels associated to each leg, None for non-named labels.

#### rank

The rank or "number of dimensions", equivalent to len (shape).

```
Type int
```

## shape

The number of indices for each of the legs.

```
Type tuple(int)
```

## dtype

The data type of the entries.

Type np.dtype

#### chinfo

The nature of the charge.

```
Type ChargeInfo
```

## qtotal

The total charge of the tensor.

```
Type 1D array
```

# legs

The leg charges for each of the legs.

```
Type list of LegCharge
```

#### \_labels

Labels for the different legs, None for non-labeled legs.

```
Type list of { str | None }
```

# \_data

The actual entries of the tensor.

**Type** list of arrays

#### \_qdata

For each of the \_data entries the qindices of the different legs.

Type 2D array (len(\_data), rank), dtype np.intp

### \_qdata\_sorted

Whether self.\_qdata is lexsorted. Defaults to *True*, but *must* be set to *False* by algorithms changing \_qdata.

Type Bool

# test\_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

```
copy (deep=True)
```

Return a (deep or shallow) copy of self.

Both deep and shallow copies will share chinfo and the LegCharges in legs.

In contrast to a deep copy, the shallow copy will also share the tensor entries, namely the *same* instances of \_qdata and \_data and labels (and other 'immutable' properties like the shape or dtype).

**Note:** Shallow copies are *not* recommended unless you know the consequences! See the following examples illustrating some of the pitfalls.

#### **Examples**

Be (very!) careful when making non-deep copies: In the following example, the original a is changed if and only if the corresponding block existed in a before. >>> b = a.copy(deep=False) # shallow copy >>> <math>b[1, 2] = 4.

Other *inplace* operations might have no effect at all (although we don't guarantee that):

```
>>> a *= 2 # has no effect on `b`
>>> b.iconj() # nor does this change `a`
```

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

Specifically, it saves <code>chinfo</code>, <code>legs</code>, <code>dtype</code> under these names, <code>qtotal</code> as "total\_charge", <code>\_data</code> as "blocks", <code>\_qdata</code> as :block\_inds", the labels in the list-form (as returned by <code>get\_leg\_labels()</code>). Moreover, it saves <code>rank</code>, <code>shape</code> and <code>\_qdata\_sorted</code> (under the name "block\_inds\_sorted") as HDF5 attributes.

## **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

classmethod from\_ndarray\_trivial (data\_flat, dtype=None, labels=None) convert a flat numpy ndarray to an Array with trivial charge conservation.

## **Parameters**

- **data\_flat** (*array\_like*) The data to be converted to a Array.
- **dtype** (np.dtype) The data type of the array entries. Defaults to dtype of *data\_flat*.
- labels (list of {str | None}) Labels associated to each leg, None for nonnamed labels.

**Returns** res – An Array with data of data\_flat.

**Return type** Array

classmethod from\_ndarray (data\_flat, legcharges, dtype=None, qtotal=None, cutoff=None, labels=None)
convert a flat (numpy) ndarray to an Array.

#### **Parameters**

- data\_flat (array\_like) The flat ndarray which should be converted to a npc *Array*. The shape has to be compatible with legcharges.
- **legcharges** (list of LegCharge) The leg charges for each of the legs. The ChargeInfo is read out from it.
- **dtype** (np.dtype) The data type of the array entries. Defaults to dtype of *data\_flat*.
- **qtotal** (None | charges) The total charge of the new array.
- **cutoff** (*float*) Blocks with np.max(np.abs(block)) > cutoff are considered as zero. Defaults to *QCUTOFF*.
- labels (list of {str / None}) Labels associated to each leg, None for nonnamed labels.

**Returns** res – An Array with data of *data\_flat*.

Return type Array

See also:

detect\_qtotal() used to detect qtotal if not given.

This function creates an array and fills the blocks *compatible* with the charges using *func*, where *func* is a function returning a *array\_like* when given a shape, e.g. one of np.ones or np.random. standard\_normal.

#### **Parameters**

- func (callable) A function-like object which is called to generate the data blocks. We expect that func returns a flat array of the given shape convertible to dtype. If no shape\_kw is given, it is called as func(shape, \*func\_args, \*\*func\_kwargs), otherwise as func(\*func\_args, `shape\_kw`=shape, \*\*func\_kwargs). shape is a tuple of int.
- **legcharges** (list of LegCharge) The leg charges for each of the legs. The ChargeInfo is read out from it.
- **dtype** (*None* / type / string) The data type of the output entries. Defaults to np.float64. Defaults to *None*: obtain it from the return value of the function. Note that this argument is not given to func, but rather a type conversion is performed afterwards. You might want to set a *dtype* in *func\_kwargs* as well.

- qtotal (None / charges) The total charge of the new array. Defaults to charge 0.
- **func\_args** (*iterable*) Additional arguments given to *func*.
- func\_kwargs (dict) Additional keyword arguments given to func.
- **shape\_kw** (*None | str*) If given, the keyword with which shape is given to *func*.
- labels (list of {str | None}) Labels associated to each leg, None for nonnamed labels.

**Returns** res – An Array with blocks filled using *func*.

Return type Array

Create an Array from a (numpy) function.

This function creates an array and fills the blocks *compatible* with the charges using *func*, where *func* is a function returning a *array\_like* when given a shape, e.g. one of np.ones or np.random.standard\_normal or the functions defined in *random\_matrix*.

#### **Parameters**

- **func** (callable) A function-like object which is called to generate the data blocks. We expect that *func* returns a flat array of the given *shape* convertible to *dtype*. If no *shape\_kw* is given, it is called like func (shape, \*fargs, \*\*fkwargs), otherwise as func (\*fargs, `shape\_kw`=shape, \*\*fkwargs). *shape* is a tuple of int.
- leg (LegCharge) The leg charges for the first leg; the second leg is set to leg. conj(). The ChargeInfo is read out from it.
- **dtype** (*None* / *type* / *string*) The data type of the output entries. Defaults to *None*: obtain it from the return value of the function. Note that this argument is not given to func, but rather a type conversion is performed afterwards. You might want to set a *dtype* in *func\_kwargs* as well.
- func\_args (iterable) Additional arguments given to func.
- func\_kwargs (dict) Additional keyword arguments given to func.
- **shape\_kw** (None | str) If given, the keyword with which shape is given to func.
- labels (list of {str | None}) Labels associated to each leg, None for nonnamed labels.

**Returns** res – An Array with blocks filled using *func*.

Return type Array

#### zeros like()

Return a copy of self with only zeros as entries, containing no data.

#### property size

The number of dtype-objects stored.

# property stored\_blocks

The number of (non-zero) blocks stored in \_data.

#### property ndim

Alias for rank or len (self.shape).

## get\_leg\_index(label)

translate a leg-index or leg-label to a leg-index.

```
Parameters label (int | string) - The leg-index directly or a label (string) set before.
         Returns leg_index – The index of the label.
         Return type int
     See also:
     get_leg_indices() calls get_leg_index for a list of labels.
     iset_leg_labels() set the labels of different legs.
get_leg_indices (labels)
     Translate a list of leg-indices or leg-labels to leg indices.
         Parameters labels (iterable of string/int) - The leg-labels (or directly indices)
             to be translated in leg-indices.
         Returns leg_indices – The translated labels.
         Return type list of int
     See also:
     get_leg_index() used to translate each of the single entries.
     iset_leg_labels() set the labels of different legs.
iset_leg_labels(labels)
     Set labels for the different axes/legs; in place.
     Introduction to leg labeling can be found in Charge conservation with np_conserved.
         Parameters labels (iterable (strings | None), len=self.rank) - One la-
             bel for each of the legs. An entry can be None for an anonymous leg.
     See also:
     get_leg() translate the labels to indices.
get leg labels()
    Return list of the leg labels, with None for anonymous legs.
has label (label)
     Check whether a given label exists.
get_leg(label)
     Return self.legs[self.get_leg_index(label)].
     Convenient function returning the leg corresponding to a leg label/index.
ireplace_label (old_label, new_label)
     Replace the leg label old_label with new_label; in place.
replace_label (old_label, new_label)
     Return a shallow copy with the leg label old_label replaced by new_label.
ireplace_labels (old_labels, new_labels)
     Replace leg label old_labels[i] with new_labels[i]; in place.
replace_labels (old_labels, new_labels)
     Return a shallow copy with old_labels[i] replaced by new_labels[i].
```

```
idrop labels(old labels=None)
```

Remove leg labels from self; in place.

**Parameters old\_labels** (list of str/int) – The leg labels/indices for which the label should be removed. By default (None), remove all labels.

#### sparse\_stats()

Returns a string detailing the sparse statistics.

#### to ndarray()

Convert self to a dense numpy ndarray.

```
get_block (qindices, insert=False)
```

Return the ndarray in \_data representing the block corresponding to *qindices*.

#### **Parameters**

- qindices (1D array of np.intp) The qindices, for which we need to look in \_qdata.
- insert (bool) If True, insert a new (zero) block, if *qindices* is not existent in self. \_data. Otherwise just return None.

**Returns** block – The block in \_data corresponding to qindices. If *insert*`=*False and there is not block with qindices, return* ``*None*`.

Return type ndarray | None

**Raises** IndexError – If *qindices* are incompatible with charge and *raise\_incomp\_q*.

#### take slice (indices, axes)

Return a copy of self fixing indices along one or multiple axes.

```
For a rank-4 Array A.take_slice([i, j], [1,2]) is equivalent to A[:, i, j, :].
```

## **Parameters**

- indices ((iterable of) int) The (flat) index for each of the legs specified by axes.
- axes ((iterable of) str/int) Leg labels or indices to specify the legs for which the indices are given.

**Returns** sliced\_self – A copy of self, equivalent to taking slices with indices inserted in axes.

```
Return type Array
```

#### See also:

add leg() opposite action of inserting a new leg.

## add\_trivial\_leg (axis=0, label=None, qconj=1)

Add a trivial leg (with just one entry) to self.

#### **Parameters**

- **axis** (*int*) The new leg is inserted before index *axis*.
- label (str | None) If not None, use it as label for the new leg.
- qconj (+1 / -1) The direction of the new leg.

**Returns extended** – A (possibly) *shallow* copy of self with an additional leg of ind\_len 1 and charge 0.

Return type Array

```
add leg(leg, i, axis=0, label=None)
```

Add a leg to *self*, setting the current array as slice for a given index.

#### **Parameters**

- leg (LegCharge) The charge data of the leg to be added.
- i (int) Index within the leg for which the data of *self* should be set.
- **axis** (axis) The new leg is inserted before this current axis.
- label (str | None) If not None, use it as label for the new leg.

**Returns extended** — A copy of self with the new *leg* at axis *axis*, such that extended. take\_slice(i, axis) returns a copy of *self*.

```
Return type Array
```

See also:

take\_slice() opposite action reducing the number of legs.

```
extend (axis, extra)
```

Increase the dimension of a given axis, filling the values with zeros.

#### **Parameters**

- axis (int / str) The axis (or axis-label) to be extended.
- **extra** (LegCharge | int) By what to extend, i.e. the charges to be appended to the leg of *axis*. An int stands for extending the length of the array by a single new block of that size with zero charges.

**Returns extended** – A copy of self with the specified axis increased.

```
Return type Array
```

```
gauge_total_charge (axis, newqtotal=None, new_qconj=None)
```

Changes the total charge by adjusting the charge on a certain leg.

The total charge is given by finding a nonzero entry [i1, i2, ...] and calculating:

Thus, the total charge can be changed by redefining (= shifting) the LegCharge of a single given leg. This is exactly what this function does.

#### **Parameters**

- axis (int or string) The new leg (index or label), for which the charge is changed.
- newqtotal (charge values, defaults to 0) The new total charge.
- new\_qconj ({+1, -1, None}) Whether the new LegCharge points inward (+1) or outward (-1) afterwards. By default (None) use the previous self.legs[leg]. qconj.

**Returns copy** - A shallow copy of self with copy.qtotal == newqtotal and new copy.legs[leg]. The new leg will be a :class`LegCharge`, even if the old leg was a LegPipe.

Return type Array

add\_charge (add\_legs, chinfo=None, qtotal=None)
 Add charges.

#### **Parameters**

- add\_legs (iterable of LegCharge) One *LegCharge* for each axis of *self*, to be added to the one in *legs*.
- chargeinfo (ChargeInfo) The ChargeInfo for all charges; create new if None.
- **qtotal** (None / charges) The total charge with respect to add\_legs. If None, derive it from non-zero entries of self.

**Returns charges\_added** – A copy of *self*, where the LegCharges *add\_legs* where added to *self.legs*. Note that the LegCharges are neither bunched or sorted; you might want to use <code>sort\_legcharge()</code>.

Return type Array

drop\_charge (charge=None, chinfo=None)

Drop (one of) the charges.

#### **Parameters**

- **charge** (int / str) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.
- **chinfo** (ChargeInfo) The ChargeInfo with *charge* dropped; create a new one if None.

**Returns dropped** – A copy of *self*, where the specified *charge* has been removed. Note that the LegCharges are neither bunched or sorted; you might want to use *sort\_legcharge()*.

Return type Array

change\_charge (charge, new\_qmod, new\_name=", chinfo=None)

Change the *qmod* of one charge in *chinfo*.

#### **Parameters**

- **charge** (int / str) Number or *name* of the charge (within *chinfo*) which is to be changed. None means dropping all charges.
- **new\_qmod** (*int*) The new *qmod* to be set.
- **new\_name** (str) The new name of the charge.
- chinfo (ChargeInfo) The ChargeInfo with qmod of charge changed; create a new one if None.

**Returns changed** – A copy of *self*, where the *qmod* of the specified *charge* has been changed. Note that the LegCharges are neither bunched or sorted; you might want to use <code>sort\_legcharge()</code>.

Return type Array

### is\_completely\_blocked()

Return bool whether all legs are blocked by charge.

sort\_legcharge (sort=True, bunch=True)

Return a copy with one or all legs sorted by charges.

Sort/bunch one or multiple of the LegCharges. Legs which are sorted *and* bunched are guaranteed to be blocked by charge.

#### **Parameters**

- **sort** (*True* | *False* | *list* of {*True*, *False*, *perm*}) A single bool holds for all legs, default=True. Else, *sort* should contain one entry for each leg, with a bool for sort/don't sort, or a 1D array perm for a given permuation to apply to a leg.
- bunch (True | False | list of {True, False}) A single bool holds for all legs, default=True. Whether or not to bunch at each leg, i.e. combine contiguous blocks with equal charges.

#### Returns

- **perm** (*tuple of 1D arrays*) The permutation applied to each of the legs, such that cp. to\_ndarray() = self.to\_ndarray() [np.ix\_(\*perm)].
- **result** (*Array*) A shallow copy of self, with legs sorted/bunched.

## isort\_qdata()

(Lexiographically) sort self.\_qdata; in place.

Lexsort self.\_qdata and self.\_data and set self.\_qdata\_sorted = True.

## make\_pipe (axes, \*\*kwargs)

Generates a LegPipe for specified axes.

#### **Parameters**

- axes (iterable of str/int) The leg labels for the axes which should be combined. Order matters!
- \*\*kwargs Additional keyword arguments given to LegPipe.

**Returns** pipe – A pipe of the legs specified by axes.

Return type LegPipe

combine\_legs (combine\_legs, new\_axes=None, pipes=None, qconj=None)

Reshape: combine multiple legs into multiple pipes. If necessary, transpose before.

#### **Parameters**

- **combine\_legs** ((iterable of) iterable of {str/int}) Bundles of leg indices or labels, which should be combined into a new output pipes. If multiple pipes should be created, use a list fore each new pipe.
- new\_axes (None | (iterable of) int) The leg-indices, at which the combined legs should appear in the resulting array. Default: for each pipe the position of its first pipe in the original array, (taking into account that some axes are 'removed' by combining). Thus no transposition is performed if combine\_legs contains only contiguous ranges.
- pipes (None | (iterable of) {LegPipes | None}) Optional: provide one or multiple of the resulting LegPipes to avoid overhead of computing new leg pipes for the same legs multiple times. The LegPipes are conjugated, if that is necessary for compatibility with the legs.
- **qconj**  $((iterable \ of) \ \{+1, -1\})$  Specify whether new created pipes point inward or outward. Defaults to +1. Ignored for given *pipes*, which are not newly calculated.

**Returns reshaped** – A copy of self, whith some legs combined into pipes as specified by the arguments.

Return type Array

See also:

split\_legs () inverse reshaping splitting LegPipes.

#### **Notes**

Labels are inherited from self. New pipe labels are generated as '(' + '.'.join(\*leglabels) + ')'. For these new labels, previously unlabeled legs are replaced by '?#', where # is the leg-index in the original tensor *self*.

#### **Examples**

```
>>> oldarray.iset_leg_labels(['a', 'b', 'c', 'd', 'e'])
>>> c1 = oldarray.combine_legs([1, 2], qconj=-1) # only single output pipe
>>> c1.get_leg_labels()
['a', '(b.c)', 'd', 'e']
```

Indices of *combine\_legs* refer to the original array. If transposing is necessary, it is performed automatically:

#### **split** legs (axes=None, cutoff=0.0)

Reshape: opposite of combine\_legs: split (some) legs which are LegPipes.

Reverts <code>combine\_legs()</code> (except a possibly performed <code>transpose</code>). The splited legs are replacing the LegPipes at their position, see the examples below. Labels are split reverting what was done in <code>combine\_legs()</code>. '?#' labels are replaced with <code>None</code>.

#### **Parameters**

- axes ((iterable of) int/str) Leg labels or indices determining the axes to split. The corresponding entries in self.legs must be LegPipe instances. Defaults to all legs, which are LegPipe instances.
- **cutoff** (*float*) Splitted data blocks with np.max(np.abs(block)) > cutoff are considered as zero. Defaults to 0.

**Returns** reshaped – A copy of self where the specified legs are splitted.

```
Return type Array
```

## See also:

combine\_legs() this is reversed by split\_legs.

#### **Examples**

Given a rank-5 Array *old\_array*, you can combine it and split it again:

```
>>> old_array.iset_leg_labels(['a', 'b', 'c', 'd', 'e'])
>>> comb_array = old_array.combine_legs([[0, 3], [2, 4]] )
>>> comb_array.get_leg_labels()
['(a.d)', 'b', '(c.e)']
>>> split_array = comb_array.split_legs([0, 2])
>>> split_array.get_leg_labels()
['a', 'd', 'b', 'c', 'e']
```

### as\_completely\_blocked()

Gives a version of self which is completely blocked by charges.

Functions like svd() or eigh() require a complete blocking by charges. This can be achieved by encapsulating each leg which is not completely blocked into a LegPipe (containing only that single leg). The LegPipe will then contain all necessary information to revert the blocking.

#### **Returns**

- encapsulated\_axes (list of int) The leg indices which have been encapsulated into Pipes.
- blocked\_self(Array) Self(iflen(encapsulated\_axes) = 0) or a copy of self, which is completely blocked.

# squeeze(axes=None)

Like np. squeeze.

If a squeezed leg has non-zero charge, this charge is added to qtotal.

**Parameters axes** (None | (iterable of) {int|str}) - Labels or indices of the legs which should be 'squeezed', i.e. the legs removed. The corresponding legs must be trivial, i.e., have ind\_len 1.

**Returns** squeezed – A scalar of self.dtype, if all axes were squeezed. Else a copy of self with reduced rank as specified by *axes*.

**Return type** :class:Array | scalar

```
astype (dtype, copy=True)
```

Return copy with new dtype, upcasting all blocks in \_data.

# **Parameters**

- **dtype** (convertible to a np.dtype) The new data type. If None, deduce the new dtype as common type of self.\_data.
- copy (bool) Whether to make a copy of the blocks even if the type didn't change.

**Returns** copy – Deep copy of self with new dtype.

Return type Array

```
ipurge_zeros (cutoff=2.220446049250313e-15, norm_order=None)
```

Removes self.\_data blocks with *norm* less than cutoff; in place.

#### **Parameters**

- **cutoff** (float) Blocks with norm <= cutoff are removed. defaults to QCUTOFF.
- norm\_order A valid *ord* argument for *np.linalg.norm*. Default None gives the Frobenius norm/2-norm for matrices/everything else. Note that this differs from other methods, e.g. *from\_ndarray()*, which use the maximum norm.

### iproject (mask, axes)

Applying masks to one or multiple axes; in place.

This function is similar as *np.compress* with boolean arrays For each specified axis, a boolean 1D array *mask* can be given, which chooses the indices to keep.

**Warning:** Although it is possible to use an 1D int array as a mask, the order is ignored! If you need to permute an axis, use permute () or sort\_legcharge().

#### **Parameters**

- mask ((list of) 1D array (bool/int)) For each axis specified by axes a mask, which indices of the axes should be kept. If mask is a bool array, keep the indices where mask is True. If mask is an int array, keep the indices listed in the mask, ignoring the order or multiplicity.
- axes ((list of) int | string) The i'th entry in this list specifies the axis for the 'i'th entry of 'mask, either as an int, or with a leg label. If axes is just a single int/string, specify just a single mask.

### Returns

- map\_qind (list of 1D arrays) The mapping of qindices for each of the specified axes.
- block\_masks (list of lists of 1D bool arrays) block\_masks[a] [qind] is a boolen mask which indices to keep in block qindex of axes[a].

#### permute (perm, axis)

Apply a permutation in the indices of an axis.

Similar as np.take with a 1D array. Roughly equivalent to res[:, ...] = self[perm, ...] for the corresponding *axis*. Note: This function is quite slow, and usually not needed!

## **Parameters**

- **perm** (array\_like 1D int) The permutation which should be applied to the leg given by axis.
- axis (str / int) A leg label or index specifying on which leg to take the permutation.

**Returns res** – A copy of self with leg *axis* permuted, such that res[i, ...] = self[perm[i], ...] for i along *axis*.

Return type Array

### See also:

sort\_legcharge () can also be used to perform a general permutation. Preferable, since it is faster for permutations which don't mix charge blocks.

## itranspose (axes=None)

Transpose axes like np.transpose; in place.

**Parameters axes** (iterable (intlstring), len rank | None) – The new order of the axes. By default (None), reverse axes.

## transpose (axes=None)

Like *itranspose* (), but on a deep copy.

# iswapaxes (axis1, axis2)

Similar as np. swapaxes; in place.

```
iscale axis(s, axis=-1)
```

Scale with varying values along an axis; in place.

```
Rescale to new_self[i1, ..., i_axis, ...] = s[i_axis] * self[i1, ..., i_axis, ...]
```

### **Parameters**

- **s** (1D array, len=self.shape[axis]) The vector with which the axis should be scaled.
- axis (str/int) The leg label or index for the axis which should be scaled.

#### See also:

iproject () can be used to discard indices for which s is zero.

```
scale_axis(s, axis=-1)
```

Same as iscale\_axis(), but return a (deep) copy.

```
iunary blockwise(func, *args, **kwargs)
```

Roughly self = f(self), block-wise; in place.

Applies an unary function func to the non-zero blocks in self.\_data.

**Note:** Assumes implicitly that func (np.zeros(...), \*args, \*\*kwargs) gives 0, since we don't let *func* act on zero blocks!

#### **Parameters**

- func (function) A function acting on flat arrays, returning flat arrays. It is called like new\_block = func(block, \*args, \*\*kwargs).
- \*args Additional arguments given to function *after* the block.
- \*\*kwargs Keyword arguments given to the function.

## **Examples**

```
unary_blockwise (func, *args, **kwargs)
```

Roughly return func(self), block-wise. Copies.

Same as <code>iunary\_blockwise()</code>, but makes a **shallow** copy first.

## iconj(complex\_conj=True)

Wraper around self.conj() with inplace=True.

# conj (complex\_conj=True, inplace=False)

Conjugate: complex conjugate data, conjugate charge data.

Conjugate all legs, set negative qtotal.

Labeling: takes 'a' -> 'a\*', 'a\*'-> 'a' and '(a,(b\*,c))' -> '(a\*,(b,c\*))'

#### **Parameters**

- complex\_conj (bool) Whether the data should be complex conjugated.
- **inplace** (bool) Whether to apply changes to *self*, or to return a *deep* copy.

## complex\_conj()

Return copy which is complex conjugated without conjugating the charge data.

```
norm (ord=None, convert to float=True)
```

Norm of flattened data.

See norm() for details.

#### ibinary\_blockwise (func, other, \*args, \*\*kwargs)

Roughly self = func(self, other), block-wise; in place.

Applies a binary function 'block-wise' to the non-zero blocks of self.\_data and other.\_data, storing result in place. Assumes that *other* is an *Array* as well, with the same shape and compatible legs. If leg labels of *other* and *self* are same up to permutations, *other* gets transposed accordingly before the action.

**Note:** Assumes implicitly that func(np.zeros(...), np.zeros(...), \*args, \*\*kwargs) gives 0, since we don't let *func* act on zero blocks!

#### **Parameters**

- **func** (function) Binary function, called as new\_block = func(block\_self, block\_other, \*args, \*\*kwargs) for blocks (=Numpy arrays) of equal shape.
- other (Array) Other Array from which to take blocks. Should have the same leg structure as self.
- \*args Extra arguments given to *func*.
- \*\*kwargs Extra keyword arguments given to func.

#### **Examples**

```
>>> a.ibinary_blockwise(np.add, b) # equivalent to ``a += b``, if ``b`` is_

an `Array`.
>>> a.ibinary_blockwise(np.max, b) # overwrites ``a`` to ``a = max(a, b)``
```

## binary\_blockwise (func, other, \*args, \*\*kwargs)

Roughly return func (self, other), block-wise. Copies.

Same as ibinary\_blockwise(), but makes a shallow copy first.

## matvec(other)

This function is used by the Lanczos algorithm needed for DMRG.

It is supposed to calculate the matrix - vector - product for a rank-2 matrix self and a rank-1 vector other.

## iadd\_prefactor\_other (prefactor, other)

```
self += prefactor * other for scalar prefactor and Array other.
```

Note that we allow the type of *self* to change if necessary. Moreover, if *self* and *other* have the same labels in different order, other gets **transposed** before the action.

# iscale\_prefactor(prefactor)

self \*= prefactor for scalar prefactor.

Note that we allow the type of *self* to change if necessary.

# **Functions**

concatenate(arrays[, axis, copy])	Stack arrays along a given axis, similar as np.concatenate.
detect_grid_outer_legcharge(grid,	Derive a LegCharge for a grid used for
grid_legs)	grid_outer().
detect_legcharge(flat_array, chargeinfo,)	Calculate a missing <i>LegCharge</i> by looking for nonzero
detect_regenarge(nat_array, chargenno,)	• • • •
7 (0.4 1 1 1 ( (0.1)	entries of a flat array.
detect_qtotal(flat_array, legcharges[, cutoff])	Returns the total charge (w.r.t <i>legs</i> ) of first non-zero sec-
	tor found in <i>flat_array</i> .
diag(s, leg[, dtype, labels])	Returns a square, diagonal matrix of entries s.
eig(a[, sort])	Calculate eigenvalues and eigenvectors for a non-
	hermitian matrix.
eigh(a[, UPLO, sort])	Calculate eigenvalues and eigenvectors for a hermitian
	matrix.
eigvals(a[, sort])	Calculate eigenvalues for a hermitian matrix.
eigvalsh(a[, UPLO, sort])	Calculate eigenvalues for a hermitian matrix.
expm(a)	Use scipy.linalg.expm to calculate the matrix exponen-
	tial of a square matrix.
eye_like(a[, axis, labels])	Return an identity matrix contractible with the leg axis
	of the Array a.
<pre>grid_concat(grid, axes[, copy])</pre>	Given an np.array of npc.Arrays, performs a multi-
	dimensional concatentation along 'axes'.
<pre>grid_outer(grid, grid_legs[, qtotal,])</pre>	Given an np.array of npc.Arrays, return the correspond-
	ing higher-dimensional Array.
inner(a, b[, axes, do_conj])	Contract all legs in <i>a</i> and <i>b</i> , return scalar.
norm(a[, ord, convert_to_float])	Norm of flattened data.
ones(legcharges[, dtype, qtotal, labels])	Short-hand for Array.from_func() with function
ones (regeninges), at pe, quein, mosts,	numpy.ones().
outer(a, b)	Forms the outer tensor product, equivalent to
( <b>u,</b> 0)	tensordot(a, b, axes=0).
pinv(a[, cutoff])	Compute the (Moore-Penrose) pseudo-inverse of a ma-
	trix.
qr(a[, mode, inner_labels, cutoff])	Q-R decomposition of a matrix.
speigs(a, charge_sector, k, *args, **kwargs)	Sparse eigenvalue decomposition $w$ , $v$ of square $a$ in a
sperys(a, charge_sector, k, raigs, rekwargs)	sparse eigenvalue decomposition $w$ , $\vee$ of square $u$ in a given charge sector.
svd(a[, full_matrices, compute_uv, cutoff,])	Singualar value decomposition of an Array <i>a</i> .
tensordot(a, b[, axes])	Similar as np. tensordot but for Array.
to_iterable_arrays(array_list)	Similar as to_iterable(), but also enclose npc Ar-
( [ 1	rays in a list.
trace(a[, leg1, leg2])	Trace of a, summing over leg1 and leg2.
zeros(legcharges[, dtype, qtotal, labels])	Create a npc array full of zeros (with no _data).

#### concatenate

• full name: tenpy.linalg.np\_conserved.concatenate

• parent module: tenpy.linalg.np\_conserved

• type: function

tenpy.linalg.np\_conserved.concatenate(arrays, axis=0, copy=True)

Stack arrays along a given axis, similar as np.concatenate.

Stacks the qind of the array, without sorting/blocking. Labels are inherited from the first array only.

### **Parameters**

- **arrays** (iterable of *Array*) The arrays to be stacked. They must have the same shape and charge data except on the specified axis.
- **axis** (int / str) Leg index or label of the first array. Defines the axis along which the arrays are stacked.
- **copy** (bool) Whether to copy the data blocks.

**Returns** stacked – Concatenation of the given arrays along the specified axis.

Return type Array

See also:

**Array**. sort\_legcharge() can be used to block by charges along the axis.

### detect grid outer legcharge

- full name: tenpy.linalg.np\_conserved.detect\_grid\_outer\_legcharge
- parent module: tenpy.linalg.np\_conserved
- type: function

tenpy.linalg.np\_conserved.detect\_grid\_outer\_legcharge(grid, grid\_legs, qtotal=None, qconj=1, bunch=False)

Derive a LegCharge for a grid used for grid\_outer().

Note: The resulting LegCharge is *not* bunched.

### **Parameters**

- grid (array like of {Array | None}) The grid as it will be given to grid outer().
- grid\_legs (list of {LegCharge | None}) One LegCharge for each dimension of the grid, except for one entry which is None. This missing entry is to be calculated.
- **qtotal** (*charge*) The desired total charge of the array. Defaults to 0.

**Returns** new\_grid\_legs – A copy of the given *grid\_legs* with the None replaced by a compatible LegCharge. The new LegCharge is neither bunched nor sorted!

Return type list of LegCharge

See also:

detect\_legcharge() similar functionality for a flat numpy array instead of a grid.

### detect\_legcharge

- full name: tenpy.linalg.np\_conserved.detect\_legcharge
- parent module: tenpy.linalg.np\_conserved
- type: function

tenpy.linalg.np\_conserved.detect\_legcharge (flat\_array, chargeinfo, legcharges, qto-tal=None, qconj=1, cutoff=None)

Calculate a missing *LegCharge* by looking for nonzero entries of a flat array.

#### **Parameters**

- flat\_array (ndarray) A flat array, in which we look for non-zero entries.
- **chargeinfo** (*ChargeInfo*) The nature of the charge.
- **legcharges** (list of LegCharge) One LegCharge for each dimension of flat\_array, except for one entry which is None. This missing entry is to be calculated.
- qconj  $(\{+1, -1\})$  qconj for the new calculated LegCharge.
- **qtotal** (*charges*) Desired total charge of the array. Defaults to zeros.
- **cutoff** (*float*) **Blocks** with np.max (np.abs(block)) > cutoff are considered as zero. Defaults to *OCUTOFF*.

**Returns** new\_legcharges – A copy of the given *legcharges* with the None replaced by a compatible LegCharge. The new legcharge is 'bunched', but not sorted!

Return type list of LegCharge

#### See also:

detect\_grid\_outer\_legcharge() similar functionality if the flat array is given by a 'grid'.
detect\_qtotal() detects the total charge, if all legs are known.

### detect qtotal

- full name: tenpy.linalg.np\_conserved.detect\_qtotal
- parent module: tenpy.linalg.np\_conserved
- · type: function

tenpy.linalg.np\_conserved.detect\_qtotal (flat\_array, legcharges, cutoff=None)
Returns the total charge (w.r.t legs) of first non-zero sector found in flat\_array.

#### **Parameters**

- **flat\_array** (array) The flat numpy array from which you want to detect the charges.
- **legcharges** (list of LegCharge) For each leg the LegCharge.
- **cutoff** (*float*) **Blocks** with np.max(np.abs(block)) > cutoff are considered as zero. Defaults to *QCUTOFF*.

**Returns** qtotal – The total charge fo the first non-zero (i.e. > cutoff) charge block.

Return type charge

See also:

```
detect_legcharge() detects the charges of one missing LegCharge if qtotal is known.
detect_grid_outer_legcharge() similar functionality if the flat array is given by a 'grid'.
```

### diag

- full name: tenpy.linalg.np\_conserved.diag
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.diag(s, leg, dtype=None, labels=None)
```

Returns a square, diagonal matrix of entries s.

The resulting matrix has legs (leg, leg.conj()) and charge 0.

### **Parameters**

- **s** (scalar | 1D array) The entries to put on the diagonal. If scalar, all diagonal entries are the same.
- leg (LegCharge) The first leg of the resulting matrix.
- **dtype** (*None* / *type*) The data type to be used for the result. By default, use dtype of *s*.
- labels (list of {str | None}) Labels associated to each leg, None for non-named labels.

**Returns diagonal** – A square matrix with diagonal entries s.

**Return type** *Array* 

#### See also:

```
Array.scale_axis() similar as tensordot(diag(s), ...), but faster.
```

### eig

- full name: tenpy.linalg.np\_conserved.eig
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.eig(a, sort=None)
```

Calculate eigenvalues and eigenvectors for a non-hermitian matrix.

```
V, V = eig(a) yields aV = V diag(w).
```

#### **Parameters**

- $\mathbf{a}$  (Array) The hermitian square matrix to be diagonalized.
- **sort** ({'m>', 'm<', '>', '<', None}) How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

#### Returns

- W (1D ndarray) The eigenvalues, sorted within the same charge blocks according to sort.
- **V**(*Array*) Unitary matrix; V[:, i] is normalized eigenvector with eigenvalue W[i]. The first label is inherited from *A*, the second label is 'eig'.

### **Notes**

Requires the legs to be contractible. If a is not blocked by charge, a blocked copy is made via a permutation P, :math:` a' = P a P = V' W' (V')^{dagger}`. The eigenvectors V are then obtained by the reverse permutation,  $V = P^{-1}V'$  such that  $A = VWV^{dagger}$ .

### eigh

- full name: tenpy.linalg.np\_conserved.eigh
- parent module: tenpy.linalg.np\_conserved
- · type: function

```
tenpy.linalg.np_conserved.eigh(a, UPLO='L', sort=None)
```

Calculate eigenvalues and eigenvectors for a hermitian matrix.

```
W, V = eigh(a) yields a = V diag(w)V^{\dagger}. Assumes that a is hermitian, a.conj().transpose() == a.
```

#### **Parameters**

- a (Array) The hermitian square matrix to be diagonalized.
- **UPLO** ({ 'L', 'U'}) Whether to take the lower ('L', default) or upper ('U') triangular part of *a*.
- **sort** ({'m>', 'm<', '>', '<', None}) How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

#### Returns

- W (1D ndarray) The eigenvalues, sorted within the same charge blocks according to sort.
- **V** (*Array*) Unitary matrix; V[:, i] is normalized eigenvector with eigenvalue W[i]. The first label is inherited from *A*, the second label is 'eig'.

### **Notes**

Requires the legs to be contractible. If a is not blocked by charge, a blocked copy is made via a permutation P, :math:` a' = P a P = V' W'  $(V')^{dagger}$ . The eigenvectors V are then obtained by the reverse permutation,  $V = P^{-1}V'$  such that A = VW' dagger.

### eigvals

- full name: tenpy.linalg.np\_conserved.eigvals
- $\bullet \ \ parent \ module: \ \textit{tenpy.linalg.np\_conserved}$
- type: function

```
tenpy.linalg.np_conserved.eigvals(a, sort=None)
```

Calculate eigenvalues for a hermitian matrix.

#### **Parameters**

- a (Array) The hermitian square matrix to be diagonalized.
- **sort** ({'m>', 'm<', '>', '<', None}) How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

**Returns** W – The eigenvalues, sorted within the same charge blocks according to *sort*.

Return type 1D ndarray

### **Notes**

The eigenvalues are sorted within blocks of the completely blocked legs.

### eigvalsh

- full name: tenpy.linalg.np\_conserved.eigvalsh
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.eigvalsh(a, UPLO='L', sort=None) Calculate eigenvalues for a hermitian matrix.
```

**Assumes** that a is hermitian, a.conj().transpose() == a.

### **Parameters**

- a (Array) The hermitian square matrix to be diagonalized.
- **UPLO** ({ 'L', 'U'}) Whether to take the lower ('L', default) or upper ('U') triangular part of *a*.
- **sort** ({'m>', 'm<', '>', '<', None}) How the eigenvalues should are sorted *within* each charge block. Defaults to None, which is same as '<'. See argsort () for details.

**Returns** W – The eigenvalues, sorted within the same charge blocks according to *sort*.

Return type 1D ndarray

### **Notes**

The eigenvalues are sorted within blocks of the completely blocked legs.

### expm

- full name: tenpy.linalg.np\_conserved.expm
- parent module: tenpy.linalg.np\_conserved
- · type: function

```
tenpy.linalg.np_conserved.expm(a)
```

Use scipy.linalg.expm to calculate the matrix exponential of a square matrix.

**Parameters a** (Array) – A square matrix to be exponentiated.

**Returns** exp\_a - The matrix exponential expm(a), calculated using scipy.linalg.expm. Same legs/labels as a.

**Return type** Array

### eye\_like

- full name: tenpy.linalg.np\_conserved.eye\_like
- parent module: tenpy.linalg.np\_conserved
- type: function

tenpy.linalg.np\_conserved.**eye\_like** (*a*, *axis=0*, *labels=None*)

Return an identity matrix contractible with the leg *axis* of the *Array a*.

### grid concat

- full name: tenpy.linalg.np\_conserved.grid\_concat
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.grid_concat(grid, axes, copy=True)
```

Given an np.array of npc.Arrays, performs a multi-dimensional concatentation along 'axes'.

Similar to numpy.block(), but only for uniform blocking.

Stacks the gind of the array, without sorting/blocking.

#### **Parameters**

- grid (array\_like of Array) The grid of arrays.
- **axes** (list of int) The axes along which to concatenate the arrays, same len as the dimension of the grid. Concatenate arrays of the *i*'th axis of the grid along the axis ``axes[i]'
- copy (bool) Whether the \_data blocks are copied.

### **Examples**

Assume we have rank 2 Arrays A, B, C, D of shapes (1, 2), (1, 4), (3, 2), (3, 4) sharing the legs of equal sizes. Then the following grid will result in a (1+3, 2+4) shaped array:

```
>>> g = grid_concat([[A, B], [C, D]], axes=[0, 1])
>>> g.shape
(4, 6)
```

If A, B, C, D were rank 4 arrays, with the first and last leg as before, and sharing *common* legs 1 and 2 of dimensions 1, 2, then you would get a rank-4 array:

```
>>> g = grid_concat([[A, B], [C, D]], axes=[0, 3])
>>> g.shape
(4, 1, 2, 6)
```

### See also:

Array.sort\_legcharge() can be used to block by charges.

### grid outer

- full name: tenpy.linalg.np\_conserved.grid\_outer
- parent module: tenpy.linalg.np\_conserved
- type: function

tenpy.linalg.np\_conserved.grid\_outer (grid, grid\_legs, qtotal=None, grid\_labels=None)
Given an np.array of npc.Arrays, return the corresponding higher-dimensional Array.

#### **Parameters**

- **grid** (array\_like of {Array | None}) The grid gives the first part of the axes of the resulting array. Entries have to have all the same shape and charge-data, giving the remaining axes. None entries in the grid are interpreted as zeros.
- grid\_legs (list of LegCharge) One LegCharge for each dimension of the grid along the grid.
- **qtotal** (*charge*) The total charge of the Array. By default (None), derive it out from a non-trivial entry of the grid.
- grid\_labels (list of {str | None}) One label associated to each of the grid axes. None for non-named labels.

**Returns res** – An Array with shape grid.shape + nontrivial\_grid\_entry.shape. Constructed such that res[idx] == grid[idx] for any index idx of the *grid* the *grid* entry is not trivial (None).

Return type Array

### See also:

detect\_grid\_outer\_legcharge() can calculate one missing LegCharge of the grid.

# **Examples**

A typical use-case for this function is the generation of an MPO. Say you have npc.Arrays Splus, Sminus, Sz, Id, each with legs [phys.conj(), phys]. Further, you have to define appropriate LegCharges *l\_left* and *l\_right*. Then one 'matrix' of the MPO for a nearest neighbour Heisenberg Hamiltonian could look like:

#### inner

• full name: tenpy.linalg.np\_conserved.inner

• parent module: tenpy.linalg.np\_conserved

· type: function

tenpy.linalg.np\_conserved.inner(a, b, axes=None, do\_conj=False)
Contract all legs in a and b, return scalar.

#### **Parameters**

- b(a,) The arrays for which to calculate the product. Must have same rank, and compatible LegCharges.
- axes ((axes\_a, axes\_b) | 'range', 'labels') axes\_a and axes\_b specifiy the legs of a and b, respectively, which should be contracted. Legs can be specified with leg labels or indices. We contract leg axes\_a[i] of a with leg axes\_b[i] of b. The default axes='range' is equivalent to (range(rank), range(rank)).axes='labels' is equivalent to either (a.get\_leg\_labels(), a.get\_leg\_labels()) for do\_conj=True, or to (a.get\_leg\_labels(), conj\_labels(a.get\_leg\_labels())) for do\_conj=False. In other words, axes='labels' requires a and b to have the same/conjugated labels up to a possible transposition, which is then reverted.
- do\_conj(bool) If False (Default), ignore it. if True, conjugate a before, i.e., return inner(a.conj(), b, axes)

**Returns inner\_product** – A scalar (of common dtype of a and b) giving the full contraction of a and b.

Return type dtype

### norm

• full name: tenpy.linalg.np conserved.norm

• parent module: tenpy.linalg.np\_conserved

• type: function

tenpy.linalg.np\_conserved.norm(a, ord=None, convert\_to\_float=True)
Norm of flattened data.

Equivalent to np.linalg.norm(a.to\_ndarray().flatten(), ord).

In contrast to numpy, we don't distinguish between matrices and vectors, but simply calculate the norm for the **flat** (block) data. The usual *ord*-norm is defined as  $(\sum_i |a_i|^{ord})^{1/ord}$ .

ord	norm
None/'fro'	Frobenius norm (same as 2-norm)
np.inf	max(abs(x))
-np.inf	min(abs(x))
0	<pre>sum(a != 0) == np.count_nonzero(x)</pre>
other	ususal <i>ord</i> -norm

#### **Parameters**

• a (Array | np.ndarray) – The array of which the norm should be calculated.

- ord The order of the norm. See table above.
- **convert\_to\_float** Convert integer to float before calculating the norm, avoiding int overflow.

**Returns norm** – The norm over the *flat* data of the array.

Return type float

#### ones

- full name: tenpy.linalg.np\_conserved.ones
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.ones(legcharges, dtype=<class 'numpy.float64'>, qtotal=None, la-
bels=None)
Short-hand for Array.from_func() with function numpy.ones().
```

Warning: For non-trivial charges, only blocks with compatible charges are filled with ones!

#### outer

- full name: tenpy.linalg.np\_conserved.outer
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.outer(a, b)
```

Forms the outer tensor product, equivalent to tensordot (a, b, axes=0).

Labels are inherited from a and b. In case of a collision (same label in both a and b), they are both dropped.

**Parameters**  $\mathbf{b}(a_{1})$  – The arrays for which to form the product.

#### Returns

**c** –

Array of rank a.rank + b.rank such that (for Ra = a.rank; Rb = b.rank):

```
c[i_1, ..., i_Ra, j_1, ... j_R] = a[i_1, ..., i_Ra] * b[j_1, ..., j_ \rightarrowrank_b]
```

Return type Array

### pinv

- full name: tenpy.linalg.np\_conserved.pinv
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.pinv(a, cutoff=1e-15)
```

Compute the (Moore-Penrose) pseudo-inverse of a matrix.

Equivalent to the following procedure: Perform a SVD, U, S, VH = svd(a, cutoff=cutoff) with a cutoff > 0, calculate P = U \* diag(1/S) \* VH (with \* denoting tensordot) and return P.conj. transpose().

#### **Parameters**

- **a** ((M, N) Array) Matrix to be pseudo-inverted.
- **cuttof** (*float*) Cutoff for small singular values, as given to *svd*(). (Note: different convetion than numpy.)

**Returns**  $\mathbf{B}$  – The pseudo-inverse of a.

Return type (N, M) Array

### qr

- full name: tenpy.linalg.np\_conserved.qr
- parent module: tenpy.linalg.np\_conserved
- · type: function

tenpy.linalg.np\_conserved.**qr**(*a*, *mode='reduced'*, *inner\_labels=[None*, *None]*, *cutoff=None*) Q-R decomposition of a matrix.

Decomposition such that A == npc.tensordot(q, r, axes=1) up to numerical rounding errors.

### **Parameters**

- a (Array) A square matrix to be exponentiated, shape (M, N).
- mode ('reduced', 'complete') 'reduced': return q and r with shapes (M,K) and (K,N), where K=min(M,N) 'complete': return q with shape (M,M).
- inner\_labels ([{str/None}, {str/None}]) The first label is used for Q. legs[1], the second for R.legs[0].
- **cutoff** (None or float) If not None, discard linearly dependent vectors to given precision, which might reduce *K* of the 'reduced' mode even further.

### Returns

- $\mathbf{q}$  (Array) If mode is 'complete', a unitary matrix. For mode 'reduced' such that Otherwise such that  $q_{i,i}^*q_{j,k}=\delta_{i,k}$
- **r** (*Array*) Upper triangular matrix if both legs of A are sorted by charges; Otherwise a simple transposition (performed when sorting by charges) brings it to upper triangular form.

#### speigs

- full name: tenpy.linalg.np\_conserved.speigs
- parent module: tenpy.linalg.np\_conserved
- · type: function

```
tenpy.linalg.np_conserved.speigs (a, charge_sector, k, *args, **kwargs)
Sparse eigenvalue decomposition w, v of square a in a given charge sector.
```

Finds k right eigenvectors (chosen by kwargs['which']) in a given charge sector, tensordot(A, V[i], axes=1) = W[i] \* V[i].

#### **Parameters**

- a (Array) A square array with contractible legs and vanishing total charge.
- **charge\_sector** (*charges*) *ndim* charges to select the block.
- **k** (*int*) How many eigenvalues/vectors should be calculated. If the block of *charge\_sector* is smaller than *k*, *k* may be reduced accordingly.
- \*args Additional arguments given to scipy.sparse.linalg.eigs.
- \*\*kwargs Additional keyword arguments given to scipy.sparse.linalg.eigs.

#### Returns

- $\mathbf{W}$  (*ndarray*) k (or less) eigenvalues
- V (list of Array) k (or less) right eigenvectors of A with total charge *charge\_sector*. Note that when interpreted as a matrix, this is the transpose of what np.eigs normally gives.

#### svd

- full name: tenpy.linalg.np\_conserved.svd
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.\mathbf{svd}(a, full\_matrices=False, compute\_uv=True, cutoff=None, qtotal\_LR=[None, None], inner\_labels=[None, None], inner\_qconj=1)
```

Singualar value decomposition of an Array  $\hat{a}$ .

Factorizes U, S, VH = svd(a), such that a = U\*diag(S)\*VH (where \* stands for a tensordot() and diag creates an correctly shaped Array with S on the diagonal). For a non-zero cutoff this holds only approximately.

There is a gauge freedom regarding the charges, see also  $Array.gauge\_total\_charge()$ . We ensure contractibility by setting U.legs[1] = VH.legs[0].conj(). Further, we gauge the LegCharge such that U and V have the desired  $qtotal\_LR$ .

#### **Parameters**

- a (Array, shape (M, N)) The matrix to be decomposed.
- full\_matrices (bool) If False (default), U and V have shapes (M, K) and (K, N), where K=len(S). If True, U and V are full square unitary matrices with shapes (M, M) and (N, N). Note that the arrays are not directly contractible in that case; diag(S) would need to be a rectangluar (M, N) matrix.

- **compute\_uv** (bool) Whether to compute and return U and V.
- **cutoff** (None | float) Keep only singular values which are (strictly) greater than *cutoff*. (Then the factorization holds only approximately). If None (default), ignored.
- qtotal\_LR ([{charges|None}, {charges|None}]) The desired qtotal for U and VH, respectively. [None, None] (Default) is equivalent to [None, a.qtotal].
   A single None entry is replaced the unique charge satisfying the requirement U.qtotal + VH.qtotal = a.qtotal (modulo qmod).
- inner\_labels\_LR ([{str/None}, {str/None}]) The first label corresponds to U.leqs[1], the second to VH.leqs[0].
- inner\_qconj ({+1, -1}) Direction of the charges for the new leg. Default +1. The new LegCharge is constructed such that VH.legs[0].qconj = qconj.

#### Returns

- U (Array) Matrix with left singular vectors as columns. Shape (M, M) or (M, K) depending on *full\_matrices*.
- **S** (1D ndarray) The singluar values of the array. If no *cutoff* is given, it has length min (M, N).
- VH (Array) Matrix with right singular vectors as rows. Shape (N, N) or (K, N) depending on *full\_matrices*.

#### tensordot

- full name: tenpy.linalg.np\_conserved.tensordot
- parent module: tenpy.linalg.np\_conserved
- type: function

```
tenpy.linalg.np_conserved.tensordot (a, b, axes=2) Similar as np.tensordot but for Array.
```

Builds the tensor product of a and b and sums over the specified axes. Does not require complete blocking of the charges.

Labels are inherited from a and b. In case of a collision (= the same label would be inherited from a and b after the contraction), both labels are dropped.

Detailed implementation notes are available in the doc-string of \_tensordot\_worker().

### **Parameters**

- $\mathbf{b}(a_1)$  The first and second npc Array for which axes are to be contracted.
- **axes** ((axes\_a, axes\_b) | int) A single integer is equivalent to (range(-axes, 0), range(axes)). Alternatively, *axes\_a* and *axes\_b* specifiy the legs of *a* and *b*, respectively, which should be contracted. Legs can be specified with leg labels or indices. Contract leg axes\_a[i] of *a* with leg axes\_b[i] of *b*.

**Returns** a\_dot\_b – The tensorproduct of *a* and *b*, summed over the specified axes. Returns a scalar in case of a full contraction.

**Return type** *Array* 

### to\_iterable\_arrays

```
• full name: tenpy.linalg.np_conserved.to_iterable_arrays
```

```
• parent module: tenpy.linalg.np_conserved
```

• type: function

```
tenpy.linalg.np_conserved.to_iterable_arrays (array_list)
Similar as to_iterable(), but also enclose npc Arrays in a list.
```

#### trace

• full name: tenpy.linalg.np\_conserved.trace

```
• parent module: tenpy.linalg.np_conserved
```

type: function

```
tenpy.linalg.np_conserved.trace(a, leg1=0, leg2=1)
```

Trace of a, summing over leg1 and leg2.

Requires that the contracted legs are contractible (i.e. have opposite charges). Labels are inherited from a.

**Parameters 1eg2** (1eg1,) – The leg label or index for the two legs which should be contracted (i.e. summed over).

```
Returns traced - A scalar if a.rank == 2, else an Array of rank a.rank - 2. Equiv-
alent to sum([a.take_slice([i, i], [leg1, leg2]) for i in range(a.
shape[leg1])]).
```

Return type Array la.dtype

#### zeros

- full name: tenpy.linalg.np\_conserved.zeros
- parent module: tenpy.linalg.np\_conserved
- · type: function

```
tenpy.linalg.np_conserved.zeros(legcharges, dtype=<class 'numpy.float64'>, qtotal=None, la-
bels=None)
```

Create a npc array full of zeros (with no data).

This is just a wrapper around Array (...), detailed documentation can be found in the class doc-string of Array.

#### Module description

A module to handle charge conservation in tensor networks.

A detailed introduction to this module (including notations) can be found in *Charge conservation with np\_conserved*.

This module  $np\_conserved$  implements a class Array designed to make use of charge conservation in tensor networks. The idea is that the Array class is used in a fashion very similar to the numpy.ndarray, e.g you can call the functions tensordot() or svd() (of this module) on them. The structure of the algorithms (as DMRG) is thus the same as with basic numpy ndarrays.

Internally, an *Array* saves charge meta data to keep track of blocks which are nonzero. All possible operations (e.g. tensordot, svd, ...) on such arrays preserve the total charge structure. In addition, these operations make use of the charges to figure out which of the blocks it has to use/combine - this is the basis for the speed-up.

```
tenpy.linalg.np_conserved.QCUTOFF = 2.220446049250313e-15
   A cutoff to ignore machine precision rounding errors when determining charges
tenpy.linalg.np_conserved.QTYPE = <class 'numpy.int64'>
   the type used for charges
```

### **Overview**

### **Classes**

Array(legcharges[, dtype, qtotal, labels])	A multidimensional array (=tensor) for using charge
	conservation.
ChargeInfo([mod, names])	Meta-data about the charge of a tensor.
LegCharge(chargeinfo, slices, charges[, qconj])	Save the charge data associated to a leg of a tensor.
LegPipe(legs[, qconj, sort, bunch])	A <i>LegPipe</i> combines multiple legs of a tensor to one.

# **Array creation**

Array.from_ndarray_trivial(data_flat[,])	convert a flat numpy ndarray to an Array with trivial
	charge conservation.
Array.from_ndarray(data_flat, legcharges[,])	convert a flat (numpy) ndarray to an Array.
Array.from_func(func, legcharges[, dtype,])	Create an Array from a numpy func.
Array.from_func_square(func, leg[, dtype,	Create an Array from a (numpy) function.
])	
zeros(legcharges[, dtype, qtotal, labels])	Create a npc array full of zeros (with no _data).
eye_like(a[, axis, labels])	Return an identity matrix contractible with the leg axis
	of the Array a.
diag(s, leg[, dtype, labels])	Returns a square, diagonal matrix of entries s.

# Concatenation

concatenate(arrays[, axis, copy])	Stack arrays along a given axis, similar as
	np.concatenate.
<pre>grid_concat(grid, axes[, copy])</pre>	Given an np.array of npc.Arrays, performs a multi-
	dimensional concatentation along 'axes'.
<pre>grid_outer(grid, grid_legs[, qtotal,])</pre>	Given an np.array of npc.Arrays, return the correspond-
	ing higher-dimensional Array.

# **Detecting charges of flat arrays**

detect_qtotal(flat_array, legcharges[, cutoff])	Returns the total charge (w.r.t <i>legs</i> ) of first non-zero sector found in <i>flat_array</i> .
<pre>detect_legcharge(flat_array, chargeinfo,)</pre>	Calculate a missing <i>LegCharge</i> by looking for nonzero
	entries of a flat array.
detect_grid_outer_legcharge(grid,	Derive a LegCharge for a grid used for
grid_legs)	<pre>grid_outer().</pre>

# **Contraction of some legs**

tensordot(a, b[, axes])	Similar as np.tensordot but for Array.
outer(a, b)	Forms the outer tensor product, equivalent to
	tensordot(a, b, axes=0).
inner(a, b[, axes, do_conj])	Contract all legs in a and b, return scalar.
trace(a[, leg1, leg2])	Trace of a, summing over leg1 and leg2.

# Linear algebra

svd(a[, full_matrices, compute_uv, cutoff,])	Singualar value decomposition of an Array a.
pinv(a[, cutoff])	Compute the (Moore-Penrose) pseudo-inverse of a ma-
	trix.
norm(a[, ord, convert_to_float])	Norm of flattened data.
qr(a[, mode, inner_labels, cutoff])	Q-R decomposition of a matrix.
expm(a)	Use scipy.linalg.expm to calculate the matrix exponen-
	tial of a square matrix.

# **Eigen systems**

eigh(a[, UPLO, sort])	Calculate eigenvalues and eigenvectors for a hermitian matrix.
eig(a[, sort])	Calculate eigenvalues and eigenvectors for a non-
	hermitian matrix.
eigvalsh(a[, UPLO, sort])	Calculate eigenvalues for a hermitian matrix.
eigvals(a[, sort])	Calculate eigenvalues for a hermitian matrix.
speigs(a, charge_sector, k, *args, **kwargs)	Sparse eigenvalue decomposition $w$ , $v$ of square $a$ in a
	given charge sector.

# 7.8.2 charges

• full name: tenpy.linalg.charges

• parent module: tenpy.linalg

• type: module

### **Classes**



ChargeInfo([mod, names])	Meta-data about the charge of a tensor.
LegCharge(chargeinfo, slices, charges[, qconj])	Save the charge data associated to a leg of a tensor.
LegPipe(legs[, qconj, sort, bunch])	A <i>LegPipe</i> combines multiple legs of a tensor to one.

# ChargeInfo

• full name: tenpy.linalg.charges.ChargeInfo

• parent module: tenpy.linalg.charges

• type: class

# **Inheritance Diagram**

ChargeInfo

### Methods

ChargeInfoinit([mod, names])	Initialize self.
ChargeInfo.add(chinfos)	Create a ChargeInfo combining multiple charges.
ChargeInfo.change(chinfo, charge, new_qmod)	Change the <i>qmod</i> of a given charge.
ChargeInfo.check_valid(charges)	Check, if <i>charges</i> has all entries as expected from
	self.mod.
ChargeInfo.drop(chinfo[, charge])	Remove a charge from a ChargeInfo.
ChargeInfo.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
ChargeInfo.make_valid([charges])	Take charges modulo self.mod.
ChargeInfo.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
ChargeInfo.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

# **Class Attributes and Properties**

ChargeInfo.mod	Modulo how much each of the charges is taken.
ChargeInfo.qnumber	The number of charges.

class tenpy.linalg.charges.ChargeInfo(mod=[], names=None)

Bases: object

Meta-data about the charge of a tensor.

Saves info about the nature of the charge of a tensor. Provides  $make\_valid()$  for taking modulo m.

(This class is implemented in tenpy.linalg.charges but also imported in tenpy.linalg. np\_conserved for convenience.)

### **Parameters**

- **mod** (*iterable of QTYPE*) The len gives the number of charges, *qnumber*. For each charge one entry *m*: the charge is conserved modulo *m*. Defaults to trivial, i.e., no charge.
- names (list of str) Descriptive names for the charges. Defaults to ['']\*qnumber.

#### names

A descriptive name for each of the charges. May have "entries.

**Type** list of strings

### \_mask

mask (mod == 1), to speed up *make\_valid* in pure python.

**Type** 1D array bool

#### \_mod\_masked

Equivalent to self.mod[self.\_maks\_mod1]

**Type** 1D array QTYPE

### \_qnumber, \_mod

Storage of gnumber and mod.

### **Notes**

Instances of this class can (should) be shared between different LegCharge and Array's.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

It stores the names under the path "names", and mod as dataset "U1\_ZN".

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

The "U1\_ZN" dataset is mandatory, 'names' are optional.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

### classmethod add(chinfos)

Create a Charge Info combining multiple charges.

**Parameters** chinfos (iterable of *ChargeInfo*) – ChargeInfo instances to be combined into a single one (in the given order).

**Returns** chinfo – ChargeInfo combining all the given charges.

Return type ChargeInfo

# classmethod drop(chinfo, charge=None)

Remove a charge from a ChargeInfo.

### **Parameters**

- **chinfo** (*ChargeInfo*) The ChargeInfo from where to drop/remove a charge.
- **charge** (*int* / *str*) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.

**Returns** chinfo – ChargeInfo where the specified charge is dropped.

Return type ChargeInfo

classmethod change (chinfo, charge, new\_qmod, new\_name=")

Change the *qmod* of a given charge.

### **Parameters**

• **chinfo** (ChargeInfo) – The ChargeInfo for which qmod of charge should be changed.

```
• new_qmod (int) – The new qmod to be set.
```

• **new\_name** (str) – The new name of the charge.

**Returns** chinfo – ChargeInfo where *qmod* of the specified charge was changed.

```
Return type ChargeInfo
```

#### test sanity()

Sanity check, raises ValueErrors, if something is wrong.

### property qnumber

The number of charges.

### property mod

Modulo how much each of the charges is taken.

```
1 for a U(1) charge, N for a Z_N symmetry.
```

### make\_valid(charges=None)

Take charges modulo self.mod.

Parameters charges (array\_like or None) - 1D or 2D array of charges, last dimension self.qnumber None defaults to trivial charges np.zeros(qnumber, dtype=QTYPE).

**Returns** A copy of *charges* taken modulo *mod*, but with x % 1 := x

Return type charges

### check\_valid(charges)

Check, if *charges* has all entries as expected from self.mod.

**Parameters** charges (2D ndarray QTYPE\_t) - Charge values to be checked.

**Returns** res – True, if all 0 <= charges <= self.mod (wherever self.mod != 1)

Return type bool

# LegCharge

- full name: tenpy.linalg.charges.LegCharge
- parent module: tenpy.linalg.charges
- type: class

### **Inheritance Diagram**

LegCharge

# Methods

LegChargeinit(chargeinfo, slices, charges)	Initialize self.
LegCharge.bunch()	Return a copy with bunched self.charges: form blocks
	for contiguous equal charges.
LegCharge.charge_sectors()	Return unique rows of self.charges.
LegCharge.conj()	Return a (shallow) copy with opposite self.qconj.
LegCharge.copy()	Return a (shallow) copy of self.
LegCharge.extend(extra)	Return a new LegCharge, which extends self with
	futher charges.
LegCharge.flip_charges_qconj()	Return a copy with both negative qconj and charges.
LegCharge.from_add_charge(legs[, charge-	Add the (independent) charges of two or more legs to
info])	get larger qnumber.
LegCharge.from_change_charge(leg, charge,	Remove a charge from a LegCharge.
)	
LegCharge.from_drop_charge(leg[, charge,	Remove a charge from a LegCharge.
])	
LegCharge.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
LegCharge.from_qdict(chargeinfo, qdict[,	Create a LegCharge from qdict form.
qconj])	
LegCharge.from_qflat(chargeinfo, qflat[,	Create a LegCharge from qflat form.
qconj])	
LegCharge.from_qind(chargeinfo, slices,	Just a wrapper around selfinit(), see class doc-
charges)	string for parameters.
$LegCharge.from\_trivial(ind\_len[,])$	Create trivial (qnumber=0) LegCharge for given len of
	indices ind_len.
LegCharge.get_block_sizes()	Return the sizes of the individual blocks.
LegCharge.get_charge(qindex)	Return charge self.charges[qindex] *
	self.qconj for a given qindex.
LegCharge.get_qindex(flat_index)	Find qindex containing a flat index.
LegCharge.get_qindex_of_charges(charges)	Return the slice selecting the block for given charge val-
<del></del>	ues.
LegCharge.get_slice(qindex)	Return slice selecting the block for a given <i>qindex</i> .
LegCharge.is_blocked()	Returns whether self is blocked, i.e.
LegCharge.is_bunched()	Checks whether bunch () would change something.
LegCharge.is_sorted()	Returns whether <i>self.charges</i> is sorted lexiographically.
LegCharge.perm_flat_from_perm_qind(perm	Qind) ert a permutation of qind (acting on self) into a flat
	permutation.
LegCharge.perm_qind_from_perm_flat(perm	
LegCharge.project(mask)	Return copy keeping only the indices specified by <i>mask</i> .
LegCharge.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
LegCharge.sort([bunch])	Return a copy of <i>self</i> sorted by charges (but maybe not bunched).
LegCharge.test_contractible(other)	Raises a ValueError if charges are incompatible for con-
	traction with other.
LegCharge.test_equal(other)	Test if charges are <i>equal</i> including <i>qconj</i> .
LegCharge.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
LegCharge.to_qdict()	Return charges in <i>qdict</i> form.
LegCharge.to_qflat()	Return charges in <i>qflat</i> form.
	C 20

```
 \textbf{class} \texttt{ tenpy.linalg.charges.LegCharge} (\textit{chargeinfo}, \textit{slices}, \textit{charges}, \textit{qconj} = 1) \\ \textbf{Bases: object}
```

Save the charge data associated to a leg of a tensor.

This class is more or less a wrapper around a 2D numpy array *charges* and a 1D array *slices*. See *Charge conservation with np conserved* for more details.

(This class is implemented in tenpy.linalg.charges but also imported in tenpy.linalg. np conserved for convenience.)

#### **Parameters**

- chargeinfo (ChargeInfo) The nature of the charge.
- slices (1D array\_like, len(block\_number+1)) A block with 'qindex' qi correspondes to the leg indices in slice(slices[qi], slices[qi+1]).
- charges (2D array\_like, shape(block\_number, chargeinfo. qnumber)) charges [qi] gives the charges for a block with 'qindex' qi.
- qconj ({+1, -1}) A flag telling whether the charge points inwards (+1, default) or outwards (-1).

#### ind len

The number of indices for this leg.

Type int

#### block number

The number of blocks, i.e., a 'qindex' for this leg is in range (block\_number).

### chinfo

The nature of the charge. Can be shared between LegCharges.

Type ChargeInfo instance

#### slices

A block with 'qindex' qi correspondes to the leg indices in slice(self.slices[qi], self. slices[qi+1]). See  $get\_slice()$ .

**Type** ndarray[np.intp\_t,ndim=1] (block\_number+1)

### charges

charges [qi] gives the charges for a block with 'qindex' qi. Note: the sign might be changed by qconj. See also get\_charge().

Type ndarray[QTYPE\_t,ndim=1] (block\_number, chinfo.qnumber)

#### qconj

A flag telling whether the charge points inwards (+1) or outwards (-1). Whenever charges are added, they should be multiplied with their *qconj* value.

**Type** {-1, 1}

### sorted

Whether the charges are guaranteed to be sorted.

Type bool

### bunched

Whether the charges are guaranteed to be bunched.

Type bool

### **Notes**

Instances of this class can be shared between different *npc.Array*. Thus, functions changing self.slices or self.charges *must* always make copies. Further they *must* set *sorted* and *bunched* to False (if they might not preserve them).

#### copy()

Return a (shallow) copy of self.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

Checks format for an ouput format key "LegCharge". Possible choices are:

"blocks" (default) Store slices and charges directly as datasets, and block\_number, sorted, bunched as further attributes.

"compact" A single array np.hstack([self.slices[:-1], self.slices[1:], self.charges]) as dataset "blockcharges", and block\_number, sorted, bunched as further attributes.

"flat" Insufficient (!) to recover the exact blocks; saves only the array returned by to\_flat() as
 dataset 'charges'.

The ind\_len, qconj, and the format parameter are saved as group attributes under the same names. chinfo is always saved as subgroup.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group `) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns obj** – Newly generated class instance containing the required data.

Return type cls

```
classmethod from_trivial(ind_len, chargeinfo=None, qconj=1)
```

Create trivial (qnumber=0) LegCharge for given len of indices ind\_len.

```
classmethod from_qflat (chargeinfo, qflat, qconj=1)
```

Create a LegCharge from qflat form.

Does *neither* bunch *nor* sort. We recommend to sort (and bunch) afterwards, if you expect that tensors using the LegCharge have entries at all positions compatible with the charges.

### **Parameters**

• **chargeinfo** (*ChargeInfo*) – The nature of the charge.

- **qflat** (array\_like (ind\_len, *qnumber*)) *qnumber* charges for each index of the leg on entry.
- **qconj** ({-1, 1}) A flag telling whether the charge points inwards (+1) or outwards (-1).

### See also:

```
sort () sorts by charges
```

bunch () bunches contiguous blocks of the same charge.

### classmethod from\_qind(chargeinfo, slices, charges, qconj=1)

Just a wrapper around self.\_\_init\_\_(), see class doc-string for parameters.

#### See also:

```
sort () sorts by charges
```

bunch () bunches contiguous blocks of the same charge.

### classmethod from\_qdict (chargeinfo, qdict, qconj=1)

Create a LegCharge from qdict form.

#### **Parameters**

- **chargeinfo** (*ChargeInfo*) The nature of the charge.
- **qdict** (dict) A dictionary mapping a tuple of charges to slices.

### classmethod from\_add\_charge(legs, chargeinfo=None)

Add the (independent) charges of two or more legs to get larger qnumber.

### **Parameters**

- **legs** (iterable of *LegCharge*) The legs for which the charges are to be combined/added.
- chargeinfo (ChargeInfo) The ChargeInfo for all charges; create new if None.

**Returns combined** – A LegCharge with the charges of both legs. Is neither sorted nor bunched!

```
Return type LegCharge
```

classmethod from\_drop\_charge (leg, charge=None, chargeinfo=None)

Remove a charge from a LegCharge.

#### **Parameters**

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (*int* / *str*) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.
- chargeinfo (ChargeInfo) The ChargeInfo with charge dropped; create new if None.

**Returns dropped** – A LegCharge with the specified charge dropped. Is neither sorted nor bunched!

Return type LegCharge

classmethod from\_change\_charge (leg, charge, new\_qmod, new\_name=", chargeinfo=None)
 Remove a charge from a LegCharge.

#### **Parameters**

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (int / str) Number or *name* of the charge (within *chinfo*) for which *mod* is to be changed.
- **new\_qmod** (int) The new *mod* to be set for *charge* in the *ChargeInfo*.
- **new\_name** (str) The new name for *charge*.
- chargeinfo (ChargeInfo) The ChargeInfo with charge changed; create new if

**Returns** leg – A LegCharge with the specified charge changed. Is neither sorted nor bunched!

Return type LegCharge

### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

### conj()

Return a (shallow) copy with opposite self.gconj.

**Returns conjugated** – Shallow copy of *self* with flipped *qconj*. *test\_contractible()* of *self* with *conjugated* will not raise an error.

Return type LegCharge

#### flip\_charges\_qconj()

Return a copy with both negative qconj and charges.

**Returns conj\_charges** – (Shallow) copy of self with negative *qconj* and *charges*, thus representing the very same charges. test\_equal() of self with conj\_charges will not raise an error.

Return type LegCharge

### to\_qflat()

Return charges in qflat form.

### to\_qdict()

Return charges in *qdict* form.

Raises ValueError, if not blocked.

### is blocked()

Returns whether self is blocked, i.e. qindex map 1:1 to charge values.

#### is sorted()

Returns whether self.charges is sorted lexiographically.

### is\_bunched()

Checks whether bunch () would change something.

# $\verb|test_contractible| (other)$

Raises a ValueError if charges are incompatible for contraction with other.

**Parameters other** (LegCharge) – The LegCharge of the other leg condsidered for contraction.

Raises ValueError – If the charges are incompatible for direct contraction.

#### **Notes**

This function checks that two legs are *ready* for contraction. This is the case, if all of the following conditions are met:

- the ChargeInfo is equal
- the *slices* are equal
- the *charges* are the same up to *opposite* signs qconj:

```
self.charges * self.qconj = - other.charges * other.qconj
```

In general, there could also be a change of the total charge, see *Charge conservation with np\_conserved* This special case is not considered here - instead use <code>gauge\_total\_charge()</code>, if a change of the charge is desired.

If you are sure that the legs should be contractable, check whether the charges are actually valid or whether self and other are blocked or should be sorted.

#### See also:

```
test_equal() self.test_contractible(other) just performs self.
test_equal(other.conj()).
```

#### test\_equal (other)

Test if charges are equal including qconj.

Check that all of the following conditions are met:

- the ChargeInfo is equal
- · the slices are equal
- the *charges* are the same up to the signs qconj:

```
self.charges * self.qconj = other.charges * other.qconj
```

### See also:

```
test_contractible() self.test_equal(other) is equivalent to self.
test_contractible(other.conj()).
```

### get block sizes()

Return the sizes of the individual blocks.

**Returns sizes** - The sizes of the individual blocks; sizes[i] = slices[i+1] - slices[i].

**Return type** ndarray, shape (block\_number,)

### get\_slice (qindex)

Return slice selecting the block for a given qindex.

#### get\_qindex (flat\_index)

Find qindex containing a flat index.

Given a flat index, to find the corresponding entry in an Array, we need to determine the block it is saved in. For example, if slices = [[0, 3], [3, 7], [7, 12]], the flat index 5 corresponds to the second entry, qindex = 1 (since 5 is in [3:7]), and the index within the block would be 2 = 5 - 3.

**Parameters** flat\_index (int) – A flat index of the leg. Negative index counts from behind.

#### Returns

- qindex (int) The qindex, i.e. the index of the block containing flat\_index.
- index\_within\_block (int) The index of flat\_index within the block given by qindex.

### get\_qindex\_of\_charges (charges)

Return the slice selecting the block for given charge values.

Inverse function of get\_charge().

**Parameters charges** (1D array\_like) - Charge values for which the slice of the block is to be determined.

**Returns** slice(i, j) – Slice of the charge values for

Return type slice

:raises ValueError : if the answer is not unique (because self is not blocked).:

### get\_charge (qindex)

Return charge self.charges[qindex] \* self.qconj for a given qindex.

#### sort (bunch=True)

Return a copy of *self* sorted by charges (but maybe not bunched).

If bunch=True, the returned copy is completely blocked by charge.

**Parameters bunch** (bool) – Whether *self.bunch* is called after sorting. If True, the leg is guaranteed to be fully blocked by charge.

#### Returns

- perm\_qind (array (self.block\_len,)) The permutation of the qindices (before bunching) used for the sorting. To obtain the flat permuation such that sorted\_array[..., :] = unsorted\_array[..., perm\_flat], use perm\_flat = unsorted\_leg.perm\_flat\_from\_perm\_qind(perm\_qind)
- **sorted\_copy** (*LegCharge*) A shallow copy of self, with new qind sorted (and thus blocked if bunch) by charges.

## See also:

bunch () enlarge blocks for contiguous qind of the same charges.

**numpy.take()** can apply *perm\_flat* to a given axis

tenpy.tools.misc.inverse\_permutation() returns inverse of a permutation

### bunch()

Return a copy with bunched self.charges: form blocks for contiguous equal charges.

#### Returns

- idx (1D array) idx[:-1] are the indices of the old qind which are kept, idx[-1] = old\_block\_number.
- **cp** (*LegCharge*) A new LegCharge with the same charges at given indices of the leg, but (possibly) shorter self.charges and self.slices.

### See also:

sort () sorts by charges, thus enforcing complete blocking in combination with bunch.

#### project (mask)

Return copy keeping only the indices specified by mask.

Parameters mask (1D array (bool)) - Whether to keep of the indices.

#### Returns

- map\_qind (*1D array*) Map of qindices, such that qind\_new = map\_qind[qind\_old], and map\_qind[qind\_old] = -1 for qindices projected out.
- **block\_masks** (1D array) The bool mask for each of the remaining blocks.
- projected\_copy (LegCharge) Copy of self with the qind projected by mask.

#### extend (extra)

Return a new LegCharge, which extends self with futher charges.

This is needed to formally increase the dimension of an Array.

**Parameters** extra (LegCharge | int) – By what to extend, i.e. the charges to be appended to *self*. An int stands for extending the length of the array by a single new block of that size and zero charges.

**Returns** extended\_leg – Copy of *self* extended by the charge blocks of the *extra* leg.

Return type LegCharge

#### charge\_sectors()

Return unique rows of self.charges.

**Returns** charges – Rows are the rows of self.charges lexsorted and without duplicates.

**Return type** array[QTYPE, ndim=2]

### perm\_flat\_from\_perm\_qind(perm\_qind)

Convert a permutation of qind (acting on self) into a flat permutation.

### perm\_qind\_from\_perm\_flat (perm\_flat)

Convert flat permutation into qind permutation.

**Parameters** perm\_flat (1D array) - A permutation acting on self, which doesn't mix the blocks of qind.

**Returns** perm\_qind – The permutation of self.qind described by perm\_flat.

Return type 1D array

**Raises** ValueError – If perm\_flat mixes blocks of different qindex.

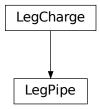
## LegPipe

• full name: tenpy.linalg.charges.LegPipe

• parent module: tenpy.linalg.charges

• type: class

# Inheritance Diagram



# Methods

LegPipeinit(legs[, qconj, sort, bunch])	Initialize self.		
LegPipe.bunch(*args, **kwargs)	Convert to LegCharge and call LegCharge.		
	bunch().		
LegPipe.charge_sectors()	Return unique rows of self.charges.		
LegPipe.conj()	Return a shallow copy with opposite self.qconj.		
LegPipe.copy()	Return a (shallow) copy of self.		
LegPipe.extend(extra)	Return a new LegCharge, which extends self with		
	futher charges.		
LegPipe.flip_charges_qconj()	Return a copy with both negative <i>qconj</i> and <i>charges</i> .		
LegPipe.from_add_charge(legs[, chargeinfo])	Add the (independent) charges of two or more legs to		
	get larger qnumber.		
LegPipe.from_change_charge(leg, charge,	Remove a charge from a LegCharge.		
new_qmod)			
LegPipe.from_drop_charge(leg[, charge,])	Remove a charge from a LegCharge.		
<pre>LegPipe.from_hdf5(hdf5_loader, h5gr, subpath)</pre>	Load instance from a HDF5 file.		
<pre>LegPipe.from_qdict(chargeinfo, qdict[, qconj])</pre>	Create a LegCharge from qdict form.		
<pre>LegPipe.from_qflat(chargeinfo, qflat[, qconj])</pre>	Create a LegCharge from qflat form.		
LegPipe.from_qind(chargeinfo, slices, charges)	Just a wrapper around selfinit(), see class doc-		
	string for parameters.		
LegPipe.from_trivial(ind_len[, chargeinfo,	Create trivial (qnumber=0) LegCharge for given len of		
])	indices ind_len.		
LegPipe.get_block_sizes()	Return the sizes of the individual blocks.		
LegPipe.get_charge(qindex)	Return charge self.charges[qindex] *		
	self.qconj for a given qindex.		
LegPipe.get_qindex(flat_index)	Find qindex containing a flat index.		
LegPipe.get_qindex_of_charges(charges)	Return the slice selecting the block for given charge val-		
	ues.		
LegPipe.get_slice(qindex)	Return slice selecting the block for a given <i>qindex</i> .		
LegPipe.is_blocked()	Returns whether self is blocked, i.e.		
LegPipe.is_bunched()	Checks whether bunch () would change something.		
LegPipe.is_sorted()	Returns whether <i>self.charges</i> is sorted lexiographically.		
	continues on next page		

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LegPipe.map_incoming_flat(incoming_indices)	Map (flat) incoming indices to an index in the outgoing			
3 1 1 = 3 · · · · · · · · · · · · · · · · · ·	pipe.			
LegPipe.outer_conj()	Like conj(), but don't change qconj for incoming			
	legs.			
LegPipe.perm_flat_from_perm_qind(perm_qind)onvert a permutation of qind (acting on self) into a flat				
	permutation.			
LegPipe.perm_qind_from_perm_flat(perm_flat)	at Convert flat permutation into qind permutation.			
LegPipe.project(*args, **kwargs)	Convert self to LegCharge and call LegCharge.			
	project().			
LegPipe.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.			
LegPipe.sort(*args, **kwargs)	Convert to LegCharge and call LegCharge.sort().			
LegPipe.test_contractible(other)	Raises a ValueError if charges are incompatible for con-			
	traction with other.			
LegPipe.test_equal(other)	Test if charges are <i>equal</i> including <i>qconj</i> .			
LegPipe.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.			
LegPipe.to_LegCharge()	Convert self to a LegCharge, discarding the information			
	how to split the legs.			
LegPipe.to_qdict()	Return charges in <i>qdict</i> form.			
LegPipe.to_qflat()	Return charges in <i>qflat</i> form.			

class tenpy.linalg.charges.LegPipe (legs, qconj=1, sort=True, bunch=True)

Bases: tenpy.linalg.charges.LegCharge

A *LegPipe* combines multiple legs of a tensor to one.

Often, it is necessary to "combine" multiple legs into one: for example to perfom a SVD, the tensor needs to be viewed as a matrix.

This class does exactly this job: it combines multiple LegCharges ('incoming legs') into one 'pipe' (the 'outgoing leg'). The pipe itself is a LegCharge, with indices running from 0 to the product of the individual legs' ind\_len, corresponding to all possible combinations of input leg indices.

(This class is implemented in tenpy.linalg.charges but also imported in tenpy.linalg.  $np\_conserved$  for convenience.)

#### **Parameters**

- legs (list of LegCharge) The legs which are to be combined.
- $qconj(\{+1, -1\})$  A flag telling whether the charge of the *resulting* pipe points inwards (+1, default) or outwards (-1).
- **sort** (bool) Whether the outgoing pipe should be sorted. Default True; recommended. Note: calling *sort* () after initialization converts to a LegCharge.
- **bunch** (bool) Whether the outgoing pipe should be bunched. Default True; recommended. Note: calling bunch () after initialization converts to a LegCharge.

### nlegs

The number of legs.

Type int

### legs

The original legs, which were combined in the pipe.

**Type** tuple of *LegCharge* 

#### subshape

ind\_len for each of the incoming legs.

```
Type tuple of int
```

### subqshape

block\_number for each of the incoming legs.

```
Type tuple of int
```

### q\_map

Shape ( $block\_number$ , 3 + nlegs). Rows: [  $b_j$ ,  $b_{j+1}$ ,  $I_s$ ,  $i_1$ , ...,  $i_{nlegs}$ ], See Notes below for details.

```
Type array[np.intp, ndim=2]
```

### q\_map\_slices

```
Defined such that the row indices of in range (q_map_slices[I_s], q_map_slices[I_s+1]) have q_map[:, 2] == I_s.
```

```
Type array[np.intp, ndim=1]
```

### \_perm

A permutation such that g map [ perm, 3:] is sorted by i l.

```
Type 1D array
```

### strides

Strides for mapping incoming qindices i l to the index of q map [ perm, :].

```
Type 1D array
```

#### **Notes**

For np.reshape, taking, for example,  $i, j, ... \to k$  amounted to  $k = s_1 * i + s_2 * j + ...$  for appropriate strides  $s_1, s_2$ .

In the charged case, however, we want to block k by charge, so we must implicitly permute as well. This reordering is encoded in  $q_map$ .

Each qindex combination of the *nlegs* input legs  $(i_1, ..., i_{nlegs})$ , will end up getting placed in some slice  $a_j: a_{j+1}$  of the outgoing pipe. Within this slice, the data is simply reshaped in usual row-major fashion ('C'-order), i.e., with strides  $s_1 > s_2 > ...$ .

It will be a subslice of a new total block labeled by qindex  $I_s$ . Because many charge combinations fuse to the same total charge, in general there will be many tuples  $(i_1,...,i_{nlegs})$  belonging to the same  $I_s$ . The rows of  $q\_map$  are precisely the collections of  $[b\_j, b\_\{j+1\}, I\_s, i\_1, ..., i\_\{nlegs\}]$ . Here,  $b_j:b_{j+1}$  denotes the slice of this qindex combination within the total block  $I\_s$ , i.e.,  $b\_j=a\_j-self$ . slices  $[I\_s]$ .

The rows of  $q\_map$  are lex-sorted first by  $\mathbb{I}\_s$ , then the  $\mathbb{i}$ . Each  $\mathbb{I}\_s$  will have multiple rows, and the order in which they are stored in q map is the order the data is stored in the actual tensor, i.e., it might look like

```
[ ...,
 [ b_j, b_{j+1}, I_s, i_1, ..., i_{nlegs} ],
 [ b_{j+1}, b_{j+2}, I_s, i'_1, ..., i'_{nlegs} ],
 [ 0, b_{j+3}, I_s + 1, i''_1, ..., i''_{nlegs} ],
 [ b_{j+3}, b_{j+4}, I_s + 1, i'''_1, ..., i'''_{nlegs}],
 ...]
```

The charge fusion rule is:

```
self.charges[Qi]*self.gcon; == sum([1.charges[qi_1]*1.gcon; for 1 in self.legs])
→mod amod
Here the qindex Qi of the pipe corresponds to qindices qi_l on the individual legs.
copy()
     Return a (shallow) copy of self.
save_hdf5 (hdf5_saver, h5gr, subpath)
     Export self into a HDF5 file.
     This method saves all the data it needs to reconstruct self with from_hdf5().
     In addition to the data saved for the LegCharge, it just saves the legs as subgroup.
         Parameters
             • hdf5_saver (Hdf5Saver) - Instance of the saving engine.
             • h5gr (:class`Group`) – HDF5 group which is supposed to represent self.
             • subpath (str) – The name of h5gr with a '/' in the end.
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
     Load instance from a HDF5 file.
     This method reconstructs a class instance from the data saved with save_hdf5().
         Parameters
             • hdf5_loader (Hdf5Loader) - Instance of the loading engine.
             • h5gr (Group) – HDF5 group which is represent the object to be constructed.
             • subpath (str) – The name of h5gr with a '/' in the end.
         Returns obj – Newly generated class instance containing the required data.
         Return type cls
test_sanity()
     Sanity check, raises ValueErrors, if something is wrong.
to_LegCharge()
     Convert self to a LegCharge, discarding the information how to split the legs.
     Usually not needed, but called by functions, which are not implemented for a LegPipe.
conj()
     Return a shallow copy with opposite self.qconj.
         Returns conjugated – Shallow copy of self with flipped qconj. Whenever we contract two
             legs, they need to be conjugated to each other. The incoming legs of the pipe are also conju-
             gated.
         Return type LegCharge
     Like conj(), but don't change qconj for incoming legs.
sort (*args, **kwargs)
     Convert to LegCharge and call LegCharge.sort().
bunch (*args, **kwargs)
     Convert to LegCharge and call LegCharge.bunch().
```

```
project (*args, **kwargs)
```

Convert self to LegCharge and call LegCharge.project().

In general, this could be implemented for a LegPipe, but would make <code>split\_legs()</code> more complicated, thus we keep it simple. If you really want to project and split afterwards, use the following work-around, which is for example used in <code>exact\_diagonalization</code>:

- 1) Create the full pipe and save it separetely.
- 2) Convert the Pipe to a Leg & project the array with it.
- 3) [... do calculations ...]
- 4) To split the 'projected pipe' of A, create and empty array B with the legs of A, but replace the projected leg by the full pipe. Set A as a slice of B. Finally split the pipe.

### map\_incoming\_flat (incoming\_indices)

Map (flat) incoming indices to an index in the outgoing pipe.

**Parameters incoming\_indices** (iterable of int) - One (flat) index on each of the incoming legs.

**Returns outgoing\_index** – The index in the outgoing leg.

Return type int

### charge\_sectors()

Return unique rows of self.charges.

**Returns** charges – Rows are the rows of self.charges lexsorted and without duplicates.

**Return type** array[QTYPE, ndim=2]

### extend(extra)

Return a new LegCharge, which extends self with futher charges.

This is needed to formally increase the dimension of an Array.

**Parameters** extra (LegCharge | int) – By what to extend, i.e. the charges to be appended to *self*. An int stands for extending the length of the array by a single new block of that size and zero charges.

**Returns extended\_leg** – Copy of *self* extended by the charge blocks of the *extra* leg.

Return type LegCharge

## flip\_charges\_qconj()

Return a copy with both negative qconj and charges.

**Returns conj\_charges** – (Shallow) copy of self with negative *qconj* and *charges*, thus representing the very same charges. test\_equal() of self with conj\_charges will not raise an error.

Return type LegCharge

# $\verb|classmethod from_add_charge| (legs, charge info=None)|$

Add the (independent) charges of two or more legs to get larger qnumber.

### **Parameters**

- **legs** (iterable of *LegCharge*) The legs for which the charges are to be combined/added.
- chargeinfo (ChargeInfo) The ChargeInfo for all charges; create new if None.

**Returns combined** – A LegCharge with the charges of both legs. Is neither sorted nor bunched!

### Return type LegCharge

**classmethod from\_change\_charge** (*leg*, *charge*, *new\_qmod*, *new\_name=''*, *chargeinfo=None*) Remove a charge from a LegCharge.

#### **Parameters**

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (int / str) Number or *name* of the charge (within *chinfo*) for which *mod* is to be changed.
- new\_qmod (int) The new mod to be set for charge in the ChargeInfo.
- **new\_name** (str) The new name for *charge*.
- chargeinfo (ChargeInfo) The ChargeInfo with charge changed; create new if None.

Returns leg – A LegCharge with the specified charge changed. Is neither sorted nor bunched!

Return type LegCharge

classmethod from\_drop\_charge (leg, charge=None, chargeinfo=None)

Remove a charge from a LegCharge.

#### **Parameters**

- **leg** (*LegCharge*) The leg from which to drop/remove a charge.
- **charge** (int / str) Number or *name* of the charge (within *chinfo*) which is to be dropped. None means dropping all charges.
- **chargeinfo** (*ChargeInfo*) The ChargeInfo with *charge* dropped; create new if None.

**Returns dropped** – A LegCharge with the specified charge dropped. Is neither sorted nor bunched!

Return type LegCharge

classmethod from\_qdict(chargeinfo, qdict, qconj=1)

Create a LegCharge from qdict form.

### **Parameters**

- **chargeinfo** (*ChargeInfo*) The nature of the charge.
- **qdict** (dict) A dictionary mapping a tuple of charges to slices.

 $\verb|classmethod from_qflat| (\textit{chargeinfo}, \textit{qflat}, \textit{qconj} = 1)$ 

Create a LegCharge from qflat form.

Does *neither* bunch *nor* sort. We recommend to sort (and bunch) afterwards, if you expect that tensors using the LegCharge have entries at all positions compatible with the charges.

#### **Parameters**

- chargeinfo (ChargeInfo) The nature of the charge.
- qflat (array\_like (ind\_len, qnumber)) qnumber charges for each index of the leg on entry.
- qconj ({-1, 1}) A flag telling whether the charge points inwards (+1) or outwards (-1).

#### See also:

```
sort () sorts by charges
     bunch () bunches contiguous blocks of the same charge.
classmethod from_qind(chargeinfo, slices, charges, qconj=1)
     Just a wrapper around self.__init__(), see class doc-string for parameters.
     See also:
     sort () sorts by charges
     bunch () bunches contiguous blocks of the same charge.
classmethod from_trivial(ind_len, chargeinfo=None, qconj=1)
     Create trivial (qnumber=0) LegCharge for given len of indices ind_len.
get_block_sizes()
     Return the sizes of the individual blocks.
         Returns sizes - The sizes of the individual blocks; sizes[i] = slices[i+1] -
             slices[i].
         Return type ndarray, shape (block number,)
get_charge (qindex)
     Return charge self.charges[qindex] * self.qconj for a given qindex.
get_qindex (flat_index)
     Find qindex containing a flat index.
     Given a flat index, to find the corresponding entry in an Array, we need to determine the block it is saved
     in. For example, if slices = [[0, 3], [3, 7], [7, 12]], the flat index 5 corresponds to the
     second entry, qindex = 1 (since 5 is in [3:7]), and the index within the block would be 2 = 5 - 3.
         Parameters flat_index (int) – A flat index of the leg. Negative index counts from behind.
         Returns
             • qindex (int) – The qindex, i.e. the index of the block containing flat_index.
             • index_within_block (int) – The index of flat_index within the block given by qindex.
get_qindex_of_charges (charges)
     Return the slice selecting the block for given charge values.
     Inverse function of get_charge().
         Parameters charges (1D array_like) - Charge values for which the slice of the block is
             to be determined.
         Returns slice(i, j) – Slice of the charge values for
         Return type slice
     :raises ValueError : if the answer is not unique (because self is not blocked).:
get_slice (qindex)
     Return slice selecting the block for a given qindex.
is_blocked()
     Returns whether self is blocked, i.e. qindex map 1:1 to charge values.
is bunched()
     Checks whether bunch () would change something.
```

#### is sorted()

Returns whether *self.charges* is sorted lexiographically.

### perm\_flat\_from\_perm\_qind(perm\_qind)

Convert a permutation of qind (acting on self) into a flat permutation.

```
perm_qind_from_perm_flat (perm_flat)
```

Convert flat permutation into gind permutation.

**Parameters** perm\_flat (1D array) - A permutation acting on self, which doesn't mix the blocks of qind.

**Returns** perm\_qind – The permutation of self.qind described by perm\_flat.

**Return type** 1D array

**Raises** ValueError – If perm\_flat mixes blocks of different qindex.

#### test\_contractible(other)

Raises a ValueError if charges are incompatible for contraction with other.

Parameters other (LegCharge) - The LegCharge of the other leg condsidered for contraction

Raises ValueError – If the charges are incompatible for direct contraction.

#### **Notes**

This function checks that two legs are *ready* for contraction. This is the case, if all of the following conditions are met:

- the ChargeInfo is equal
- the *slices* are equal
- the *charges* are the same up to *opposite* signs qconj:

```
self.charges * self.qconj = - other.charges * other.qconj
```

In general, there could also be a change of the total charge, see *Charge conservation with np\_conserved* This special case is not considered here - instead use <code>gauge\_total\_charge()</code>, if a change of the charge is desired.

If you are sure that the legs should be contractable, check whether the charges are actually valid or whether self and other are blocked or should be sorted.

### See also:

```
test_equal() self.test_contractible(other) just performs self.
test_equal(other.conj()).
```

### test\_equal (other)

Test if charges are *equal* including *qconj*.

Check that all of the following conditions are met:

- the ChargeInfo is equal
- the slices are equal
- the *charges* are the same up to the signs qconj:

```
self.charges * self.qconj = other.charges * other.qconj
```

#### See also:

```
test_contractible() self.test_equal(other) is equivalent to self.
test_contractible(other.conj()).
```

#### to\_qdict()

Return charges in qdict form.

Raises ValueError, if not blocked.

#### to\_qflat()

Return charges in qflat form.

# **Module description**

Basic definitions of a charge.

This module contains implementations for handling the quantum numbers ("charges") of the Array.

In particular, the classes Charge Info, LegCharge and LegPipe are implemented here.

**Note:** The contents of this module are imported in *np\_conserved*, so you usually don't need to import this module in your application.

A detailed introduction to np\_conserved can be found in Charge conservation with np\_conserved.

In this module, some functions have the python decorator @use\_cython. Functions with this decorator are replaced by the ones written in Cython, implemented in the file tenpy/linalg/\_npc\_helper.pyx. For further details, see the definition of use\_cython().

```
tenpy.linalg.charges.QTYPE = <class 'numpy.int64'> Numpy data type for the charges.
```

# 7.8.3 svd\_robust

• full name: tenpy.linalg.svd\_robust

• parent module: tenpy.linalg

• type: module

#### **Functions**

svd(a[, full_matrices, compute_uv,])	Wrapper around scipy.linalg.svd() with gesvd
	backup plan.
svd_gesvd(a[, full_matrices, compute_uv,])	svd with LAPACK's '#gesvd' (with # = d/z for
	float/complex).

#### svd

```
• full name: tenpy.linalg.svd_robust.svd
```

• parent module: tenpy.linalg.svd\_robust

• type: function

```
\label{linalg.svd_robust.svd} \begin{subarray}{ll} tenpy.linalg.svd\_robust.svd (a, & full\_matrices=True, & compute\_uv=True, & overwrite\_a=False, & check\_finite=True, & lapack\_driver='gesdd', & warn=True) \\ & Wrapper around scipy.linalg.svd() & with & gesvd & backup plan. \\ \end{subarray}
```

Tries to avoid raising an LinAlgError by using using the lapack\_driver gesvd, if gesdd failed.

Parameters not described below are as in scipy.linalg.svd()

#### **Parameters**

- overwrite\_a (bool) Ignored (i.e. set to False) if lapack\_driver='gesdd'. Otherwise described in scipy.linalg.svd().
- lapack\_driver ({'gesdd', 'gesvd'}, optional) Whether to use the more efficient divide-and-conquer approach ('gesdd') or general rectangular approach ('gesvd') to compute the SVD. MATLAB and Octave use the 'gesvd' approach. Default is 'gesdd'. If 'gesdd' fails, 'gesvd' is used as backup.
- warn (bool) Whether to create a warning when the SVD failed.

**Returns U, S, Vh** – As described in doc-string of scipy.linalg.svd().

Return type ndarray

# svd\_gesvd

- full name: tenpy.linalg.svd\_robust.svd\_gesvd
- parent module: tenpy.linalg.svd\_robust
- type: function

```
tenpy.linalg.svd_robust.svd_gesvd(a, full_matrices=True, compute_uv=True, check_finite=True)
svd with LAPACK's '#gesvd' (with # = d/z for float/complex).
```

Similar as numpy.linalg.svd(), but use LAPACK 'gesvd' driver. Works only with 2D arrays. Outer part is based on the code of *numpy.linalg.svd*.

#### **Parameters**

- full\_matrices, compute\_uv(a,)-See numpy.linalg.svd() for details.
- **check finite** check whether input arrays contain 'NaN' or 'inf'.

**Returns U, S, Vh – See** numpy.linalg.svd() for details.

Return type ndarray

# **Module description**

(More) robust version of singular value decomposition.

We often need to perform an SVD. In general, an SVD is a matrix factorization that is always well defined and should also work for ill-conditioned matrices. But sadly, both numpy.linalg.svd() and scipy.linalg.svd() fail from time to time, raising LinalgError("SVD did not converge"). The reason is that both of them call the LAPACK function #gesdd (where # depends on the data type), which takes an iterative approach that can fail. However, it is usually much faster than the alternative (and robust) #gesvd.

Our workaround is as follows: we provide a function svd() with call signature as scipy's svd. This function is basically just a wrapper around scipy's svd, i.e., we keep calling the faster dgesdd. But if that fails, we can still use dgesvd as a backup.

Sadly, *dgesvd* and *zgesvd* were not included into scipy until version '0.18.0' (nor in numpy), which is as the time of this writing the latest stable scipy version. For scipy version newer than '0.18.0', we make use of the new keyword 'lapack\_driver' for svd, otherwise we (try to) load *dgesvd* and *zgesvd* from shared LAPACK libraries.

The tribute for the dgesvd wrapper code goes to 'jgarcke', originally posted at http://projects.scipy.org/numpy/ticket/990, which is now hosted at https://github.com/numpy/numpy/issues/1588 He explains a bit more in detail what fails.

The include of dgesvd to scipy was done in https://github.com/scipy/scipy/pull/5994.

# **Examples**

The idea is that you just import the *svd* from this module and use it as replacement for np.linalg.svd or scipy. linalg.svd:

```
>>> from svd_robust import svd
>>> U, S, VT = svd([[1., 1.], [0., [1.]])
```

# 7.8.4 random matrix

• full name: tenpy.linalg.random\_matrix

• parent module: tenpy.linalq

• type: module

### **Functions**

COE(size)	Circular orthogonal ensemble (COE).
CRE(size)	Circular real ensemble (CRE).
CUE(size)	Circular unitary ensemble (CUE).
GOE(size)	Gaussian orthogonal ensemble (GOE).
GUE(size)	Gaussian unitary ensemble (GUE).
O_close_1(size[, a])	return an random orthogonal matrix 'close' to the Iden-
	tity.
U_close_1(size[, a])	return an random orthogonal matrix 'close' to the iden-
	tity.
box(size[, W])	return random number uniform in (-W, W].
standard_normal_complex(size)	return (R + 1. $j*I$ ) for independent R and I from
	np.random.standard_normal.

# COE

```
• full name: tenpy.linalg.random_matrix.COE
   • parent module: tenpy.linalg.random_matrix
   • type: function
tenpy.linalg.random_matrix.COE(size)
     Circular orthogonal ensemble (COE).
          Parameters size (tuple) – (n, n), where n is the dimension of the output matrix.
          Returns U - Unitary, symmetric (complex) matrix drawn from the COE (=Haar measure on this
              space).
          Return type ndarray
CRE
   • full name: tenpy.linalg.random_matrix.CRE
   • parent module: tenpy.linalg.random_matrix
   • type: function
tenpy.linalg.random_matrix.CRE(size)
     Circular real ensemble (CRE).
          Parameters size (tuple) – (n, n), where n is the dimension of the output matrix.
          Returns U – Orthogonal matrix drawn from the CRE (=Haar measure on O(n)).
          Return type ndarray
CUE
   • full name: tenpy.linalg.random_matrix.CUE
   • parent module: tenpy.linalg.random_matrix
   • type: function
tenpy.linalg.random_matrix.CUE(size)
     Circular unitary ensemble (CUE).
          Parameters size (tuple) – (n, n), where n is the dimension of the output matrix.
          Returns U – Unitary matrix drawn from the CUE (=Haar measure on U(n)).
          Return type ndarray
```

# **GOE**

```
full name: tenpy.linalg.random_matrix.GOEparent module: tenpy.linalg.random_matrix
```

· type: function

```
\texttt{tenpy.linalg.random\_matrix.GOE} \ (\textit{size})
```

Gaussian orthogonal ensemble (GOE).

**Parameters** size (tuple) – (n, n), where n is the dimension of the output matrix.

**Returns H** – Real, symmetric numpy matrix drawn from the GOE, i.e.  $p(H) = 1/Zexp(-n/4tr(H^2))$ 

Return type ndarray

### **GUE**

- full name: tenpy.linalg.random\_matrix.GUE
- parent module: tenpy.linalg.random\_matrix
- type: function

```
tenpy.linalg.random_matrix.GUE(size)
```

Gaussian unitary ensemble (GUE).

**Parameters** size (tuple) – (n, n), where n is the dimension of the output matrix.

**Returns H** – Hermitian (complex) numpy matrix drawn from the GUE, i.e.  $p(H) = 1/Zexp(-n/4tr(H^2))$ .

Return type ndarray

# O\_close\_1

- full name: tenpy.linalg.random\_matrix.O\_close\_1
- parent module: tenpy.linalg.random\_matrix
- type: function

```
tenpy.linalg.random_matrix.O_close_1 (size, a=0.01) return an random orthogonal matrix 'close' to the Identity.
```

# **Parameters**

- **size** (tuple) (n, n), where n is the dimension of the output matrix.
- **a** (float) Parameter determining how close the result is on O;  $\lim_{a\to 0} < |O-E| >_a = 0$  (where E is the identity).

**Returns O** – Orthogonal matrix close to the identity (for small *a*).

**Return type** ndarray

### U close 1

- full name: tenpy.linalg.random\_matrix.U\_close\_1
- parent module: tenpy.linalg.random\_matrix
- type: function

tenpy.linalg.random\_matrix.**U\_close\_1** (*size*, *a*=0.01) return an random orthogonal matrix 'close' to the identity.

#### **Parameters**

- **size** (tuple) (n, n), where n is the dimension of the output matrix.
- **a** (float) Parameter determining how close the result is to the identity.  $\lim_{a\to 0} < |O-E|>_a=0$  (where E is the identity).

**Returns U** – Unitary matrix close to the identity (for small a). Eigenvalues are chosen i.i.d. as  $\exp(1.j*a*x)$  with x uniform in [-1, 1].

Return type ndarray

#### box

- full name: tenpy.linalg.random\_matrix.box
- parent module: tenpy.linalg.random\_matrix
- type: function

tenpy.linalg.random\_matrix.**box** (size, W=1.0) return random number uniform in (-W, W].

# standard\_normal\_complex

- full name: tenpy.linalg.random\_matrix.standard\_normal\_complex
- parent module: tenpy.linalg.random\_matrix
- type: function

```
tenpy.linalg.random_matrix.standard_normal_complex (size) return (R + 1.j*I) for independent R and I from np.random.standard_normal.
```

### **Module description**

Provide some random matrix ensembles for numpy.

The implemented ensembles are:

ensemble	matrix class drawn from	measure	invariant under	beta
GOE	real, symmetric	$\sim \exp(-n/4 \operatorname{tr}(H^2))$	orthogonal O	1
GUE	hermitian	$\sim \exp(-n/2 \operatorname{tr}(H^2))$	unitary U	2
CRE	O(n)	Haar	orthogonal O	/
COE	U in U(n) with $U = U^T$	Haar	orthogonal O	1
CUE	U(n)	Haar	unitary U	2
O_close_1	O(n)	?	/	/
U_close_1	U(n)	?	/	/

All functions in this module take a tuple (n, n) as first argument, such that we can use the function  $from\_func()$  to generate a block diagonal Array with the block from the corresponding ensemble, for example:

```
npc.Array.from_func_square(GOE, [leg, leg.conj()])
```

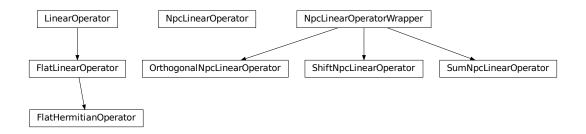
# **7.8.5** sparse

• full name: tenpy.linalg.sparse

• parent module: tenpy.linalg

• type: module

#### **Classes**



FlatHermitianOperator(npc_matvec, leg,	Hermitian variant of FlatLinearOperator.
dtype)	
FlatLinearOperator(npc_matvec, leg, dtype[,	Square Linear operator acting on numpy arrays based
])	on a <i>matvec</i> acting on npc Arrays.
NpcLinearOperator	Prototype for a Linear Operator acting on Array.
NpcLinearOperatorWrapper(orig_operator)	Base class for wrapping around another
	NpcLinearOperator.
OrthogonalNpcLinearOperator(orig_operator,	Replace H -> P H P with the projector P = 1 -
)	sum_o  o> <o .< td=""></o .<>
ShiftNpcLinearOperator(orig_operator, shift)	Representes original_operator + shift *
	identity.
SumNpcLinearOperator(orig_operator,)	Sum of two linear operators.

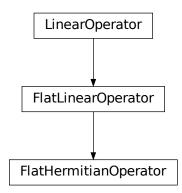
# FlatHermitianOperator

 $\bullet \ \ full \ name: tenpy.linalg.sparse.FlatHermitianOperator$ 

• parent module: tenpy.linalg.sparse

• type: class

# **Inheritance Diagram**



# **Methods**

FlatHermitianOperator.	Initialize this LinearOperator.
init(npc_matvec,)	
FlatHermitianOperator.adjoint()	Hermitian adjoint.
FlatHermitianOperator.dot(x)	Matrix-matrix or matrix-vector multiplication.
FlatHermitianOperator.flat_to_npc(vec)	Convert flat vector of selected charge sector into npc Ar-
	ray.
FlatHermitianOperator.	Convert flat vector of undetermined charge sectors into
flat_to_npc_None_sector(vec)	npc Array.
FlatHermitianOperator.	Convert flat vector of all charge sectors into npc Array
flat_to_npc_all_sectors(vec)	with extra "charge" leg.
FlatHermitianOperator.	Create a FlatLinearOperator from a square Array.
$from\_NpcArray(mat[,])$	
FlatHermitianOperator.	Create a FlatLinearOperator` from a matvec function
from_guess_with_pipe $(\dots)$	acting on multiple legs.
FlatHermitianOperator.matmat(X)	Matrix-matrix multiplication.
FlatHermitianOperator.matvec(x)	Matrix-vector multiplication.
FlatHermitianOperator.	Convert npc Array with qtotal = self.charge_sector into
npc_to_flat(npc_vec)	ndarray.
FlatHermitianOperator.	Convert npc Array with qtotal = self.charge_sector into
<pre>npc_to_flat_all_sectors(npc_vec)</pre>	ndarray.
	continues on next name

continues on next page

Table 62 – continued from previous page

FlatHermitianOperator.rmatmat( $X$ )	Adjoint matrix-matrix multiplication.
FlatHermitianOperator.rmatvec $(x)$	Adjoint matrix-vector multiplication.
FlatHermitianOperator.transpose()	Transpose this linear operator.

### **Class Attributes and Properties**

FlatHermitianOperator.H	Hermitian adjoint.
FlatHermitianOperator.T	Transpose this linear operator.
FlatHermitianOperator.charge_sector	Charge sector of the vector which is acted on.
FlatHermitianOperator.ndim	

Bases: tenpy.linalg.sparse.FlatLinearOperator

Hermitian variant of FlatLinearOperator.

Note that we don't check matvec() to return a hermitian result, we only define an adjoint to be self.

# property H

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self.H instead of self.adjoint().

**Returns** A\_H – Hermitian adjoint of self.

Return type LinearOperator

### property T

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

### adjoint()

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self. H instead of self.adjoint().

**Returns** A\_H – Hermitian adjoint of self.

Return type LinearOperator

# property charge\_sector

Charge sector of the vector which is acted on.

#### dot(x)

Matrix-matrix or matrix-vector multiplication.

**Parameters x** (array\_like) – 1-d or 2-d array, representing a vector or matrix.

**Returns** Ax - 1-d or 2-d array (depending on the shape of x) that represents the result of applying this linear operator on x.

#### **Return type** array

### flat\_to\_npc(vec)

Convert flat vector of selected charge sector into npc Array.

**Parameters vec** (1D ndarray) – Numpy vector to be converted. Should have the entries according to self.charge\_sector.

**Returns npc\_vec** – Same as *vec*, but converted into a npc array.

Return type Array

#### flat\_to\_npc\_None\_sector(vec)

Convert flat vector of undetermined charge sectors into npc Array.

The charge sector to be used is chosen as the block with the maximal norm, not by *self.charge\_sector* (which might be *None*).

Parameters vec (1D ndarray) - Numpy vector to be converted.

**Returns** npc\_vec – Same as *vec*, but converted into a npc array.

Return type Array

#### flat\_to\_npc\_all\_sectors (vec)

Convert flat vector of all charge sectors into npc Array with extra "charge" leg.

**Parameters** vec (1D ndarray) – Numpy vector to be converted.

**Returns** npc\_vec – Same as *vec*, but converted into a npc array.

Return type Array

# classmethod from\_NpcArray (mat, charge\_sector=0)

Create a *FlatLinearOperator* from a square *Array*.

### **Parameters**

- mat (Array) A square matrix, with contractable legs.
- **charge\_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., *assumes* the dominant eigenvector is in charge sector 0.

Create a *FlatLinearOperator*` from a *matvec* function acting on multiple legs.

This function creates a wrapper *matvec* function to allow acting on a "vector" with multiple legs. The wrapper combines the legs into a *LegPipe* before calling the actual *matvec* function, and splits them again in the end.

#### **Parameters**

- npc\_matvec (function) Function to calculate the action of the linear operator on an npc vector with the given split labels *labels\_split*. Has to return an npc vector with the same legs.
- **v0\_guess** (*Array*) Initial guess/starting vector which can be applied to *npc\_matvec*.
- labels\_split (None | list of str) Labels of v0\_guess in the order in which they are to be combined into a LegPipe. None defaults to v0\_guess.get\_leg\_labels().
- **dtype** (*np.dtype* / *None*) The data type of the arrays. None defaults to dtype of  $v0\_guess$  (!).

#### Returns

- lin\_op (cls) Instance of the class to be used as linear operator
- **guess\_flat** (*np.ndarray*) Numpy vector representing the guess v0\_guess.

#### matmat(X)

Matrix-matrix multiplication.

Performs the operation y=A\*X where A is an MxN linear operator and X dense N\*K matrix or ndarray.

```
Parameters X ({matrix, ndarray}) – An array with shape (N,K).
```

**Returns** Y – A matrix or ndarray with shape (M,K) depending on the type of the X argument.

**Return type** {matrix, ndarray}

#### **Notes**

This matmat wraps any user-specified matmat routine or overridden \_matmat method to ensure that y has the correct type.

#### matvec(x)

Matrix-vector multiplication.

Performs the operation y=A\*x where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters x (\{matrix, ndarray\}) – An array with shape (N,) or (N,1).
```

**Returns** y - A matrix or ndarray with shape (M,) or (M,1) depending on the type and shape of the x argument.

**Return type** {matrix, ndarray}

### **Notes**

This matvec wraps the user-specified matvec routine or overridden \_matvec method to ensure that y has the correct shape and type.

```
npc_to_flat (npc_vec)
```

Convert npc Array with qtotal = self.charge\_sector into ndarray.

**Parameters** npc\_vec (Array) – Npc Array to be converted. Should only have entries in *self.charge\_sector*.

**Returns** vec – Same as *npc\_vec*, but converted into a flat Numpy array.

Return type 1D ndarray

#### npc\_to\_flat\_all\_sectors (npc\_vec)

Convert npc Array with qtotal = self.charge\_sector into ndarray.

**Parameters** npc\_vec (Array) - Npc Array to be converted. Should only have entries in self.charge\_sector.

**Returns** vec – Same as *npc\_vec*, but converted into a flat Numpy array.

Return type 1D ndarray

#### rmatmat(X)

Adjoint matrix-matrix multiplication.

Performs the operation  $y = A^H * x$  where A is an MxN linear operator and x is a column vector or 1-d array, or 2-d array. The default implementation defers to the adjoint.

```
Parameters X ({matrix, ndarray}) – A matrix or 2D array.
```

**Returns** Y – A matrix or 2D array depending on the type of the input.

Return type {matrix, ndarray}

#### **Notes**

This rmatmat wraps the user-specified rmatmat routine.

### rmatvec(x)

Adjoint matrix-vector multiplication.

Performs the operation  $y = A^H * x$  where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters x (\{matrix, ndarray\}) – An array with shape (M,) or (M,1).
```

**Returns** y - A matrix or ndarray with shape (N,) or (N,1) depending on the type and shape of the x argument.

**Return type** {matrix, ndarray}

#### **Notes**

This rmatvec wraps the user-specified rmatvec routine or overridden \_rmatvec method to ensure that y has the correct shape and type.

#### transpose()

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

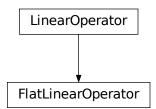
# FlatLinearOperator

• full name: tenpy.linalg.sparse.FlatLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

### **Inheritance Diagram**



# Methods

FlatLinearOperatorinit(npc_matvec,	Initialize this LinearOperator.
leg,)	
FlatLinearOperator.adjoint()	Hermitian adjoint.
FlatLinearOperator.dot(x)	Matrix-matrix or matrix-vector multiplication.
FlatLinearOperator.flat_to_npc(vec)	Convert flat vector of selected charge sector into npc Ar-
	ray.
FlatLinearOperator.	Convert flat vector of undetermined charge sectors into
flat_to_npc_None_sector(vec)	npc Array.
FlatLinearOperator.	Convert flat vector of all charge sectors into npc Array
flat_to_npc_all_sectors(vec)	with extra "charge" leg.
FlatLinearOperator.from_NpcArray(mat[,	Create a FlatLinearOperator from a square Array.
])	
FlatLinearOperator.	Create a FlatLinearOperator` from a matvec function
from_guess_with_pipe()	acting on multiple legs.
FlatLinearOperator. $\mathtt{matmat}(X)$	Matrix-matrix multiplication.
FlatLinearOperator.matvec(x)	Matrix-vector multiplication.
FlatLinearOperator.npc_to_flat(npc_vec)	Convert npc Array with qtotal = self.charge_sector into
	ndarray.
FlatLinearOperator.	Convert npc Array with qtotal = self.charge_sector into
<pre>npc_to_flat_all_sectors(npc_vec)</pre>	ndarray.
FlatLinearOperator.rmatmat( $X$ )	Adjoint matrix-matrix multiplication.
FlatLinearOperator.rmatvec(x)	Adjoint matrix-vector multiplication.
FlatLinearOperator.transpose()	Transpose this linear operator.

# **Class Attributes and Properties**

FlatLinearOperator.H	Hermitian adjoint.
FlatLinearOperator.T	Transpose this linear operator.
FlatLinearOperator.charge_sector	Charge sector of the vector which is acted on.
FlatLinearOperator.ndim	

Bases: scipy.sparse.linalg.interface.LinearOperator

Square Linear operator acting on numpy arrays based on a *matvec* acting on npc Arrays.

Note that this class represents a square linear operator. In terms of charges, this means it has legs [self.leg.conj(), self.leg] and trivial (zero) qtotal.

#### **Parameters**

- npc\_matvec (function) Function to calculate the action of the linear operator on an npc vector (with the specified *leg*). Has to return an npc vector with the same leg.
- **leg** (*LegCharge*) Leg of the vector on which *npc\_matvec* can act on.
- **dtype** (np. dtype) The data type of the arrays.
- **charge\_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., *assumes* the dominant eigenvector is in charge sector 0.

• **vec\_label** (*None* / *str*) – Label to be set to the npc vector before acting on it with *npc\_matvec*. Ignored if *None*.

# possible\_charge\_sectors

Each row corresponds to one possible choice for *charge\_sector*.

**Type** ndarray[QTYPE, ndim=2]

#### shape

The dimensions for the selected charge sector.

Type (int, int)

#### dtype

The data type of the arrays.

Type np.dtype

#### leg

Leg of the vector on which npc\_matvec can act on.

Type LegCharge

#### vec label

Label to be set to the npc vector before acting on it with *npc\_matvec*. Ignored if *None*.

Type None | str

### npc\_matvec

Function to calculate the action of the linear operator on an npc vector (with one *leg*).

Type function

### matvec\_count

The number of times *npc\_matvec* was called.

Type int

#### mask

The indices of *leg* corresponding to the *charge\_sector* to be diagonalized.

**Type** ndarray[ndim=1, bool]

# \_npc\_matvec\_multileg

Only set if initalized with <code>from\_guess\_with\_pipe()</code>. The <code>npc\_matvec</code> function to be wrapped around. Takes the npc Array in multidimensional form and returns it that way.

Type function | None

#### labels split

Only set if initalized with from\_guess\_with\_pipe(). Labels of the guess before combining them into a pipe (stored as leg).

Type list of str

# classmethod from\_NpcArray(mat, charge\_sector=0)

Create a *FlatLinearOperator* from a square *Array*.

#### **Parameters**

- mat (Array) A square matrix, with contractable legs.
- **charge\_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., *assumes* the dominant eigenvector is in charge sector 0.

Create a FlatLinearOperator` from a matvec function acting on multiple legs.

This function creates a wrapper *matvec* function to allow acting on a "vector" with multiple legs. The wrapper combines the legs into a *LegPipe* before calling the actual *matvec* function, and splits them again in the end.

### **Parameters**

- **npc\_matvec** (function) Function to calculate the action of the linear operator on an npc vector with the given split labels *labels\_split*. Has to return an npc vector with the same legs.
- **v0\_guess** (Array) Initial guess/starting vector which can be applied to *npc\_matvec*.
- labels\_split (None | list of str) Labels of v0\_guess in the order in which they are to be combined into a LegPipe. None defaults to v0\_guess.get\_leg\_labels().
- **dtype** (*np.dtype* / *None*) The data type of the arrays. None defaults to dtype of  $v\theta\_guess$  (!).

#### Returns

- lin\_op (cls) Instance of the class to be used as linear operator
- **guess\_flat** (*np.ndarray*) Numpy vector representing the guess *v0\_guess*.

#### property charge\_sector

Charge sector of the vector which is acted on.

#### flat to npc(vec)

Convert flat vector of selected charge sector into npc Array.

**Parameters vec** (1D ndarray) – Numpy vector to be converted. Should have the entries according to self.charge\_sector.

**Returns** npc\_vec – Same as *vec*, but converted into a npc array.

Return type Array

### npc\_to\_flat (npc\_vec)

Convert npc Array with qtotal = self.charge\_sector into ndarray.

**Parameters** npc\_vec (Array) - Npc Array to be converted. Should only have entries in self.charge\_sector.

**Returns** vec – Same as *npc\_vec*, but converted into a flat Numpy array.

**Return type** 1D ndarray

#### flat to npc all sectors(vec)

Convert flat vector of all charge sectors into npc Array with extra "charge" leg.

**Parameters vec** (1D ndarray) – Numpy vector to be converted.

**Returns npc\_vec** – Same as *vec*, but converted into a npc array.

Return type Array

### flat\_to\_npc\_None\_sector(vec)

Convert flat vector of undetermined charge sectors into npc Array.

The charge sector to be used is chosen as the block with the maximal norm, not by *self.charge\_sector* (which might be *None*).

**Parameters vec** (1D ndarray) – Numpy vector to be converted.

**Returns** npc\_vec – Same as *vec*, but converted into a npc array.

Return type Array

### npc\_to\_flat\_all\_sectors (npc\_vec)

Convert npc Array with qtotal = self.charge\_sector into ndarray.

**Parameters** npc\_vec (Array) – Npc Array to be converted. Should only have entries in *self.charge\_sector*.

**Returns vec** – Same as *npc\_vec*, but converted into a flat Numpy array.

Return type 1D ndarray

#### property H

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self.H instead of self.adjoint().

**Returns** A\_H – Hermitian adjoint of self.

Return type LinearOperator

#### property T

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

### adjoint()

Hermitian adjoint.

Returns the Hermitian adjoint of self, aka the Hermitian conjugate or Hermitian transpose. For a complex matrix, the Hermitian adjoint is equal to the conjugate transpose.

Can be abbreviated self. H instead of self. adjoint().

**Returns** A\_H – Hermitian adjoint of self.

Return type LinearOperator

# dot(x)

Matrix-matrix or matrix-vector multiplication.

**Parameters x** (*array\_like*) − 1-d or 2-d array, representing a vector or matrix.

**Returns** Ax - 1-d or 2-d array (depending on the shape of x) that represents the result of applying this linear operator on x.

Return type array

#### matmat(X)

Matrix-matrix multiplication.

Performs the operation y=A\*X where A is an MxN linear operator and X dense N\*K matrix or ndarray.

**Parameters X** ({matrix, ndarray}) - An array with shape (N,K).

**Returns** Y – A matrix or ndarray with shape (M,K) depending on the type of the X argument.

**Return type** {matrix, ndarray}

#### **Notes**

This matmat wraps any user-specified matmat routine or overridden \_matmat method to ensure that y has the correct type.

#### matvec(x)

Matrix-vector multiplication.

Performs the operation y=A\*x where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters \mathbf{x} ({matrix, ndarray}) – An array with shape (N,) or (N,1).
```

**Returns** y - A matrix or ndarray with shape (M,) or (M,1) depending on the type and shape of the x argument.

**Return type** {matrix, ndarray}

### **Notes**

This matvec wraps the user-specified matvec routine or overridden \_matvec method to ensure that y has the correct shape and type.

#### rmatmat(X)

Adjoint matrix-matrix multiplication.

Performs the operation  $y = A^H * x$  where A is an MxN linear operator and x is a column vector or 1-d array, or 2-d array. The default implementation defers to the adjoint.

```
Parameters X ({matrix, ndarray}) – A matrix or 2D array.
```

**Returns** Y – A matrix or 2D array depending on the type of the input.

**Return type** {matrix, ndarray}

# Notes

This rmatmat wraps the user-specified rmatmat routine.

#### rmatvec(x)

Adjoint matrix-vector multiplication.

Performs the operation  $y = A^H * x$  where A is an MxN linear operator and x is a column vector or 1-d array.

```
Parameters x (\{matrix, ndarray\}) – An array with shape (M,) or (M,1).
```

**Returns** y - A matrix or ndarray with shape (N,) or (N,1) depending on the type and shape of the x argument.

**Return type** {matrix, ndarray}

#### **Notes**

This rmatvec wraps the user-specified rmatvec routine or overridden \_rmatvec method to ensure that y has the correct shape and type.

### transpose()

Transpose this linear operator.

Returns a LinearOperator that represents the transpose of this one. Can be abbreviated self.T instead of self.transpose().

# **NpcLinearOperator**

• full name: tenpy.linalg.sparse.NpcLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

### **Inheritance Diagram**

**NpcLinearOperator** 

### Methods

NpcLinearOperatorinit	Initialize self.
NpcLinearOperator.adjoint()	Return the hermitian conjugate of <i>self</i>
NpcLinearOperator.matvec(vec)	Calculate the action of the operator on a vector <i>vec</i> .
NpcLinearOperator.to_matrix()	Contract self to a matrix.

# **Class Attributes and Properties**

NpcLinearOperator.acts\_on

# class tenpy.linalg.sparse.NpcLinearOperator

Bases: object

Prototype for a Linear Operator acting on Array.

Note that an Array implements a matvec function. Thus you can use any (square) npc Array as an NpcLinear-Operator.

# dtype

The data type of its action.

### Type np.type

# acts\_on

Labels of the state on which the operator can act. NB: Class attribute.

Type list of str

#### matvec(vec)

Calculate the action of the operator on a vector vec.

Note that we don't require *vec* to be one-dimensional. However, for square operators we require that the result of *matvec* has the same legs (in the same order) as *vec* such that they can be added. Note that this excludes a non-trivial *qtotal* for square operators.

#### to\_matrix()

Contract self to a matrix.

If *self* represents an operator with very small shape, e.g. because the MPS bond dimension is very small, an algorithm might choose to contract *self* to a single tensor.

**Returns** matrix – Contraction of the represented operator.

**Return type** Array

### adjoint()

Return the hermitian conjugate of self

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

# **NpcLinearOperatorWrapper**

• full name: tenpy.linalg.sparse.NpcLinearOperatorWrapper

• parent module: tenpy.linalg.sparse

• type: class

# **Inheritance Diagram**

**NpcLinearOperatorWrapper** 

### Methods

NpcLinearOperatorWrapper.	Initialize self.
init(orig_operator)	
NpcLinearOperatorWrapper.adjoint()	Return the hermitian conjugate of <i>self</i> .
NpcLinearOperatorWrapper.to_matrix()	Contract <i>self</i> to a matrix.
NpcLinearOperatorWrapper.unwrapped()	Return to the original NpcLinearOperator.

# class tenpy.linalg.sparse.NpcLinearOperatorWrapper(orig\_operator)

Bases: object

Base class for wrapping around another NpcLinearOperator.

Attributes not explicitly set with self.attribute = value (or by defining methods) default to the attributes of the wrapped  $orig\_operator$ .

**Warning:** If there are multiple levels of wrapping operators, the order might be critical to get correct results; e.g. <code>OrthogonalNpcLinearOperator</code> needs to be the outer-most wrapper to produce correct results and/or be efficient.

**Parameters orig\_operator** (NpcLinearOperator) - The original operator implementing the *matvec*.

#### orig\_operator

The original operator implementing the *matvec*.

**Type** NpcLinearOperator

### unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

#### to\_matrix()

Contract self to a matrix.

### adjoint()

Return the hermitian conjugate of self.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

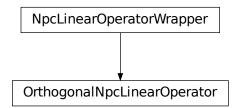
# OrthogonalNpcLinearOperator

• full name: tenpy.linalg.sparse.OrthogonalNpcLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

# **Inheritance Diagram**



# **Methods**

OrthogonalNpcLinearOperatorinit()	Initialize self.
OrthogonalNpcLinearOperator.adjoint()	Return the hermitian conjugate of <i>self</i> .
OrthogonalNpcLinearOperator.	
matvec(vec)	
OrthogonalNpcLinearOperator.	Contract <i>self</i> to a matrix.
to_matrix()	
OrthogonalNpcLinearOperator.	Return to the original NpcLinearOperator.
unwrapped()	

class tenpy.linalg.sparse.OrthogonalNpcLinearOperator(orig\_operator, ortho\_vecs)
 Bases: tenpy.linalg.sparse.NpcLinearOperatorWrapper

buses. cempy. ilmaig. spaise. wpellinear operator wrapper

Replace H  $\rightarrow$  P H P with the projector P = 1  $\rightarrow$  sum\_o  $\mid$ o>  $\triangleleft$ o|.

Here, | o > are the vectors from ortho\_vecs.

# **Parameters**

- orig\_operator (EffectiveH) The original *EffectiveH* instance to wrap around.
- ortho\_vecs (list of Array) The vectors to orthogonalize against.

### to\_matrix()

Contract self to a matrix.

# adjoint()

Return the hermitian conjugate of self.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

# unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

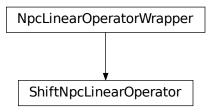
# **ShiftNpcLinearOperator**

• full name: tenpy.linalg.sparse.ShiftNpcLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

# **Inheritance Diagram**



### **Methods**

ShiftNpcLinearOperatorinit()	Initialize self.
ShiftNpcLinearOperator.adjoint()	Return the hermitian conjugate of <i>self</i> .
ShiftNpcLinearOperator.matvec(vec)	
ShiftNpcLinearOperator.to_matrix()	Contract <i>self</i> to a matrix.
ShiftNpcLinearOperator.unwrapped()	Return to the original NpcLinearOperator.

```
class tenpy.linalg.sparse.ShiftNpcLinearOperator(orig_operator, shift)
```

Bases: tenpy.linalg.sparse.NpcLinearOperatorWrapper

Representes original\_operator + shift \* identity.

This can be useful e.g. for better Lanczos convergence.

# to\_matrix()

Contract self to a matrix.

# adjoint()

Return the hermitian conjugate of self.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

### unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

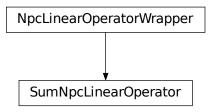
# **SumNpcLinearOperator**

• full name: tenpy.linalg.sparse.SumNpcLinearOperator

• parent module: tenpy.linalg.sparse

• type: class

# **Inheritance Diagram**



### **Methods**

SumNpcLinearOperator.	Initialize self.
init(orig_operator,)	
SumNpcLinearOperator.adjoint()	Return the hermitian conjugate of <i>self</i> .
SumNpcLinearOperator.matvec(vec)	
SumNpcLinearOperator.to_matrix()	Contract <i>self</i> to a matrix.
SumNpcLinearOperator.unwrapped()	Return to the original NpcLinearOperator.

```
class tenpy.linalg.sparse.SumNpcLinearOperator(orig_operator, other_operator)
```

Bases: tenpy.linalg.sparse.NpcLinearOperatorWrapper

Sum of two linear operators.

### to\_matrix()

Contract self to a matrix.

# adjoint()

Return the hermitian conjugate of self.

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

# unwrapped()

Return to the original NpcLinearOperator.

If multiple levels of wrapping were used, this returns the most unwrapped one.

# **Module description**

Providing support for sparse algorithms (using matrix-vector products only).

Some linear algebra algorithms, e.g. Lanczos, do not require the full representations of a linear operator, but only the action on a vector, i.e., a matrix-vector product *matvec*. Here we define the structure of such a general operator, *NpcLinearOperator*, as it is used in our own implementations of these algorithms (e.g., *lanczos*). Moreover, the *FlatLinearOperator* allows to use all the scipy sparse methods by providing functionality to convert flat numpy arrays to and from np\_conserved arrays.

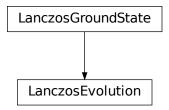
# 7.8.6 lanczos

• full name: tenpy.linalg.lanczos

• parent module: tenpy.linalg

• type: module

### **Classes**



LanczosEvolution(H, psi0, options)	Calculate $exp(deltaH) psi0>$ using Lanczos.
LanczosGroundState(H, psi0, options[,])	Lanczos algorithm working on npc arrays.

# **Functions**

<pre>gram_schmidt(vecs[, rcond, verbose])</pre>	In place Gram-Schmidt Orthogonalization and normal-
	ization for npc Arrays.
lanczos(H, psi[, options, orthogonal_to])	Simple wrapper calling LanczosGroundState(H,
	<pre>psi, options, orthogonal_to).run()</pre>
<pre>lanczos_arpack(H, psi[, options, orthogonal_to])</pre>	Use scipy.sparse.linalg.eigsh() to find the
	ground state of <i>H</i> .
plot_stats(ax, Es)	Plot the convergence of the energies.

# gram\_schmidt

```
• full name: tenpy.linalg.lanczos.gram_schmidt
```

```
• parent module: tenpy.linalg.lanczos
```

• type: function

```
tenpy.linalg.lanczos.gram_schmidt(vecs, rcond=1e-14, verbose=0)
```

In place Gram-Schmidt Orthogonalization and normalization for npc Arrays.

#### **Parameters**

- **vecs** (list of *Array*) The vectors which should be orthogonalized. All with the same *order* of the legs. Entries are modified *in place*. if a norm < rcond, the entry is set to *None*.
- rcond (float) Vectors of norm < rcond (after projecting out previous vectors) are discarded.
- **verbose** (*int*) Print additional output if verbose >= 1.

#### Returns

- vecs (list of Array) The ortho-normalized vectors (without any None).
- ov  $(2D \ Array)$  For  $j \ge i$ , ov $[j, i] = npc.inner(vecs[j], vecs[i], range', do_conj=True) (where vecs[j] was orthogonalized to all vecs[k], k < i).$

#### **lanczos**

- full name: tenpy.linalg.lanczos.lanczos
- parent module: tenpy.linalg.lanczos
- type: function

```
tenpy.linalg.lanczos.lanczos(H, psi, options={}, orthogonal_to=[])
Simple wrapper calling LanczosGroundState(H, psi, options, orthogonal_to).run()
```

Deprecated since version 0.6.0: Going to remove the *orthogonal\_to* argument. Instead, replace H with *OrthogonalNpcLinearOperator(H, orthogonal\_to)* using the *OrthogonalNpcLinearOperator*.

```
Parameters psi, options, orthogonal_to(H,)-See LanczosGroundState.
```

Returns See LanczosGroundState.run().

Return type E0, psi0, N

# lanczos\_arpack

- full name: tenpy.linalg.lanczos.lanczos\_arpack
- parent module: tenpy.linalg.lanczos
- type: function

```
tenpy.linalg.lanczos.lanczos_arpack (H, psi, options={}), orthogonal\_to={}[]) Use scipy.sparse.linalg.eigsh() to find the ground state of H.
```

This function has the same call/return structure as <code>lanczos()</code>, but uses the ARPACK package through the functions <code>speigsh()</code> instead of the custom lanczos implementation in <code>LanczosGroundState</code>. This function is mostly intended for debugging, since it requires to convert the vector from np\_conserved <code>Array</code> into a flat numpy array and back during <code>each matvec-operation!</code>

Deprecated since version 0.6.0: Going to remove the *orthogonal\_to* argument. Instead, replace H with *OrthogonalNpcLinearOperator(H, orthogonal\_to)* using the *OrthogonalNpcLinearOperator*.

**Parameters psi, options, orthogonal\_to**(H,)-See LanczosGroundState. H and psi should have/use labels.

#### Returns

- **E0** (*float*) Ground state energy.
- **psi0** (*Array*) Ground state vector.

# plot\_stats

• full name: tenpy.linalg.lanczos.plot\_stats

• parent module: tenpy.linalg.lanczos

· type: function

tenpy.linalg.lanczos.plot\_stats(ax, Es)

Plot the convergence of the energies.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **Es** (list of ndarray.) The energies Lanczos.Es.

### **Module description**

Lanczos algorithm for np\_conserved arrays.

# 7.9 models

• full name: tenpy.models

• parent module: tenpy

• type: module

# **Module description**

Definition of the various models.

For an introduction to models see Models.

The module tenpy.models.model contains base classes for models. The module tenpy.models.lattice contains base classes and implementations of lattices. All other modules in this folder contain model classes derived from these base classes.

# **Submodules**

lattice	Classes to define the lattice structure of a model.
model	This module contains some base classes for models.

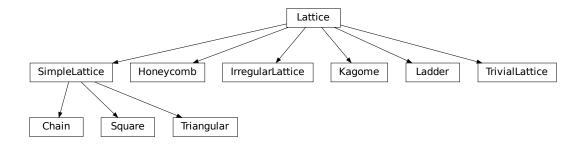
# **7.9.1 lattice**

• full name: tenpy.models.lattice

• parent module: tenpy.models

• type: module

# **Classes**



Chain(L, site, **kwargs)	A chain of L equal sites.
Honeycomb(Lx, Ly, sites, **kwargs)	A honeycomb lattice.
<pre>IrregularLattice(regular_lattice[, remove,])</pre>	A variant of a regular lattice, where we might have extra
	sites or sites missing.
Kagome(Lx, Ly, sites, **kwargs)	A Kagome lattice.
Ladder(L, sites, **kwargs)	A ladder coupling two chains.
Lattice(Ls, unit_cell[, order, bc, bc_MPS,])	A general, regular lattice.
SimpleLattice(Ls, site, **kwargs)	A lattice with a unit cell consiting of just a single site.
Square(Lx, Ly, site, **kwargs)	A square lattice.
Triangular(Lx, Ly, site, **kwargs)	A triangular lattice.
TrivialLattice(mps_sites, **kwargs)	Trivial lattice consisting of a single (possibly large) unit
	cell in 1D.

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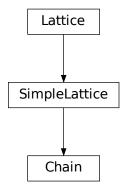
# Chain

• full name: tenpy.models.lattice.Chain

• parent module: tenpy.models.lattice

• type: class

# **Inheritance Diagram**



# **Methods**

Chaininit(L, site, **kwargs)	Initialize self.
Chain.count_neighbors([u, key])	Count e.g.
Chain.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Chain.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Chain.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Chain.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
Chain.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
Chain.mps2lat_values(A[, axes, u])	same as Lattice.mps2lat_values(), but ignore
	u, setting it to 0.
Chain.mps2lat_values_masked(A[, axes,])	Reshape/reorder an array A to replace an MPS index by
	lattice indices.
Chain.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
Chain.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Chain.mps_sites()	Return a list of sites for all MPS indices.
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Table 76 – continued from previous page

	mem brenene beige
Chain.multi_coupling_shape( $dx$ )	Calculate correct shape of the strengths for a
	multi_coupling.
Chain.number_nearest_neighbors([u])	Deprecated.
Chain.number_next_nearest_neighbors([u])	Deprecated.
Chain.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
Chain.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.
Chain.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-
])	tions.
Chain.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.
Chain.plot_order(ax[, order, textkwargs])	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
Chain.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
Chain.position(lat_idx)	return 'space' position of one or multiple sites.
Chain.possible_couplings(u1, u2, dx)	Find possible MPS indices for two-site couplings.
Chain.possible_multi_couplings(ops)	Generalization of possible_couplings() to cou-
	plings with more than 2 sites.
Chain.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
Chain.site(i)	return Site instance corresponding to an MPS index i
Chain.test_sanity()	Sanity check.

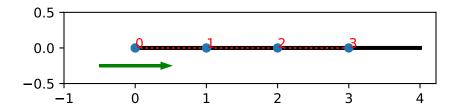
# **Class Attributes and Properties**

Chain.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
Chain.dim	
Chain.nearest_neighbors	
Chain.next_nearest_neighbors	
Chain.next_next_nearest_neighbors	
Chain.order	Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

 ${\bf class} \ {\bf tenpy.models.lattice.Chain} \ (L, site, **kwargs)$ 

 $Bases: \ \textit{tenpy.models.lattice.SimpleLattice}$ 

A chain of L equal sites.



# **Parameters**

- L (int) The lenght of the chain.
- **site** (Site) The local lattice site. The *unit\_cell* of the Lattice is just [site].

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• \*\*kwargs - Additional keyword arguments given to the Lattice. pairs are initialize with [next\_]next\_]nearest\_neighbors. positions can be specified as a single vector.

### ordering(order)

Provide possible orderings of the *N* lattice sites.

The following orders are defined in this method compared to Lattice.ordering():

or-	Resulting order	
der		
'def	uOLt'1, 2, 3, 4, ,L-1	
'fol	$1 \in \mathbb{R}$ , L-1, 1, L-2,, L//2. This order might be usefull if you want to consider a	
ring with periodic boundary conditions with a finite MPS: It avoids the ultra-long range of the		
	coupling from site 0 to L present in the default order.	

#### property boundary conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

### count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

### Return type int

### $coupling\_shape(dx)$

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

#### Returns

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (array) Translation vector from origin to the lower left corner of box spanned by dx.

### enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (int) – The new number of sites in the MPS unit cell will be increased from  $N\_sites$  to factor\*N $\_sites\_per\_ring$ . Since MPS unit cells are repeated in

```
the x-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx*factor, Ly, ..., Lu).
```

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns obj** – Newly generated class instance containing the required data.

Return type cls

# lat2mps\_idx (lat\_idx)

Translate lattice indices  $(x_0, \ldots, x_{D-1})$ , u) to MPS index i.

**Parameters lat\_idx** ( $array\_like$  [.., dim+1]) – The last dimension corresponds to lattice indices ( $x\_0$ , ...,  $x\_\{D-1\}$ , u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc\_MPS$ , an  $x\_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

### $mps2lat_idx(i)$

Translate MPS index i to lattice indices (x\_0, ..., x\_{dim-1}, u).

Parameters i (int / array\_like of int) - MPS index/indices.

**Returns** lat\_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``( $x_0, \ldots, x_{dim-1}, u$ )` corresponding to i. For i accross the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

Return type array

### $mps2lat\_values(A, axes=0, u=None)$

```
same as Lattice.mps2lat_values(), but ignore u, setting it to 0.
```

```
{\tt mps2lat\_values\_masked} \ (A, axes =-1, mps\_inds = None, include\_u = None)
```

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values()$  allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.

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• include\_u((list of) bool) - Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

**Return type** np.ma.MaskedArray

### mps\_idx\_fix\_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit\_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

Return type array

### mps\_lat\_idx\_fix\_u (u=None)

Similar as mps\_idx\_fix\_u(), but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

#### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

### mps\_sites()

Return a list of sites for all MPS indices.

Equivalent to [self.site(i) for i in range(self.N\_sites)].

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

### multi coupling shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

#### Returns

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

# $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

# $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

#### property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

plot\_bc\_identified (ax, direction=- 1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

plot\_coupling(ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (*i0*, *i1*, ..., *u1*) to the site (*i0*+dx[0], *i1*+dx[1], ..., *u2*), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

plot\_order(ax, order=None, textkwargs={}, \*\*kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

# **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.

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- **textkwargs** (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

#### position (lat\_idx)

return 'space' position of one or multiple sites.

```
Parameters lat idx (ndarray, (..., dim+1)) – Lattice indices.
```

**Returns pos** – The position of the lattice sites specified by *lat\_idx* in real-space.

```
Return type ndarray, (..., dim)
```

### $possible\_couplings(u1, u2, dx)$

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

# **Parameters**

- **u2** (*u*1,) Indices within the unit cell; the *u*1 and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

#### Returns

- mps1, mps2 (array) For each possible two-site coupling the MPS indices for the u1 and u2.
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat indices* has only rows within this shape.

#### possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

#### **Returns**

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,... for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.

• **coupling\_shape** (*tuple of int*) – Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

### site(i)

return Site instance corresponding to an MPS index i

#### test\_sanity()

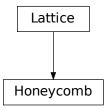
Sanity check.

Raises ValueErrors, if something is wrong.

# Honeycomb

- full name: tenpy.models.lattice.Honeycomb
- parent module: tenpy.models.lattice
- type: class

# **Inheritance Diagram**



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# Methods

Honeycombinit(Lx, Ly, sites, **kwargs)	Initialize self.
Honeycomb.count_neighbors([u, key])	Count e.g.
Honeycomb.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Honeycomb.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Honeycomb.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
Honeycomb.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
Honeycomb.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
Honeycomb.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice
	indices.
Honeycomb.mps2lat_values_masked(A[,	Reshape/reorder an array A to replace an MPS index by
axes,])	lattice indices.
Honeycomb.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
Honeycomb.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Honeycomb.mps_sites()	Return a list of sites for all MPS indices.
Honeycomb.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
Honeycomb.number_nearest_neighbors([u])	Deprecated.
Honeycomb.number_next_nearest_neighbor	
Honeycomb.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
<pre>Honeycomb.plot_basis(ax[, origin, shade])</pre>	Plot arrows indicating the basis vectors of the lattice.
Honeycomb.plot_bc_identified( $ax[,]$ )	Mark two sites indified by periodic boundary condi-
	tions.
Honeycomb.plot_coupling(ax[, coupling,	Plot lines connecting nearest neighbors of the lattice.
wrap])	
<pre>Honeycomb.plot_order(ax[, order, textkwargs])</pre>	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
Honeycomb.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
Honeycomb.position(lat_idx)	return 'space' position of one or multiple sites.
Honeycomb.possible_couplings(u1, u2, dx)	Find possible MPS indices for two-site couplings.
Honeycomb.possible_multi_couplings(ops)	Generalization of possible_couplings() to cou-
	plings with more than 2 sites.
Honeycomb.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
Honeycomb.site(i)	return Site instance corresponding to an MPS index i
Honeycomb.test_sanity()	Sanity check.

### **Class Attributes and Properties**

Honeycomb.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
Honeycomb.dim	
Honeycomb.fifth_nearest_neighbors	
Honeycomb.fourth_nearest_neighbors	
Honeycomb.nearest_neighbors	
Honeycomb.next_nearest_neighbors	
Honeycomb.next_next_nearest_neighbors	
Honeycomb.order	Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

**class** tenpy.models.lattice.**Honeycomb** (*Lx*, *Ly*, *sites*, \*\*kwargs)

Bases: tenpy.models.lattice.Lattice

A honeycomb lattice.

#### **Parameters**

- Ly  $(Lx_{,})$  The length in each direction.
- **sites** ((list of) *Site*) The two local lattice sites making the *unit\_cell* of the *Lattice*. If only a single *Site* is given, it is used for both sites.
- \*\*kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs are set accordingly. For the Honeycomb lattice 'fourth\_nearest\_neighbors', 'fifth\_nearest\_neighbors' are set in pairs.

### ordering(order)

Provide possible orderings of the *N* lattice sites.

The following orders are defined in this method compared to Lattice.ordering():

order	equivalent <i>priority</i>	equivalent snake_winding
'default'	(0, 2, 1)	(False, False, False)
'snake'	(0, 2, 1)	(False, True, False)

### property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

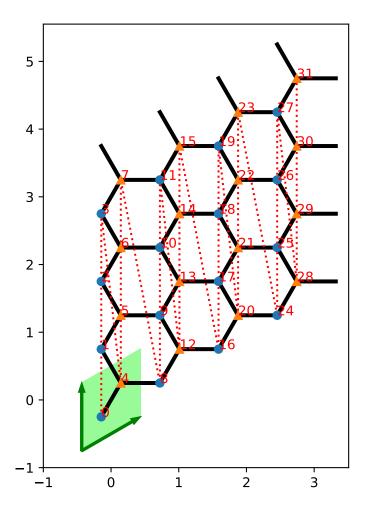
Return type list of str

count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- key(str) Key of pairs to select what to count.



**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

### Return type int

### $coupling\_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

#### **Returns**

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

### enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

### Return type cls

## $lat2mps\_idx(lat\_idx)$

Translate lattice indices  $(x_0, \ldots, x_{D-1})$ , u) to MPS index i.

**Parameters lat\_idx** ( $array_like[..,dim+1]$ ) – The last dimension corresponds to lattice indices ( $x_0, ..., x_{D-1}, u$ ). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_MPS$ , an  $x_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

**Return type** array\_like

### $mps2lat_idx(i)$

Translate MPS index i to lattice indices (x\_0, ..., x\_{dim-1}, u).

Parameters i (int / array\_like of int) - MPS index/indices.

**Returns** lat\_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``( $x_0, \ldots, x_{dim-1}, u$ )` corresponding to i. For i accross the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

### Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder A to replace an MPS index by lattice indices.

#### **Parameters**

- A (ndarray) Some values. Must have A.shape[axes] = self.N\_sites if u is None, or A.shape[axes] = self.N\_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit\_cell[u], as returned by mps\_idx\_fix\_u(). The resulting array will not have the additional dimension(s) of u.

**Returns res\_A** – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

### Examples

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat\_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[lat.mps2lat_idx(5)] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use  $mps\_idx\_fix\_u()$  to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

**Todo:** make sure this function is used for expectation values...

### mps2lat\_values\_masked(A, axes=-1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values$  () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool)—Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

**Return type** np.ma.MaskedArray

### $mps_idx_fix_u(u=None)$

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit\_cell[u].

**Parameters u** (None / int) – Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

Return type array

#### mps lat idx fix u(u=None)

Similar as mps\_idx\_fix\_u(), but return also the corresponding lattice indices.

**Parameters u** (None / int) – Selects a site of the unit cell. None (default) means all sites.

### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

#### mps\_sites()

Return a list of sites for all MPS indices.

Equivalent to [self.site(i) for i in range(self.N\_sites)].

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

#### multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

#### Returns

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

### $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors () instead.

#### $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors () instead.

#### property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

```
plot_basis (ax, origin=0.0, 0.0, shade=None, **kwargs)
```

Plot arrows indicating the basis vectors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **\*\*kwargs** Keyword arguments for ax.arrow.

```
plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)
```

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

### plot\_coupling (ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_order (ax, order=None, textkwargs={}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

### position(lat\_idx)

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

**Returns pos** – The position of the lattice sites specified by *lat\_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible couplings (u1, u2, dx)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

#### **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u1* and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

### Returns

• mps1, mps2 (array) – For each possible two-site coupling the MPS indices for the u1 and u2.

- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

### possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

#### **Returns**

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

### site(i)

return Site instance corresponding to an MPS index i

## test\_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

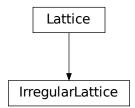
### **IrregularLattice**

• full name: tenpy.models.lattice.IrregularLattice

• parent module: tenpy.models.lattice

• type: class

# Inheritance Diagram



## Methods

IrregularLatticeinit(regular_lattice[,	Initialize self.
])	
IrregularLattice.count_neighbors([u,	Count e.g.
key])	
IrregularLattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
IrregularLattice.	Repeat the unit cell for infinite MPS boundary condi-
enlarge_mps_unit_cell([factor])	tions; in place.
IrregularLattice.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
IrregularLattice.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index <i>i</i> .
IrregularLattice.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
IrregularLattice.mps2lat_values(A[,	Reshape/reorder A to replace an MPS index by lattice
axes, u])	indices.
IrregularLattice.	Reshape/reorder an array A to replace an MPS index by
${\it mps2lat\_values\_masked}(A[,\dots])$	lattice indices.
<pre>IrregularLattice.mps_idx_fix_u([u])</pre>	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
IrregularLattice.	Similar as mps_idx_fix_u(), but return also the
${\it mps\_lat\_idx\_fix\_u([u])}$	corresponding lattice indices.
<pre>IrregularLattice.mps_sites()</pre>	Return a list of sites for all MPS indices.
IrregularLattice.	Calculate correct shape of the strengths for a
multi_coupling_shape(dx)	multi_coupling.
IrregularLattice.	Deprecated.
$number\_nearest\_neighbors  extbf{([u])}$	
IrregularLattice.	Deprecated.
$number\_next\_nearest\_neighbors$ ([u])	
IrregularLattice.ordering(order)	Provide possible orderings of the lattice sites.
IrregularLattice.plot_basis(ax[, origin,	Plot arrows indicating the basis vectors of the lattice.
shade])	
IrregularLattice.	Mark two sites indified by periodic boundary condi-
$plot\_bc\_identified(ax[,])$	tions.
	continues on next page

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$IrregularLattice.plot\_coupling(ax[,])$	Plot lines connecting nearest neighbors of the lattice.
IrregularLattice.plot_order(ax[, order,	Plot a line connecting sites in the specified "order" and
])	text labels enumerating them.
<pre>IrregularLattice.plot_sites(ax[, markers])</pre>	Plot the sites of the lattice with markers.
IrregularLattice.position(lat_idx)	return 'space' position of one or multiple sites.
IrregularLattice.	Find possible MPS indices for two-site couplings.
possible_couplings(u1, u2, dx)	
IrregularLattice.	Generalization of possible_couplings() to cou-
possible_multi_couplings(ops)	plings with more than 2 sites.
IrregularLattice.save_hdf5(hdf5_saver,	Export self into a HDF5 file.
h5gr,)	
IrregularLattice.site(i)	return Site instance corresponding to an MPS index i
IrregularLattice.test_sanity()	Sanity check.

### **Class Attributes and Properties**

IrregularLattice.boundary_conditions	Human-readable list of boundary conditions from bc
	and bc_shift.
IrregularLattice.dim	The dimension of the lattice.
IrregularLattice.nearest_neighbors	
IrregularLattice.	
next_nearest_neighbors	
IrregularLattice.	
next_next_nearest_neighbors	
IrregularLattice.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

Bases: tenpy.models.lattice.Lattice

A variant of a regular lattice, where we might have extra sites or sites missing.

**Note:** The lattice defines only the geometry of the sites, not the couplings; you can have position-dependent couplings/onsite terms despite having a regular lattice.

By adjusting the *order* and a few private attributes and methods, we can make functions like *possible\_couplings()* work with a more "irregular" lattice structure, where some of the sites are missing and other sites added instead.

### **Parameters**

- regular\_lattice (Lattice) The lattice this is based on.
- remove (2D array | None) Each row is a lattice index (x\_0, ..., x\_{dim-1}, u) of a site to be removed. If None, don't remove something.
- add (Tuple[2D array, list] / None) Each row of the 2D array is a lattice index (x\_0, ..., x\_{dim-1}, u) specifying where a site is to be added; u is the index of the site within the final unit\_cell of the irregular lattice. For each row of the 2D array, there is one entry in the list specifying where the site is inserted in the MPS; the values are compared to the MPS indices of the regular lattice and sorted into it, so "2.5"

goes between what was site 2 and 3 in the regular lattice. An entry *None* indicates that the site should be inserted after the lattice site  $(x_0, \ldots, x_{\min-1}, -1)$  of the *regular lattice*.

- add\_unit\_cell (list of Site) Extra sites to be added to the unit cell.
- add\_positions (iterable of 1D arrays) For each extra site in add\_unit\_cell the position within the unit cell. Defaults to np.zeros((len(add\_unit\_cell), dim)).

### regular\_lattice

The lattice this is based on.

```
Type Lattice
```

#### remove, add

See above. Used in ordering () only.

**Type** 2D array | None

### **Examples**

Let's imagine that we have two different sites; for concreteness we can thing of a fermion site, which we represent with 'F', and a spin site 'S'.

You could now imagine that to have fermion chain with spins on the "bonds". In the periodic/infinite case, you would simply define

```
>>> lat = Lattice([2], unit_cell=['F', 'S'], bc='periodic', bc_MPS='infinite')
>>> lat.mps_sites()
['F', 'S', 'F', 'S']
```

For a finite system, you don't want to terminate with a spin on the right, so you need to remove the very last site by specifying the lattice index [L-1, 1] of that site:

```
>>> L = 4
>>> reg_lat = Lattice([L], unit_cell=['F', 'S'], bc='open', bc_MPS='finite')
>>> irr_lat = IrregularLattice(reg_lat, remove=[[L-1, 1]])
>>> irr_lat.mps_sites()
['F', 'S', 'F', 'S', 'F', 'S', 'F']
```

Another simple example would be to add a spin in the center of a fermion chain. In that case, we add another site to the unit cell and specify the lattice index as [(L-1)//2, 1] (where the 1 is the index of 'S' in the unit cell ['F', 'S'] of the irregular lattice). The *None* for the MPS index is equivalent to (L-1)/2 in this case.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

#### **Parameters**

- hdf5 saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

#### classmethod from hdf5 (hdf5 loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

### ordering(order)

Provide possible orderings of the lattice sites.

**Parameters order** – Argument for the *Lattice.ordering()* of the *regular\_lattice*, or 2D ndarray providing the order of the *regular lattice*.

**Returns order** – The order to be used for *order*, i.e. *order* with added/removed sites as specified by remove and add.

**Return type** array, shape (N, D+1)

### property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

### mps\_idx\_fix\_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit\_cell[u].

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

**Returns mps\_idx** - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

Return type array

### property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

**Return type** list of str

#### count neighbors (u=0, $key='nearest\ neighbors'$ )

Count e.g. the number of nearest neighbors for a site in the bulk.

#### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

### Return type int

### $coupling_shape(dx)$

Calculate correct shape of the strengths for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

#### Returns

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

### property dim

The dimension of the lattice.

### enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

### lat2mps\_idx (lat\_idx)

Translate lattice indices (x 0, ..., x  $\{D-1\}$ , u) to MPS index i.

**Parameters lat\_idx** ( $array_1ike\ [...,\ dim+1]$ ) – The last dimension corresponds to lattice indices ( $x_0$ , ...,  $x_{D-1}$ , u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_MPS$ , an x 0 outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

**Return type** array\_like

#### mps2lat idx(i)

Translate MPS index i to lattice indices  $(x_0, \dots, x_{\min-1})$ , u).

Parameters i (int | array\_like of int) - MPS index/indices.

**Returns** lat\_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``( $x_0, \ldots, x_{dim-1}, u$ )` corresponding to i. For i accross the MPS unit cell and "infinite" bc MPS, we shift x 0 accordingly.

### Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder A to replace an MPS index by lattice indices.

#### **Parameters**

- A (ndarray) Some values. Must have A.shape[axes] = self.N\_sites if u is None, or A.shape[axes] = self.N\_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit\_cell[u], as returned by mps\_idx\_fix\_u(). The resulting array will not have the additional dimension(s) of u.

**Returns res\_A** – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

### **Examples**

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat\_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[lat.mps2lat_idx(5)] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use  $mps\_idx\_fix\_u$  () to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

**Todo:** make sure this function is used for expectation values...

### mps2lat\_values\_masked (A, axes=- 1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values$  () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns res\_A** – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

**Return type** np.ma.MaskedArray

### mps\_lat\_idx\_fix\_u (u=None)

Similar as  $mps_idx_fix_u()$ , but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

### mps sites()

Return a list of sites for all MPS indices.

Equivalent to [self.site(i) for i in range(self.N\_sites)].

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

#### multi\_coupling\_shape (dx)

Calculate correct shape of the strengths for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx [i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of tenpy.models.model.MultiCouplingModel.add\_multi\_coupling().

#### Returns

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

#### $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

### $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use <code>count\_neighbors()</code> instead.

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **\*\*kwargs** Keyword arguments for ax.arrow.

plot\_bc\_identified (ax, direction=- 1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

plot\_coupling(ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

plot\_order (ax, order=None, textkwargs={}, \*\*kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- **textkwargs** (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().

• \*\*kwargs - Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

### position (lat\_idx)

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

**Returns pos** – The position of the lattice sites specified by *lat\_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible_couplings (u1, u2, dx)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

#### **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u*1 and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

#### Returns

- mps1, mps2 (array) For each possible two-site coupling the MPS indices for the u1 and u2.
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

### possible\_multi\_couplings(ops)

Generalization of possible couplings () to couplings with more than 2 sites.

Parameters ops (list of (opname, dx, u)) - Same as the argument ops of add\_multi\_coupling().

### Returns

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat indices* has only rows within this shape.

```
site(i)
    return Site instance corresponding to an MPS index i

test_sanity()
    Sanity check.
    Raises ValueErrors, if something is wrong.
```

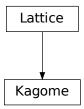
## Kagome

• full name: tenpy.models.lattice.Kagome

• parent module: tenpy.models.lattice

• type: class

## **Inheritance Diagram**



### **Methods**

Kagomeinit(Lx, Ly, sites, **kwargs)	Initialize self.
Kagome.count_neighbors([u, key])	Count e.g.
Kagome.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Kagome.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Kagome.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Kagome.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
Kagome.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
Kagome.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice
	indices.
Kagome.mps2lat_values_masked(A[, axes,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
Kagome.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
	continues on next page

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Kagome.mps_lat_idx_fix_u([u])       Similar as mps_idx_fix_u(), but return al corresponding lattice indices.         Kagome.mps_sites()       Return a list of sites for all MPS indices.         Kagome.multi_coupling_shape(dx)       Calculate correct shape of the strengths	
Kagome.mps_sites()Return a list of sites for all MPS indices.Kagome.multi_coupling_shape(dx)Calculate correct shape of the strengths	
Kagome.multi_coupling_shape(dx) Calculate correct shape of the strengths	
	for a
multi_coupling.	
Kagome.number_nearest_neighbors([u]) Deprecated.	
Kagome.number_next_nearest_neighbors([u])Deprecated.	
Kagome.ordering(order) Provide possible orderings of the N lattice sites.	
Kagome.plot_basis(ax[, origin, shade]) Plot arrows indicating the basis vectors of the latt	ice.
Kagome.plot_bc_identified(ax[, direction, Mark two sites indified by periodic boundary	condi-
$\ldots$ ]) tions.	
Kagome.plot_coupling(ax[, coupling, wrap]) Plot lines connecting nearest neighbors of the latt	ice.
Kagome.plot_order(ax[, order, textkwargs]) Plot a line connecting sites in the specified "order"	r" and
text labels enumerating them.	
Kagome.plot_sites(ax[, markers]) Plot the sites of the lattice with markers.	
Kagome.position(lat_idx) return 'space' position of one or multiple sites.	
Kagome.possible_couplings(u1, u2, dx) Find possible MPS indices for two-site couplings	
Kagome.possible_multi_couplings(ops) Generalization of possible_couplings() t	o cou-
plings with more than 2 sites.	
Kagome.save_hdf5(hdf5_saver, h5gr, subpath) Export self into a HDF5 file.	
Kagome.site(i) return Site instance corresponding to an MPS in	ndex i

### **Class Attributes and Properties**

Kagome.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
Kagome.dim	
Kagome.nearest_neighbors	
Kagome.next_nearest_neighbors	
Kagome.next_next_nearest_neighbors	
Kagome.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.Kagome (Lx, Ly, sites, \*\*kwargs)
 Bases: tenpy.models.lattice.Lattice

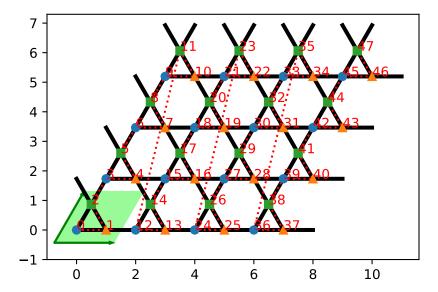
A Kagome lattice.

### **Parameters**

- Ly (Lx,) The length in each direction.
- **sites** ((list of) *Site*) The two local lattice sites making the *unit\_cell* of the *Lattice*. If only a single *Site* is given, it is used for both sites.
- \*\*kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs
  are set accordingly.

### property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.



**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

**Return type** list of str

## count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

#### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

### Return type int

### $coupling\_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

### Returns

• **coupling\_shape** (*tuple of int*) – Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

• **shift\_lat\_indices** (*array*) – Translation vector from origin to the lower left corner of box spanned by *dx*.

### enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from  $N\_sites$  to factor\*N $\_sites\_per\_ring$ . Since MPS unit cells are repeated in the x-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

#### lat2mps\_idx (lat\_idx)

Translate lattice indices  $(x_0, \ldots, x_{D-1})$ , u) to MPS index i.

**Parameters lat\_idx** ( $array_1ike\ [...,\ dim+1]$ ) – The last dimension corresponds to lattice indices ( $x_0, ..., x_{D-1}, u$ ). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_MPS$ , an  $x_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

#### mps2lat\_idx(i)

Translate MPS index i to lattice indices  $(x_0, \ldots, x_{\min-1}, u)$ .

**Parameters i** (int | array\_like of int) – MPS index/indices.

**Returns lat\_idx** – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``( $x_0$ , ...,  $x_i$ dim-1}, u)` corresponding to i. For i accross the MPS unit cell and "infinite" bc MPS, we shift x 0 accordingly.

**Return type** array

### $mps2lat\_values(A, axes=0, u=None)$

Reshape/reorder A to replace an MPS index by lattice indices.

#### **Parameters**

- A (ndarray) Some values. Must have A. shape [axes] = self.N\_sites if u is None, or A. shape [axes] = self.N cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit\_cell[u], as returned by

```
mps\_idx\_fix\_u (). The resulting array will not have the additional dimension(s) of u.
```

**Returns res\_A** – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

### **Examples**

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by  $self.mps2lat_idx(i)$ . Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[lat.mps2lat_idx(5)] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use  $mps\_idx\_fix\_u()$  to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

**Todo:** make sure this function is used for expectation values...

### mps2lat\_values\_masked(A, axes=-1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values()$  allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.

- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

Return type np.ma.MaskedArray

#### mps\_idx\_fix\_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit\_cell[u].

**Parameters u** (None / int) – Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

**Return type** array

### mps lat idx fix u(u=None)

Similar as mps\_idx\_fix\_u(), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

### mps\_sites()

Return a list of sites for all MPS indices.

Equivalent to [self.site(i) for i in range(self.N\_sites)].

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

#### multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument *ops* of tenpy.models.model.MultiCouplingModel.add\_multi\_coupling().

### **Returns**

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

```
number_nearest_neighbors(u=0)
```

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors () instead.

### $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors () instead.

### property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through through the lattice.

You can visualize the order with plot\_order().

#### ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Parameters order (str | ('standard', snake\_winding, priority) | ('grouped', groups)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get\_order() and 'grouped' for get\_order\_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

**Returns** order – the order to be used for order.

Return type array, shape (N, D+1), dtype np.intp

#### See also:

```
get_order() generates the order from equivalent priority and snake_winding.
get_order_grouped() variant of get_order.
plot_order() visualizes the resulting order.
```

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

plot\_bc\_identified (ax, direction=- 1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

```
plot_coupling (ax, coupling=None, wrap=False, **kwargs)
```

Plot lines connecting nearest neighbors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_order(ax, order=None, textkwargs={}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

### position(lat idx)

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) – Lattice indices.
```

**Returns** pos – The position of the lattice sites specified by *lat idx* in real-space.

```
Return type ndarray, (..., dim)
```

### possible\_couplings (u1, u2, dx)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le x_$ 

#### **Parameters**

- **u2** (u1,) Indices within the unit cell; the u1 and u2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

#### **Returns**

- mps1, mps2 (array) For each possible two-site coupling the MPS indices for the u1 and u2.
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

### possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

### Returns

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save hdf5 (hdf5 saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

```
site(i)
```

return Site instance corresponding to an MPS index i

## ${\tt test\_sanity}\,(\,)$

Sanity check.

Raises ValueErrors, if something is wrong.

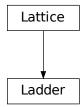
### Ladder

• full name: tenpy.models.lattice.Ladder

• parent module: tenpy.models.lattice

• type: class

## **Inheritance Diagram**



### **Methods**

Ladderinit(L, sites, **kwargs)	Initialize self.
Ladder.count_neighbors([u, key])	Count e.g.
Ladder.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Ladder.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Ladder.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Ladder.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
Ladder.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
Ladder.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice
	indices.
Ladder.mps21at_values_masked(A[, axes,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
Ladder.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
	continues on next page

Table 84 – continued from previous page

Ladder.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
Ladder.mps_rat_rux_rrx_u([u])	<del>_</del>
	corresponding lattice indices.
Ladder.mps_sites()	Return a list of sites for all MPS indices.
Ladder.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
Ladder.number_nearest_neighbors([u])	Deprecated.
Ladder.number_next_nearest_neighbors([u])Deprecated.	
Ladder.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
Ladder.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.
Ladder.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-
])	tions.
<pre>Ladder.plot_coupling(ax[, coupling, wrap])</pre>	Plot lines connecting nearest neighbors of the lattice.
<pre>Ladder.plot_order(ax[, order, textkwargs])</pre>	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
Ladder.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
Ladder.position(lat_idx)	return 'space' position of one or multiple sites.
Ladder.possible_couplings(u1, u2, dx)	Find possible MPS indices for two-site couplings.
Ladder.possible_multi_couplings(ops)	Generalization of possible_couplings() to cou-
	plings with more than 2 sites.
Ladder.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
Ladder.site(i)	return Site instance corresponding to an MPS index i
Ladder.test_sanity()	Sanity check.

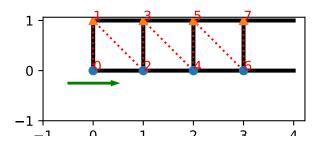
## **Class Attributes and Properties**

Ladder.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
Ladder.dim	
Ladder.nearest_neighbors	
Ladder.next_nearest_neighbors	
Ladder.next_next_nearest_neighbors	
Ladder.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.Ladder(L, sites, \*\*kwargs)

Bases: tenpy.models.lattice.Lattice

A ladder coupling two chains.



## **Parameters**

• L (int) – The length of each chain, we have 2\*L sites in total.

- **sites** ((list of) *Site*) The two local lattice sites making the *unit\_cell* of the *Lattice*. If only a single *Site* is given, it is used for both chains.
- \*\*kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs
  are set accordingly.

### property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

### count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

#### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

### Return type int

### $coupling\_shape(dx)$

Calculate correct shape of the strengths for a coupling.

Parameters dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

#### Returns

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

### enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns obj** – Newly generated class instance containing the required data.

**Return type** cls

```
lat2mps_idx (lat_idx)
```

Translate lattice indices  $(x_0, \ldots, x_{D-1})$ , u) to MPS index i.

**Parameters** lat\_idx ( $array_like[..,dim+1]$ ) - The last dimension corresponds to lattice indices (x\_0, ..., x\_{D-1}, u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_mPS$ , an  $x_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

```
mps2lat_idx(i)
```

Translate MPS index i to lattice indices  $(x_0, \dots, x_{dim-1}, u)$ .

**Parameters i** (int | array\_like of int) - MPS index/indices.

**Returns** lat\_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices  $(x_0, \ldots, x_{dim-1}, u)$  corresponding to i. For i accross the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder *A* to replace an MPS index by lattice indices.

#### **Parameters**

- A (ndarray) Some values. Must have A. shape[axes] = self.N\_sites if u is None, or A. shape[axes] = self.N\_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit\_cell[u], as returned by mps\_idx\_fix\_u(). The resulting array will not have the additional dimension(s) of u.

**Returns res\_A** – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., y, ...].

Return type ndarray

#### **Examples**

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by  $self.mps2lat_idx(i)$ . Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[lat.mps2lat_idx(5)] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use  $mps\_idx\_fix\_u()$  to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

**Todo:** make sure this function is used for expectation values...

### mps2lat\_values\_masked (A, axes=- 1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values$  () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

Return type np.ma.MaskedArray

### mps\_idx\_fix\_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit\_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

### Return type array

### mps\_lat\_idx\_fix\_u (u=None)

Similar as  $mps_idx_fix_u()$ , but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

#### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

#### mps\_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

### multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

#### **Returns**

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

### $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

### $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

#### property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

#### ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Parameters order (str | ('standard', snake\_winding, priority) |
 ('grouped', groups)) - Specifies the desired ordering using one of the strings
 of the above tables. Alternatively, an ordering is specified by a tuple with first en try specifying a function, 'standard' for get\_order() and 'grouped' for
 get\_order\_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

**Returns order** – the order to be used for order.

**Return type** array, shape (N, D+1), dtype np.intp

#### See also:

```
get_order() generates the order from equivalent priority and snake_winding.
get_order_grouped() variant of get_order.
plot_order() visualizes the resulting order.
```

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

plot\_bc\_identified (ax, direction=-1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- cylinder\_axis (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

plot\_coupling (ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

#### **Parameters**

• ax (matplotlib.axes.Axes) - The axes on which we should plot.

- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_order(ax, order=None, textkwargs={}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

```
position(lat_idx)
```

return 'space' position of one or multiple sites.

Parameters lat\_idx (ndarray, (..., dim+1)) - Lattice indices.

**Returns pos** – The position of the lattice sites specified by *lat\_idx* in real-space.

**Return type** ndarray, (..., dim)

```
possible couplings (u1, u2, dx)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

### **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u*1 and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

### Returns

• mps1, mps2 (array) – For each possible two-site coupling the MPS indices for the u1 and u2.

- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

### possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

#### **Returns**

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

### site(i)

return Site instance corresponding to an MPS index i

### test\_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

#### Lattice

• full name: tenpy.models.lattice.Lattice

 $\bullet \ \ parent \ module: \ \textit{tenpy.models.lattice}$ 

• type: class

# Inheritance Diagram

Lattice

## Methods

Latticeinit(Ls, unit_cell[, order, bc,])	Initialize self.
Lattice.count_neighbors([u, key])	Count e.g.
Lattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Lattice.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Lattice.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Lattice.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
Lattice.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
Lattice.mps2lat_values(A[, axes, u])	Reshape/reorder A to replace an MPS index by lattice
	indices.
Lattice.mps2lat_values_masked( $A[$ , axes,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
Lattice.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
$Lattice.mps\_lat\_idx\_fix\_u([u])$	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Lattice.mps_sites()	Return a list of sites for all MPS indices.
Lattice.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
Lattice.number_nearest_neighbors([u])	Deprecated.
Lattice.number_next_nearest_neighbors(	
Lattice.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
Lattice.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.
Lattice.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-
])	tions.
Lattice.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.
Lattice.plot_order(ax[, order, textkwargs])	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
Lattice.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
Lattice.position(lat_idx)	return 'space' position of one or multiple sites.
Lattice.position(lat_idx) Lattice.possible_couplings(u1, u2, dx)	return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings.
Lattice.position(lat_idx)	return 'space' position of one or multiple sites. Find possible MPS indices for two-site couplings. Generalization of possible_couplings() to cou-
Lattice.position(lat_idx)  Lattice.possible_couplings(u1, u2, dx)  Lattice.possible_multi_couplings(ops)	return 'space' position of one or multiple sites.  Find possible MPS indices for two-site couplings.  Generalization of possible_couplings() to couplings with more than 2 sites.
Lattice.position(lat_idx)  Lattice.possible_couplings(u1, u2, dx)  Lattice.possible_multi_couplings(ops)  Lattice.save_hdf5(hdf5_saver, h5gr, subpath)	return 'space' position of one or multiple sites.  Find possible MPS indices for two-site couplings.  Generalization of possible_couplings() to couplings with more than 2 sites.  Export self into a HDF5 file.
Lattice.position(lat_idx)  Lattice.possible_couplings(u1, u2, dx)  Lattice.possible_multi_couplings(ops)	return 'space' position of one or multiple sites.  Find possible MPS indices for two-site couplings.  Generalization of possible_couplings() to couplings with more than 2 sites.

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Lattice.test_sanity()	Sanity check.

## **Class Attributes and Properties**

Lattice.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
Lattice.dim	The dimension of the lattice.
Lattice.nearest_neighbors	
Lattice.next_nearest_neighbors	
Lattice.next_next_nearest_neighbors	
Lattice.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.Lattice(Ls,  $unit\_cell$ , order='default', bc='open',  $bc\_MPS='finite'$ , basis=None, positions=None,  $near-est\_neighbors=None$ ,  $next\_nearest\_neighbors=None$ 

Bases: object

A general, regular lattice.

The lattice consists of a **unit cell** which is repeated in *dim* different directions. A site of the lattice is thus identified by **lattice indices** ( $x_0$ , ...,  $x_{\dim-1}$ , u), where  $0 \le x_1 \le Ls[1]$  pick the position of the unit cell in the lattice and  $0 \le u \le len(unit_cell)$  picks the site within the unit cell. The site is located in 'space' at  $sum_1 x_1 *basis[1] + unit_cell_positions[u]$  (see position()). (Note that the position in space is only used for plotting, not for defining the couplings.)

In addition to the pure geometry, this class also defines an *order* of all sites. This order maps the lattice to a finite 1D chain and defines the geometry of MPSs and MPOs. The **MPS index** i corresponds thus to the lattice sites given by  $(x_0, \ldots, x_{\dim-1}, u) = \text{tuple}(\text{self.order}[i])$ . Infinite boundary conditions of the MPS repeat in the first spatial direction of the lattice, i.e., if the site at  $(x_0, x_1, \ldots, x_{\dim-1}, u)$  has MPS index i, the site at at  $(x_0 + \text{Ls}[0], x_1, \ldots, x_{\dim-1}, u)$  corresponds to MPS index i + N\_sites. Use  $mps2lat_idx()$  and  $lat2mps_idx()$  for conversion of indices. The function  $mps2lat_values()$  performs the necessary reshaping and re-ordering from arrays indexed in MPS form to arrays indexed in lattice form.

Deprecated since version 0.5.0: The parameters and attributes *nearest\_neighbors*, *next\_nearest\_neighbors* and *next\_next\_nearest\_neighbors* are deprecated. Instead, we use a dictionary *pairs* with those names as keys and the corresponding values as specified before.

## **Parameters**

- **Ls** (list of int) the length in each direction
- unit\_cell (list of Site) The sites making up a unit cell of the lattice. If you want to specify it only after initialization, use None entries in the list.
- **order** (str | ('standard', snake\_winding, priority) | ('grouped', groups)) A string or tuple specifying the order, given to *ordering* ().
- **bc** ((iterable of) {'open' | 'periodic' | int}) Boundary conditions in each direction of the lattice. A single string holds for all directions. An integer shift means that we have periodic boundary conditions along this direction, but shift/tilt by -shift\*lattice.basis[0] (~cylinder axis for bc\_MPS='infinite') when going around the boundary along this direction.

- bc\_MPS ('finite' | 'segment' | 'infinite') Boundary conditions for an MPS/MPO living on the ordered lattice. If the system is 'infinite', the infinite direction is always along the first basis vector (justifying the definition of N\_rings and N\_sites\_per\_ring).
- **basis** (*iterable of 1D arrays*) For each direction one translation vectors shifting the unit cell. Defaults to the standard ONB np.eye (dim).
- **positions** (*iterable of 1D arrays*) For each site of the unit cell the position within the unit cell. Defaults to np.zeros((len(unit\_cell), dim)).
- nearest\_neighbors (None | list of (u1, u2, dx)) Deprecated. Specify as pairs ['nearest\_neighbors'] instead.
- next\_nearest\_neighbors (None | list of (u1, u2, dx)) Deprecated. Specify as pairs ['next\_nearest\_neighbors'] instead.
- next\_next\_nearest\_neighbors (None | list of (u1, u2, dx)) Deprecated. Specify as pairs ['next\_next\_nearest\_neighbors'] instead.
- pairs (dict) Of the form {'nearest\_neighbors': [(u1, u2, dx), ...], ...}. Typical keys are 'nearest\_neighbors', 'next\_nearest\_neighbors'. For each of them, it specifies a list of tuples (u1, u2, dx) which can be used as parameters for add\_coupling() to generate couplings over each pair of ,e.g., 'nearest\_neighbors'. Note that this adds couplings for each pair only in one direction!

#### Ls

the length in each direction.

**Type** tuple of int

## shape

the 'shape' of the lattice, same as Ls + (len(unit\_cell), )

Type tuple of int

## N\_cells

the number of unit cells in the lattice, np.prod(self.Ls).

Type int

## **N\_sites**

the number of sites in the lattice, np.prod(self.shape).

Type int

#### N sites per ring

Defined as N\_sites / Ls[0], for an infinite system the number of cites per "ring".

Type int

## N\_rings

Alias for Ls [0], for an infinite system the number of "rings" in the unit cell.

Type int

### unit cell

the sites making up a unit cell of the lattice.

**Type** list of Site

#### bc

Boundary conditions of the couplings in each direction of the lattice, translated into a bool array with the global *bc\_choices*.

Type bool ndarray

## bc shift

The shift in x-direction when going around periodic boundaries in other directions.

**Type** None | ndarray(int)

## bc\_MPS

Boundary conditions for an MPS/MPO living on the ordered lattice. If the system is 'infinite', the infinite direction is always along the first basis vector (justifying the definition of  $N\_rings$  and  $N\_sites\_per\_ring$ ).

Type 'finite' | 'segment' | 'infinite'

### basis

translation vectors shifting the unit cell. The row i gives the vector shifting in direction i.

**Type** ndarray (dim, Dim)

### unit\_cell\_positions

for each site in the unit cell a vector giving its position within the unit cell.

**Type** ndarray, shape (len(unit\_cell), Dim)

### pairs

See above.

Type dict

## \_order

The place where *order* is stored.

Type ndarray (N\_sites, dim+1)

## \_strides

necessary for lat2mps\_idx().

**Type** ndarray (dim, )

### \_perm

permutation needed to make order lexsorted, \_perm = np.lexsort (\_order.T).

Type ndarray (N, )

## \_mps2lat\_vals\_idx

index array for reshape/reordering in mps2lat\_vals()

Type ndarray shape

## \_mps\_fix\_u

for each site of the unit cell an index array selecting the mps indices of that site.

Type tuple of ndarray (N\_cells, ) np.intp

### \_mps2lat\_vals\_idx\_fix\_u

similar as \_mps2lat\_vals\_idx, but for a fixed u picking a site from the unit cell.

**Type** tuple of ndarray of shape Ls

## test\_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

## property dim

The dimension of the lattice.

## property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

## ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	$(\dim -1,, 1, 0, \dim)$	(False,, False, False)

Parameters order (str | ('standard', snake\_winding, priority) | ('grouped', groups)) - Specifies the desired ordering using one of the strings

of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for  $get\_order()$  and 'grouped' for  $get\_order\_grouped()$ , and other arguments in the tuple as specified in the documentation of these functions.

**Returns** order – the order to be used for order.

**Return type** array, shape (N, D+1), dtype np.intp

#### See also:

```
get_order() generates the order from equivalent priority and snake_winding.
get_order_grouped() variant of get_order.
plot_order() visualizes the resulting order.
```

### property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

**Return type** list of str

```
enlarge_mps_unit_cell(factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

```
position (lat_idx)
```

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

**Returns** pos – The position of the lattice sites specified by *lat\_idx* in real-space.

**Return type** ndarray, (..., dim)

## site(i)

return Site instance corresponding to an MPS index i

## mps\_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

## ${\tt mps2lat\_idx}\,(i)$

Translate MPS index i to lattice indices (x\_0, ..., x\_{dim-1}, u).

```
Parameters i (int | array_like of int) - MPS index/indices.
```

**Returns** lat\_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices ``( $x_0, \ldots, x_d$ ) corresponding to i. For i across the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

**Return type** array

#### lat2mps idx(lat idx)

Translate lattice indices  $(x_0, \dots, x_{D-1})$ , u) to MPS index i.

**Parameters lat\_idx** ( $array_like[..,dim+1]$ ) – The last dimension corresponds to lattice indices ( $x_0, ..., x_{D-1}, u$ ). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_MPS$ , an  $x_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

## mps\_idx\_fix\_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit\_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

Return type array

## mps\_lat\_idx\_fix\_u (u=None)

Similar as  $mps_idx_fix_u$  (), but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

### **Returns**

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- **lat\_idx** (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

### $mps2lat\_values(A, axes=0, u=None)$

Reshape/reorder A to replace an MPS index by lattice indices.

## **Parameters**

- A (ndarray) Some values. Must have A.shape[axes] = self.N\_sites if u is None, or A.shape[axes] = self.N\_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit\_cell[u], as returned by  $mps_idx_fix_u()$ . The resulting array will not have the additional dimension(s) of u.

**Returns res\_A** – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., j, ...].

Return type ndarray

## **Examples**

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat\_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[lat.mps2lat_idx(5)] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use  $mps\_idx\_fix\_u()$  to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

**Todo:** make sure this function is used for expectation values...

mps2lat\_values\_masked (A, axes=- 1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps21at\_values()$  allowing for the case of an arbitrary set of MPS indices present in each axis of A.

## **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

**Return type** np.ma.MaskedArray

```
count_neighbors (u=0, key='nearest_neighbors')
```

Count e.g. the number of nearest neighbors for a site in the bulk.

#### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of *pairs* to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

## Return type int

## $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

### $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count neighbors () instead.

## $coupling\_shape(dx)$

Calculate correct shape of the strengths for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

## **Returns**

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

## $possible\_couplings(u1, u2, dx)$

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le x_a \le x_a$  and  $0 \le x_a + dx[a] \le x_a \le x_a$ .

#### **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u*1 and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

## Returns

• **mps1**, **mps2** (*array*) – For each possible two-site coupling the MPS indices for the *u1* and *u2*.

- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

## multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi-coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.multiCouplingModel.add_multi_coupling().$ 

### **Returns**

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

## possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add multi coupling().

### **Returns**

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

plot\_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], \*\*kwargs)

Plot the sites of the lattice with markers.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

plot\_order (ax, order=None, textkwargs={}, \*\*kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

## **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().

• \*\*kwargs - Further keyword arguments given to ax.plot().

```
plot_coupling (ax, coupling=None, wrap=False, **kwargs)
```

Plot lines connecting nearest neighbors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_basis (ax, origin=0.0, 0.0, shade=None, **kwargs)
```

Plot arrows indicating the basis vectors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

```
plot_bc_identified (ax, direction=-1, origin=None, cylinder_axis=False, **kwargs)

Mark two sites indified by periodic boundary conditions.
```

Works only for lattice with a 2-dimensional basis.

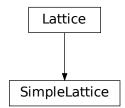
### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

## **SimpleLattice**

- full name: tenpy.models.lattice.SimpleLattice
- parent module: tenpy.models.lattice
- type: class

# Inheritance Diagram



## Methods

SimpleLatticeinit(Ls, site, **kwargs)	Initialize self.
SimpleLattice.count_neighbors([u, key])	Count e.g.
SimpleLattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
	etarppeat the unit cell for infinite MPS boundary condi-
21p2-21a00100,011a190p2_a112_0011([1	tions; in place.
SimpleLattice.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	Educ instance from a fibro inc.
SimpleLattice.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index $i$ .
SimpleLattice.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	x_{dim-1}, u).
SimpleLattice.mps2lat_values(A[, axes, u])	same as Lattice.mps2lat_values(), but ignore
	u, setting it to 0.
SimpleLattice.mps2lat_values_masked(A[,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
SimpleLattice.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
SimpleLattice.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
SimpleLattice.mps_sites()	Return a list of sites for all MPS indices.
SimpleLattice.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
SimpleLattice.number_nearest_neighbors	([D]eprecated.
SimpleLattice.number_next_nearest_neig	
SimpleLattice.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
SimpleLattice.plot_basis(ax[, origin,	Plot arrows indicating the basis vectors of the lattice.
shade])	
SimpleLattice.plot_bc_identified(ax[,	Mark two sites indified by periodic boundary condi-
])	tions.
SimpleLattice.plot_coupling(ax[, coupling,	Plot lines connecting nearest neighbors of the lattice.
wrap])	
SimpleLattice.plot_order(ax[, order, textk-	Plot a line connecting sites in the specified "order" and
wargs])	text labels enumerating them.
	continues on next page

Table 88 – continued from previous pa	age
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SimpleLattice.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
SimpleLattice.position(lat_idx)	return 'space' position of one or multiple sites.
SimpleLattice.possible_couplings(u1,	Find possible MPS indices for two-site couplings.
u2, dx)	
SimpleLattice.possible_multi_couplings	(openeralization of possible_couplings() to cou-
	plings with more than 2 sites.
SimpleLattice.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	
SimpleLattice.site(i)	return $Site$ instance corresponding to an MPS index $i$
SimpleLattice.test_sanity()	Sanity check.

## **Class Attributes and Properties**

SimpleLattice.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
SimpleLattice.dim	The dimension of the lattice.
SimpleLattice.nearest_neighbors	
SimpleLattice.next_nearest_neighbors	
SimpleLattice.next_next_nearest_neighb	oors
SimpleLattice.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.SimpleLattice(Ls, site, \*\*kwargs)

Bases: tenpy.models.lattice.Lattice

A lattice with a unit cell consiting of just a single site.

In many cases, the unit cell consists just of a single site, such that the the last entry of u of an 'lattice index' can only be 0. From the point of internal algorithms, we handle this class like a Lattice – in that way we don't need to distinguish special cases in the algorithms.

Yet, from the point of a tenpy user, for example if you measure an expectation value on each site in a *SimpleLattice*, you expect to get an ndarray of dimensions self.Ls, not self.shape. To avoid that problem, *SimpleLattice* overwrites just the meaning of u=None in mps2lat\_values() to be the same as u=0.

## **Parameters**

- **Ls** (list of int) the length in each direction
- **site** (*Site*) the lattice site. The *unit\_cell* of the *Lattice* is just [site].
- \*\*kwargs Additional keyword arguments given to the Lattice. If order is specified in
  the form ('standard', snake\_windingi, priority), the snake\_winding and
  priority should only be specified for the spatial directions. Similarly, positions can be specified as a single vector.

## $mps2lat\_values(A, axes=0, u=None)$

same as Lattice.mps2lat\_values(), but ignore u, setting it to 0.

## property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

### count neighbors (u=0, $key='nearest\ neighbors'$ )

Count e.g. the number of nearest neighbors for a site in the bulk.

#### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

## Return type int

## $coupling\_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

#### Returns

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (array) Translation vector from origin to the lower left corner of box spanned by dx.

## property dim

The dimension of the lattice.

## enlarge\_mps\_unit\_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

### **Parameters**

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

## Return type cls

## lat2mps\_idx (lat\_idx)

Translate lattice indices  $(x_0, \ldots, x_{D-1})$ , u) to MPS index i.

**Parameters** lat\_idx ( $array_1ike[.., dim+1]$ ) – The last dimension corresponds to lattice indices ( $x_0, ..., x_{D-1}, u$ ). All lattice indices should be positive and

smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc\_MPS$ , an  $x \ \theta$  outside indicates shifts across the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

**Return type** array\_like

## $mps2lat_idx(i)$

Translate MPS index i to lattice indices  $(x \ 0, \ldots, x \ \{dim-1\}, u)$ .

Parameters i (int | array\_like of int) - MPS index/indices.

**Returns** lat\_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``( $x_0, \ldots, x_{dim-1}, u$ )` corresponding to i. For i across the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

Return type array

```
mps2lat_values_masked(A, axes=-1, mps_inds=None, include_u=None)
```

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values()$  allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

Return type np.ma.MaskedArray

## mps\_idx\_fix\_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit\_cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

Return type array

## mps\_lat\_idx\_fix\_u (u=None)

Similar as  $mps\_idx\_fix\_u$  (), but return also the corresponding lattice indices.

**Parameters u** (None / int) – Selects a site of the unit cell. None (default) means all sites.

## **Returns**

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

### mps\_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

## multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

#### Returns

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

## number nearest neighbors (u=0)

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

## $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use <code>count\_neighbors()</code> instead.

## property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

## ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Parameters order (str | ('standard', snake\_winding, priority) |
 ('grouped', groups)) - Specifies the desired ordering using one of the strings
 of the above tables. Alternatively, an ordering is specified by a tuple with first en try specifying a function, 'standard' for get\_order() and 'grouped' for
 get\_order\_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

**Returns** order – the order to be used for order.

**Return type** array, shape (N, D+1), dtype np.intp

### See also:

```
get_order() generates the order from equivalent priority and snake_winding.
get_order_grouped() variant of get_order.
plot_order() visualizes the resulting order.
```

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

**plot\_bc\_identified** (ax, direction=-1, origin=None, cylinder\_axis=False, \*\*kwargs) Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

## **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

plot\_coupling (ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot order(ax, order=None, textkwargs={}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

### position (lat\_idx)

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

**Returns pos** – The position of the lattice sites specified by *lat\_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible_couplings (u1, u2, dx)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

#### **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u*1 and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

### **Returns**

- mps1, mps2 (array) For each possible two-site coupling the MPS indices for the u1 and u2.
- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

## possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) – Same as the argument *ops* of  $add\_multi\_coupling()$ .

#### Returns

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

## **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

#### site(i)

return Site instance corresponding to an MPS index i

## test\_sanity()

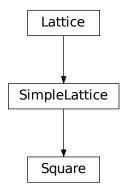
Sanity check.

Raises ValueErrors, if something is wrong.

## **Square**

- full name: tenpy.models.lattice.Square
- parent module: tenpy.models.lattice
- type: class

# **Inheritance Diagram**



## Methods

Squareinit(Lx, Ly, site, **kwargs)	Initialize self.
Square.count_neighbors([u, key])	Count e.g.
Square.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Square.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Square.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Square.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
Square.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
Square.mps2lat_values(A[, axes, u])	<pre>same as Lattice.mps2lat_values(), but ignore</pre>
	u, setting it to 0.
Square.mps2lat_values_masked(A[, axes,	Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
Square.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is <i>u</i> .
Square.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Square.mps_sites()	Return a list of sites for all MPS indices.
$Square.multi\_coupling\_shape(dx)$	Calculate correct shape of the strengths for a
	multi_coupling.
Square.number_nearest_neighbors([u])	Deprecated.
Square.number_next_nearest_neighbors([u	
Square.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
Square.plot_basis(ax[, origin, shade])	Plot arrows indicating the basis vectors of the lattice.
Square.plot_bc_identified(ax[, direction,	Mark two sites indified by periodic boundary condi-
])	tions.
Square.plot_coupling(ax[, coupling, wrap])	Plot lines connecting nearest neighbors of the lattice.
	continues on next page

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Square.plot_order(ax[, order, textkwargs])	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
Square.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
Square.position(lat_idx)	return 'space' position of one or multiple sites.
Square.possible_couplings(u1, u2, dx)	Find possible MPS indices for two-site couplings.
Square.possible_multi_couplings(ops)	Generalization of possible_couplings() to cou-
	plings with more than 2 sites.
Square.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
Square.site(i)	return $Site$ instance corresponding to an MPS index $i$
Square.test_sanity()	Sanity check.

## **Class Attributes and Properties**

Square.boundary_conditions	Human-readable list of boundary conditions from bc
	and bc_shift.
Square.dim	
Square.nearest_neighbors	
Square.next_nearest_neighbors	
Square.next_next_nearest_neighbors	
Square.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.lattice.Square(Lx, Ly, site, \*\*kwargs)

Bases: tenpy.models.lattice.SimpleLattice

A square lattice.

## **Parameters**

- Ly (Lx,) The length in each direction.
- **site** (Site) The local lattice site. The *unit\_cell* of the *Lattice* is just [site].
- \*\*kwargs Additional keyword arguments given to the Lattice. pairs are set accordingly. If order is specified in the form ('standard', snake\_winding, priority), the snake\_winding and priority should only be specified for the spatial directions. Similarly, positions can be specified as a single vector.

## property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

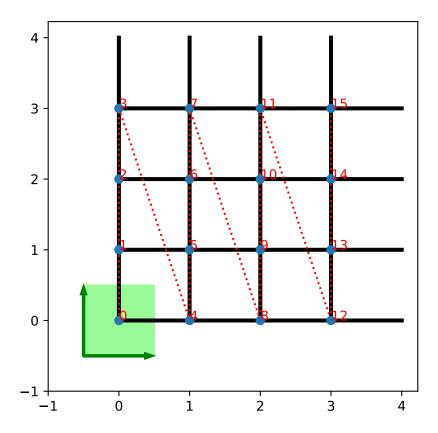
**Return type** list of str

count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.



**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

## Return type int

## $coupling\_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

### **Returns**

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

## enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

## **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

## Return type cls

## lat2mps\_idx (lat\_idx)

Translate lattice indices  $(x_0, \ldots, x_{D-1})$ , u) to MPS index i.

**Parameters lat\_idx** ( $array_like[..,dim+1]$ ) – The last dimension corresponds to lattice indices ( $x_0, ..., x_{D-1}, u$ ). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_MPS$ , an  $x_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

## mps2lat\_idx(i)

Translate MPS index i to lattice indices (x\_0, ..., x\_{dim-1}, u).

Parameters i (int / array\_like of int) - MPS index/indices.

**Returns lat\_idx** – First dimensions like i, last dimension has len dim +1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

## Return type array

```
mps2lat\_values(A, axes=0, u=None)
```

same as Lattice.mps2lat\_values(), but ignore u, setting it to 0.

## mps2lat\_values\_masked (A, axes=- 1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values()$  allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u ((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

Return type np.ma.MaskedArray

## mps\_idx\_fix\_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit\_cell[u].

**Parameters u** (None / int) – Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

**Return type** array

## mps\_lat\_idx\_fix\_u (u=None)

Similar as mps\_idx\_fix\_u(), but return also the corresponding lattice indices.

**Parameters u** (None / int) – Selects a site of the unit cell. None (default) means all sites.

## **Returns**

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

#### mps sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

## multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

### Returns

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

## $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

### $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count neighbors () instead.

## property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

## ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Parameters order (str | ('standard', snake\_winding, priority) | ('grouped', groups)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get\_order() and 'grouped' for get\_order\_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

**Returns** order – the order to be used for order.

**Return type** array, shape (N, D+1), dtype np.intp

#### See also:

```
get_order() generates the order from equivalent priority and snake_winding.
get_order_grouped() variant of get_order.
plot_order() visualizes the resulting order.
```

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

plot\_bc\_identified (ax, direction=- 1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

plot\_coupling (ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

## **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (*i0*, *i1*, ..., *u1*) to the site (*i0*+dx[0], *i1*+dx[1], ..., *u2*), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

plot\_order (ax, order=None, textkwargs={}, \*\*kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.

- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

#### position (lat\_idx)

return 'space' position of one or multiple sites.

```
Parameters lat idx (ndarray, (..., dim+1)) – Lattice indices.
```

**Returns pos** – The position of the lattice sites specified by *lat\_idx* in real-space.

```
Return type ndarray, (..., dim)
```

```
possible\_couplings(u1, u2, dx)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

## **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u*1 and *u*2 of add coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

#### **Returns**

- mps1, mps2 (array) For each possible two-site coupling the MPS indices for the u1 and u2.
- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat indices* has only rows within this shape.

### possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

### **Returns**

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,... for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.

• **coupling\_shape** (*tuple of int*) – Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

## site(i)

return Site instance corresponding to an MPS index i

### test\_sanity()

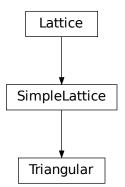
Sanity check.

Raises ValueErrors, if something is wrong.

## **Triangular**

- full name: tenpy.models.lattice.Triangular
- parent module: tenpy.models.lattice
- type: class

## **Inheritance Diagram**



## Methods

Triangularinit(Lx, Ly, site, **kwargs)	Initialize self.
Triangular.count_neighbors([u, key])	Count e.g.
Triangular.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
Triangular.enlarge_mps_unit_cell([factor])	
	tions; in place.
Triangular.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
Triangular.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ , u) to MPS index <i>i</i> .
$ extit{Triangular.mps2}  extit{lat\_idx}  extit{(i)}$	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
Triangular.mps2lat_values(A[, axes, u])	<pre>same as Lattice.mps2lat_values(), but ignore</pre>
	u, setting it to 0.
Triangular.mps2lat_values_masked(A[,	Reshape/reorder an array A to replace an MPS index by
axes,])	lattice indices.
Triangular.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is <i>u</i> .
${\it Triangular.mps\_lat\_idx\_fix\_u([u])}$	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
Triangular.mps_sites()	Return a list of sites for all MPS indices.
Triangular.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
Triangular.number_nearest_neighbors([u])	
Triangular.number_next_nearest_neighbo.	
Triangular.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
<pre>Triangular.plot_basis(ax[, origin, shade])</pre>	Plot arrows indicating the basis vectors of the lattice.
Triangular.plot_bc_identified( $ax[,]$ )	Mark two sites indified by periodic boundary condi-
	tions.
Triangular.plot_coupling(ax[, coupling,	Plot lines connecting nearest neighbors of the lattice.
wrap])	
Triangular.plot_order(ax[, order, textk-	Plot a line connecting sites in the specified "order" and
wargs])	text labels enumerating them.
Triangular.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
Triangular.position(lat_idx)	return 'space' position of one or multiple sites.
Triangular.possible_couplings(u1, u2, dx)	Find possible MPS indices for two-site couplings.
Triangular.possible_multi_couplings(ops)	
	plings with more than 2 sites.
Triangular.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
Triangular.site(i)	return $Site$ instance corresponding to an MPS index $i$
Triangular.test_sanity()	Sanity check.

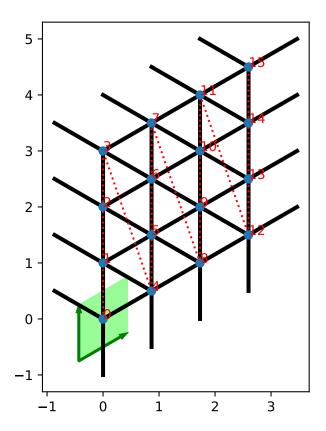
## **Class Attributes and Properties**

Triangular.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
Triangular.dim	
Triangular.nearest_neighbors	
Triangular.next_nearest_neighbors	
Triangular.next_next_nearest_neighbors	
Triangular.order	Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

class tenpy.models.lattice.Triangular(Lx, Ly, site, \*\*kwargs)

Bases: tenpy.models.lattice.SimpleLattice

A triangular lattice.



## **Parameters**

- Ly (Lx,) The length in each direction.
- **site** (*Site*) The local lattice site. The *unit\_cell* of the *Lattice* is just [site].
- \*\*kwargs Additional keyword arguments given to the Lattice. pairs are set ac-

cordingly. If *order* is specified in the form ('standard', snake\_windingi, priority), the *snake\_winding* and *priority* should only be specified for the spatial directions. Similarly, *positions* can be specified as a single vector.

## property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

## count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

#### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

## Return type int

## $coupling\_shape(dx)$

Calculate correct shape of the strengths for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

#### Returns

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

## enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns obj** – Newly generated class instance containing the required data.

## Return type cls

### lat2mps idx(lat idx)

Translate lattice indices  $(x_0, \ldots, x_{D-1})$ , u) to MPS index i.

**Parameters** lat\_idx ( $array_1ike[..,dim+1]$ ) - The last dimension corresponds to lattice indices ( $x_0, ..., x_{D-1}, u$ ). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_MPS$ , an  $x_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

### $mps2lat_idx(i)$

Translate MPS index i to lattice indices  $(x_0, \dots, x_{\dim -1})$ , u).

Parameters i (int | array\_like of int) - MPS index/indices.

**Returns lat\_idx** – First dimensions like i, last dimension has len dim +1 and contains the lattice indices `` $(x_0, \ldots, x_{dim-1}, u)$ ` corresponding to i. For i accross the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

Return type array

## $mps2lat\_values(A, axes=0, u=None)$

same as Lattice.mps2lat\_values(), but ignore u, setting it to 0.

## mps21at\_values\_masked(A, axes=-1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values$  () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

## **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

Return type np.ma.MaskedArray

#### mps\_idx\_fix\_u (u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit cell[u].

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

Return type array

## mps\_lat\_idx\_fix\_u (u=None)

Similar as  $mps_idx_fix_u$  (), but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

#### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

## mps\_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

## multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx [i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

## Returns

- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

## $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use *count\_neighbors()* instead.

## ${\tt number\_next\_nearest\_neighbors}\;(u{=}\theta)$

Deprecated.

Deprecated since version 0.5.0: Use <code>count\_neighbors()</code> instead.

### property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

## ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Parameters order (str | ('standard', snake\_winding, priority) |
 ('grouped', groups)) - Specifies the desired ordering using one of the strings
 of the above tables. Alternatively, an ordering is specified by a tuple with first en try specifying a function, 'standard' for get\_order() and 'grouped' for
 get\_order\_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

**Returns order** – the order to be used for order.

**Return type** array, shape (N, D+1), dtype np.intp

### See also:

```
get_order() generates the order from equivalent priority and snake_winding.
get_order_grouped() variant of get_order.
plot_order() visualizes the resulting order.
```

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

## **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

plot\_bc\_identified (ax, direction=- 1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- cylinder\_axis (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

plot\_coupling (ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

### **Parameters**

• ax (matplotlib.axes.Axes) - The axes on which we should plot.

- coupling (list of (u1, u2, dx)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (i0, i1, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_order(ax, order=None, textkwargs={}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

```
position(lat_idx)
```

return 'space' position of one or multiple sites.

Parameters lat\_idx (ndarray, (..., dim+1)) - Lattice indices.

**Returns pos** – The position of the lattice sites specified by *lat\_idx* in real-space.

**Return type** ndarray, (..., dim)

```
possible couplings (u1, u2, dx)
```

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

## **Parameters**

- **u2** (*u*1,) Indices within the unit cell; the *u*1 and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

## Returns

• mps1, mps2 (array) – For each possible two-site coupling the MPS indices for the u1 and u2.

- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

## possible\_multi\_couplings(ops)

Generalization of possible couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

### **Returns**

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len dim. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

## **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

## site(i)

return Site instance corresponding to an MPS index i

## test\_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

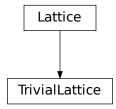
## **TrivialLattice**

• full name: tenpy.models.lattice.TrivialLattice

 $\bullet \ \ parent \ module: \ \textit{tenpy.models.lattice}$ 

• type: class

# Inheritance Diagram



## Methods

TrivialLatticeinit(mps_sites, **kwargs)	Initialize self.
TrivialLattice.count_neighbors([u, key])	Count e.g.
TrivialLattice.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
TrivialLattice.enlarge_mps_unit_cell([fa	ncRepleat the unit cell for infinite MPS boundary condi-
	tions; in place.
<pre>TrivialLattice.from_hdf5(hdf5_loader, h5gr,)</pre>	Load instance from a HDF5 file.
TrivialLattice.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
TrivialLattice.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
TrivialLattice.mps2lat_values(A[, axes,	Reshape/reorder A to replace an MPS index by lattice
u])	indices.
TrivialLattice.mps2lat_values_masked(A	, Reshape/reorder an array A to replace an MPS index by
])	lattice indices.
$TrivialLattice.mps\_idx\_fix\_u([u])$	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
TrivialLattice.mps_lat_idx_fix_u([u])	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
TrivialLattice.mps_sites()	Return a list of sites for all MPS indices.
TrivialLattice.multi_coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a
	multi_coupling.
TrivialLattice.number_nearest_neighbor	
TrivialLattice.number_next_nearest_nei	2 1 12 27
TrivialLattice.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
TrivialLattice.plot_basis(ax[, origin,	Plot arrows indicating the basis vectors of the lattice.
shade])	
TrivialLattice.plot_bc_identified(ax[,	Mark two sites indified by periodic boundary condi-
])	tions.
TrivialLattice.plot_coupling(ax[, cou-	Plot lines connecting nearest neighbors of the lattice.
pling, ])	
	continues on next page

Table 94 – continued from previous page	
TrivialLattice.plot_order(ax[, order,])	Plot a line connecting sites in the specified "order" and
	text labels enumerating them.
TrivialLattice.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
TrivialLattice.position(lat_idx)	return 'space' position of one or multiple sites.
TrivialLattice.possible_couplings(u1,	Find possible MPS indices for two-site couplings.
u2, dx)	
TrivialLattice.possible_multi_coupling	rs@pseralization of possible_couplings() to cou-
	plings with more than 2 sites.
TrivialLattice.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	
TrivialLattice.site(i)	return $Site$ instance corresponding to an MPS index $i$
TrivialLattice.test_sanity()	Sanity check.

# **Class Attributes and Properties**

TrivialLattice.boundary_conditions	Human-readable list of boundary conditions from bc and bc_shift.
TrivialLattice.dim	The dimension of the lattice.
TrivialLattice.nearest_neighbors	
TrivialLattice.next_nearest_neighbors	
TrivialLattice.next_next_nearest_neigh	abors
TrivialLattice.order	Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

# class tenpy.models.lattice.TrivialLattice(mps\_sites, \*\*kwargs)

 $Bases: \ \textit{tenpy.models.lattice.Lattice}$ 

Trivial lattice consisting of a single (possibly large) unit cell in 1D.

This is usefull if you need a valid Lattice with given mps\_sites() and don't care about the actual geometry, e.g, because you don't intend to use the CouplingModel.

#### **Parameters**

- mps\_sites (list of Site) The sites making up a unit cell of the lattice.
- \*\*kwargs Further keyword arguments given to Lattice.

# property boundary\_conditions

Human-readable list of boundary conditions from bc and bc\_shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

# count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

## **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

# Return type int

## $coupling\_shape(dx)$

Calculate correct shape of the *strengths* for a coupling.

**Parameters** dx (tuple of int) - Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

## **Returns**

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

## property dim

The dimension of the lattice.

## enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

# classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

## lat2mps idx(lat idx)

Translate lattice indices  $(x_0, \dots, x_{D-1})$ , u) to MPS index i.

**Parameters lat\_idx** (array\_like [.., dim+1]) - The last dimension corresponds to lattice indices (x\_0, ..., x\_{D-1}, u). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite" bc\_MPS, an x\_0 outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

# $mps2lat_idx(i)$

Translate MPS index i to lattice indices  $(x \ 0, \ldots, x \ \{dim-1\}, u)$ .

Parameters i (int / array\_like of int) - MPS index/indices.

**Returns** lat\_idx – First dimensions like i, last dimension has len dim + 1 and contains the lattice indices  $(x_0, \ldots, x_{dim-1}, u)$  corresponding to i. For i accross the MPS unit cell and "infinite"  $bc_MPS$ , we shift  $x_0$  accordingly.

**Return type** array

```
mps2lat\_values(A, axes=0, u=None)
```

Reshape/reorder A to replace an MPS index by lattice indices.

#### **Parameters**

- A (ndarray) Some values. Must have A. shape[axes] = self.N\_sites if u is None, or A. shape[axes] = self.N cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit\_cell[u], as returned by mps\_idx\_fix\_u(). The resulting array will not have the additional dimension(s) of u.

**Returns res\_A** – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., j, ...].

**Return type** ndarray

## **Examples**

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by self.mps2lat\_idx(i). Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[lat.mps2lat_idx(5)] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use  $mps\_idx\_fix\_u()$  to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
```

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```
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

**Todo:** make sure this function is used for expectation values...

## mps2lat\_values\_masked (A, axes=- 1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values$  () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u((list of) bool)—Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

Return type np.ma.MaskedArray

# mps\_idx\_fix\_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self. unit\_cell[u].

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

Return type array

## mps\_lat\_idx\_fix\_u (u=None)

Similar as mps\_idx\_fix\_u(), but return also the corresponding lattice indices.

Parameters u (None / int) – Selects a site of the unit cell. None (default) means all sites.

#### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

#### mps sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

#### multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

#### Returns

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

## $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

## $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count neighbors () instead.

## property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

## ordering(order)

Provide possible orderings of the *N* lattice sites.

This function can be overwritten by derived lattices to define additional orderings. The following orders are defined in this method:

order	equivalent <i>priority</i>	equivalent snake_winding
'Cstyle'	$(0, 1, \ldots, \dim -1, \dim)$	(False,, False, False)
'default'		
'snake'	$(0, 1, \ldots, \dim -1, \dim)$	(True,, True, True)
'snakeCstyle'		
'Fstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)
'snakeFstyle'	(dim-1,, 1, 0, dim)	(False,, False, False)

Parameters order (str | ('standard', snake\_winding, priority) | ('grouped', groups)) - Specifies the desired ordering using one of the strings of the above tables. Alternatively, an ordering is specified by a tuple with first entry specifying a function, 'standard' for get\_order() and 'grouped' for get\_order\_grouped(), and other arguments in the tuple as specified in the documentation of these functions.

**Returns** order – the order to be used for order.

**Return type** array, shape (N, D+1), dtype np.intp

#### See also:

```
get_order() generates the order from equivalent priority and snake_winding.
get_order_grouped() variant of get_order.
plot_order() visualizes the resulting order.
```

plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

plot\_bc\_identified (ax, direction=-1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- **cylinder\_axis** (bool) Whether to plot the cylinder axis as well.
- origin (None / np.ndarray) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

plot\_coupling (ax, coupling=None, wrap=False, \*\*kwargs)

Plot lines connecting nearest neighbors of the lattice.

# **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (*i0*, *i1*, ..., *u1*) to the site (*i0*+dx[0], *i1*+dx[1], ..., *u2*), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

plot\_order (ax, order=None, textkwargs={}, \*\*kwargs)

Plot a line connecting sites in the specified "order" and text labels enumerating them.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.

- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

## position (lat\_idx)

return 'space' position of one or multiple sites.

```
Parameters lat idx (ndarray, (..., dim+1)) – Lattice indices.
```

**Returns** pos – The position of the lattice sites specified by *lat\_idx* in real-space.

```
Return type ndarray, (..., dim)
```

## $possible\_couplings(u1, u2, dx)$

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le Ls[a]$  and  $0 \le x_a + dx[a] \le lat.Ls[a]$ .

## **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u*1 and *u*2 of add coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

## **Returns**

- **mps1**, **mps2** (*array*) For each possible two-site coupling the MPS indices for the *u1* and *u2*.
- **lat\_indices** (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat indices* has only rows within this shape.

## possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

## **Returns**

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,... for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.

• **coupling\_shape** (*tuple of int*) – Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

## **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5qr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

## site(i)

return Site instance corresponding to an MPS index i

## test\_sanity()

Sanity check.

Raises ValueErrors, if something is wrong.

## **Functions**

<pre>get_lattice(lattice_name)</pre>	Given the name of a Lattice class, get the lattice class
	itself.
<pre>get_order(shape, snake_winding[, priority])</pre>	Built the Lattice.order in (Snake-) C-Style for a
	given lattice shape.
get_order_grouped(shape, groups)	Variant of get_order(), grouping some sites of the
	unit cell.

## get\_lattice

- full name: tenpy.models.lattice.get\_lattice
- parent module: tenpy.models.lattice
- type: function

tenpy.models.lattice.get\_lattice(lattice\_name)

Given the name of a Lattice class, get the lattice class itself.

Parameters lattice\_name (str) - Name of a Lattice class defined in the module lattice, for example "Chain", "Square", "Honeycomb", ....

**Returns** LatticeClass – The lattice class (type, not instance) specified by *lattice\_name*.

Return type Lattice

## get order

• full name: tenpy.models.lattice.get\_order

• parent module: tenpy.models.lattice

· type: function

tenpy.models.lattice.get\_order(shape, snake\_winding, priority=None)
Built the Lattice.order in (Snake-) C-Style for a given lattice shape.

In this function, the word 'direction' referst to a physical direction of the lattice or the index u of the unit cell as an "artificial direction".

#### **Parameters**

- **shape** (tuple of int) The shape of the lattice, i.e., the length in each direction.
- **snake\_winding** (tuple of bool) For each direction one bool, whether we should wind as a "snake" (True) in that direction (i.e., going forth and back) or simply repeat ascending (False)
- **priority** (None | tuple of float) If None (default), use C-Style ordering. Otherwise, this defines the priority along which direction to wind first; the direction with the highest priority increases fastest. For example, "C-Style" order is enforced by priority=(0, 1, 2, ...), and Fortrans F-style order is enforced by priority=(dim, dim-1, ..., 1, 0)
- group (None | tuple of tuple) If None (default), ignore it. Otherwise, it specifies that we group the fastests changing dimension

**Returns order** – An order of the sites for Lattice.order in the specified ordering.

**Return type** ndarray (np.prod(shape), len(shape))

## See also:

```
Lattice.ordering() method in Lattice to obtain the order from parameters.

Lattice.plot_order() visualizes the resulting order in a Lattice.

get order grouped() a variant grouping sites of the unit cell.
```

## get order grouped

• full name: tenpy.models.lattice.get order grouped

• parent module: tenpy.models.lattice

· type: function

```
tenpy.models.lattice.get_order_grouped(shape, groups)
Variant of get_order(), grouping some sites of the unit cell.
```

In this function, the word 'direction' referst to a physical direction of the lattice or the index u of the unit cell as an "artificial direction". This function is usefull for lattices with a unit cell of more than 2 sites (e.g. Kagome). The argument *group* is a To explain the order, assume we have a 3-site unit cell in a 2D lattice with shape (Lx, Ly, Lu). Calling this function with groups=((1, 1), (2, 0)) returns an order of the following form:

```
# columns: [x, y, u]
[0, 0, 1] # first for u = 1 along y
[0, 1, 1]
    :
[0, Ly-1, 1]
[0, 0, 2] # then for u = 2 and 0
[0, 0, 0]
[0, 1, 2]
[0, 1, 0]
    :
[0, Ly-1, 2]
[0, Ly-1, 0]
# and then repeat the above for increasing `x`.
```

#### **Parameters**

- **shape** (tuple of int) The shape of the lattice, i.e., the length in each direction.
- **groups** (tuple of tuple of int) A partition and reordering of range(shape[-1]) into smaller groups. The ordering goes first within a group, then along the last spatial dimensions, then changing between different groups and finally in Cstyle order along the remaining spatial dimensions.

**Returns order** – An order of the sites for Lattice.order in the specified ordering.

**Return type** ndarray (np.prod(shape), len(shape))

#### See also:

```
Lattice.ordering() method in Lattice to obtain the order from parameters.

Lattice.plot_order() visualizes the resulting order in a Lattice.
```

# **Module description**

Classes to define the lattice structure of a model.

The base class Lattice defines the general structure of a lattice, you can subclass this to define you own lattice. The SimpleLattice is a slight simplification for lattices with a single-site unit cell. Further, we have some predefined lattices, namely Chain, Ladder in 1D and Square, Triangular, Honeycomb, and Kagome in 2D.

The IrregularLattice provides a way to remove or add sites to an existing, regular lattice.

See also the Models.

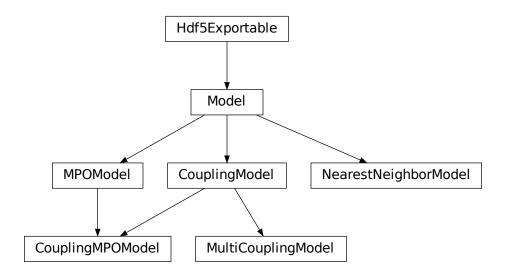
## 7.9.2 model

• full name: tenpy.models.model

• parent module: tenpy.models

• type: module

# Classes



CouplingMPOModel(model_params)	Combination of the CouplingModel and
	MPOModel.
CouplingModel(lattice[, bc_coupling,])	Base class for a general model of a Hamiltonian consist-
	ing of two-site couplings.
MPOMode1(lattice, H_MPO)	Base class for a model with an MPO representation of
	the Hamiltonian.
Model(lattice)	Base class for all models.
MultiCouplingModel(lattice[, bc_coupling,])	Generalizes CouplingModel to allow couplings in-
	volving more than two sites.
NearestNeighborModel(lattice, H_bond)	Base class for a model of nearest neighor interactions
	w.r.t.

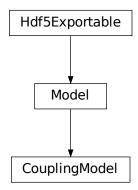
# CouplingModel

• full name: tenpy.models.model.CouplingModel

• parent module: tenpy.models.model

• type: class

# Inheritance Diagram



# Methods

	T '.' 1' 10
CouplingModelinit(lattice[,])	Initialize self.
CouplingModel.add_coupling(strength, u1,	Add twosite coupling terms to the Hamiltonian, sum-
)	ming over lattice sites.
CouplingModel.add_coupling_term(strength,	Add a two-site coupling term on given MPS sites.
i,)	
CouplingModel.add_local_term(strength,	Add a single term to <i>self</i> .
term)	
CouplingModel.add_onsite(strength, u, op-	Add onsite terms to onsite_terms.
name)	
CouplingModel.add_onsite_term(strength, i,	Add an onsite term on a given MPS site.
op)	
CouplingModel.all_coupling_terms()	Sum of all coupling_terms.
CouplingModel.all_onsite_terms()	Sum of all onsite_terms.
CouplingModel.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
CouplingModel.calc_H_bond([tol_zero])	<pre>calculate H_bond from coupling_terms and</pre>
	onsite_terms.
CouplingModel.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
CouplingModel.coupling_strength_add_ex	tAdd an external flux to the coupling strength.
CouplingModel.enlarge_mps_unit_cell([fac	etoRepeat the unit cell for infinite MPS boundary condi-
	tions; in place.
CouplingModel.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
CouplingModel.group_sites([n,	Modify <i>self</i> in place to group sites.
grouped_sites])	
CouplingModel.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	
CouplingModel.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

Bases: tenpy.models.model.Model

Base class for a general model of a Hamiltonian consisting of two-site couplings.

In this class, the terms of the Hamiltonian are specified explicitly as OnsiteTerms or CouplingTerms.

Deprecated since version 0.4.0:  $bc\_coupling$  will be removed in 1.0.0. To specify the full geometry in the lattice, use the bc parameter of the Lattice.

## **Parameters**

- lattice (Lattice) The lattice defining the geometry and the local Hilbert space(s).
- bc\_coupling ((iterable of) {'open' | 'periodic' | int}) Boundary conditions of the couplings in each direction of the lattice. Defines how the couplings are added in add\_coupling(). A single string holds for all directions. An integer shift means that we have periodic boundary conditions along this direction, but shift/tilt by -shift\*lattice.basis[0] (~cylinder axis for bc\_MPS='infinite') when going around the boundary along this direction.
- **explicit\_plus\_hc** (bool) If True, the Hermitian conjugate of the MPO is computed at runtime, rather than saved in the MPO.

#### onsite terms

The OnsiteTerms ordered by category.

```
Type {'category': OnsiteTerms}
```

## coupling\_terms

The CouplingTerms ordered by category. In a MultiCouplingModel, values may also be MultiCouplingTerms.

```
Type {'category': CouplingTerms}
```

## explicit\_plus\_hc

If *True*, *self* represents the terms in *onsite\_terms* and *coupling\_terms* and their hermitian conjugate added. The flag will be carried on the MPO, which will have a reduced bond dimension if self.add\_coupling(..., plus\_hc=True) was used. Note that *add\_onsite()* and *add\_coupling()* respect this flag, ensuring that the *represented* Hamiltonian is indepentent of the *explicit\_plus\_hc* flag.

Type bool

## test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

```
add_local_term (strength, term, category=None, plus_hc=False)
```

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to *onsite\_terms* or *coupling\_terms*, respectively.

## **Parameters**

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array\_like*)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice

index  $lat\_idx$ . Here,  $lat\_idx$  is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.

- category Descriptive name used as key for onsite\_terms or coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

```
add_onsite (strength, u, opname, category=None, plus_hc=False)
```

Add onsite terms to onsite terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat.unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to <code>onsite\_terms</code>; doesn't rebuild the MPO.

## **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit\_cell[u] out of the unit cell.
- **opname** (*str*) valid operator name of an onsite operator in lat.unit\_cell[u].
- category (str) Descriptive name used as key for onsite\_terms. Defaults to opname.
- plus  $hc(b \circ 0)$  If *True*, the hermitian conjugate of the terms is added automatically.

#### See also:

```
add_coupling() Add a terms acting on two sites.
```

add\_onsite\_term() Add a single term without summing over vecx.

# $\verb"add_onsite_term" (strength, i, op, category=None, plus\_hc=False)"$

Add an onsite term on a given MPS site.

Wrapper for self.onsite\_terms[category].add\_onsite\_term(...).

# **Parameters**

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < I.
- op (str) Name of the involved operator.
- category (str) Descriptive name used as key for onsite\_terms. Defaults to op.
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

# all\_onsite\_terms()

Sum of all onsite\_terms.

Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}, u1), and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site (x\_0+dx[0], ...,

 $x_{\dim-1}+dx[\dim-1]$ , u1). Possible combinations  $x_0$ , ...,  $x_{\dim-1}$  are determined from the boundary conditions in  $possible\_couplings()$ .

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling\_shape* returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to <code>coupling\_terms</code>; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str\_on\_first* and *raise\_op2\_left* will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit\_cell[u1] for OP1.
- **op1** (*str*) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit\_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (*iterable of int*) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str / None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

## **Examples**

When initializing a model, you can add a term  $J\sum_{\langle i,j\rangle} S_i^z S_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx) ` is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

Gdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

## See also:

```
add_onsite() Add terms acting on one site only.
MultiCouplingModel.add_multi_coupling_term() for terms on more than two sites.
add_coupling_term() Add a single term without summing over vecx.
```

add\_coupling\_term(strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling\_terms[category].add\_coupling\_term(...).

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

# **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}(i,)$  The MPS indices of the two sites on which the operator acts. We require  $0 <= i < N_sites$  and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_sites$ , it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.

- op\_string (str) The operator to be inserted between i and j.
- **category** (*str*) Descriptive name used as key for *coupling\_terms*. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

#### all\_coupling\_terms()

Sum of all coupling\_terms.

## calc\_H\_onsite(tol\_zero=1e-15)

Calculate *H\_onsite* from *self.onsite\_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take <code>explicit\_plus\_hc</code> into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are
 considered to be zero.</pre>

#### Returns

- **H\_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If <code>explicit\_plus\_hc</code> is True, Hermitian conjugates of the onsite terms will be included.

## calc\_H\_bond(tol\_zero=1e-15)

calculate H\_bond from coupling\_terms and onsite\_terms.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns** H\_bond – Bond terms as required by the constructor of <code>NearestNeighborModel</code>. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

## calc\_H\_MPO(tol\_zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite\_terms and coupling\_terms to build an MPO graph (and then an MPO).

Parameters tol\_zero (float) - Prefactors with abs(strength) < tol\_zero are
 considered to be zero.</pre>

**Returns H MPO** – MPO representation of the Hamiltonian.

Return type MPO

## coupling\_strength\_add\_ext\_flux (strength, dx, phase)

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i\ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in add\_coupling().

## **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add\_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

**Returns strength** – The strength array to be used as *strength* in  $add\_coupling()$  with the given dx.

**Return type** complex array

## **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

# enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

# **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns obj** – Newly generated class instance containing the required data.

# Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each *n* sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

#### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

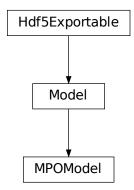
# **MPOModel**

• full name: tenpy.models.model.MPOModel

• parent module: tenpy.models.model

· type: class

# **Inheritance Diagram**



## **Methods**

MPOModelinit(lattice, H_MPO)	Initialize self.
MPOModel.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
MPOModel.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
MPOModel.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
<pre>MPOModel.group_sites([n, grouped_sites])</pre>	Modify <i>self</i> in place to group sites.
MPOModel.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
MPOModel.test_sanity()	

class tenpy.models.model.MPOModel(lattice, H\_MPO)

Bases: tenpy.models.model.Model

Base class for a model with an MPO representation of the Hamiltonian.

In this class, the Hamiltonian gets represented by an MPO. Thus, instances of this class are suitable for MPO-based algorithms like DMRG <code>dmrg</code> and MPO time evolution.

**Todo:** implement MPO for time evolution...

Parameters H\_MPO (MPO) – The Hamiltonian rewritten as an MPO.

## H\_MPO

MPO representation of the Hamiltonian. If the  $explicit\_plus\_hc$  flag of the MPO is True, the represented Hamiltonian is  $H\_MPO + hermitian\_cojugate(H\_MPO)$ .

Type tenpy.networks.mpo.MPO

```
enlarge mps unit cell(factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

## **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of *GroupedSite*) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

```
calc H bond from MPO(tol zero=1e-15)
```

Calculate the bond Hamiltonian from the MPO Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

```
Returns H_bond – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0*', 'p1', 'p1*']
```

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

## **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with <code>save\_dict\_content()</code>, storing the format under the attribute 'format'.

#### **Parameters**

• hdf5 saver (Hdf5Saver) – Instance of the saving engine.

- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

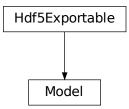
# Model

• full name: tenpy.models.model.Model

• parent module: tenpy.models.model

• type: class

# **Inheritance Diagram**



## Methods

Modelinit(lattice)	Initialize self.
Model.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
Model.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
<pre>Model.group_sites([n, grouped_sites])</pre>	Modify self in place to group sites.
Model.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.

class tenpy.models.model.Model(lattice)

 $Bases: \ \textit{tenpy.tools.hdf5\_io.Hdf5Exportable}$ 

Base class for all models.

The common base to all models is the underlying Hilbert space and geometry, specified by a Lattice.

**Parameters lattice** (Lattice) – The lattice defining the geometry and the local Hilbert space(s).

lat

The lattice defining the geometry and the local Hilbert space(s).

Type Lattice

enlarge\_mps\_unit\_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

#### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with <code>save\_dict\_content()</code>, storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

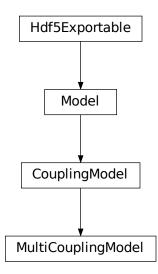
# MultiCouplingModel

 $\bullet \ \ full \ name: tenpy.models.model.MultiCouplingModel\\$ 

• parent module: tenpy.models.model

• type: class

# **Inheritance Diagram**



# **Methods**

MultiCouplingModelinit(lattice[,])	Initialize self.
MultiCouplingModel.	Add twosite coupling terms to the Hamiltonian, sum-
add_coupling(strength,)	ming over lattice sites.
MultiCouplingModel.	Add a two-site coupling term on given MPS sites.
add_coupling_term $([,])$	
MultiCouplingModel.	Add a single term to self.
<pre>add_local_term(strength, term)</pre>	
MultiCouplingModel.	Add multi-site coupling terms to the Hamiltonian, sum-
add_multi_coupling $(\ldots [,\ldots ])$	ming over lattice sites.
MultiCouplingModel.	Add a general M-site coupling term on given MPS sites.
add_multi_coupling_term $(\dots)$	
MultiCouplingModel.add_onsite(strength, u,	Add onsite terms to onsite_terms.
)	
MultiCouplingModel.	Add an onsite term on a given MPS site.
<pre>add_onsite_term(strength,)</pre>	
	continues on next page

table 101 Continued from previous page	
MultiCouplingModel.	Sum of all coupling_terms.
all_coupling_terms()	
MultiCouplingModel.all_onsite_terms()	Sum of all onsite_terms.
MultiCouplingModel.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
MultiCouplingModel.	calculate H_bond from coupling_terms and
calc_H_bond([tol_zero])	onsite_terms.
MultiCouplingModel.	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
<pre>calc_H_onsite([tol_zero])</pre>	
MultiCouplingModel.	Add an external flux to the coupling strength.
${\it coupling\_strength\_add\_ext\_flux}(\ldots)$	
MultiCouplingModel.	Repeat the unit cell for infinite MPS boundary condi-
<pre>enlarge_mps_unit_cell([])</pre>	tions; in place.
MultiCouplingModel.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
MultiCouplingModel.group_sites([n,])	Modify <i>self</i> in place to group sites.
MultiCouplingModel.save_hdf5(hdf5_saver,	Export self into a HDF5 file.
)	
MultiCouplingModel.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

Table 101 – continued from previous page

Bases: tenpy.models.model.CouplingModel

Generalizes CouplingModel to allow couplings involving more than two sites.

The corresponding couplings can be added with add\_multi\_coupling() and add\_multi\_coupling\_term() and are saved in coupling\_terms, which can now contain instances of MultiCouplingTerms.

 $\begin{tabular}{ll} {\bf add\_multi\_coupling} (strength, & ops, & \_deprecate\_l='DEPRECATED', & \_deprecate\_2='DEPRECATED', op\_string=None, category=None, plus\_hc=False) \\ {\bf Add multi-site coupling terms to the Hamiltonian, summing over lattice sites.} \\ \end{tabular}$ 

Represents couplings of the form  $sum_{\vec{x}} strength[shift(\vec{x})] * OP_0 * OP_1 * ... * OP_{M-1}$ , involving M operators. Here,  $OP_m$  stands for the operator defined by the m-th tuple (opname, dx, u) given in the argument ops, which determines the position  $\vec{x} + \vec{dx}$  and unit-cell index u of the site it acts on; the actual operator is given by  $self.lat.unit\_cell[u].get\_op(opname)$ .

The coupling strength may vary spatially if the given strength is a numpy array. The correct shape of this array is the  $coupling\_shape$  returned by tenpy.models.lattice. possible\_multi\_couplings() and depends on the boundary conditions. The shift(...) depends on the dx entries of ops and is chosen such that the first entry strength[0, 0, ...] of strength is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.6.0: We switched from the three arguments u0, op0 and  $other_op$  with other\_ops=[(u1, op1, dx1), (op2, u2, dx2), ...] to a single, equivalent argment ops which should now read ops=[(op0, dx0, u0), (op1, dx1, u1), (op2, dx2, u2), ...], where dx0 = [0]\*self.lat.dim. Note the changed order inside the tuples!

## **Parameters**

• **strength** (*scalar* / *array*) – Prefactor of the coupling. May vary spatially, and is tiled to the required shape.

- **ops** (list of (opname, dx, u)) Each tuple determines one operator of the coupling, see the description above. *opname* (str) is the name of the operator, dx (list of length lat.dim) is a translation vector, and u (int) is the index of  $lat.unit\_cell$  on which the operator acts. The first entry of ops corresponds to  $OP_0$  and acts last in the physical sense.
- op\_string(str | None) If a string is given, we use this as the name of an operator to be used inbetween the operators, excluding the sites on which any operators act. This operator should be defined on all sites in the unit cell.
  - If None, auto-determine whether a Jordan-Wigner string is needed (using  $op\_needs\_JW()$ ) for each of the segments inbetween the operators and also on the sites of the left operators.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op0}\_i {other\_ops[0]}\_j {other\_ops[1]}\_k ...".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

## **Examples**

A call to add\_coupling() with arguments add\_coupling(strength, u1, 'A', u2, 'B', dx) is equivalent to the following:

```
>>> dx_0 = [0] * self.lat.dim # = [0] for a 1D lattice, [0, 0] in 2D 
>>> self.add_coupling(strength, [('A', dx_0, u1), ('B', dx, u2)])
```

To explicitly add the hermitian conjugate, you need to take the complex conjugate of the *strength*, reverse the order of the operators and take the hermitian conjugates of the individual operator names:

```
>>> self.add_coupling(np.conj(strength), [(hc('B'), dx, u2), (hc('A'), dx_0, u1)]) # h.c.
```

## See also:

```
add_onsite() Add terms acting on one site only.
add coupling() Add terms acting on two sites.
```

add\_multi\_coupling\_term() Add a single term, not summing over the possible  $\vec{x}$ .

add\_multi\_coupling\_term (strength, ijkl, ops\_ijkl, op\_string, category=None, plus\_hc=False)
Add a general M-site coupling term on given MPS sites.

Wrapper for self.coupling\_terms[category].add\_multi\_coupling\_term(...).

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

## **Parameters**

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With i, j, k, ... = ijkl, we require that they are ordered ascending, i < j < k < ... and that 0 <= i < N\_sites. Inidees >= N\_sites indicate couplings between different unit cells of an infinite MPS.

- ops\_ijkl (list of str) Names of the involved operators on sites  $i, j, k, \ldots$
- op\_string (list of str) Names of the operator to be inserted between the operators, e.g., op\_string[0] is inserted between i and j.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op0}\_i {op1}\_j {op2}\_k ...".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add\_coupling (strength, u1, op1, u2, op2, dx, op\_string=None, str\_on\_first=True, raise\_op2\_left=False, category=None, plus\_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}, u1), and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site (x\_0+dx[0], ..., x\_{dim-1}+dx[dim-1], u1). Possible combinations x\_0, ..., x\_{dim-1} are determined from the boundary conditions in possible\_couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling\_shape* returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str\_on\_first* and *raise\_op2\_left* will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- **u1** (*int*) Picks the site lat.unit\_cell[u1] for OP1.
- **op1** (*str*) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- u2 (int) Picks the site lat.unit\_cell[u2] for OP2.
- **op2** (str) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.

- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

## **Examples**

When initializing a model, you can add a term  $J \sum_{\langle i,j \rangle} S_i^z S_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx)` is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

--Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

## See also:

```
add_onsite() Add terms acting on one site only.
```

MultiCouplingModel.add\_multi\_coupling\_term() for terms on more than two sites.

add\_coupling\_term() Add a single term without summing over vecx.

add\_coupling\_term (strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling\_terms[category].add\_coupling\_term(...).

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

#### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}$  (i,) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N\_sites and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_s$ ites, it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.
- op\_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add\_local\_term (strength, term, category=None, plus\_hc=False)

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite\_terms or coupling\_terms, respectively.

## Parameters

- **strength** (*float/complex*) The prefactor of the term.
- **term** (list of (str, array\_like)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice index lat\_idx. Here, lat\_idx is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite\_terms or coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add\_onsite (strength, u, opname, category=None, plus\_hc=False)

Add onsite terms to onsite terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat.unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to onsite\_terms; doesn't rebuild the MPO.

#### **Parameters**

• **strength** (*scalar* / *array*) – Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.

```
• u (int) - Picks a Site lat.unit_cell[u] out of the unit cell.
             • opname (str) - valid operator name of an onsite operator in lat.unit_cell[u].
             • category (str) - Descriptive name used as key for onsite_terms. Defaults to
              opname.
             • plus hc (b \circ \circ 1) – If True, the hermitian conjugate of the terms is added automatically.
    See also:
    add_coupling() Add a terms acting on two sites.
    add_onsite_term() Add a single term without summing over vecx.
add_onsite_term(strength, i, op, category=None, plus_hc=False)
    Add an onsite term on a given MPS site.
    Wrapper for self.onsite_terms[category].add_onsite_term(...).
        Parameters
             • strength (float) – The strength of the term.
             • i (int) – The MPS index of the site on which the operator acts. We require 0 <= i <
              L.
             • op (str) – Name of the involved operator.
             • category (str) - Descriptive name used as key for onsite_terms. Defaults to op.
             • plus_hc (bool) - If True, the hermitian conjugate of the term is added automatically.
all_coupling_terms()
    Sum of all coupling_terms.
all_onsite_terms()
    Sum of all onsite_terms.
calc_H_MPO (tol_zero=1e-15)
    Calculate MPO representation of the Hamiltonian.
    Uses onsite_terms and coupling_terms to build an MPO graph (and then an MPO).
        Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are
            considered to be zero.
         Returns H_MPO – MPO representation of the Hamiltonian.
        Return type MPO
calc H bond (tol zero=1e-15)
    calculate H_bond from coupling_terms and onsite_terms.
        Parameters tol_zero (float) - prefactors with abs(strength) < tol_zero are</pre>
            considered to be zero.
```

:raises ValueError : if the Hamiltonian contains longer-range terms.:

Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

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**Returns H\_bond** – Bond terms as required by the constructor of *NearestNeighborModel*.

```
calc H onsite (tol zero=1e-15)
```

Calculate *H\_onsite* from *self.onsite\_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take explicit\_plus\_hc into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

#### Returns

- **H\_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit\_plus\_hc is True, Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i \ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in add\_coupling().

## **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (iterable of int) Translation vector (of the unit cell) between op1 and op2 in add\_coupling().
- phase (iterable of float) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase phi when hopping around the cylinder.

**Returns** strength – The strength array to be used as strength in add\_coupling() with the given dx.

Return type complex array

## **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

```
>>> strength = 1. # hopping strength without external flux
>>> phi = np.pi/4 # determines the external flux strength
>>> strength_with_flux = self.coupling_strength_add_ext_flux(strength, dx, [0, ophi])
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
```

(continues on next page)

(continued from previous page

```
self.add_coupling(strength_with_flux, u1, 'Cd', u2, 'C', dx)
self.add_coupling(np.conj(strength_with_flux), u2, 'Cd', u1, 'C', -dx)
```

```
enlarge_mps_unit_cell (factor=2)
```

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

# classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each *n* sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

## **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

# test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

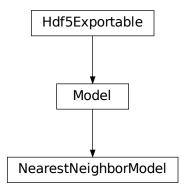
# NearestNeighborModel

• full name: tenpy.models.model.NearestNeighborModel

• parent module: tenpy.models.model

• type: class

# **Inheritance Diagram**



# **Methods**

NearestNeighborModel. init (lattice,	Initialize self.
H_bond)	
NearestNeighborModel.	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
bond_energies(psi)	
NearestNeighborModel.	Calculate the MPO Hamiltonian from the bond Hamil-
$calc_H_{MPO\_from\_bond}([])$	tonian.
NearestNeighborModel.	Repeat the unit cell for infinite MPS boundary condi-
$enlarge\_mps\_unit\_cell([])$	tions; in place.
NearestNeighborModel.	Initialize a NearestNeighborModel from a model class
<pre>from_MPOModel(mpo_model)</pre>	defining an MPO.
NearestNeighborModel.	Load instance from a HDF5 file.
<pre>from_hdf5(hdf5_loader,)</pre>	
NearestNeighborModel.group_sites([n,	Modify <i>self</i> in place to group sites.
])	
NearestNeighborModel.	Export self into a HDF5 file.
save_hdf5(hdf5_saver,)	
NearestNeighborModel.test_sanity()	
	continues on next page

## Table 102 – continued from previous page

NearestNeighborModel.	Return a NearestNeighborModel with same lattice, but
$trivial\_like\_NNModel()$	trivial (H=0) bonds.

```
class tenpy.models.model.NearestNeighborModel(lattice, H_bond)
```

```
Bases: tenpy.models.model.Model
```

Base class for a model of nearest neigbor interactions w.r.t. the MPS index.

In this class, the Hamiltonian  $H = \sum_i H_{i,i+1}$  is represented by "bond terms"  $H_{i,i+1}$  acting only on two neighboring sites i and i+1, where i is an integer. Instances of this class are suitable for  $t \in \mathcal{Bd}$ .

Note that the "nearest-neighbor" in the name refers to the MPS index, not the lattice. In short, this works only for 1-dimensional (1D) nearest-neighbor models: A 2D lattice is internally mapped to a 1D MPS "snake", and even a nearest-neighbor coupling in 2D becomes long-range in the MPS chain.

#### **Parameters**

- lattice (tenpy.model.lattice.Lattice) The lattice defining the geometry and the local Hilbert space(s).
- H\_bond (list of {Array | None}) The Hamiltonian rewritten as sum\_i H\_bond[i] for MPS indices i. H\_bond[i] acts on sites (i-1, i); we require len (H\_bond) == lat.N\_sites. Legs of each H\_bond[i] are ['p0', 'p0\*', 'p1', 'p1\*'].

#### H bond

The Hamiltonian rewritten as sum\_i H\_bond[i] for MPS indices i. H\_bond[i] acts on sites (i-1, i), None represents 0. Legs of each H\_bond[i] are ['p0', 'p0\*', 'p1', 'p1\*']. H\_bond is not affected by the explicit\_plus\_hc flag of a CouplingModel.

```
Type list of {Array | None}
```

## classmethod from\_MPOModel(mpo\_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group\_sites().

**Parameters** mpo\_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

## **Examples**

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
parameter 'L'=20 for SpinChainNNN2
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define  $H\_bond$ . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
```

# trivial\_like\_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

#### bond energies (psi)

Calculate bond energies <psilH\_bondlpsi>.

**Parameters** psi(MPS) – The MPS for which the bond energies should be calculated.

**Returns**  $E\_bond - List$  of bond energies: for finite bc,  $E\_Bond[i]$  is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc  $E\_bond[i]$  is the energy of bond i-1, i.

**Return type** 1D ndarray

# enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## group\_sites(n=2, grouped\_sites=None)

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

#### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of *GroupedSite*) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

## calc\_H\_MPO\_from\_bond(tol\_zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm <  $tol_zero$  are considered to be zero.

**Returns H\_MPO** – MPO representation of the Hamiltonian.

Return type MPO

# classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

## **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

## **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

## **Module description**

This module contains some base classes for models.

A 'model' is supposed to represent a Hamiltonian in a generalized way. The Lattice specifies the geometry and underlying Hilbert space, and is thus common to all models. It is needed to intialize the common base class Model of all models.

Different algorithms require different representations of the Hamiltonian. For example for DMRG, the Hamiltonian needs to be given as an MPO, while TEBD needs the Hamiltonian to be represented by 'nearest neighbor' bond terms. This module contains the base classes defining these possible representations, namley the MPOModel and NearestNeighborModel.

A particular model like the XXZChain should then yet another class derived from these classes. In it's \_\_init\_\_, it needs to explicitly call the MPOModel.\_\_init\_\_(self, lattice, H\_MPO), providing an MPO representation of H, and also the NearestNeighborModel.\_\_init\_\_(self, lattice, H\_bond), providing a representation of H by bond terms  $H_bond$ .

The CouplingModel is the attempt to generalize the representation of H by explicitly specifying the couplings in a general way, and providing functionality for converting them into  $H\_MPO$  and  $H\_bond$ . This allows to quickly generate new model classes for a very broad class of Hamiltonians.

For simplicity, the <code>CouplingModel</code> is limited to interactions involving only two sites. Yet, we also provide the <code>MultiCouplingModel</code> to generate Models for Hamiltonians involving couplings between multiple sites.

The CouplingMPOModel aims at structuring the initialization for most models and is used as base class in (most of) the predefined models in TeNPy.

See also the introduction in *Models*.

## Specific models

tf_ising	Prototypical example of a quantum model: the trans-
	verse field Ising model.
xxz_chain	Prototypical example of a 1D quantum model: the spin-
	1/2 XXZ chain.
spins	Nearest-neighbour spin-S models.
spins_nnn	Next-Nearest-neighbour spin-S models.
	continues on next page

continues on next page

Table 103 – continued from previous page

fermions_spinless	Spinless fermions with hopping and interaction.
hubbard	Bosonic and fermionic Hubbard models.
hofstadter	Cold atomic (Harper-)Hofstadter model on a strip or
	cylinder.
haldane	Bosonic and fermionic Haldane models.
toric_code	Kitaev's exactly solvable toric code model.

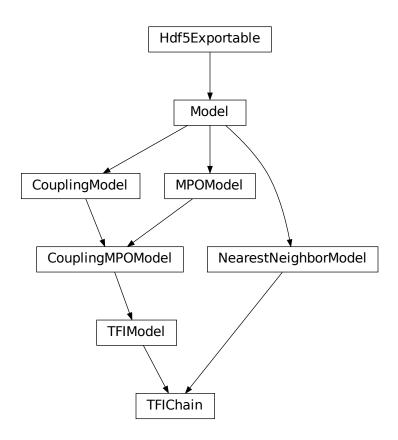
# 7.9.3 tf\_ising

• full name: tenpy.models.tf\_ising

• parent module: tenpy.models

• type: module

### Classes



TFIChair.	(model_	_params)	
-----------	---------	----------	--

The TFIModel on a Chain, suitable for TEBD.

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## Table 104 – continued from previous page

TFIModel(model\_params)

Transverse field Ising model on a general lattice.

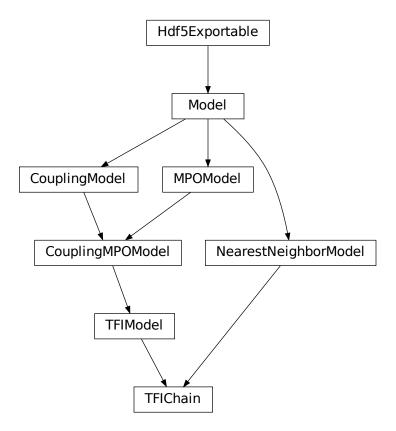
### **TFIChain**

• full name: tenpy.models.tf\_ising.TFIChain

• parent module: tenpy.models.tf\_ising

• type: class

## **Inheritance Diagram**



### Methods

TFIChaininit(model_params)	Initialize self.
TFIChain.add_coupling(strength, u1, op1, u2,	Add twosite coupling terms to the Hamiltonian, sum-
)	ming over lattice sites.
TFIChain.add_coupling_term(strength, i, j,	Add a two-site coupling term on given MPS sites.
)	
TFIChain.add_local_term(strength, term[,	Add a single term to <i>self</i> .
])	
<pre>TFIChain.add_onsite(strength, u, opname[,])</pre>	Add onsite terms to onsite_terms.
TFIChain.add_onsite_term(strength, i, op[,	Add an onsite term on a given MPS site.
])	
TFIChain.all_coupling_terms()	Sum of all coupling_terms.
TFIChain.all_onsite_terms()	Sum of all onsite_terms.
TFIChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
TFIChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
TFIChain.calc_H_MPO_from_bond([tol_zero])	Calculate the MPO Hamiltonian from the bond Hamil-
	tonian.
TFIChain.calc_H_bond([tol_zero])	calculate $H\_bond$ from coupling_terms and
	onsite_terms.
TFIChain.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
TFIChain.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
TFIChain.coupling_strength_add_ext_flu	
TFIChain.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
TFIChain.from_MPOModel(mpo_model)	Initialize a NearestNeighborModel from a model class
	defining an MPO.
TFIChain.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
<pre>TFIChain.group_sites([n, grouped_sites])</pre>	Modify <i>self</i> in place to group sites.
TFIChain.init_lattice(model_params)	Initialize a lattice for the given model parameters.
TFIChain.init_sites(model_params)	Define the local Hilbert space and operators; needs to
	be implemented in subclasses.
TFIChain.init_terms(model_params)	Add the onsite and coupling terms to the model; sub-
	classes should implement this.
TFIChain.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
TFIChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
TFIChain.trivial_like_NNModel()	Return a NearestNeighborModel with same lattice, but
	trivial (H=0) bonds.

```
class tenpy.models.tf_ising.TFIChain (model_params)
```

Bases: tenpy.models.tf\_ising.TFIModel, tenpy.models.model.
NearestNeighborModel

The TFIModel on a Chain, suitable for TEBD.

See the  ${\tt TFIModel}$  for the documentation of parameters.

add\_coupling (strength, u1, op1, u2, op2, dx, op\_string=None, str\_on\_first=True, raise\_op2\_left=False, category=None, plus\_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}), u1),

and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site  $(x_0+dx[0], \ldots, x_{\dim-1}+dx[\dim-1], u1)$ . Possible combinations  $x_0, \ldots, x_{\dim-1}$  are determined from the boundary conditions in possible couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling\_shape* returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str\_on\_first* and *raise\_op2\_left* will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- **u1** (*int*) Picks the site lat.unit\_cell[u1] for OP1.
- op1 (str) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- **u2** (int) Picks the site lat.unit\_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (*iterable of int*) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

#### **Examples**

When initializing a model, you can add a term  $J\sum_{\langle i,j\rangle} S_i^z S_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx) is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c.

→Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

#### See also:

```
add_onsite() Add terms acting on one site only.
```

MultiCouplingModel.add\_multi\_coupling\_term() for terms on more than two sites.

add\_coupling\_term() Add a single term without summing over vecx.

add\_coupling\_term (strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling\_terms[category].add\_coupling\_term(...).

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

### **Parameters**

• **strength** (*float*) – The strength of the coupling term.

- $\mathbf{j}$  (*i*,) The MPS indices of the two sites on which the operator acts. We require  $0 <= i < N_sites$  and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_sites$ , it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.
- op\_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

```
add_local_term (strength, term, category=None, plus_hc=False)
```

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite\_terms or coupling\_terms, respectively.

#### **Parameters**

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array\_like*)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat\_idx*. Here, *lat\_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite\_terms of coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

### add\_onsite (strength, u, opname, category=None, plus\_hc=False)

Add onsite terms to onsite\_terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat.unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to onsite\_terms; doesn't rebuild the MPO.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- **u** (*int*) Picks a Site lat.unit\_cell[u] out of the unit cell.
- **opname** (*str*) valid operator name of an onsite operator in lat.unit\_cell[u].
- category (str) Descriptive name used as key for onsite\_terms. Defaults to opname.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

### See also:

```
add_coupling() Add a terms acting on two sites.
add_onsite_term() Add a single term without summing over vecx.
```

```
add_onsite_term (strength, i, op, category=None, plus_hc=False)
    Add an onsite term on a given MPS site.
    Wrapper for self.onsite_terms[category].add_onsite_term(...).
        Parameters
             • strength (float) – The strength of the term.
             • i (int) - The MPS index of the site on which the operator acts. We require 0 <= i <
              L.
             • op (str) – Name of the involved operator.
             • category (str) - Descriptive name used as key for onsite_terms. Defaults to op.
             • plus_hc (bool) – If True, the hermitian conjugate of the term is added automatically.
all_coupling_terms()
    Sum of all coupling_terms.
all_onsite_terms()
    Sum of all onsite terms.
bond energies (psi)
    Calculate bond energies <psilH_bondlpsi>.
        Parameters psi (MPS) – The MPS for which the bond energies should be calculated.
         Returns E bond – List of bond energies: for finite bc, E Bond[i] is the energy of bond i,
            i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the
            energy of bond i-1, i.
         Return type 1D ndarray
calc_H_MPO(tol_zero=1e-15)
    Calculate MPO representation of the Hamiltonian.
    Uses onsite_terms and coupling_terms to build an MPO graph (and then an MPO).
        Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are</pre>
            considered to be zero.
        Returns H MPO – MPO representation of the Hamiltonian.
        Return type MPO
calc_H_MPO_from_bond(tol_zero=1e-15)
    Calculate the MPO Hamiltonian from the bond Hamiltonian.
        Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.
         Returns H MPO – MPO representation of the Hamiltonian.
        Return type MPO
calc_H_bond(tol_zero=1e-15)
```

Legs are ['p0', 'p0\*', 'p1', 'p1\*'] **Return type** list of *Array* 

**Parameters tol\_zero** (float) - prefactors with abs(strength) < tol\_zero are

Returns H\_bond - Bond terms as required by the constructor of NearestNeighborModel.

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calculate *H\_bond* from coupling\_terms and onsite\_terms.

considered to be zero.

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc H bond from MPO (tol zero=1e-15)
```

Calculate the bond Hamiltonian from the MPO Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

**Returns**  $H_{bond}$  - Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

**Return type** list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_onsite(tol_zero=1e-15)
```

Calculate *H\_onsite* from *self.onsite\_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take explicit\_plus\_hc into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

#### Returns

- **H\_onsite** (*list of npc.Array*)
- onsite terms of the Hamiltonian. If explicit\_plus\_hc is True, Hermitian conjugates of the onsite terms will be included.

```
coupling_strength_add_ext_flux (strength, dx, phase)
```

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i\ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in  $add\_coupling()$ .

### **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add\_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

**Returns strength** – The strength array to be used as *strength* in  $add\_coupling()$  with the given dx.

Return type complex array

### **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

### enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

### classmethod from\_MPOModel(mpo\_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group\_sites().

**Parameters** mpo\_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

### **Examples**

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
parameter 'L'=20 for SpinChainNNN2
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define *H\_bond*. However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
```

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

#### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

```
init_lattice (model_params)
```

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) - The model parameters given to __init__.
```

**Returns lat** – An initialized lattice.

Return type Lattice

### **Options**

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite"  $bc\_MPS$ , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on  $bc\_y$ ).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc\_MPS="finite" and "periodic" for bc\_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

#### init\_sites (model\_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init\_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

**Note:** Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

**Returns sites** – The local sites of the lattice, defining the local basis states and operators.

```
Return type (tuple of) Site
```

### init\_terms (model\_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

### **Parameters**

- hdf5 saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### trivial\_like\_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

## **Module description**

Prototypical example of a quantum model: the transverse field Ising model.

Like the XXZChain, the transverse field ising chain *TFIChain* is contained in the more general *SpinChain*; the idea is more to serve as a pedagogical example for a 'model'.

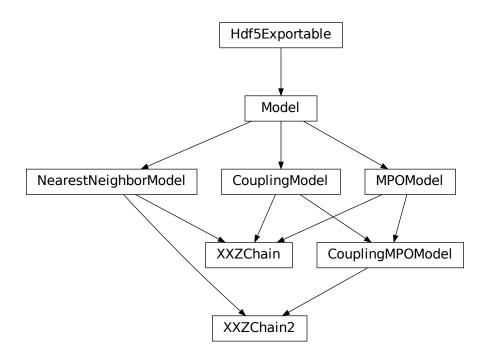
We choose the field along z to allow to conserve the parity, if desired.

# 7.9.4 xxz\_chain

full name: tenpy.models.xxz\_chainparent module: tenpy.models

• type: module

### **Classes**



XXZChain(model_params)	Spin-1/2 XXZ chain with Sz conservation.
XXZChain2(model_params)	Another implementation of the Spin-1/2 XXZ chain
	with Sz conservation.

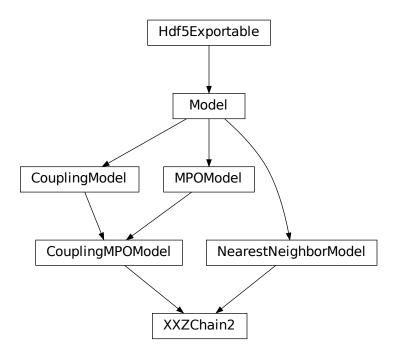
## XXZChain2

• full name: tenpy.models.xxz\_chain.XXZChain2

• parent module: tenpy.models.xxz\_chain

• type: class

## **Inheritance Diagram**



## **Methods**

XXZChain2init(model_params)	Initialize self.
XXZChain2.add_coupling(strength, u1, op1,)	Add twosite coupling terms to the Hamiltonian, sum-
	ming over lattice sites.
XXZChain2.add_coupling_term(strength, i, j,	Add a two-site coupling term on given MPS sites.
)	
XXZChain2.add_local_term(strength, term[,	Add a single term to <i>self</i> .
])	
XXZChain2.add_onsite(strength, u, opname[,	Add onsite terms to onsite_terms.
])	
XXZChain2.add_onsite_term(strength, i, op[,	Add an onsite term on a given MPS site.
])	
·	

continues on next page

Table 107 – continued from previous page

XXZChain2.all_coupling_terms()	Sum of all coupling_terms.
XXZChain2.all_onsite_terms()	Sum of all onsite_terms.
XXZChain2.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
XXZChain2.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
XXZChain2.calc_H_MPO_from_bond([tol_zero])	Calculate the MPO Hamiltonian from the bond Hamil-
	tonian.
XXZChain2.calc_H_bond([tol_zero])	calculate $H\_bond$ from coupling_terms and
	onsite_terms.
XXZChain2.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
XXZChain2.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
XXZChain2.coupling_strength_add_ext_fl	
XXZChain2.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
XXZChain2.from_MPOModel(mpo_model)	Initialize a NearestNeighborModel from a model class
	defining an MPO.
XXZChain2.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
_path)	
XXZChain2.group_sites([n, grouped_sites])	Modify <i>self</i> in place to group sites.
XXZChain2.init_lattice(model_params)	Initialize a lattice for the given model parameters.
XXZChain2.init_sites(model_params)	Define the local Hilbert space and operators; needs to
	be implemented in subclasses.
<pre>XXZChain2.init_terms(model_params)</pre>	Add the onsite and coupling terms to the model; sub-
	classes should implement this.
XXZChain2.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
XXZChain2.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
$\it XXZChain2.trivial\_like\_NNModel()$	Return a NearestNeighborModel with same lattice, but
	trivial (H=0) bonds.

class tenpy.models.xxz\_chain.XXZChain2 (model\_params)

 $Bases: \\ tenpy.models.model.Coupling MPOModel, \\ \textit{NearestNeighborModel} \\ tenpy.models.model. \\$ 

Another implementation of the Spin-1/2 XXZ chain with Sz conservation.

This implementation takes the same parameters as the XXZChain, but is implemented based on the CouplingMPOModel.

Parameters model\_params (dict | Config) - See XXZChain

### init\_sites (model\_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init\_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

**Note:** Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

**Parameters model\_params** (dict) – The model parameters given to \_\_init\_\_.

**Returns sites** – The local sites of the lattice, defining the local basis states and operators.

### Return type (tuple of) Site

#### init\_terms (model\_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}, u1), and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site (x\_0+dx[0], ..., x\_{dim-1}+dx[dim-1], u1). Possible combinations x\_0, ..., x\_{dim-1} are determined from the boundary conditions in  $possible\_couplings()$ .

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling\_shape* returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str\_on\_first* and *raise\_op2\_left* will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit\_cell[u1] for OP1.
- **op1** (*str*) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit\_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str / None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".

• plus\_hc (bool) – If *True*, the hermitian conjugate of the terms is added automatically.

### **Examples**

When initializing a model, you can add a term  $J\sum_{\langle i,j\rangle}S_i^zS_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx)` is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

--Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

### See also:

```
add_onsite() Add terms acting on one site only.
```

MultiCouplingModel.add\_multi\_coupling\_term() for terms on more than two sites.

add\_coupling\_term() Add a single term without summing over vecx.

add\_coupling\_term (strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling\_terms[category].add\_coupling\_term(...).

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

#### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}$  (i,) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N\_sites and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_s$ ites, it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.
- op\_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add\_local\_term (strength, term, category=None, plus\_hc=False)
Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite\_terms or coupling\_terms, respectively.

#### **Parameters**

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array\_like*)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat\_idx*. Here, *lat\_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite\_terms or coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add\_onsite (strength, u, opname, category=None, plus\_hc=False)

Add onsite terms to onsite\_terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat.unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to onsite\_terms; doesn't rebuild the MPO.

### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit\_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit\_cell[u].
- category (str) Descriptive name used as key for onsite\_terms. Defaults to opname.

```
• plus_hc (bool) – If True, the hermitian conjugate of the terms is added automatically.
     See also:
     add_coupling() Add a terms acting on two sites.
     add_onsite_term() Add a single term without summing over vecx.
add_onsite_term (strength, i, op, category=None, plus_hc=False)
     Add an onsite term on a given MPS site.
     Wrapper for self.onsite_terms[category].add_onsite_term(...).
         Parameters
             • strength (float) – The strength of the term.
             • \mathbf{i} (int) – The MPS index of the site on which the operator acts. We require 0 <= \mathbf{i} <
               L.
             • op (str) – Name of the involved operator.
             • category (str) - Descriptive name used as key for onsite_terms. Defaults to op.
             • plus_hc (bool) - If True, the hermitian conjugate of the term is added automatically.
all coupling terms()
     Sum of all coupling_terms.
all_onsite_terms()
     Sum of all onsite terms.
bond_energies (psi)
     Calculate bond energies <psilH_bondlpsi>.
         Parameters psi (MPS) - The MPS for which the bond energies should be calculated.
         Returns E_bond – List of bond energies: for finite bc, E_Bond[i] is the energy of bond i,
             i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the
             energy of bond i-1, i.
         Return type 1D ndarray
calc H MPO(tol zero=1e-15)
     Calculate MPO representation of the Hamiltonian.
     Uses onsite_terms and coupling_terms to build an MPO graph (and then an MPO).
         Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are</pre>
             considered to be zero.
         Returns H MPO – MPO representation of the Hamiltonian.
         Return type MPO
calc_H_MPO_from_bond(tol_zero=1e-15)
     Calculate the MPO Hamiltonian from the bond Hamiltonian.
         Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.
         Returns H_MPO – MPO representation of the Hamiltonian.
         Return type MPO
calc H bond (tol zero=1e-15)
     \label{lem:calculate} \textbf{\textit{calculate}} \ \textit{\textit{H\_bond}} \ \text{from} \ \text{coupling\_terms} \ \text{and} \ \text{onsite\_terms}.
```

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

### calc\_H\_bond\_from\_MPO (tol\_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

#### calc H onsite(tol zero=1e-15)

Calculate *H\_onsite* from *self.onsite\_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take explicit\_plus\_hc into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

#### Returns

- **H\_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit\_plus\_hc is True, Hermitian conjugates of the onsite terms will be included.

### coupling\_strength\_add\_ext\_flux (strength, dx, phase)

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i\ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in  $add\_coupling()$ .

#### **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add\_coupling().
- **phase** (*iterable of float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an

infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

**Returns strength** – The strength array to be used as *strength* in  $add\_coupling()$  with the given dx.

**Return type** complex array

### **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

### enlarge\_mps\_unit\_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

### classmethod from\_MPOModel(mpo\_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group\_sites().

**Parameters** mpo\_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

#### **Examples**

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
parameter 'L'=20 for SpinChainNNN2
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define  $H\_bond$ . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
```

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

### init\_lattice (model\_params)

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

**Parameters model\_params** (dict) - The model parameters given to \_\_init\_\_.

Returns lat – An initialized lattice.

Return type Lattice

### **Options**

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite"  $bc\_MPS$ , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on  $bc\_y$ ).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc\_MPS="finite" and "periodic" for bc\_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

#### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

### trivial\_like\_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

## **Module description**

Prototypical example of a 1D quantum model: the spin-1/2 XXZ chain.

The XXZ chain is contained in the more general SpinChain; the idea of this module is more to serve as a pedagogical example for a model.

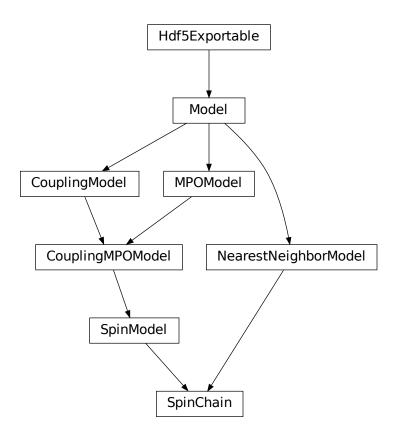
# 7.9.5 spins

• full name: tenpy.models.spins

• parent module: tenpy.models

• type: module

### Classes



SpinChain(model_params)	The SpinModel on a Chain, suitable for TEBD.
SpinModel(model_params)	Spin-S sites coupled by nearest neighbour interactions.

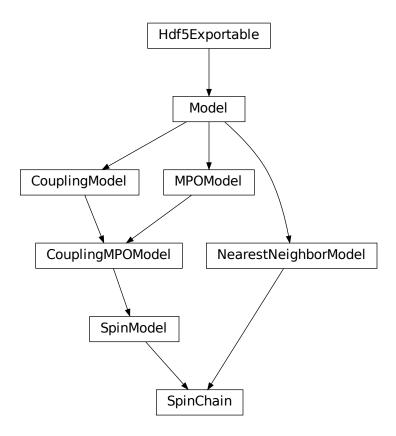
## **SpinChain**

• full name: tenpy.models.spins.SpinChain

• parent module: tenpy.models.spins

• type: class

## **Inheritance Diagram**



## Methods

SpinChaininit(model_params)	Initialize self.
SpinChain.add_coupling(strength, u1, op1,)	Add twosite coupling terms to the Hamiltonian, sum-
	ming over lattice sites.
SpinChain.add_coupling_term(strength, i, j,	Add a two-site coupling term on given MPS sites.
)	
SpinChain.add_local_term(strength, term[,	Add a single term to <i>self</i> .
])	
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Table 109 – continued from previous pag	е
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SpinChain.add_onsite(strength, u, opname[,	Add onsite terms to onsite_terms.
])	
SpinChain.add_onsite_term(strength, i, op[,	Add an onsite term on a given MPS site.
])	
SpinChain.all_coupling_terms()	Sum of all coupling_terms.
SpinChain.all_onsite_terms()	Sum of all onsite_terms.
SpinChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
SpinChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
SpinChain.calc_H_MPO_from_bond([tol_zero])	Calculate the MPO Hamiltonian from the bond Hamil-
	tonian.
SpinChain.calc_H_bond([tol_zero])	calculate $H\_bond$ from coupling_terms and
	onsite_terms.
SpinChain.calc_H_bond_from_MPO([tol_zero])	Calculate the bond Hamiltonian from the MPO Hamil-
	tonian.
SpinChain.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
SpinChain.coupling_strength_add_ext_fl	
SpinChain.enlarge_mps_unit_cell([factor])	Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
SpinChain.from_MPOModel(mpo_model)	Initialize a NearestNeighborModel from a model class
G ' G ' G ' G ' 155 1 1 1.5 1	defining an MPO.
SpinChain.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	Madification along the angularity
SpinChain.group_sites([n, grouped_sites])	Modify <i>self</i> in place to group sites.  Initialize a lattice for the given model parameters.
SpinChain.init_lattice(model_params) SpinChain.init_sites(model_params)	Define the local Hilbert space and operators; needs to
spinchain.init_sites(model_params)	be implemented in subclasses.
SpinChain.init_terms(model_params)	Add the onsite and coupling terms to the model; sub-
Spinenain: init_telms(model_params)	classes should implement this.
SpinChain.save_hdf5(hdf5_saver, h5gr, sub-	Export <i>self</i> into a HDF5 file.
path)	Export seg into a fibi 5 mc.
SpinChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
SpinChain.trivial_like_NNModel()	Return a NearestNeighborModel with same lattice, but
7	trivial (H=0) bonds.

#### class tenpy.models.spins.SpinChain(model\_params)

Bases: tenpy.models.spins.SpinModel, tenpy.models.model.NearestNeighborModel

The SpinModel on a Chain, suitable for TEBD.

See the SpinModel for the documentation of parameters.

add\_coupling(strength, u1, op1, u2, op2, dx, op\_string=None, str\_on\_first=True, raise\_op2\_left=False, category=None, plus\_hc=False)

Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}, u1), and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site (x\_0+dx[0], ..., x\_{dim-1}+dx[dim-1], u1). Possible combinations x\_0, ..., x\_{dim-1} are determined from the boundary conditions in  $possible\_couplings()$ .

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling\_shape* returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first

entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str\_on\_first* and *raise\_op2\_left* will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit\_cell[u1] for OP1.
- **op1** (str) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit\_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (iterable of int) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

### **Examples**

When initializing a model, you can add a term  $J\sum_{\langle i,j\rangle}S_i^zS_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx)` is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

--Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

#### See also:

add\_onsite() Add terms acting on one site only.

MultiCouplingModel.add\_multi\_coupling\_term() for terms on more than two sites.

add coupling term() Add a single term without summing over vecx.

add\_coupling\_term (strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling\_terms[category].add\_coupling\_term(...).

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}$  (*i*,) The MPS indices of the two sites on which the operator acts. We require 0 <=  $i < N_sites$  and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_sites$ , it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.
- op string (str) The operator to be inserted between i and j.

- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add\_local\_term (strength, term, category=None, plus\_hc=False)

Add a single term to *self*.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite\_terms or coupling\_terms, respectively.

#### **Parameters**

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array\_like*)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat\_idx*. Here, *lat\_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite\_terms or coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add\_onsite (strength, u, opname, category=None, plus\_hc=False)

Add onsite terms to onsite\_terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat. unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to onsite\_terms; doesn't rebuild the MPO.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit\_cell[u] out of the unit cell.
- **opname** (str) valid operator name of an onsite operator in lat.unit\_cell[u].
- category (str) Descriptive name used as key for onsite\_terms. Defaults to opname.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

#### See also:

```
add_coupling() Add a terms acting on two sites.
```

add\_onsite\_term() Add a single term without summing over vecx.

add\_onsite\_term(strength, i, op, category=None, plus\_hc=False)

Add an onsite term on a given MPS site.

Wrapper for self.onsite\_terms[category].add\_onsite\_term(...).

#### **Parameters**

• **strength** (*float*) – The strength of the term.

- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- op (str) Name of the involved operator.
- category (str) Descriptive name used as key for onsite\_terms. Defaults to op.
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

#### all\_coupling\_terms()

Sum of all coupling terms.

#### all\_onsite\_terms()

Sum of all onsite\_terms.

### bond\_energies (psi)

Calculate bond energies <psilH\_bondlpsi>.

Parameters psi (MPS) - The MPS for which the bond energies should be calculated.

**Returns**  $E\_bond - List$  of bond energies: for finite bc,  $E\_Bond[i]$  is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc  $E\_bond[i]$  is the energy of bond i-1, i.

Return type 1D ndarray

#### calc H MPO (tol zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite\_terms and coupling\_terms to build an MPO graph (and then an MPO).

Parameters tol\_zero (float) - Prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns H\_MPO** – MPO representation of the Hamiltonian.

Return type MPO

### calc\_H\_MPO\_from\_bond(tol\_zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

**Returns H\_MPO** – MPO representation of the Hamiltonian.

Return type MPO

### calc\_H\_bond(tol\_zero=1e-15)

calculate H\_bond from coupling\_terms and onsite\_terms.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

### calc\_H\_bond\_from\_MPO (tol\_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

**Parameters** tol\_zero (float) – Arrays with norm < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

### **Return type** list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

### calc\_H\_onsite(tol\_zero=1e-15)

Calculate *H* onsite from self.onsite terms.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take explicit\_plus\_hc into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

#### Returns

- **H\_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit\_plus\_hc is True, Hermitian conjugates of the onsite terms will be included.

### coupling\_strength\_add\_ext\_flux (strength, dx, phase)

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i\ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in  $add\_coupling()$ .

#### **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add\_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

**Returns strength** – The strength array to be used as *strength* in  $add\_coupling()$  with the given dx.

**Return type** complex array

#### **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

### enlarge\_mps\_unit\_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

#### classmethod from MPOModel(mpo model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group\_sites().

**Parameters** mpo\_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

#### **Examples**

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
parameter 'L'=20 for SpinChainNNN2
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define *H\_bond*. However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
```

#### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

### **Parameters**

• hdf5\_loader (Hdf5Loader) - Instance of the loading engine.

- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify *self* in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

#### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

**Return type** list of *GroupedSite* 

```
init_lattice (model_params)
```

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

**Returns** lat – An initialized lattice.

Return type Lattice

### **Options**

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite"  $bc\_MPS$ , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on  $bc\_y$ ).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

### option CouplingMPOModel.bc\_x: str

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc\_MPS="finite" and "periodic" for bc\_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

### init\_sites (model\_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init\_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

**Note:** Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

```
Parameters model_params (dict) - The model parameters given to __init__.
```

**Returns** sites – The local sites of the lattice, defining the local basis states and operators.

Return type (tuple of) Site

### init\_terms (model\_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5qr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### trivial\_like\_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

## **Module description**

Nearest-neighbour spin-S models.

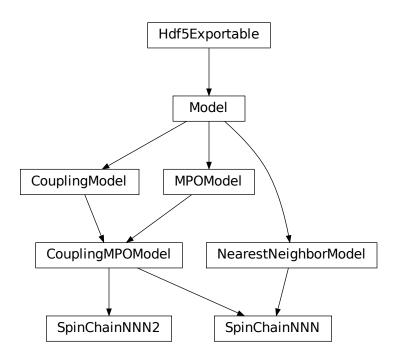
Uniform lattice of spin-S sites, coupled by nearest-neighbour interactions.

# 7.9.6 spins\_nnn

full name: tenpy.models.spins\_nnnparent module: tenpy.models

• type: module

### **Classes**



SpinChainNNN(model_params)	Spin-S sites coupled by (next-)nearest neighbour inter-
	actions on a GroupedSite.
SpinChainNNN2(model_params)	Spin-S sites coupled by next-nearest neighbour interac-
	tions.

## **Module description**

Next-Nearest-neighbour spin-S models.

Uniform lattice of spin-S sites, coupled by next-nearest-neighbour interactions. We have two variants implementing the same hamiltonian. The SpinChainNNN uses the <code>GroupedSite</code> to keep it a <code>NearestNeighborModel</code> suitable for TEBD, while the SpinChainNNN2 just involves longer-range couplings in the MPO. The latter is preferable for pure DMRG calculations and avoids having to add each of the short range couplings twice for the grouped sites.

Note that you can also get a <code>NearestNeighborModel</code> for TEBD from the latter by using <code>group\_sites()</code> and <code>from\_MPOModel()</code>. An example for such a case is given in the file <code>examples/c\_tebd.py</code>.

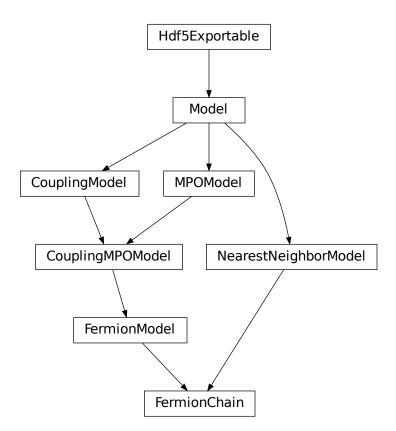
## 7.9.7 fermions spinless

• full name: tenpy.models.fermions\_spinless

• parent module: tenpy.models

• type: module

### **Classes**



FermionChain(model_params)	The FermionModel on a Chain, suitable for TEBD.
FermionModel(model_params)	Spinless fermions with particle number conservation.

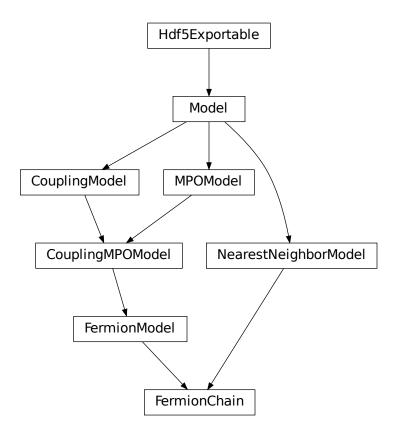
## **FermionChain**

• full name: tenpy.models.fermions\_spinless.FermionChain

• parent module: tenpy.models.fermions\_spinless

• type: class

## **Inheritance Diagram**



# Methods

FermionChaininit(model_params)	Initialize self.
FermionChain.add_coupling(strength, u1,	Add twosite coupling terms to the Hamiltonian, sum-
op1,)	ming over lattice sites.
FermionChain.add_coupling_term(strength,	Add a two-site coupling term on given MPS sites.
i,)	1 6
FermionChain.add_local_term(strength,	Add a single term to <i>self</i> .
term)	C V
FermionChain.add_onsite(strength, u, op-	Add onsite terms to onsite_terms.
name)	
FermionChain.add_onsite_term(strength, i,	Add an onsite term on a given MPS site.
op)	
FermionChain.all_coupling_terms()	Sum of all coupling_terms.
FermionChain.all_onsite_terms()	Sum of all onsite_terms.
FermionChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
FermionChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
FermionChain.calc_H_MPO_from_bond([tol_ze	er@alculate the MPO Hamiltonian from the bond Hamil-
	tonian.
FermionChain.calc_H_bond([tol_zero])	calculate H_bond from coupling_terms and
	onsite_terms.
FermionChain.calc_H_bond_from_MPO([tol_ze	er@alculate the bond Hamiltonian from the MPO Hamil-
	tonian.
FermionChain.calc_H_onsite([tol_zero])	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
FermionChain.coupling_strength_add_ext	
FermionChain.enlarge_mps_unit_cell([factors	or Repeat the unit cell for infinite MPS boundary condi-
	tions; in place.
FermionChain.from_MPOModel(mpo_model)	Initialize a NearestNeighborModel from a model class
	defining an MPO.
FermionChain.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
FermionChain.group_sites([n,	Modify <i>self</i> in place to group sites.
grouped_sites])	
FermionChain.init_lattice(model_params)	Initialize a lattice for the given model parameters.
FermionChain.init_sites(model_params)	Define the local Hilbert space and operators; needs to
	be implemented in subclasses.
FermionChain.init_terms(model_params)	Add the onsite and coupling terms to the model; sub-
	classes should implement this.
FermionChain.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
subpath)	
FermionChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
FermionChain.trivial_like_NNModel()	Return a NearestNeighborModel with same lattice, but
	trivial (H=0) bonds.

class tenpy.models.fermions\_spinless.FermionChain (model\_params)

 ${\bf Bases:} \qquad {\tt tenpy.models.fermions\_spinless.FermionModel,} \qquad {\tt tenpy.models.model.} \\ {\tt NearestNeighborModel}$ 

The FermionModel on a Chain, suitable for TEBD.

See the FermionModel for the documentation of parameters.

add\_coupling (strength, u1, op1, u2, op2, dx, op\_string=None, str\_on\_first=True, raise\_op2\_left=False, category=None, plus\_hc=False)
Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,...,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}, u1), and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site (x\_0+dx[0], ..., x\_{dim-1}+dx[dim-1], u1). Possible combinations x\_0, ..., x\_{dim-1} are determined from the boundary conditions in possible couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling\_shape* returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments  $str\_on\_first$  and  $raise\_op2\_left$  will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat.unit cell[u1] for OP1.
- op1 (str) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (*iterable of int*) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

## Examples

When initializing a model, you can add a term  $J\sum_{\langle i,j\rangle}S_i^zS_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx)` is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

→Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

#### See also:

```
add_onsite() Add terms acting on one site only.
```

MultiCouplingModel.add\_multi\_coupling\_term() for terms on more than two sites.

add\_coupling\_term() Add a single term without summing over vecx.

add\_coupling\_term (strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

Wrapper for self.coupling\_terms[category].add\_coupling\_term(...).

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}$  (i,) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N\_sites and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_s$ ites, it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.
- op\_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

# add\_local\_term (strength, term, category=None, plus\_hc=False)

Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite\_terms or coupling\_terms, respectively.

#### **Parameters**

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array\_like*)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat\_idx*. Here, *lat\_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite\_terms or coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

### add\_onsite (strength, u, opname, category=None, plus\_hc=False)

Add onsite terms to onsite\_terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat.unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to onsite\_terms; doesn't rebuild the MPO.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit\_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit\_cell[u].
- category (str) Descriptive name used as key for onsite\_terms. Defaults to opname.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

# See also:

add\_coupling() Add a terms acting on two sites.

```
add_onsite_term() Add a single term without summing over vecx.
```

```
add_onsite_term (strength, i, op, category=None, plus_hc=False)
```

Add an onsite term on a given MPS site.

Wrapper for self.onsite\_terms[category].add\_onsite\_term(...).

#### **Parameters**

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- op (str) Name of the involved operator.
- category (str) Descriptive name used as key for onsite\_terms. Defaults to op.
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

## all\_coupling\_terms()

Sum of all coupling\_terms.

#### all\_onsite\_terms()

Sum of all onsite\_terms.

#### bond energies (psi)

Calculate bond energies <psilH\_bondlpsi>.

**Parameters** psi (MPS) – The MPS for which the bond energies should be calculated.

**Returns**  $E\_bond - List$  of bond energies: for finite bc,  $E\_Bond[i]$  is the energy of bond i, i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc  $E\_bond[i]$  is the energy of bond i-1, i.

Return type 1D ndarray

## calc\_H\_MPO(tol\_zero=1e-15)

Calculate MPO representation of the Hamiltonian.

Uses onsite\_terms and coupling\_terms to build an MPO graph (and then an MPO).

Parameters tol\_zero (float) - Prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns H\_MPO** – MPO representation of the Hamiltonian.

Return type MPO

#### calc H MPO from bond (tol zero=1e-15)

Calculate the MPO Hamiltonian from the bond Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

**Returns H\_MPO** – MPO representation of the Hamiltonian.

Return type MPO

#### calc\_H\_bond(tol\_zero=1e-15)

calculate *H\_bond* from coupling\_terms and onsite\_terms.

**Parameters tol\_zero** (*float*) - prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

```
Return type list of Array
```

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_bond_from_MPO (tol_zero=1e-15)
```

Calculate the bond Hamiltonian from the MPO Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

**Return type** list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

```
calc_H_onsite(tol_zero=1e-15)
```

Calculate *H\_onsite* from *self.onsite\_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take explicit\_plus\_hc into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

#### Returns

- **H\_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit\_plus\_hc is True, Hermitian conjugates of the onsite terms will be included.

### coupling\_strength\_add\_ext\_flux (strength, dx, phase)

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i\ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as *op1* and the annihilation operator as *op2* in *add\_coupling()*.

## **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add\_coupling().
- **phase** (*iterable* of *float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

**Returns strength** – The strength array to be used as *strength* in add\_coupling() with the given dx.

**Return type** complex array

### **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

# enlarge\_mps\_unit\_cell(factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_MPOModel(mpo\_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group\_sites().

**Parameters** mpo\_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

## **Examples**

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
parameter 'L'=20 for SpinChainNNN2
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define *H\_bond*. However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
```

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

#### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

```
init_lattice (model_params)
```

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

```
Parameters model_params (dict) - The model parameters given to __init__.
```

**Returns** lat – An initialized lattice.

Return type Lattice

# **Options**

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell.

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite"  $bc\_MPS$ , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on  $bc\_y$ ).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc\_MPS="finite" and "periodic" for bc\_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

#### init\_sites (model\_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init\_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

**Note:** Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

```
Parameters model_params (dict) – The model parameters given to __init__.
```

**Returns** sites – The local sites of the lattice, defining the local basis states and operators.

```
Return type (tuple of) Site
```

## init\_terms (model\_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

### **Parameters**

- hdf5 saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

# test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### trivial\_like\_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

# **Module description**

Spinless fermions with hopping and interaction.

**Todo:** add further terms (e.g. c^dagger c^dagger + h.c.) to the Hamiltonian.

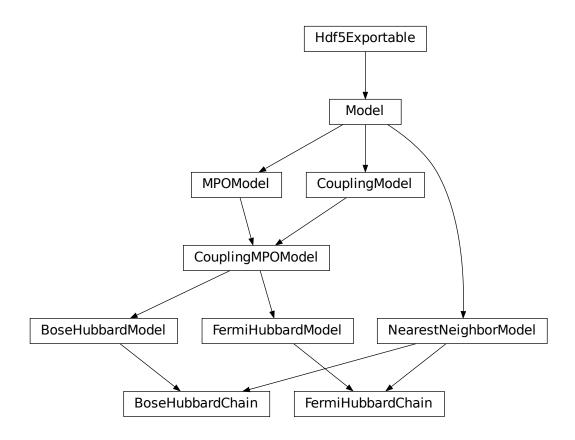
# 7.9.8 hubbard

• full name: tenpy.models.hubbard

• parent module: tenpy.models

• type: module

## **Classes**



BoseHubbardChain(model\_params)

The BoseHubbardModel on a Chain, suitable for TEBD.

continues on next page

Table 113 - continued from previous page

BoseHubbardModel(model_params)	Spinless Bose-Hubbard model.
FermiHubbardChain(model_params)	The FermiHubbardModel on a Chain, suitable for
	TEBD.
FermiHubbardModel(model_params)	Spin-1/2 Fermi-Hubbard model.

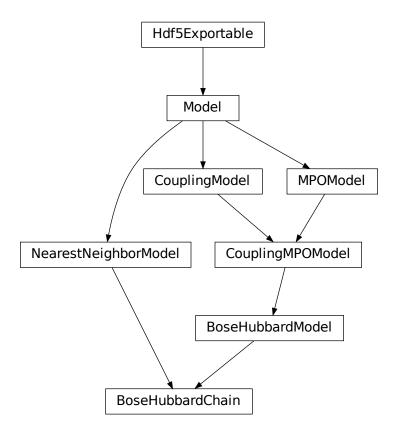
# **BoseHubbardChain**

• full name: tenpy.models.hubbard.BoseHubbardChain

• parent module: tenpy.models.hubbard

• type: class

# **Inheritance Diagram**



# Methods

BoseHubbardChaininit(model_params)	Initialize self.
BoseHubbardChain.add_coupling(strength,	Add twosite coupling terms to the Hamiltonian, sum-
u1,)	ming over lattice sites.
BoseHubbardChain.	Add a two-site coupling term on given MPS sites.
<pre>add_coupling_term(strength,)</pre>	
BoseHubbardChain.	Add a single term to <i>self</i> .
<pre>add_local_term(strength, term)</pre>	•
BoseHubbardChain.add_onsite(strength, u,	Add onsite terms to onsite_terms.
opname)	
BoseHubbardChain.	Add an onsite term on a given MPS site.
<pre>add_onsite_term(strength, i, op)</pre>	
BoseHubbardChain.all_coupling_terms()	Sum of all coupling_terms.
BoseHubbardChain.all_onsite_terms()	Sum of all onsite_terms.
BoseHubbardChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
BoseHubbardChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
BoseHubbardChain.	Calculate the MPO Hamiltonian from the bond Hamil-
calc_H_MPO_from_bond([tol_zero])	tonian.
BoseHubbardChain.calc_H_bond([tol_zero])	<pre>calculate H_bond from coupling_terms and</pre>
	onsite_terms.
BoseHubbardChain.	Calculate the bond Hamiltonian from the MPO Hamil-
<pre>calc_H_bond_from_MPO([tol_zero])</pre>	tonian.
BoseHubbardChain.	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
calc_H_onsite([tol_zero])	
BoseHubbardChain.	Add an external flux to the coupling strength.
coupling_strength_add_ext_flux()	
BoseHubbardChain.	Repeat the unit cell for infinite MPS boundary condi-
enlarge_mps_unit_cell([factor])	tions; in place.
BoseHubbardChain.	Initialize a NearestNeighborModel from a model class
from_MPOModel(mpo_model)	defining an MPO.
BoseHubbardChain.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
BoseHubbardChain.group_sites([n,	Modify <i>self</i> in place to group sites.
grouped_sites])	
BoseHubbardChain.	Initialize a lattice for the given model parameters.
<pre>init_lattice(model_params)</pre>	
BoseHubbardChain.	Define the local Hilbert space and operators; needs to
<pre>init_sites(model_params)</pre>	be implemented in subclasses.
BoseHubbardChain.	Add the onsite and coupling terms to the model; sub-
	classes should implement this
init_terms(model_params)	classes should implement this.
BoseHubbardChain.save_hdf5(hdf5_saver,	Export <i>self</i> into a HDF5 file.
BoseHubbardChain.save_hdf5(hdf5_saver, h5gr,)	Export self into a HDF5 file.
BoseHubbardChain.save_hdf5(hdf5_saver, h5gr,) BoseHubbardChain.test_sanity()	Export <i>self</i> into a HDF5 file.  Sanity check, raises ValueErrors, if something is wrong.
BoseHubbardChain.save_hdf5(hdf5_saver, h5gr,)	Export self into a HDF5 file.

# class tenpy.models.hubbard.BoseHubbardChain(model\_params)

 $\begin{tabular}{ll} \textbf{Bases:} & \texttt{tenpy.models.hubbard.BoseHubbardModel,} & \texttt{tenpy.models.model.} \\ \textit{NearestNeighborModel} & \end{tabular}$ 

The BoseHubbardModel on a Chain, suitable for TEBD.

See the BoseHubbardModel for the documentation of parameters.

add\_coupling (strength, u1, op1, u2, op2, dx, op\_string=None, str\_on\_first=True, raise\_op2\_left=False, category=None, plus\_hc=False)

Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}, u1), and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site (x\_0+dx[0], ..., x\_{dim-1}+dx[dim-1], u1). Possible combinations x\_0, ..., x\_{dim-1} are determined from the boundary conditions in possible\_couplings().

The coupling *strength* may vary spatially if the given *strength* is a numpy array. The correct shape of this array is the *coupling\_shape* returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of *strength* is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str\_on\_first* and *raise\_op2\_left* will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat .unit\_cell[u1] for OP1.
- **op1** (str) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit\_cell[u2] for OP2.
- **op2** (*str*) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (*iterable of int*) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- **category** (*str*) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

### **Examples**

When initializing a model, you can add a term  $J \sum_{\langle i,j \rangle} S_i^z S_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx) is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

→Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

# See also:

```
add_onsite() Add terms acting on one site only.
```

MultiCouplingModel.add\_multi\_coupling\_term() for terms on more than two sites.

add\_coupling\_term() Add a single term without summing over vecx.

add\_coupling\_term (strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

```
Wrapper for self.coupling_terms[category].add_coupling_term(...).
```

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

#### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}$  (i,) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N\_sites and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_s$ ites, it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.
- op\_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add\_local\_term (strength, term, category=None, plus\_hc=False)
Add a single term to self.

The represented term is strongth times the product of the

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite\_terms or coupling\_terms, respectively.

### **Parameters**

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array\_like*)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat\_idx*. Here, *lat\_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite\_terms or coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

 $\verb"add_onsite" (strength, u, opname, category = None, plus\_hc = False)$ 

Add onsite terms to onsite\_terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat.unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to onsite\_terms; doesn't rebuild the MPO.

# **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit\_cell[u] out of the unit cell.
- **opname** (str) valid operator name of an onsite operator in lat.unit\_cell[u].
- category (str) Descriptive name used as key for onsite\_terms. Defaults to opname.

```
• plus_hc (bool) – If True, the hermitian conjugate of the terms is added automatically.
     See also:
     add_coupling() Add a terms acting on two sites.
     add_onsite_term() Add a single term without summing over vecx.
add_onsite_term (strength, i, op, category=None, plus_hc=False)
     Add an onsite term on a given MPS site.
     Wrapper for self.onsite_terms[category].add_onsite_term(...).
         Parameters
             • strength (float) – The strength of the term.
             • \mathbf{i} (int) – The MPS index of the site on which the operator acts. We require 0 <= \mathbf{i} <
               L.
             • op (str) – Name of the involved operator.
             • category (str) - Descriptive name used as key for onsite_terms. Defaults to op.
             • plus_hc (bool) - If True, the hermitian conjugate of the term is added automatically.
all coupling terms()
     Sum of all coupling_terms.
all_onsite_terms()
     Sum of all onsite terms.
bond_energies (psi)
     Calculate bond energies <psilH_bondlpsi>.
         Parameters psi (MPS) - The MPS for which the bond energies should be calculated.
         Returns E_bond – List of bond energies: for finite bc, E_Bond[i] is the energy of bond i,
             i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the
             energy of bond i-1, i.
         Return type 1D ndarray
calc H MPO(tol zero=1e-15)
     Calculate MPO representation of the Hamiltonian.
     Uses onsite_terms and coupling_terms to build an MPO graph (and then an MPO).
         Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are
             considered to be zero.
         Returns H MPO – MPO representation of the Hamiltonian.
         Return type MPO
calc_H_MPO_from_bond(tol_zero=1e-15)
     Calculate the MPO Hamiltonian from the bond Hamiltonian.
         Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.
         Returns H_MPO – MPO representation of the Hamiltonian.
         Return type MPO
calc H bond (tol zero=1e-15)
     \label{lem:calculate} \textbf{\textit{calculate}} \ \textit{\textit{H\_bond}} \ \text{from} \ \text{coupling\_terms} \ \text{and} \ \text{onsite\_terms}.
```

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

# calc\_H\_bond\_from\_MPO (tol\_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

### calc H onsite(tol zero=1e-15)

Calculate *H\_onsite* from *self.onsite\_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take explicit\_plus\_hc into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

#### Returns

- **H\_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit\_plus\_hc is True, Hermitian conjugates of the onsite terms will be included.

# coupling\_strength\_add\_ext\_flux (strength, dx, phase)

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i\ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in  $add\_coupling()$ .

#### **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add\_coupling().
- **phase** (*iterable of float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an

infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

**Returns** strength – The strength array to be used as strength in add\_coupling() with the given dx.

Return type complex array

### **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

# enlarge\_mps\_unit\_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_MPOModel(mpo\_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group\_sites().

**Parameters** mpo\_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

### **Examples**

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
parameter 'L'=20 for SpinChainNNN2
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define  $H\_bond$ . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
```

# classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

### init\_lattice (model\_params)

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

Parameters model\_params (dict) - The model parameters given to \_\_init\_\_.

**Returns** lat – An initialized lattice.

Return type Lattice

### **Options**

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite"  $bc\_MPS$ , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on  $bc\_y$ ).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc\_MPS="finite" and "periodic" for bc\_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

# init sites(model params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init\_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

**Note:** Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

```
Parameters model_params (dict) - The model parameters given to __init__.
```

**Returns sites** – The local sites of the lattice, defining the local basis states and operators.

```
Return type (tuple of) Site
```

```
init_terms (model_params)
```

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

# **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

## test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

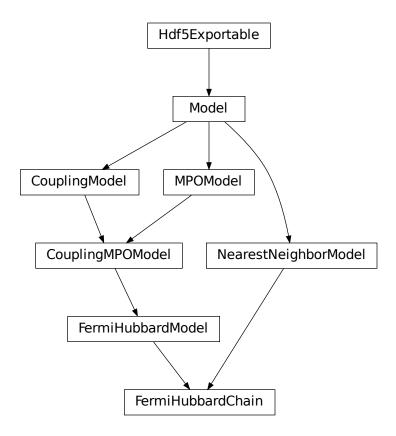
# trivial\_like\_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

# FermiHubbardChain

- full name: tenpy.models.hubbard.FermiHubbardChain
- parent module: tenpy.models.hubbard
- type: class

# **Inheritance Diagram**



# Methods

FermiHubbardChaininit(model_params)	Initialize self.
FermiHubbardChain.add_coupling(strength,	Add twosite coupling terms to the Hamiltonian, sum-
u1,)	ming over lattice sites.
FermiHubbardChain.	Add a two-site coupling term on given MPS sites.
$ extit{add\_coupling\_term}(\dots[,\dots])$	
FermiHubbardChain.	Add a single term to <i>self</i> .
<pre>add_local_term(strength, term)</pre>	C v
FermiHubbardChain.add_onsite(strength, u,	Add onsite terms to onsite_terms.
opname)	
FermiHubbardChain.	Add an onsite term on a given MPS site.
add_onsite_term(strength,)	
FermiHubbardChain.	Sum of all coupling_terms.
all_coupling_terms()	
FermiHubbardChain.all_onsite_terms()	Sum of all onsite_terms.
FermiHubbardChain.bond_energies(psi)	Calculate bond energies <psilh_bondlpsi>.</psilh_bondlpsi>
FermiHubbardChain.calc_H_MPO([tol_zero])	Calculate MPO representation of the Hamiltonian.
FermiHubbardChain.	Calculate the MPO Hamiltonian from the bond Hamil-
$calc\_H\_{MPO\_from\_bond([\dots])}$	tonian.
FermiHubbardChain.calc_H_bond([tol_zero])	calculate <i>H_bond</i> from coupling_terms and
	onsite_terms.
FermiHubbardChain.	Calculate the bond Hamiltonian from the MPO Hamil-
$calc\_H\_bond\_from\_MPO([])$	tonian.
FermiHubbardChain.	Calculate <i>H_onsite</i> from <i>self.onsite_terms</i> .
calc_H_onsite([tol_zero])	
FermiHubbardChain.	Add an external flux to the coupling strength.
${\it coupling\_strength\_add\_ext\_flux}()$	
FermiHubbardChain.	Repeat the unit cell for infinite MPS boundary condi-
enlarge_mps_unit_cell([factor])	tions; in place.
FermiHubbardChain.	Initialize a NearestNeighborModel from a model class
<pre>from_MPOModel(mpo_model)</pre>	defining an MPO.
FermiHubbardChain.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
FermiHubbardChain.group_sites([n,	Modify <i>self</i> in place to group sites.
grouped_sites])	
FermiHubbardChain.	Initialize a lattice for the given model parameters.
initlattice(model_params)	
FermiHubbardChain.	Define the local Hilbert space and operators; needs to
<pre>init_sites(model_params)</pre>	be implemented in subclasses.
FermiHubbardChain.	Add the onsite and coupling terms to the model; sub-
<pre>init_terms(model_params)</pre>	classes should implement this.
FermiHubbardChain.save_hdf5(hdf5_saver,	Export self into a HDF5 file.
)	
FermiHubbardChain.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
FermiHubbardChain.	Return a NearestNeighborModel with same lattice, but
trivial_like_NNModel()	trivial (H=0) bonds.

# class tenpy.models.hubbard.FermiHubbardChain(model\_params)

Bases: tenpy.models.hubbard.FermiHubbardModel, tenpy.models.model.
NearestNeighborModel

The Fermi Hubbard Model on a Chain, suitable for TEBD.

See the FermiHubbardModel for the documentation of parameters.

add\_coupling (strength, u1, op1, u2, op2, dx, op\_string=None, str\_on\_first=True, raise\_op2\_left=False, category=None, plus\_hc=False)

Add twosite coupling terms to the Hamiltonian, summing over lattice sites.

Represents couplings of the form  $\sum_{x_0,\dots,x_{dim-1}} strength[shift(\vec{x})] * OP0 * OP1$ , where OP0 := lat.unit\_cell[u0].get\_op(op0) acts on the site (x\_0, ..., x\_{dim-1}, u1), and OP1 := lat.unit\_cell[u1].get\_op(op1) acts on the site (x\_0+dx[0], ..., x\_{dim-1}+dx[dim-1], u1). Possible combinations x\_0, ..., x\_{dim-1} are determined from the boundary conditions in possible\_couplings().

The coupling strength may vary spatially if the given strength is a numpy array. The correct shape of this array is the  $coupling\_shape$  returned by tenpy.models.lattice.possible\_couplings() and depends on the boundary conditions. The shift(...) depends on dx, and is chosen such that the first entry strength[0, 0, ...] of strength is the prefactor for the first possible coupling fitting into the lattice if you imagine open boundary conditions.

The necessary terms are just added to coupling\_terms; this function does not rebuild the MPO.

Deprecated since version 0.4.0: The arguments *str\_on\_first* and *raise\_op2\_left* will be removed in version 1.0.0.

#### **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the coupling. May vary spatially (see above). If an array of smaller size is provided, it gets tiled to the required shape.
- u1 (int) Picks the site lat .unit\_cell[u1] for OP1.
- **op1** (str) Valid operator name of an onsite operator in lat.unit\_cell[u1] for OP1.
- **u2** (*int*) Picks the site lat.unit\_cell[u2] for OP2.
- op2 (str) Valid operator name of an onsite operator in lat.unit\_cell[u2] for OP2.
- dx (*iterable of int*) Translation vector (of the unit cell) between OP1 and OP2. For a 1D lattice, a single int is also fine.
- op\_string (str | None) Name of an operator to be used between the OP1 and OP2 sites. Typical use case is the phase for a Jordan-Wigner transformation. The operator should be defined on all sites in the unit cell. If None, auto-determine whether a Jordan-Wigner string is needed, using op\_needs\_JW().
- **str\_on\_first** (bool) Whether the provided *op\_string* should also act on the first site. This option should be chosen as True for Jordan-Wigner strings. When handling Jordan-Wigner strings we need to extend the *op\_string* to also act on the 'left', first site (in the sense of the MPS ordering of the sites given by the lattice). In this case, there is a well-defined ordering of the operators in the physical sense (i.e. which of *op1* or *op2* acts first on a given state). We follow the convention that *op2* acts first (in the physical sense), independent of the MPS ordering. Deprecated.
- raise\_op2\_left (bool) Raise an error when op2 appears left of op1 (in the sense of the MPS ordering given by the lattice). Deprecated.
- **category** (*str*) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

### **Examples**

When initializing a model, you can add a term  $J \sum_{\langle i,j \rangle} S_i^z S_j^z$  on all nearest-neighbor bonds of the lattice like this:

```
>>> J = 1. # the strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(J, u1, 'Sz', u2, 'Sz', dx)
```

The strength can be an array, which gets tiled to the correct shape. For example, in a 1D *Chain* with an even number of sites and periodic (or infinite) boundary conditions, you can add alternating strong and weak couplings with a line like:

```
>>> self.add_coupling([1.5, 1.], 0, 'Sz', 0, 'Sz', dx)
```

Make sure to use the *plus\_hc* argument if necessary, e.g. for hoppings:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx, plus_hc=True)
```

Alternatively, you can add the hermitian conjugate terms explictly. The correct way is to complex conjugate the strength, take the hermitian conjugate of the operators and swap the order (including a swap u1 <-> u2), and use the opposite direction -dx, i.e. the h.c. of add\_coupling(t, u1, 'A', u2, 'B', dx) is ``add\_coupling(np.conj(t), u2, hc('B'), u1, hc('A'), -dx), where hc takes the hermitian conjugate of the operator names, see  $get_hc_op_name()$ . For spin-less fermions (FermionSite), this would be

```
>>> t = 1. # hopping strength
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cd', u2, 'C', dx)
... self.add_coupling(np.conj(t), u2, 'Cd', u1, 'C', -dx) # h.c.
```

With spin-full fermions (SpinHalfFermions), it could be:

```
>>> for u1, u2, dx in self.lat.pairs['nearest_neighbors']:
... self.add_coupling(t, u1, 'Cdu', u2, 'Cd', dx) # Cdagger_up C_down
... self.add_coupling(np.conj(t), u2, 'Cdd', u1, 'Cu', -dx) # h.c._

→Cdagger_down C_up
```

Note that the Jordan-Wigner strings for the fermions are added automatically!

# See also:

```
add_onsite() Add terms acting on one site only.
```

MultiCouplingModel.add\_multi\_coupling\_term() for terms on more than two sites.

add\_coupling\_term() Add a single term without summing over vecx.

add\_coupling\_term (strength, i, j, op\_i, op\_j, op\_string='Id', category=None, plus\_hc=False)
Add a two-site coupling term on given MPS sites.

```
Wrapper for self.coupling_terms[category].add_coupling_term(...).
```

**Warning:** This function does not handle Jordan-Wigner strings! You might want to use add\_local\_term() instead.

#### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}$  (i,) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N\_sites and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_s$ ites, it indicates couplings between unit cells of an infinite MPS.
- op2 (op1,) Names of the involved operators.
- op\_string (str) The operator to be inserted between i and j.
- category (str) Descriptive name used as key for coupling\_terms. Defaults to a string of the form "{op1}\_i {op2}\_j".
- plus\_hc (bool) If *True*, the hermitian conjugate of the term is added automatically.

add\_local\_term (strength, term, category=None, plus\_hc=False)
Add a single term to self.

The repesented term is *strength* times the product of the operators given in *terms*. Each operator is specified by the name and the site it acts on; the latter given by a lattice index, see *Lattice*.

Depending on the length of *term*, it can add an onsite term or a coupling term to onsite\_terms or coupling\_terms, respectively.

#### **Parameters**

- **strength** (float/complex) The prefactor of the term.
- **term** (*list of* (*str*, *array\_like*)) List of tuples (opname, lat\_idx) where *opname* is a string describing the operator acting on the site given by the lattice index *lat\_idx*. Here, *lat\_idx* is for example [x, y, u] for a 2D lattice, with u being the index within the unit cell.
- category Descriptive name used as key for onsite\_terms or coupling\_terms.
- plus\_hc (bool) If *True*, the hermitian conjugate of the terms is added automatically.

add\_onsite (strength, u, opname, category=None, plus\_hc=False)

Add onsite terms to onsite\_terms.

Adds  $\sum_{\vec{x}} strength[\vec{x}] * OP$  to the represented Hamiltonian, where the operator OP=lat.unit\_cell[u].get\_op(opname) acts on the site given by a lattice index (x\_0, ..., x\_{dim-1}, u),

The necessary terms are just added to onsite\_terms; doesn't rebuild the MPO.

# **Parameters**

- **strength** (*scalar* / *array*) Prefactor of the onsite term. May vary spatially. If an array of smaller size is provided, it gets tiled to the required shape.
- u (int) Picks a Site lat.unit\_cell[u] out of the unit cell.
- opname (str) valid operator name of an onsite operator in lat.unit\_cell[u].
- category (str) Descriptive name used as key for onsite\_terms. Defaults to opname.

```
• plus_hc (bool) – If True, the hermitian conjugate of the terms is added automatically.
     See also:
     add_coupling() Add a terms acting on two sites.
     add_onsite_term() Add a single term without summing over vecx.
add_onsite_term (strength, i, op, category=None, plus_hc=False)
     Add an onsite term on a given MPS site.
     Wrapper for self.onsite_terms[category].add_onsite_term(...).
         Parameters
             • strength (float) – The strength of the term.
             • \mathbf{i} (int) – The MPS index of the site on which the operator acts. We require 0 <= \mathbf{i} <
               L.
             • op (str) – Name of the involved operator.
             • category (str) - Descriptive name used as key for onsite_terms. Defaults to op.
             • plus_hc (bool) - If True, the hermitian conjugate of the term is added automatically.
all coupling terms()
     Sum of all coupling_terms.
all_onsite_terms()
     Sum of all onsite terms.
bond_energies (psi)
     Calculate bond energies <psilH_bondlpsi>.
         Parameters psi (MPS) - The MPS for which the bond energies should be calculated.
         Returns E_bond – List of bond energies: for finite bc, E_Bond[i] is the energy of bond i,
             i+1. (i.e. we omit bond 0 between sites L-1 and 0); for infinite bc E_bond[i] is the
             energy of bond i-1, i.
         Return type 1D ndarray
calc H MPO(tol zero=1e-15)
     Calculate MPO representation of the Hamiltonian.
     Uses onsite_terms and coupling_terms to build an MPO graph (and then an MPO).
         Parameters tol_zero (float) - Prefactors with abs(strength) < tol_zero are</pre>
             considered to be zero.
         Returns H MPO – MPO representation of the Hamiltonian.
         Return type MPO
calc_H_MPO_from_bond(tol_zero=1e-15)
     Calculate the MPO Hamiltonian from the bond Hamiltonian.
         Parameters tol_zero (float) - Arrays with norm < tol_zero are considered to be zero.
         Returns H_MPO – MPO representation of the Hamiltonian.
         Return type MPO
calc H bond (tol zero=1e-15)
     \label{lem:calculate} \textbf{\textit{calculate}} \ \textit{\textit{H\_bond}} \ \text{from} \ \text{coupling\_terms} \ \text{and} \ \text{onsite\_terms}.
```

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

**Return type** list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

# calc\_H\_bond\_from\_MPO (tol\_zero=1e-15)

Calculate the bond Hamiltonian from the MPO Hamiltonian.

**Parameters** tol\_zero (float) - Arrays with norm < tol\_zero are considered to be zero.

**Returns H\_bond** – Bond terms as required by the constructor of NearestNeighborModel. Legs are ['p0', 'p0\*', 'p1', 'p1\*']

Return type list of Array

:raises ValueError : if the Hamiltonian contains longer-range terms.:

### calc H onsite(tol zero=1e-15)

Calculate *H\_onsite* from *self.onsite\_terms*.

Deprecated since version 0.4.0: This function will be removed in 1.0.0. Replace calls to this function by self.all\_onsite\_terms().remove\_zeros(tol\_zero).to\_Arrays(self.lat.mps\_sites()). You might also want to take explicit\_plus\_hc into account.

Parameters tol\_zero (float) - prefactors with abs(strength) < tol\_zero are considered to be zero.

#### Returns

- **H\_onsite** (list of npc.Array)
- onsite terms of the Hamiltonian. If explicit\_plus\_hc is True, Hermitian conjugates of the onsite terms will be included.

# coupling\_strength\_add\_ext\_flux (strength, dx, phase)

Add an external flux to the coupling strength.

When performing DMRG on a "cylinder" geometry, it might be useful to put an "external flux" through the cylinder. This means that a particle hopping around the cylinder should pick up a phase given by the external flux [Resta1997]. This is also called "twisted boundary conditions" in literature. This function adds a complex phase to the *strength* array on some bonds, such that particles hopping in positive direction around the cylinder pick up  $exp(+i\ phase)$ .

**Warning:** For the sign of *phase* it is important that you consistently use the creation operator as op1 and the annihilation operator as op2 in  $add\_coupling()$ .

#### **Parameters**

- **strength** (*scalar* / *array*) The strength to be used in *add\_coupling()*, when no external flux would be present.
- **dx** (*iterable of int*) Translation vector (of the unit cell) between *op1* and *op2* in add\_coupling().
- **phase** (*iterable of float*) The phase of the external flux for hopping in each direction of the lattice. E.g., if you want flux through the cylinder on which you have an

infinite MPS, you should give phase=[0, phi] souch that particles pick up a phase *phi* when hopping around the cylinder.

**Returns** strength – The strength array to be used as *strength* in add\_coupling() with the given dx.

**Return type** complex array

### **Examples**

Let's say you have an infinite MPS on a cylinder, and want to add nearest-neighbor hopping of fermions with the FermionSite. The cylinder axis is the x-direction of the lattice, so to put a flux through the cylinder, you want particles hopping around the cylinder to pick up a phase phi given by the external flux.

# enlarge\_mps\_unit\_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

This has to be done after finishing initialization and can not be reverted.

**Parameters factor** (int) – The new number of sites in the MPS unit cell will be increased from  $N\_sites$  to factor\*N $\_sites\_per\_ring$ . Since MPS unit cells are repeated in the x-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_MPOModel(mpo\_model)

Initialize a NearestNeighborModel from a model class defining an MPO.

This is especially usefull in combination with MPOModel.group\_sites().

**Parameters** mpo\_model (MPOModel) - A model instance implementing the MPO. Does not need to be a NearestNeighborModel, but should only have nearest-neighbor couplings.

### **Examples**

The SpinChainNNN2 has next-nearest-neighbor couplings and thus only implements an MPO:

```
>>> from tenpy.models.spins_nnn import SpinChainNNN2
>>> nnn_chain = SpinChainNNN2({'L': 20})
parameter 'L'=20 for SpinChainNNN2
>>> print(isinstance(nnn_chain, NearestNeighborModel))
False
>>> print("range before grouping:", nnn_chain.H_MPO.max_range)
range before grouping: 2
```

By grouping each two neighboring sites, we can bring it down to nearest neighbors.

```
>>> nnn_chain.group_sites(2)
>>> print("range after grouping:", nnn_chain.H_MPO.max_range)
range after grouping: 1
```

Yet, TEBD will not yet work, as the model doesn't define  $H\_bond$ . However, we can initialize a Nearest-NeighborModel from the MPO:

```
>>> nnn_chain_for_tebd = NearestNeighborModel.from_MPOModel(nnn_chain)
```

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
group_sites (n=2, grouped_sites=None)
```

Modify self in place to group sites.

Group each n sites together using the *GroupedSite*. This might allow to do TEBD with a Trotter decomposition, or help the convergence of DMRG (in case of too long range interactions).

This has to be done after finishing initialization and can not be reverted.

### **Parameters**

- **n** (*int*) Number of sites to be grouped together.
- grouped\_sites (None | list of GroupedSite) The sites grouped together.

**Returns** grouped\_sites – The sites grouped together.

Return type list of GroupedSite

### init\_lattice (model\_params)

Initialize a lattice for the given model parameters.

This function reads out the model parameter *lattice*. This can be a full *Lattice* instance, in which case it is just returned without further action. Alternatively, the *lattice* parameter can be a string giving the name of one of the predefined lattices, which then gets initialized. Depending on the dimensionality of the lattice, this requires different model parameters.

Parameters model\_params (dict) - The model parameters given to \_\_init\_\_.

Returns lat – An initialized lattice.

Return type Lattice

### **Options**

```
option CouplingMPOModel.lattice: str | Lattice
```

The name of a lattice pre-defined in TeNPy to be initialized. Alternatively, a (possibly self-defined) Lattice instance. In the latter case, no further parameters are read out.

```
option CouplingMPOModel.bc_MPS: str
```

Boundary conditions for the MPS.

```
option CouplingMPOModel.order: str
```

The order of sites within the lattice for non-trivial lattices, e.g, 'default', 'snake', see ordering(). Only used if *lattice* is a string.

```
option CouplingMPOModel.L: int
```

The length in x-direction; only read out for 1D lattices. For an infinite system the length of the unit cell

```
option CouplingMPOModel.Lx: int
option CouplingMPOModel.Ly: int
```

The length in x- and y-direction; only read out for 2D lattices. For "infinite"  $bc\_MPS$ , the system is infinite in x-direction and Lx is the number of "rings" in the infinite MPS unit cell, while Ly gives the circumference around the cylinder or width of the rung for a ladder (depending on  $bc\_y$ ).

```
option CouplingMPOModel.bc_y: str
```

"cylinder" | "ladder"; only read out for 2D lattices. The boundary conditions in y-direction.

```
option CouplingMPOModel.bc_x: str
```

"open" | "periodic". Can be used to force "periodic" boundaries for the lattice, i.e., for the couplings in the Hamiltonian, even if the MPS is finite. Defaults to "open" for bc\_MPS="finite" and "periodic" for bc\_MPS="infinite. If you are not aware of the consequences, you should probably *not* use "periodic" boundary conditions. (The MPS is still "open", so this will introduce long-range couplings between the first and last sites of the MPS!)

## init\_sites (model\_params)

Define the local Hilbert space and operators; needs to be implemented in subclasses.

This function gets called by <code>init\_lattice()</code> to get the <code>Site</code> for the lattice unit cell.

**Note:** Initializing the sites requires to define the conserved quantum numbers. All pre-defined sites accept conserve=None to disable using quantum numbers. Many models in TeNPy read out the *conserve* model parameter, which can be set to "best" to indicate the optimal parameters.

```
Parameters model_params (dict) - The model parameters given to __init__.
```

**Returns sites** – The local sites of the lattice, defining the local basis states and operators.

```
Return type (tuple of) Site
```

# init\_terms (model\_params)

Add the onsite and coupling terms to the model; subclasses should implement this.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

## test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

# trivial\_like\_NNModel()

Return a NearestNeighborModel with same lattice, but trivial (H=0) bonds.

# **Module description**

Bosonic and fermionic Hubbard models.

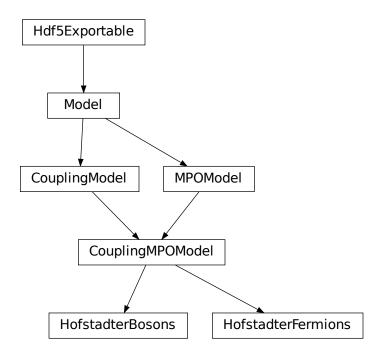
# 7.9.9 hofstadter

• full name: tenpy.models.hofstadter

• parent module: tenpy.models

• type: module

# **Classes**



HofstadterBosons(model_params)	Bosons on a square lattice with magnetic flux.
HofstadterFermions(model_params)	Fermions on a square lattice with magnetic flux.

#### **Functions**

<pre>gauge_hopping(model_params)</pre>	Compute hopping amplitudes for the Hofstadter models	
	based on a gauge choice.	

# gauge\_hopping

- full name: tenpy.models.hofstadter.gauge\_hopping
- parent module: tenpy.models.hofstadter
- · type: function

tenpy.models.hofstadter.gauge\_hopping(model\_params)

Compute hopping amplitudes for the Hofstadter models based on a gauge choice.

In the Hofstadter model, the magnetic field enters as an Aharonov-Bohm phase. This phase is dependent on a choice of gauge, which simultaneously defines a 'magnetic unit cell' (MUC).

The magnetic unit cell is the smallest set of lattice plaquettes that encloses an integer number of flux quanta. It can be user-defined by setting mx and my, but for common gauge choices is computed based on the flux density.

# The gauge choices are:

- 'landau\_x': Landau gauge along the x-axis. The magnetic unit cell will have shape :math`(mathtt{mx}, 1)`. For flux densities p/q, mx will default to q. Example: at a flux density 1/3, the magnetic unit cell will have shape (3,1), so it encloses exactly 1 flux quantum.
- 'landau\_y': Landau gauge along the y-axis. The magnetic unit cell will have shape :math`(1, mathtt{my})`. For flux densities :math`p/q`, my will default to q. Example: at a flux density 3/7, the magnetic unit cell will have shape (1, 7), so it encloses axactly 3 flux quanta.
- 'symmetric': symmetric gauge. The magnetic unit cell will have shape (mx, my), with mx = my. For flux densities p/q, mx and my will default to q Example: at a flux density 4/9, the magnetic unit cell will have shape (9,9).

#### **Parameters**

- gauge ('landau\_x' | 'landau\_y' | 'symmetric') Choice of the gauge, see table above.
- my(mx,) Dimensions of the magnetic unit cell in terms of lattice sites. None defaults to the minimal choice compatible with *gauge* and *phi\_pq*.
- **Jy** (Jx,) 'Bare' hopping amplitudes (without phase). Without any flux we have hop\_x = -Jx and hop\_y = -Jy.
- phi\_pq(tuple (int, int)) Magnetic flux as a fraction p/q, defined as (p, q)

**Returns hop\_x, hop\_y** – Hopping amplitudes to be used as prefactors for  $c_{x,y}^{\dagger}c_{x+1,y}$  (hop\_x) and  $c_{x,y}^{\dagger}c_{x,y+1}$  (hop\_x), respectively, with the necessary phases for the gauge.

Return type float | array

# **Module description**

Cold atomic (Harper-)Hofstadter model on a strip or cylinder.

**Todo:** WARNING: These models are still under development and not yet tested for correctness. Use at your own risk! Replicate known results to confirm models work correctly. Long term: implement different lattices. Long term: implement variable hopping strengths Jx, Jy.

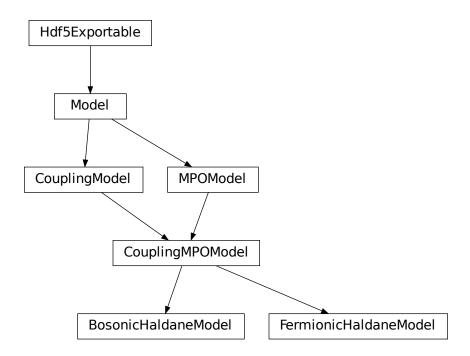
# **7.9.10** haldane

• full name: tenpy.models.haldane

• parent module: tenpy.models

• type: module

## **Classes**



BosonicHaldaneModel(model_params)	Hardcore bosonic Haldane model.
FermionicHaldaneModel(model_params)	Spinless fermionic Haldane model.

# **Module description**

Bosonic and fermionic Haldane models.

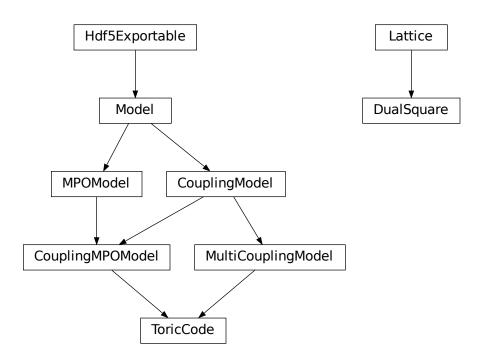
# 7.9.11 toric\_code

• full name: tenpy.models.toric\_code

• parent module: tenpy.models

• type: module

# **Classes**



DualSquare(Lx, Ly, sites, **kwargs)	The dual lattice of the square lattice (again square).
ToricCode(model_params)	Toric code model.

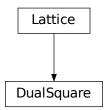
# **DualSquare**

• full name: tenpy.models.toric\_code.DualSquare

• parent module: tenpy.models.toric\_code

• type: class

# **Inheritance Diagram**



# Methods

DualSquareinit(Lx, Ly, sites, **kwargs)	Initialize self.
DualSquare.count_neighbors([u, key])	Count e.g.
DualSquare.coupling_shape(dx)	Calculate correct shape of the <i>strengths</i> for a coupling.
DualSquare.enlarge_mps_unit_cell([factor])	<u> </u>
	tions; in place.
DualSquare.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
DualSquare.lat2mps_idx(lat_idx)	Translate lattice indices $(x_0, \ldots, x_{D-1})$ ,
	u) to MPS index i.
DualSquare.mps2lat_idx(i)	Translate MPS index $i$ to lattice indices (x_0,,
	$x_{dim-1}$ , u).
$DualSquare.mps2lat\_values(A[,axes,u])$	Reshape/reorder A to replace an MPS index by lattice
	indices.
Dual $Square.mps2lat\_values\_masked$ (A[,	Reshape/reorder an array A to replace an MPS index by
axes,])	lattice indices.
DualSquare.mps_idx_fix_u([u])	return an index array of MPS indices for which the site
	within the unit cell is $u$ .
$ extit{DualSquare.mps\_lat\_idx\_fix\_u([u])}$	Similar as mps_idx_fix_u(), but return also the
	corresponding lattice indices.
DualSquare.mps_sites()	Return a list of sites for all MPS indices.
DualSquare.multi_coupling_shape(dx)	Calculate correct shape of the strengths for a
	multi_coupling.
DualSquare.number_nearest_neighbors([u])	Deprecated.
DualSquare.number_next_nearest_neighbo.	r D((pu))cated.
DualSquare.ordering(order)	Provide possible orderings of the <i>N</i> lattice sites.
	continues on next page

Table	120 -	<ul> <li>continued</li> </ul>	from	previous page	
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<pre>DualSquare.plot_basis(ax[, origin, shade])</pre>	Plot arrows indicating the basis vectors of the lattice.
DualSquare.plot_bc_identified(ax[,])	Mark two sites indified by periodic boundary condi-
	tions.
DualSquare.plot_coupling(ax[, coupling,	Plot lines connecting nearest neighbors of the lattice.
wrap])	
DualSquare.plot_order(ax[, order, textk-	Plot a line connecting sites in the specified "order" and
wargs])	text labels enumerating them.
DualSquare.plot_sites(ax[, markers])	Plot the sites of the lattice with markers.
DualSquare.position(lat_idx)	return 'space' position of one or multiple sites.
DualSquare.possible_couplings(u1, u2, dx)	Find possible MPS indices for two-site couplings.
DualSquare.possible_multi_couplings(ops	Generalization of possible_couplings() to cou-
	plings with more than 2 sites.
DualSquare.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
DualSquare.site(i)	return $Site$ instance corresponding to an MPS index $i$
DualSquare.test_sanity()	Sanity check.

# **Class Attributes and Properties**

DualSquare.boundary_conditions	Human-readable list of boundary conditions from bc
	<pre>and bc_shift.</pre>
DualSquare.dim	The dimension of the lattice.
DualSquare.nearest_neighbors	
DualSquare.next_nearest_neighbors	
DualSquare.next_next_nearest_neighbors	
DualSquare.order	Defines an ordering of the lattice sites, thus mapping the
	lattice to a 1D chain.

class tenpy.models.toric\_code.DualSquare(Lx, Ly, sites, \*\*kwargs)

Bases: tenpy.models.lattice.Lattice

The dual lattice of the square lattice (again square).

The sites in this lattice correspond to the vertical and horizontal (nearest neighbor) bonds of a common Square lattice with the same dimensions Lx, Ly.

### **Parameters**

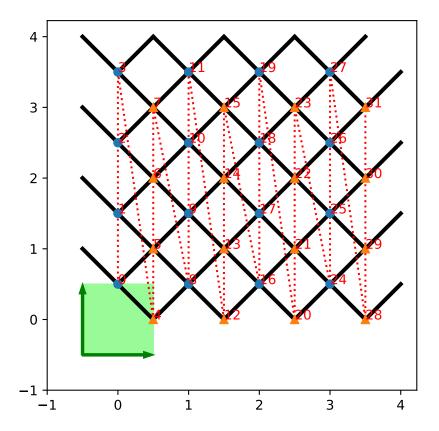
- Ly (Lx, ) Dimensions of the original lattice. This lattice has 2\*Lx\*Ly sites.
- **sites** (Site) The sites for the horizontal (first entry) and vertical (second entry) bonds.
- \*\*kwargs Additional keyword arguments given to the Lattice. basis, pos and pairs are set accordingly.

# ordering(order)

Provide possible orderings of the *N* lattice sites.

The following orders are defined in this method compared to tenpy.models.lattice.Lattice.ordering():

order	equivalent <i>priority</i>	equivalent snake_winding
'default'	(0, 2, 1)	(False, False, False)



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## property boundary\_conditions

Human-readable list of boundary conditions from bc and bc shift.

**Returns boundary\_conditions** – List of "open" or "periodic", one entry for each direction of the lattice.

Return type list of str

## count\_neighbors (u=0, key='nearest\_neighbors')

Count e.g. the number of nearest neighbors for a site in the bulk.

#### **Parameters**

- **u** (*int*) Specifies the site in the unit cell, for which we should count the number of neighbors (or whatever *key* specifies).
- **key** (*str*) Key of pairs to select what to count.

**Returns number** – Number of nearest neighbors (or whatever *key* specified) for the *u*-th site in the unit cell, somewhere in the bulk of the lattice. Note that it might not be the correct value at the edges of a lattice with open boundary conditions.

#### Return type int

## $coupling\_shape(dx)$

Calculate correct shape of the strengths for a coupling.

**Parameters dx** (tuple of int) – Translation vector in the lattice for a coupling of two operators. Corresponds to dx argument of tenpy.models.model.CouplingModel.add\_multi\_coupling().

#### Returns

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*.

#### property dim

The dimension of the lattice.

### enlarge\_mps\_unit\_cell (factor=2)

Repeat the unit cell for infinite MPS boundary conditions; in place.

**Parameters factor** (*int*) – The new number of sites in the MPS unit cell will be increased from *N\_sites* to factor\*N\_sites\_per\_ring. Since MPS unit cells are repeated in the *x*-direction in our convetion, the lattice shape goes from (Lx, Ly, ..., Lu) to (Lx\*factor, Ly, ..., Lu).

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

## **Return type** cls

#### lat2mps idx(lat idx)

Translate lattice indices  $(x_0, \dots, x_{D-1})$ , u) to MPS index i.

**Parameters lat\_idx** ( $array_like[..,dim+1]$ ) – The last dimension corresponds to lattice indices ( $x_0, ..., x_{D-1}, u$ ). All lattice indices should be positive and smaller than the corresponding entry in self.shape. Exception: for "infinite"  $bc_MPS$ , an  $x_0$  outside indicates shifts accross the boundary.

**Returns i** – MPS index/indices corresponding to *lat\_idx*. Has the same shape as *lat\_idx* without the last dimension.

Return type array\_like

## mps2lat\_idx(i)

Translate MPS index i to lattice indices  $(x_0, \ldots, x_{\min-1}, u)$ .

**Parameters i** (int | array\_like of int) - MPS index/indices.

**Returns** lat\_idx – First dimensions like i, last dimension has len dim+1 and contains the lattice indices ``( $x_0, \ldots, x_{dim-1}, u$ )` corresponding to i. For i accross the MPS unit cell and "infinite" bc MPS, we shift x 0 accordingly.

Return type array

## $mps2lat\_values(A, axes=0, u=None)$

Reshape/reorder A to replace an MPS index by lattice indices.

#### **Parameters**

- A (ndarray) Some values. Must have A.shape[axes] = self.N\_sites if u is None, or A.shape[axes] = self.N\_cells if u is an int.
- axes ((iterable of) int) chooses the axis which should be replaced.
- u (None | int) Optionally choose a subset of MPS indices present in the axes of A, namely the indices corresponding to self.unit\_cell[u], as returned by mps\_idx\_fix\_u(). The resulting array will not have the additional dimension(s) of u.

**Returns res\_A** – Reshaped and reordered verions of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., j, ...].

**Return type** ndarray

#### **Examples**

Say you measure expection values of an onsite term for an MPS, which gives you an 1D array A, where A[i] is the expectation value of the site given by  $self.mps2lat_idx(i)$ . Then this function gives you the expectation values ordered by the lattice:

```
>>> print(lat.shape, A.shape)
(10, 3, 2) (60,)
>>> A_res = lat.mps2lat_values(A)
>>> A_res.shape
(10, 3, 2)
>>> A_res[lat.mps2lat_idx(5)] == A[5]
True
```

If you have a correlation function C[i, j], it gets just slightly more complicated:

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```
>>> print(lat.shape, C.shape)
(10, 3, 2) (60, 60)
>>> lat.mps2lat_values(C, axes=[0, 1]).shape
(10, 3, 2, 10, 3, 2)
```

If the unit cell consists of different physical sites, an onsite operator might be defined only on one of the sites in the unit cell. Then you can use  $mps\_idx\_fix\_u$  () to get the indices of sites it is defined on, measure the operator on these sites, and use the argument u of this function.

```
>>> u = 0
>>> idx_subset = lat.mps_idx_fix_u(u)
>>> A_u = A[idx_subset]
>>> A_u_res = lat.mps2lat_values(A_u, u=u)
>>> A_u_res.shape
(10, 3)
>>> np.all(A_res[:, :, u] == A_u_res[:, :])
True
```

**Todo:** make sure this function is used for expectation values...

## mps2lat\_values\_masked (A, axes=-1, mps\_inds=None, include\_u=None)

Reshape/reorder an array A to replace an MPS index by lattice indices.

This is a generalization of  $mps2lat\_values$  () allowing for the case of an arbitrary set of MPS indices present in each axis of A.

#### **Parameters**

- A (ndarray) Some values.
- **axes** ((iterable of) int) Chooses the axis of A which should be replaced. If multiple axes are given, you also need to give multiple index arrays as mps\_inds.
- mps\_inds ((list of) 1D ndarray) Specifies for each axis in axes, for which MPS indices we have values in the corresponding axis of A. Defaults to [np.arange (A. shape[ax]) for ax in axes]. For indices across the MPS unit cell and "infinite" bc\_MPS, we shift x\_0 accordingly.
- include\_u ((list of) bool) Specifies for each axis in axes, whether the u index of the lattice should be included into the output array res\_A. Defaults to len(self.unit\_cell) > 1.

**Returns** res\_A – Reshaped and reordered copy of A. Such that MPS indices along the specified axes are replaced by lattice indices, i.e., if MPS index j maps to lattice site (x0, x1, x2), then res\_A[..., x0, x1, x2, ...] = A[..., mps\_inds[j], ...].

Return type np.ma.MaskedArray

## mps\_idx\_fix\_u(u=None)

return an index array of MPS indices for which the site within the unit cell is u.

If you have multiple sites in your unit-cell, an onsite operator is in general not defined for all sites. This functions returns an index array of the mps indices which belong to sites given by self.unit\_cell[u].

**Parameters u** (*None* / *int*) – Selects a site of the unit cell. None (default) means all sites.

**Returns** mps\_idx - MPS indices for which self.site(i) is self.unit\_cell[u]. Ordered ascending.

## Return type array

#### mps lat idx fix u(u=None)

Similar as  $mps_idx_fix_u()$ , but return also the corresponding lattice indices.

Parameters u (None / int) - Selects a site of the unit cell. None (default) means all sites.

#### Returns

- mps\_idx (array) MPS indices i for which self.site(i) is self. unit\_cell[u].
- lat\_idx (2D array) The row j contains the lattice index (without u) corresponding to mps\_idx[j].

#### mps\_sites()

Return a list of sites for all MPS indices.

```
Equivalent to [self.site(i) for i in range(self.N_sites)].
```

This should be used for *sites* of 1D tensor networks (MPS, MPO,...).

## multi\_coupling\_shape (dx)

Calculate correct shape of the *strengths* for a multi\_coupling.

**Parameters** dx (2D array, shape (N\_ops, dim)) – dx[i, :] is the translation vector in the lattice for the *i*-th operator. Corresponds to the dx of each operator given in the argument ops of  $tenpy.models.model.MultiCouplingModel.add_multi_coupling().$ 

#### **Returns**

- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.
- **shift\_lat\_indices** (*array*) Translation vector from origin to the lower left corner of box spanned by *dx*. (Unlike for *coupling\_shape()* it can also contain entries > 0)

## $number_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

## $number_next_nearest_neighbors(u=0)$

Deprecated.

Deprecated since version 0.5.0: Use count\_neighbors() instead.

#### property order

Defines an ordering of the lattice sites, thus mapping the lattice to a 1D chain.

Each row of the array contains the lattice indices for one site, the order of the rows thus specifies a path through the lattice, along which an MPS will wind through the lattice.

You can visualize the order with plot\_order().

#### plot\_basis (ax, origin=0.0, 0.0, shade=None, \*\*kwargs)

Plot arrows indicating the basis vectors of the lattice.

## **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- \*\*kwargs Keyword arguments for ax.arrow.

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plot\_bc\_identified (ax, direction=- 1, origin=None, cylinder\_axis=False, \*\*kwargs)

Mark two sites indified by periodic boundary conditions.

Works only for lattice with a 2-dimensional basis.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **direction** (*int*) The direction of the lattice along which we should mark the idenitified sites. If None, mark it along all directions with periodic boundary conditions.
- cylinder\_axis (bool) Whether to plot the cylinder axis as well.
- **origin** (*None* / *np.ndarray*) The origin starting from where we mark the identified sites. Defaults to the first entry of unit\_cell\_positions.
- \*\*kwargs Keyword arguments for the used ax.plot.

```
plot_coupling(ax, coupling=None, wrap=False, **kwargs)
```

Plot lines connecting nearest neighbors of the lattice.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- **coupling** (*list of* (*u1*, *u2*, *dx*)) By default (None), use self. pairs['nearest\_neighbors']. Specifies the connections to be plotted; iteating over lattice indices (*i0*, *i1*, ...), we plot a connection from the site (i0, i1, ..., u1) to the site (i0+dx[0], i1+dx[1], ..., u2), taking into account the boundary conditions.
- wrap (bool) If True, wrap
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_order(ax, order=None, textkwargs={}, **kwargs)
```

Plot a line connecting sites in the specified "order" and text labels enumerating them.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- order (None | 2D array (self.N\_sites, self.dim+1)) The order as returned by ordering(); by default (None) use order.
- textkwargs (None | dict) If not None, we add text labels enumerating the sites in the plot. The dictionary can contain keyword arguments for ax.text().
- \*\*kwargs Further keyword arguments given to ax.plot().

```
plot_sites (ax, markers=['o', '^', 's', 'p', 'h', 'D'], **kwargs)
```

Plot the sites of the lattice with markers.

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- markers (list) List of values for the keywork *marker* of ax.plot() to distinguish the different sites in the unit cell, a site u in the unit cell is plotted with a marker markers[u % len(markers)].
- \*\*kwargs Further keyword arguments given to ax.plot().

### position(lat idx)

return 'space' position of one or multiple sites.

```
Parameters lat_idx (ndarray, (..., dim+1)) - Lattice indices.
```

**Returns** pos – The position of the lattice sites specified by *lat idx* in real-space.

```
Return type ndarray, (..., dim)
```

## possible\_couplings (u1, u2, dx)

Find possible MPS indices for two-site couplings.

For periodic boundary conditions (bc[a] == False) the index x\_a is taken modulo Ls[a] and runs through range (Ls[a]). For open boundary conditions, x\_a is limited to  $0 \le x_a \le x_$ 

#### **Parameters**

- **u2** (*u*1, ) Indices within the unit cell; the *u*1 and *u*2 of add\_coupling()
- dx (array) Length dim. The translation in terms of basis vectors for the coupling.

#### **Returns**

- mps1, mps2 (array) For each possible two-site coupling the MPS indices for the u1 and u2.
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

## possible\_multi\_couplings(ops)

Generalization of possible\_couplings () to couplings with more than 2 sites.

**Parameters ops** (list of (opname, dx, u)) - Same as the argument *ops* of add\_multi\_coupling().

### Returns

- mps\_ijkl (2D int array) Each row contains MPS indices i,j,k,l,...` for each of the operators positions. The positions are defined by dx (j,k,l,... relative to i) and boundary coundary conditions of self (how much the box for given dx can be shifted around without hitting a boundary these are the different rows).
- lat\_indices (2D int array) Rows of lat\_indices correspond to rows of mps\_ijkl and contain the lattice indices of the "lower left corner" of the box containing the coupling.
- **coupling\_shape** (*tuple of int*) Len *dim*. The correct shape for an array specifying the coupling strength. *lat\_indices* has only rows within this shape.

```
save hdf5 (hdf5 saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

Specifically, it saves unit\_cell, Ls, unit\_cell\_positions, basis, boundary\_conditions, pairs under their name, bc\_MPS as "boundary\_conditions\_MPS", and order as "order\_for\_MPS". Moreover, it saves dim and N\_sites as HDF5 attributes.

## Parameters

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

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```
site(i)
    return Site instance corresponding to an MPS index i

test_sanity()
    Sanity check.

Raises ValueErrors, if something is wrong.
```

## **Module description**

Kitaev's exactly solvable toric code model.

As we put the model on a cylinder, the name "toric code" is a bit misleading, but it is the established name for this model...

# 7.10 networks

full name: tenpy.networksparent module: tenpy

• type: module

## **Module description**

Definitions of tensor networks like MPS and MPO.

Here, 'tensor network' refers just to the (parital) contraction of tensors. For example an MPS represents the contraction along the 'virtual' legs/bonds of its *B*.

## **Submodules**

site	Defines a class describing the local physical Hilbert
	space.
mps	This module contains a base class for a Matrix Product
	State (MPS).
mpo	Matrix product operator (MPO).
terms	Classes to store a collection of operator names and sites
	they act on, together with prefactors.
purification_mps	This module contains an MPS class representing an den-
	sity matrix by purification.
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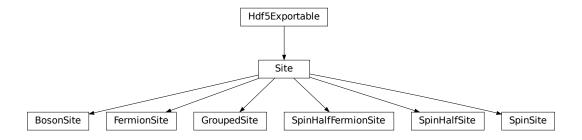
## 7.10.1 site

• full name: tenpy.networks.site

• parent module: tenpy.networks

• type: module

## **Classes**



BosonSite([Nmax, conserve, filling])	Create a Site for up to Nmax bosons.
FermionSite([conserve, filling])	Create a Site for spin-less fermions.
GroupedSite(sites[, labels, charges])	Group two or more Site into a larger one.
Site(leg[, state_labels])	Collects necessary information about a single local site
	of a lattice.
SpinHalfFermionSite([cons_N, cons_Sz, fill-	Create a Site for spinful (spin-1/2) fermions.
ing])	
SpinHalfSite([conserve])	Spin-1/2 site.
SpinSite([S, conserve])	General Spin S site.

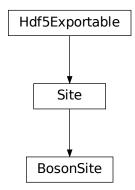
## **BosonSite**

• full name: tenpy.networks.site.BosonSite

• parent module: tenpy.networks.site

• type: class

# Inheritance Diagram



## Methods

BosonSiteinit([Nmax, conserve, filling])	Initialize self.
BosonSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
BosonSite.change_charge([new_leg_charge,	Change the charges of the site (in place).
])	
BosonSite.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
BosonSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
BosonSite.get_op(name)	Return operator of given name.
BosonSite.multiply_op_names(names)	Multiply operator names together.
BosonSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
BosonSite.remove_op(name)	Remove an added operator.
BosonSite.rename_op(old_name, new_name)	Rename an added operator.
BosonSite.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
BosonSite.state_index(label)	Return index of a basis state from its label.
BosonSite.state_indices(labels)	Same as state_index(), but for multiple labels.
BosonSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
BosonSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

## **Class Attributes and Properties**

BosonSite.dim	Dimension of the local Hilbert space.
BosonSite.onsite_ops	Dictionary of on-site operators for iteration.

**class** tenpy.networks.site.**BosonSite**(*Nmax=1*, *conserve='N'*, *filling=0.0*)

Bases: tenpy.networks.site.Site

Create a *Site* for up to *Nmax* bosons.

Local states are vac, 1, 2, ..., Nc. (Exception: for parity conservation, we sort as vac, 2, 4, ..., 1, 3, 5, ....)

operator	description	
Id, JW	Identity ⊮	
В	Annihilation operator b	
Bd	Creation operator $b^{\dagger}$	
N	Number operator $n = b^{\dagger}b$	
NN	$n^2$	
dN	$\delta n := n - filling$	
dNdN	$(\delta n)^2$	
P	Parity $Id - 2(n \mod 2)$ .	

conserve	qmod	excluded onsite operators
'N'	[1]	_
'parity'	[2]	_
None	[]	_

## Parameters

- Nmax (int) Cutoff defining the maximum number of bosons per site. The default Nmax=1 describes hard-core bosons.
- **conserve** (*str*) Defines what is conserved, see table above.
- **filling** (*float*) Average filling. Used to define dN.

## conserve

Defines what is conserved, see table above.

Type str

#### filling

Average filling. Used to define dN.

Type float

add\_op (name, op, need\_JW=False, hc=None)

Add one on-site operators.

#### **Parameters**

• name (str) – A valid python variable name, used to label the operator. The name under which *op* is added as attribute to self.

- op (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p\*'.
- need\_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need\_JW\_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc\_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc\_ops.

change\_charge (new\_leg\_charge=None, permute=None)

Change the charges of the site (in place).

#### **Parameters**

- new\_leg\_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state\_labels and perm. If you sorted the previous leg with perm\_qind, new\_leg\_charge = leg.sort(), use old\_leg. perm\_flat\_from\_perm\_qind(perm\_qind). Ignored if None.

## property dim

Dimension of the local Hilbert space.

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

## **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

## get\_hc\_op\_name (name)

Return the hermitian conjugate of a given operator.

**Parameters name** (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as get op() does.

**Returns hc\_op\_name** – Operator name for the hermi such that  $get\_op()$  of

Return type str

## get\_op (name)

Return operator of given name.

**Parameters** name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

**Returns op** – The operator given by *name*, with labels 'p', 'p\*'. If name already was an npc Array, it's directly returned.

Return type np\_conserved

## multiply\_op\_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get\_op* returns the product of the corresponding operators.

**Parameters** names (list of str) – List of valid operator labels.

**Returns combined\_opname** – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

## property onsite\_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

## op\_needs\_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

**Parameters** name (str) – The name of the operator, as in  $get\_op()$ .

**Returns needs\_JW** — Whether the operator needs a Jordan-Wigner string, judging from need\_JW\_string.

Return type bool

#### remove\_op (name)

Remove an added operator.

**Parameters** name (str) – The name of the operator to be removed.

## rename\_op (old\_name, new\_name)

Rename an added operator.

## **Parameters**

- old\_name (str) The old name of the operator.
- **new\_name** (str) The new name of the operator.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with <code>save\_dict\_content()</code>, storing the format under the attribute 'format'.

#### **Parameters**

- hdf5 saver (Hdf5Saver) Instance of the saving engine.
- **h5gr** (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

### state\_index(label)

Return index of a basis state from its label.

**Parameters label** (int | string) – eather the index directly or a label (string) set before.

Returns state\_index - the index of the basis state associated with the label.

Return type int

## state indices(labels)

Same as state index(), but for multiple labels.

#### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

## valid\_opname (name)

Check whether 'name' labels a valid onsite-operator.

**Parameters name** (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

**Returns** valid – True if *name* is a valid argument to  $get\_op()$ .

Return type bool

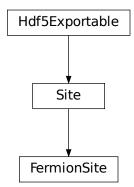
## **FermionSite**

• full name: tenpy.networks.site.FermionSite

• parent module: tenpy.networks.site

• type: class

## **Inheritance Diagram**



## **Methods**

FermionSiteinit([conserve, filling])	Initialize self.
FermionSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
FermionSite.change_charge([new_leg_charge,	Change the charges of the site (in place).
])	
FermionSite.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
FermionSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
FermionSite.get_op(name)	Return operator of given name.
FermionSite.multiply_op_names(names)	Multiply operator names together.
	continues on next page

Table 126 - continued from previous page

FermionSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
FermionSite.remove_op(name)	Remove an added operator.
<pre>FermionSite.rename_op(old_name, new_name)</pre>	Rename an added operator.
FermionSite.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
41- \	
path)	
FermionSite.state_index(label)	Return index of a basis state from its label.
	Return index of a basis state from its label.  Same as state_index(), but for multiple labels.
FermionSite.state_index(label)	

## **Class Attributes and Properties**

FermionSite.dim	Dimension of the local Hilbert space.
FermionSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.FermionSite(conserve='N', filling=0.5)

Bases: tenpy.networks.site.Site

Create a Site for spin-less fermions.

Local states are empty and full.

**Warning:** Using the Jordan-Wigner string (JW) is crucial to get correct results, otherwise you just describe hardcore bosons! Further details in *Fermions and the Jordan-Wigner transformation*.

operator	description
Id	Identity ⊮
JW	Sign for the Jordan-Wigner string.
С	Annihilation operator $c$ (up to 'JW'-string left of it)
Cd	Creation operator $c^{\dagger}$ (up to 'JW'-string left of it)
N	Number operator $n = c^{\dagger}c$
dN	$\delta n := n - filling$
dNdN	$(\delta n)^2$

conserve	qmod	exluded onsite operators
'N'	[1]	_
'parity'	[2]	_
None	[]	_

#### **Parameters**

- **conserve** (*str*) Defines what is conserved, see table above.
- **filling** (*float*) Average filling. Used to define dN.

## conserve

Defines what is conserved, see table above.

#### Type str

#### filling

Average filling. Used to define dN.

### Type float

add\_op (name, op, need\_JW=False, hc=None)

Add one on-site operators.

#### **Parameters**

- name (str) A valid python variable name, used to label the operator. The name under which op is added as attribute to self.
- **op** (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p\*'.
- need\_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need\_JW\_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc\_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc\_ops.

change\_charge (new\_leg\_charge=None, permute=None)

Change the charges of the site (in place).

#### **Parameters**

- new\_leg\_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state\_labels and perm. If you sorted the previous leg with perm\_qind, new\_leg\_charge = leg.sort(), use old\_leg. perm\_flat\_from\_perm\_qind(perm\_qind). Ignored if None.

## property dim

Dimension of the local Hilbert space.

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5 loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

## get\_hc\_op\_name (name)

Return the hermitian conjugate of a given operator.

**Parameters name** (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as  $get\_op()$  does.

**Returns** hc\_op\_name – Operator name for the hermi such that  $get\_op()$  of

#### Return type str

```
get_op (name)
```

Return operator of given name.

**Parameters** name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

**Returns op** – The operator given by *name*, with labels 'p', 'p\*'. If name already was an npc Array, it's directly returned.

Return type np\_conserved

## multiply\_op\_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get\_op* returns the product of the corresponding operators.

**Parameters** names (list of str) – List of valid operator labels.

**Returns combined\_opname** – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

## property onsite\_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

## op\_needs\_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

**Parameters** name (str) – The name of the operator, as in  $get\_op()$ .

**Returns needs\_JW** — Whether the operator needs a Jordan-Wigner string, judging from  $need\_JW\_string$ .

Return type bool

## remove\_op (name)

Remove an added operator.

**Parameters** name (str) – The name of the operator to be removed.

rename\_op (old\_name, new\_name)

Rename an added operator.

#### **Parameters**

- old\_name (str) The old name of the operator.
- **new\_name** (str) The new name of the operator.

save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

## **Parameters**

• hdf5 saver (Hdf5Saver) - Instance of the saving engine.

- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

## $state\_index(label)$

Return index of a basis state from its label.

**Parameters label** (int | string) – eather the index directly or a label (string) set before.

**Returns** state\_index – the index of the basis state associated with the label.

Return type int

## state\_indices(labels)

Same as state\_index(), but for multiple labels.

#### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

## valid\_opname (name)

Check whether 'name' labels a valid onsite-operator.

**Parameters name** (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

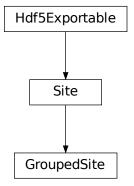
**Returns valid** – True if name is a valid argument to  $get\_op()$ .

Return type bool

## GroupedSite

- full name: tenpy.networks.site.GroupedSite
- parent module: tenpy.networks.site
- type: class

## **Inheritance Diagram**



## **Methods**

Initialize self.
Add one on-site operators.
Change the charges of the site (in place).
Load instance from a HDF5 file.
Return the hermitian conjugate of a given operator.
Return operator of given name.
Return the Kronecker product $op0\otimes op1$ of local oper-
ators.
Multiply operator names together.
Whether an (composite) onsite operator is fermionic and
needs a Jordan-Wigner string.
Remove an added operator.
Rename an added operator.
Export self into a HDF5 file.
Return index of a basis state from its label.
Same as state_index(), but for multiple labels.
Sanity check, raises ValueErrors, if something is wrong.
Check whether 'name' labels a valid onsite-operator.

## **Class Attributes and Properties**

GroupedSite.dim	Dimension of the local Hilbert space.
GroupedSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.GroupedSite(sites, labels=None, charges='same')

Bases: tenpy.networks.site.Site

Group two or more Site into a larger one.

A typical use-case is that you want a NearestNeighborModel for TEBD although you have next-nearest neighbor interactions: you just double your local Hilbertspace to consist of two original sites. Note that this is a 'hack' at the cost of other things (e.g., measurements of 'local' operators) getting more complicated/computationally expensive.

If the individual sites indicate fermionic operators (with entries in *need\_JW\_string*), we construct the new on-site oerators of *site1* to include the JW string of *site0*, i.e., we use the Kronecker product of [JW, op] instead of [Id, op] if necessary (but always [op, Id]). In that way the onsite operators of this DoubleSite automatically fulfill the expected commutation relations. See also *Fermions and the Jordan-Wigner transformation*.

## Parameters

- **sites** (list of *Site*) The individual sites being grouped together. Copied before use if charges!='same'.
- labels Include the Kronecker product of the each onsite operator *op* on sites[i] and identities on other sites with the name opname+labels[i]. Similarly, set state labels for ' '.join(state[i]+'\_'+labels[i]). Defaults to [str(i) for i in range(n\_sites)], which for example grouping two SpinSites gives operators name

like "Sz0" and sites labels like 'up\_0 down\_1'.

• **charges** ('same' | 'drop' | 'independent') - How to handle charges, defaults to 'same'. 'same' means that all *sites* have the same *ChargeInfo*, and the total charge is the sum of the charges on the individual *sites*. 'independent' means that the *sites* have possibly different *ChargeInfo*, and the charges are conserved separately, i.e., we have *n\_sites* conserved charges. For 'drop', we drop any charges, such that the remaining legcharges are trivial.

## n\_sites

The number of sites grouped together, i.e. len(sites).

```
Type int
```

#### sites

The sites grouped together into self.

```
Type list of Site
```

#### labels

The labels using which the single-site operators are added during construction.

**Type** list of str

## kroneckerproduct (ops)

Return the Kronecker product  $op0 \otimes op1$  of local operators.

**Parameters ops** (list of Array) – One operator (or operator name) on each of the ungrouped sites. Each operator should have labels ['p', 'p\*'].

**Returns prod** – Kronecker product  $ops[0] \otimes ops[1] \otimes \cdots$ , with labels ['p', 'p\*'].

Return type Array

add\_op (name, op, need\_JW=False, hc=None)

Add one on-site operators.

#### **Parameters**

- name (str) A valid python variable name, used to label the operator. The name under which op is added as attribute to self.
- **op** (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p\*'.
- need\_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need\_JW\_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc\_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc\_ops.

change\_charge (new\_leg\_charge=None, permute=None)

Change the charges of the site (in place).

#### **Parameters**

- new\_leg\_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state\_labels and perm. If you sorted the previ-

```
ous leg with perm_qind, new_leg_charge = leg.sort(), use old_leg.
perm_flat_from_perm_qind(perm_qind). Ignored if None.
```

### property dim

Dimension of the local Hilbert space.

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The name of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

#### get\_hc\_op\_name (name)

Return the hermitian conjugate of a given operator.

**Parameters name** (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as  $get\_op()$  does.

**Returns hc\_op\_name** – Operator name for the hermi such that  $get\_op()$  of

Return type str

## get\_op (name)

Return operator of given name.

**Parameters** name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

**Returns op** – The operator given by *name*, with labels 'p', 'p\*'. If name already was an npc Array, it's directly returned.

Return type np\_conserved

## multiply\_op\_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get\_op* returns the product of the corresponding operators.

**Parameters names** (list of str) – List of valid operator labels.

**Returns combined\_opname** – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

### property onsite\_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

### op\_needs\_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

**Parameters** name (str) – The name of the operator, as in get op().

```
Returns needs_JW — Whether the operator needs a Jordan-Wigner string, judging from need_JW_string.
```

## Return type bool

## remove\_op (name)

Remove an added operator.

**Parameters** name (str) – The name of the operator to be removed.

## rename\_op (old\_name, new\_name)

Rename an added operator.

#### **Parameters**

- old\_name (str) The old name of the operator.
- **new\_name** (str) The new name of the operator.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

## $state\_index(label)$

Return index of a basis state from its label.

**Parameters label** (int / string) – eather the index directly or a label (string) set before.

Returns state\_index - the index of the basis state associated with the label.

Return type int

## state\_indices(labels)

Same as state\_index(), but for multiple labels.

## test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### valid opname(name)

Check whether 'name' labels a valid onsite-operator.

**Parameters name** (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

**Returns** valid – True if *name* is a valid argument to  $get\_op()$ .

Return type bool

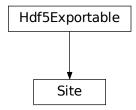
## Site

• full name: tenpy.networks.site.Site

• parent module: tenpy.networks.site

• type: class

# **Inheritance Diagram**



## Methods

Siteinit(leg[, state_labels])	Initialize self.
Site.add_op(name, op[, need_JW, hc])	Add one on-site operators.
Site.change_charge([new_leg_charge, per-	Change the charges of the site (in place).
mute])	
Site.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
Site.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
Site.get_op(name)	Return operator of given name.
Site.multiply_op_names(names)	Multiply operator names together.
Site.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
Site.remove_op(name)	Remove an added operator.
Site.rename_op(old_name, new_name)	Rename an added operator.
Site.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
Site.state_index(label)	Return index of a basis state from its label.
Site.state_indices(labels)	Same as state_index(), but for multiple labels.
Site.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
Site.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

## **Class Attributes and Properties**

Site.dim	Dimension of the local Hilbert space.
Site.onsite_ops	Dictionary of on-site operators for iteration.

```
class tenpy.networks.site.Site(leg, state_labels=None, **site_ops)
    Bases: tenpy.tools.hdf5_io.Hdf5Exportable
```

Collects necessary information about a single local site of a lattice.

This class defines what the local basis states are: it provides the leg defining the charges of the physical leg for this site. Moreover, it stores (local) on-site operators, which are directly available as attribute, e.g., self.Sz is the Sz operator for the SpinSite. Alternatively, operators can be obtained with  $get\_op()$ . The operator names Id and JW are reserved for the identy and Jordan-Wigner strings.

**Warning:** The order of the local basis can change depending on the charge conservation! This is a *necessary* feature since we need to sort the basis by charges for efficiency. We use the <code>state\_labels</code> and <code>perm</code> to keep track of these permutations.

#### **Parameters**

- **leg** (*LegCharge*) Charges of the physical states, to be used for the physical leg of MPS.
- **state\_labels** (*None | list of str*) Optionally a label for each local basis states. None entries are ignored / not set.
- \*\*site\_ops Additional keyword arguments of the form name=op given to add\_op(). The identity operator 'Id' is automatically included. If no 'JW' for the Jordan-Wigner string is given, 'JW' is set as an alias to 'Id'.

## leg

Charges of the local basis states.

```
Type LegCharge
```

#### state labels

(Optional) labels for the local basis states.

```
Type {str: int}
```

#### opnames

Labels of all onsite operators (i.e. self.op exists if 'op' in self.opnames). Note that  $get\_op()$  allows arbitrary concatenations of them.

```
Type set
```

### need\_JW\_string

Labels of all onsite operators that need a Jordan-Wigner string. Used in <code>op\_needs\_JW()</code> to determine whether an operator anticommutes or commutes with operators on other sites.

```
Type set
```

## ops

Onsite operators are added directly as attributes to self. For example after self.add\_op('Sz', Sz) you can use self.Sz for the Sz operator. All onsite operators have labels 'p', 'p\*'.

```
Type Array
```

#### perm

```
Type 1D array
```

## JW\_exponent

```
Exponents of the 'JW' operator, such that self.JW.to_ndarray() = np.diag(np.exp(1.j*np.pi*JW_exponent))
```

```
Type 1D array
```

#### hc\_ops

Mapping from operator names to their hermitian conjugates. Use qet\_hc\_op\_name() to obtain entries.

```
Type dict(str->str)
```

## **Examples**

The following generates a site for spin-1/2 with Sz conservation. Note that Sx = (Sp + Sm)/2 violates Sz conservation and is thus not a valid on-site operator.

```
>>> chinfo = npc.ChargeInfo([1], ['Sz'])
>>> ch = npc.LegCharge.from_qflat(chinfo, [1, -1])
>>> Sp = [[0, 1.], [0, 0]]
>>> Sm = [[0, 0], [1., 0]]
>>> Sz = [[0.5, 0], [0, -0.5]]
>>> site = Site(ch, ['up', 'down'], Splus=Sp, Sminus=Sm, Sz=Sz)
>>> print(site.Splus.to_ndarray())
array([[ 0., 1.],
       [ 0., 0.]])
>>> print(site.get_op('Sminus').to_ndarray())
array([[ 0., 0.],
       [ 1., 0.]])
>>> print(site.get_op('Splus Sminus').to_ndarray())
array([[ 1., 0.],
       [ 0.,
              0.11)
```

### change\_charge (new\_leg\_charge=None, permute=None)

Change the charges of the site (in place).

#### **Parameters**

- new\_leg\_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state\_labels and perm. If you sorted the previous leg with perm\_qind, new\_leg\_charge = leg.sort(), use old\_leg. perm\_flat\_from\_perm\_qind(perm\_qind). Ignored if None.

#### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### property dim

Dimension of the local Hilbert space.

#### property onsite\_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

```
add_op (name, op, need_JW=False, hc=None)
```

Add one on-site operators.

#### **Parameters**

- name (str) A valid python variable name, used to label the operator. The name under which *op* is added as attribute to self.
- op (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p\*'.
- need\_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need\_JW\_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc\_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc\_ops.

## rename\_op (old\_name, new\_name)

Rename an added operator.

#### **Parameters**

- old\_name (str) The old name of the operator.
- **new\_name** (str) The new name of the operator.

## remove\_op (name)

Remove an added operator.

**Parameters** name (str) – The name of the operator to be removed.

```
state_index(label)
```

Return index of a basis state from its label.

**Parameters label** (int | string) – eather the index directly or a label (string) set before.

**Returns** state\_index – the index of the basis state associated with the label.

Return type int

## state\_indices (labels)

Same as state\_index(), but for multiple labels.

#### get\_op (name)

Return operator of given name.

**Parameters** name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

**Returns op** – The operator given by *name*, with labels 'p', 'p\*'. If name already was an npc Array, it's directly returned.

Return type np\_conserved

## get\_hc\_op\_name (name)

Return the hermitian conjugate of a given operator.

**Parameters name** (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as  $get\_op()$  does.

**Returns hc\_op\_name** – Operator name for the hermi such that  $get\_op()$  of

Return type str

#### op\_needs\_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

**Parameters** name (str) – The name of the operator, as in  $get\_op()$ .

**Returns needs\_JW** – Whether the operator needs a Jordan-Wigner string, judging from need\_JW\_string.

Return type bool

#### valid\_opname (name)

Check whether 'name' labels a valid onsite-operator.

**Parameters name** (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

**Returns valid** – True if *name* is a valid argument to  $get\_op()$ .

Return type bool

## multiply\_op\_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get\_op* returns the product of the corresponding operators.

**Parameters** names (list of str) – List of valid operator labels.

**Returns combined\_opname** – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

## **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

## save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

## **Parameters**

• hdf5 saver (Hdf5Saver) - Instance of the saving engine.

- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

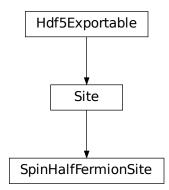
## **SpinHalfFermionSite**

• full name: tenpy.networks.site.SpinHalfFermionSite

• parent module: tenpy.networks.site

• type: class

# **Inheritance Diagram**



## Methods

SpinHalfFermionSiteinit([cons_N,	Initialize self.
])	
SpinHalfFermionSite.add_op(name, op[,])	Add one on-site operators.
SpinHalfFermionSite.	Change the charges of the site (in place).
change_charge([])	
SpinHalfFermionSite.	Load instance from a HDF5 file.
<pre>from_hdf5(hdf5_loader,)</pre>	
SpinHalfFermionSite.	Return the hermitian conjugate of a given operator.
<pre>get_hc_op_name(name)</pre>	
SpinHalfFermionSite.get_op(name)	Return operator of given name.
SpinHalfFermionSite.	Multiply operator names together.
multiply_op_names(names)	
SpinHalfFermionSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
SpinHalfFermionSite.remove_op(name)	Remove an added operator.
	continues on novt page

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Table 132 – continued from previous page

SpinHalfFermionSite.rename_op(old_name,	Rename an added operator.
new_name)	
SpinHalfFermionSite.	Export self into a HDF5 file.
save_hdf5(hdf5_saver,)	
SpinHalfFermionSite.state_index(label)	Return index of a basis state from its label.
SpinHalfFermionSite.	Same as state_index(), but for multiple labels.
state_indices(labels)	
SpinHalfFermionSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
SpinHalfFermionSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

## **Class Attributes and Properties**

SpinHalfFermionSite.dim	Dimension of the local Hilbert space.
SpinHalfFermionSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.SpinHalfFermionSite( $cons_N='N'$ ,  $cons_Sz='Sz'$ , filling=1.0)
Bases: tenpy.networks.site.Site

Create a Site for spinful (spin-1/2) fermions.

Local states are: empty (vacuum), up (one spin-up electron), down (one spin-down electron), and full (both electrons)

Local operators can be built from creation operators.

**Warning:** Using the Jordan-Wigner string (JW) in the correct way is crucial to get correct results, otherwise you just describe hardcore bosons!

operator	description
Id	Identity ⊮
JW	Sign for the Jordan-Wigner string $(-1)^{n_{\uparrow}+n_{\downarrow}}$
JWu	Partial sign for the Jordan-Wigner string $(-1)^{n_{\uparrow}}$
JWd	Partial sign for the Jordan-Wigner string $(-1)^{n_{\downarrow}}$
Cu	Annihilation operator spin-up $c_{\uparrow}$ (up to 'JW'-string on sites left of it).
Cdu	Creation operator spin-up $c^{\dagger}_{\uparrow}$ (up to 'JW'-string on sites left of it).
Cd	Annihilation operator spin-down $c_{\downarrow}$ (up to 'JW'-string on sites left of it). Includes JWu such
	that it anti-commutes onsite with Cu, Cdu.
Cdd	Creation operator spin-down $c_{\perp}^{\dagger}$ (up to 'JW'-string on sites left of it). Includes JWu such that it
	anti-commutes onsite with Cu, Cdu.
Nu	Number operator $n_{\uparrow} = c_{\uparrow}^{\dagger} c_{\uparrow}$
Nd	Number operator $n_{\downarrow} = c_{\downarrow}^{\dagger} c_{\downarrow}$
NuNd	Dotted number operators $n_{\uparrow}n_{\downarrow}$
Ntot	Total number operator $n_t = n_{\uparrow} + n_{\downarrow}$
dN	Total number operator compared to the filling $\Delta n = n_t - filling$
Sx,	Spin operators $S^{x,y,z}$ , in particular $S^z=\frac{1}{2}(n_{\uparrow}-n_{\downarrow})$
Sy, Sz	
Sp, Sm	Spin flips $S^{\pm}=S^x\pm iS^y$ , e.g. $S^+=c^{\dagger}_{\uparrow}c_{\downarrow}$

The spin operators are defined as  $S^{\gamma}=(c_{\uparrow}^{\dagger},c_{\downarrow}^{\dagger})\sigma^{\gamma}(c_{\uparrow},c_{\downarrow})^{T}$ , where  $\sigma^{\gamma}$  are spin-1/2 matrices (i.e. half the pauli matrices).

cons_N	cons_Sz	qmod	excluded onsite operators
'N'	'Sz'	[1, 1]	Sx, Sy
'N'	'parity'	[1, 2]	_
'N'	None	[1]	_
'parity'	'Sz'	[2, 1]	Sx, Sy
'parity'	'parity'	[2, 2]	_
'parity'	None	[2]	_
None	'Sz'	[1]	Sx, Sy
None	'parity'	[2]	_
None	None	[]	_

**Todo:** Check if Jordan-Wigner strings for 4x4 operators are correct.

#### **Parameters**

- cons\_N('N' | 'parity' | None) Whether particle number is conserved, c.f. table above.
- cons\_Sz('Sz' | 'parity' | None) Whether spin is conserved, c.f. table above.
- **filling** (*float*) Average filling. Used to define dN.

#### cons N

Whether particle number is conserved, c.f. table above.

### cons\_Sz

Whether spin is conserved, c.f. table above.

## filling

Average filling. Used to define dN.

add\_op (name, op, need\_JW=False, hc=None)

Add one on-site operators.

#### **Parameters**

- name (str) A valid python variable name, used to label the operator. The name under which op is added as attribute to self.
- **op** (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p\*'.
- need\_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need\_JW\_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc\_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc\_ops.

change\_charge (new\_leg\_charge=None, permute=None)

Change the charges of the site (in place).

#### **Parameters**

- new\_leg\_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state\_labels and perm. If you sorted the previous leg with perm\_qind, new\_leg\_charge = leg.sort(), use old\_leg. perm\_flat\_from\_perm\_qind(perm\_qind). Ignored if None.

#### property dim

Dimension of the local Hilbert space.

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
get_hc_op_name (name)
```

Return the hermitian conjugate of a given operator.

**Parameters name** (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as  $qet\_op()$  does.

**Returns hc\_op\_name** – Operator name for the hermi such that  $get\_op()$  of

Return type str

```
get_op (name)
```

Return operator of given name.

**Parameters** name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

**Returns op** – The operator given by *name*, with labels 'p', 'p\*'. If name already was an npc Array, it's directly returned.

Return type np\_conserved

## multiply\_op\_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get\_op* returns the product of the corresponding operators.

**Parameters** names (list of str) – List of valid operator labels.

**Returns combined\_opname** – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

#### property onsite\_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

### op\_needs\_JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

**Parameters** name (str) – The name of the operator, as in  $get\_op()$ .

**Returns needs\_JW** — Whether the operator needs a Jordan-Wigner string, judging from need\_JW\_string.

## Return type bool

#### remove\_op (name)

Remove an added operator.

**Parameters** name (str) – The name of the operator to be removed.

### rename\_op (old\_name, new\_name)

Rename an added operator.

#### **Parameters**

- **old\_name** (*str*) The old name of the operator.
- **new\_name** (str) The new name of the operator.

## save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

## state\_index(label)

Return index of a basis state from its label.

**Parameters label** (int | string) – eather the index directly or a label (string) set before.

**Returns state index** – the index of the basis state associated with the label.

#### Return type int

## state\_indices(labels)

Same as state\_index(), but for multiple labels.

### test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### valid\_opname (name)

Check whether 'name' labels a valid onsite-operator.

**Parameters name** (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

**Returns valid** – True if *name* is a valid argument to  $get\_op()$ .

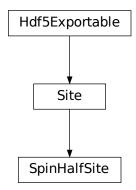
# Return type bool

# **SpinHalfSite**

• full name: tenpy.networks.site.SpinHalfSite • parent module: tenpy.networks.site

• type: class

# **Inheritance Diagram**



## **Methods**

ChinHalfCita init ([concernal)	Initialize self.
SpinHalfSiteinit([conserve])	
SpinHalfSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
SpinHalfSite.change_charge([new_leg_charge,	Change the charges of the site (in place).
])	
SpinHalfSite.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
SpinHalfSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
SpinHalfSite.get_op(name)	Return operator of given name.
SpinHalfSite.multiply_op_names(names)	Multiply operator names together.
SpinHalfSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
SpinHalfSite.remove_op(name)	Remove an added operator.
SpinHalfSite.rename_op(old_name,	Rename an added operator.
new_name)	
SpinHalfSite.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
subpath)	
SpinHalfSite.state_index(label)	Return index of a basis state from its label.
SpinHalfSite.state_indices(labels)	Same as state_index(), but for multiple labels.
	continues on next page

## Table 134 – continued from previous page

SpinHalfSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
SpinHalfSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

## **Class Attributes and Properties**

SpinHalfSite.dim	Dimension of the local Hilbert space.
SpinHalfSite.onsite_ops	Dictionary of on-site operators for iteration.

class tenpy.networks.site.SpinHalfSite(conserve='Sz')

Bases: tenpy.networks.site.Site

Spin-1/2 site.

Local states are up (0) and down (1). Local operators are the usual spin-1/2 operators, e.g. Sz = [[0.5, 0.], [0., -0.5]],  $Sx = 0.5 \times sigma_x$  for the Pauli matrix  $sigma_x$ .

operator	description
Id, JW	Identity ⊮
Sx, Sy, Sz	Spin components $S^{x,y,z}$ , equal to half the Pauli matrices.
Sigmax, Sigmay, Sigmaz	Pauli matrices $\sigma^{x,y,z}$
Sp, Sm	Spin flips $S^{\pm} = S^x \pm iS^y$

conserve	qmod	excluded onsite operators
'Sz'	[1]	Sx, Sy, Sigmax, Sigmay
'parity'	[2]	_
None	[]	_

**Parameters** conserve (str) – Defines what is conserved, see table above.

### conserve

Defines what is conserved, see table above.

Type str

add\_op (name, op, need\_JW=False, hc=None)

Add one on-site operators.

## **Parameters**

- name (str) A valid python variable name, used to label the operator. The name under which op is added as attribute to self.
- **op** (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p\*'.
- need\_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need\_JW\_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc\_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc\_ops.

change\_charge (new\_leg\_charge=None, permute=None)

Change the charges of the site (in place).

#### **Parameters**

- new\_leg\_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state\_labels and perm. If you sorted the previous leg with perm\_qind, new\_leg\_charge = leg.sort(), use old\_leg. perm\_flat\_from\_perm\_qind(perm\_qind). Ignored if None.

#### property dim

Dimension of the local Hilbert space.

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
get_hc_op_name (name)
```

Return the hermitian conjugate of a given operator.

**Parameters name** (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as  $qet\_op()$  does.

**Returns hc\_op\_name** – Operator name for the hermi such that  $get\_op()$  of

Return type str

```
get_op (name)
```

Return operator of given name.

**Parameters** name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

**Returns op** – The operator given by *name*, with labels 'p', 'p\*'. If name already was an npc Array, it's directly returned.

Return type np\_conserved

## multiply\_op\_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get\_op* returns the product of the corresponding operators.

**Parameters** names (list of str) – List of valid operator labels.

**Returns combined\_opname** – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

#### property onsite\_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

#### op needs JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

**Parameters** name (str) – The name of the operator, as in  $get\_op()$ .

**Returns needs\_JW** — Whether the operator needs a Jordan-Wigner string, judging from need\_JW\_string.

## Return type bool

#### remove\_op (name)

Remove an added operator.

**Parameters** name (str) – The name of the operator to be removed.

### rename\_op (old\_name, new\_name)

Rename an added operator.

#### **Parameters**

- **old\_name** (*str*) The old name of the operator.
- $new_name(str)$  The new name of the operator.

## save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with <code>save\_dict\_content()</code>, storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

## state\_index(label)

Return index of a basis state from its label.

**Parameters label** (int / string) – eather the index directly or a label (string) set before.

**Returns state index** – the index of the basis state associated with the label.

#### Return type int

## state\_indices(labels)

Same as state\_index(), but for multiple labels.

## test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### valid\_opname (name)

Check whether 'name' labels a valid onsite-operator.

**Parameters name** (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

**Returns valid** – True if *name* is a valid argument to *get\_op()*.

# Return type bool

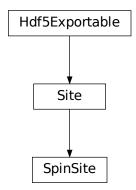
# **SpinSite**

• full name: tenpy.networks.site.SpinSite

• parent module: tenpy.networks.site

• type: class

# **Inheritance Diagram**



## **Methods**

SpinSiteinit([S, conserve])	Initialize self.
SpinSite.add_op(name, op[, need_JW, hc])	Add one on-site operators.
SpinSite.change_charge([new_leg_charge,	Change the charges of the site (in place).
permute])	
SpinSite.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
SpinSite.get_hc_op_name(name)	Return the hermitian conjugate of a given operator.
SpinSite.get_op(name)	Return operator of given name.
SpinSite.multiply_op_names(names)	Multiply operator names together.
SpinSite.op_needs_JW(name)	Whether an (composite) onsite operator is fermionic and
	needs a Jordan-Wigner string.
SpinSite.remove_op(name)	Remove an added operator.
SpinSite.rename_op(old_name, new_name)	Rename an added operator.
SpinSite.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
SpinSite.state_index(label)	Return index of a basis state from its label.
SpinSite.state_indices(labels)	Same as state_index(), but for multiple labels.
SpinSite.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.
SpinSite.valid_opname(name)	Check whether 'name' labels a valid onsite-operator.

## **Class Attributes and Properties**

SpinSite.dim	Dimension of the local Hilbert space.
SpinSite.onsite_ops	Dictionary of on-site operators for iteration.

```
class tenpy.networks.site.SpinSite(S=0.5, conserve='Sz')
```

Bases: tenpy.networks.site.Site

General Spin S site.

There are 2S+1 local states range from down (0) to up (2S+1), corresponding to Sz=-S, -S+1, ..., S-1, S. Local operators are the spin-S operators, e.g. Sz = [[0.5, 0.], [0., -0.5]],  $Sx = 0.5*sigma\_x$  for the Pauli matrix  $sigma\_x$ .

operator	description
Id, JW	Identity ⊮
Sx, Sy, Sz	Spin components $S^{x,y,z}$ , equal to half the Pauli matrices.
Sp, Sm	Spin flips $S^{\pm} = S^x \pm iS^y$

conserve	qmod	excluded onsite operators
'Sz'	[1]	Sx, Sy
'parity'	[2]	_
None	[]	_

**Parameters** conserve (str) – Defines what is conserved, see table above.

s

The 2S+1 states range from m = -S, -S+1, ... +S.

## conserve

Defines what is conserved, see table above.

 $\verb"add_op" (name, op, need\_JW=False, hc=None")$ 

Add one on-site operators.

## **Parameters**

- name (str) A valid python variable name, used to label the operator. The name under which *op* is added as attribute to self.
- op (np.ndarray | Array) A matrix acting on the local hilbert space representing the local operator. Dense numpy arrays are automatically converted to Array. LegCharges have to be [leg, leg.conj()]. We set labels 'p', 'p\*'.
- need\_JW (bool) Whether the operator needs a Jordan-Wigner string. If True, add name to need\_JW\_string.
- hc (None | False | str) The name for the hermitian conjugate operator, to be used for hc\_ops. By default (None), try to auto-determine it. If False, disable adding antries to hc\_ops.

change\_charge (new\_leg\_charge=None, permute=None)

Change the charges of the site (in place).

#### **Parameters**

- new\_leg\_charge (LegCharge | None) The new charges to be used. If None, use trivial charges.
- permute (ndarray | None) The permuation applied to the physical leg, which gets used to adjust state\_labels and perm. If you sorted the previous leg with perm\_qind, new\_leg\_charge = leg.sort(), use old\_leg. perm\_flat\_from\_perm\_qind(perm\_qind). Ignored if None.

## property dim

Dimension of the local Hilbert space.

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

**Return type** cls

## get\_hc\_op\_name (name)

Return the hermitian conjugate of a given operator.

**Parameters name** (str) – The name of the operator to be returned. Multiple operators separated by whitespace are interpreted as an operator product, exactly as  $get\_op()$  does.

**Returns hc\_op\_name** – Operator name for the hermi such that  $get\_op()$  of

Return type str

#### get\_op (name)

Return operator of given name.

**Parameters** name (str) – The name of the operator to be returned. In case of multiple operator names separated by whitespace, we multiply them together to a single on-site operator (with the one on the right acting first).

**Returns op** – The operator given by *name*, with labels 'p', 'p\*'. If name already was an npc Array, it's directly returned.

Return type np\_conserved

#### multiply\_op\_names (names)

Multiply operator names together.

Join the operator names in *names* such that *get\_op* returns the product of the corresponding operators.

**Parameters** names (list of str) – List of valid operator labels.

**Returns combined\_opname** – A valid operator name Operatorname representing the product of operators in *names*.

Return type str

#### property onsite\_ops

Dictionary of on-site operators for iteration.

Single operators are accessible as attributes.

#### op needs JW (name)

Whether an (composite) onsite operator is fermionic and needs a Jordan-Wigner string.

**Parameters** name (str) – The name of the operator, as in  $get\_op()$ .

**Returns needs\_JW** — Whether the operator needs a Jordan-Wigner string, judging from need\_JW\_string.

## Return type bool

#### remove\_op (name)

Remove an added operator.

**Parameters** name (str) – The name of the operator to be removed.

### rename\_op (old\_name, new\_name)

Rename an added operator.

#### **Parameters**

- **old\_name** (*str*) The old name of the operator.
- **new\_name** (str) The new name of the operator.

## save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

## state\_index(label)

Return index of a basis state from its label.

**Parameters** label (int | string) – eather the index directly or a label (string) set before.

**Returns state index** – the index of the basis state associated with the label.

#### Return type int

## state\_indices(labels)

Same as state\_index(), but for multiple labels.

## test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

#### valid\_opname (name)

Check whether 'name' labels a valid onsite-operator.

**Parameters name** (str) – Label for the operator. Can be multiple operator(labels) separated by whitespace, indicating that they should be multiplied together.

**Returns valid** – True if *name* is a valid argument to *get\_op()*.

## Return type bool

## **Functions**

<pre>group_sites(sites[, n, labels, charges])</pre>	Given a list of sites, group each <i>n</i> sites together.
multi_sites_combine_charges(sites[,])	Adjust the charges of the given sites (in place) such that
	they can be used together.

## group\_sites

- full name: tenpy.networks.site.group\_sites
- parent module: tenpy.networks.site
- type: function

tenpy.networks.site.group\_sites (sites, n=2, labels=None, charges='same') Given a list of sites, group each n sites together.

#### **Parameters**

- **sites** (list of *Site*) The sites to be grouped together.
- n (int) We group each n consecutive sites from sites together in a GroupedSite.
- charges (labels,) See GroupedSites.

**Returns grouped\_sites** – The grouped sites. Has length (len(sites)-1)//n + 1.

Return type list of GroupedSite

## multi\_sites\_combine\_charges

- full name: tenpy.networks.site.multi sites combine charges
- parent module: tenpy.networks.site
- type: function

tenpy.networks.site.multi\_sites\_combine\_charges(sites, same\_charges=[])

Adjust the charges of the given sites (in place) such that they can be used together.

When we want to contract tensors corresponding to different Site instances, these sites need to share a single Charge Info. This function adjusts the charges of these sites such that they can be used together.

## **Parameters**

- sites (list of Site) The sites to be combined. Modified in place.
- same\_charges ([[(int, int|str), (int, int|str), ...], ...]) Defines which charges actually are the same, i.e. their quantum numbers are added up. Each charge is specified by a tuple (s, i) = (int, int|str), where s gives the index of the site within sites and i the index or name of the charge in the ChargeInfo of this site.

**Returns** perms – For each site the permutation performed on the physical leg to sort by charges.

Return type list of ndarray

## **Examples**

```
>>> ferm = SpinHalfFermionSite(cons_N='N', cons_Sz='Sz')
>>> spin = SpinSite(1.0, 'Sz')
>>> ferm.leg.chinfo is spin.leg.chinfo
False
>>> print(spin.leg)
+1
0 [[-1]
1 [1]]
>>> multi_sites_combine_charges([ferm, spin], same_charges=[[(0, 'Sz'), (1, 0)]])
[array([0, 1, 2, 3]), array([0, 1])]
>>> # no permutations where needed
>>> ferm.leg.chinfo is spin.leg.chinfo
>> ferm.leg.chinfo.names
['N', 'Sz']
>>> print(spin.leg)
+1
0 [[ 0 -1]
1 [ 0 1]]
```

## **Module description**

Defines a class describing the local physical Hilbert space.

The Site is the prototype, read it's docstring.

## 7.10.2 mps

• full name: tenpy.networks.mps

• parent module: tenpy.networks

• type: module

## **Classes**



MPS(sites, Bs, SVs[, bc, form, norm])	A Matrix Product State, finite (MPS) or infinite (iMPS).
<pre>MPSEnvironment(bra, ket[, init_LP, init_RP,])</pre>	Stores partial contractions of $< bra Op ket >$ for local
	operators <i>Op</i> .
<pre>TransferMatrix(bra, ket[, shift_bra,])</pre>	Transfer matrix of two MPS (bra & ket).

## **MPSEnvironment**

• full name: tenpy.networks.mps.MPSEnvironment

• parent module: tenpy.networks.mps

• type: class

# **Inheritance Diagram**

MPSEnvironment

## **Methods**

MPSEnvironmentinit(bra, ket[, init_LP,	Initialize self.
])	
MPSEnvironment.del_LP(i)	Delete stored part strictly to the left of site <i>i</i> .
MPSEnvironment.del_RP(i)	Delete storde part scrictly to the right of site <i>i</i> .
MPSEnvironment.expectation_value(ops[,	Expectation value <pre></pre>
])	tor(s).
MPSEnvironment.full_contraction(i0)	Calculate the overlap by a full contraction of the net-
	work.
MPSEnvironment.get_LP(i[, store])	Calculate LP at given site from nearest available one (in-
	cluding <i>i</i> ).
MPSEnvironment.get_LP_age(i)	Return number of physical sites in the contractions of
	get_LP(i).
$ extit{MPSEnvironment.get\_RP(i[, store])}$	Calculate RP at given site from nearest available one
	(including <i>i</i> ).
MPSEnvironment.get_RP_age $(i)$	Return number of physical sites in the contractions of
	get_RP(i).
MPSEnvironment.get_initialization_data	
MPSEnvironment.init_LP(i)	Build initial left part LP.
MPSEnvironment.init_RP(i)	Build initial right part RP for an
	MPS/MPOEnvironment.
MPSEnvironment.set_LP(i, LP, age)	Store part to the left of site <i>i</i> .
MPSEnvironment.set_RP(i, RP, age)	Store part to the right of site <i>i</i> .
MPSEnvironment.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

Bases: object

Stores partial contractions of < bra|Op|ket > for local operators Op.

The network for a contraction  $\langle bra|Op|ket \rangle$  of a local operator Op, say exemplary at sites i, i+1 looks like:

Of course, we can also calculate the overlap  $\langle bra|ket \rangle$  by using the special case Op = Id.

We use the following label convention (where arrows indicate *qconj*):

To avoid recalculations of the whole network e.g. in the DMRG sweeps, we store the contractions up to some site index in this class. For bc='finite', 'segment', the very left and right part LP[0] and RP[-1] are trivial and don't change, but for bc='infinite' they are might be updated (by inserting another unit cell to the left/right).

The MPS bra and ket have to be in canonical form. All the environments are constructed without the singular values on the open bond. In other words, we contract left-canonical A to the left parts LP and right-canonical B to the right parts RP. Thus, the special case ket=bra should yield identity matrices for LP and RP.

## **Parameters**

- **bra** (MPS) The MPS to project on. Should be given in usual 'ket' form; we call *conj()* on the matrices directly. Stored in place, without making copies. If necessary to match charges, we call <code>gauge\_total\_charge()</code>.
- **ket** (MPO | None) The MPS on which the local operator acts. Stored in place, without making copies. If None, use *bra*.
- init\_LP (None | Array) Initial very left part LP. If None, build trivial one with init\_LP().
- init\_RP (None | Array) Initial very right part RP. If None, build trivial one with init\_RP().
- age\_LP (int) The number of physical sites involved into the contraction yielding firstLP.
- age\_RP (int) The number of physical sites involved into the contraction yielding lastRP.

L

Number of physical sites involved into the Environment, i.e. the least common multiple of bra.L and ket.L.

Type int

## bra, ket

The two MPS for the contraction.

```
Type MPS
dtype
     The data type.
         Type type
finite
     Whether the boundary conditions of the MPS are finite.
         Type bool
LΡ
     Left parts of the environment, len L. LP [i] contains the contraction strictly left of site i (or None, if we
     don't have it calculated).
         Type list of {None | Array}
_RP
     Right parts of the environment, len L. RP [i] contains the contraction strictly right of site i (or None, if
     we don't have it calculated).
         Type list of {None | Array}
_LP_age
     Used for book-keeping, how large the DMRG system grew: _LP_age[i] stores the number of physical
     sites invovled into the contraction network which yields self._LP[i].
         Type list of int | None
RP age
     Used for book-keeping, how large the DMRG system grew: _RP_age[i] stores the number of physical
     sites invovled into the contraction network which yields self._RP[i].
         Type list of int | None
test_sanity()
     Sanity check, raises ValueErrors, if something is wrong.
init_LP(i)
     Build initial left part LP.
         Parameters i (int) – Build LP left of site i.
         Returns init LP – Identity contractible with the vL leg of ket.get B(i), labels 'vR*',
              'vR'.
         Return type Array
init RP(i)
     Build initial right part RP for an MPS/MPOEnvironment.
         Parameters i(int) – Build RP right of site i.
         Returns init_RP – Identity contractible with the vR leg of ket.get_B(i), labels 'vL*',
              'vL'.
         Return type Array
get_LP (i, store=True)
     Calculate LP at given site from nearest available one (including i).
```

form:

The returned LP\_i corresponds to the following contraction, where the M's and the N's are in the 'A'

#### **Parameters**

- i(int) The returned *LP* will contain the contraction *strictly* left of site *i*.
- **store** (bool) Wheter to store the calculated *LP* in *self* (True) or discard them (False).

**Returns** LP\_i – Contraction of everything left of site i, with labels ' $\forall R*$ ', ' $\forall R$ ' for bra, ket.

Return type Array

## get\_RP (i, store=True)

Calculate RP at given site from nearest available one (including *i*).

The returned RP\_i corresponds to the following contraction, where the M's and the N's are in the 'B' form:

## **Parameters**

- i (int) The returned RP will contain the contraction strictly right of site i.
- **store** (bool) Wheter to store the calculated *RP* in *self* (True) or discard them (False).

**Returns RP\_i** – Contraction of everything left of site *i*, with labels 'vL\*', 'vL' for *bra*, *ket*.

Return type Array

## $get_LP_age(i)$

Return number of physical sites in the contractions of get\_LP(i).

Might be None.

## $get_RP_age(i)$

Return number of physical sites in the contractions of get\_RP(i).

Might be None.

## $set\_LP(i, LP, age)$

Store part to the left of site i.

## $set_RP(i, RP, age)$

Store part to the right of site i.

### $del_LP(i)$

Delete stored part strictly to the left of site i.

## $\mathtt{del}_{\mathtt{RP}}\left(i\right)$

Delete storde part scrictly to the right of site *i*.

#### get initialization data()

Return data for (re-)initialization.

The returned parameters are collected in a dictionary with the following names.

#### Returns

- init\_LP, init\_RP (Array) LP on the left of site 0 and RP on the right of site L-1, which can be used as *init* LP and *init* RP for the initialization of a new environment.
- age\_LP, age\_RP (*int*) The number of physical sites involved into the contraction yielding *init\_LP* and *init\_RP*, respectively.

#### full\_contraction (i0)

Calculate the overlap by a full contraction of the network.

The full contraction of the environments gives the overlap <br/>bra|ket>, taking into account MPS.norm of both bra and ket. For this purpose, this function contracts get\_LP(i0+1, store=False) and get\_RP(i0, store=False) with appropriate singular values in between.

**Parameters** iO (int) – Site index.

#### expectation\_value (ops, sites=None, axes=None)

Expectation value <br/> <br/>bra | ops | ket > of (n-site) operator(s).

Calculates n-site expectation values of operators sandwiched between bra and ket. For examples the contraction for a two-site operator on site *i* would look like:

Here, the B are taken from ket, the  $B^*$  from bra. The call structure is the same as for MPS. expectation\_value().

## **Parameters**

- **ops** ((list of) { Array | str }) The operators, for wich the expectation value should be taken, All operators should all have the same number of legs (namely 2 n). If less than len(sites) operators are given, we repeat them periodically. Strings (like 'Id', 'Sz') are translated into single-site operators defined by sites.
- **sites** (list) List of site indices. Expectation values are evaluated there. If None (default), the entire chain is taken (clipping for finite b.c.)
- axes (None | (list of str, list of str)) Two lists of each n leg labels giving the physical legs of the operator used for contraction. The first n legs are contracted with conjugated B, the second n legs with the non-conjugated B. None defaults to (['p'], ['p\*']) for single site (n=1), or (['p0', 'p1', ... 'p{n-1}'], ['p0\*', 'p1\*', ... 'p{n-1}\*']) for n > 1.

**Returns exp\_vals** - Expectation values, exp\_vals[i] = <br/>bra|ops[i]|ket>, where ops[i] acts on site(s) j, j+1, ..., j+{n-1} with j=sites[i].

**Return type** 1D ndarray

## **Examples**

One site examples (n=1):

```
>>> env.expectation_value('Sz')
[Sz0, Sz1, ..., Sz{L-1}]
>>> env.expectation_value(['Sz', 'Sx'])
[Sz0, Sx1, Sz2, Sx3, ...]
>>> env.expectation_value('Sz', sites=[0, 3, 4])
[Sz0, Sz3, Sz4]
```

Two site example (n=2), assuming homogeneous sites:

```
>>> SzSx = npc.outer(psi.sites[0].Sz.replace_labels(['p', 'p*'], ['p0', 'p0*

→']),

psi.sites[1].Sx.replace_labels(['p', 'p*'], ['p1', 'p1*

→']))

>>> env.expectation_value(SzSx)

[Sz0Sx1, Sz1Sx2, Sz2Sx3, ...] # with len L-1 for finite bc, or L for

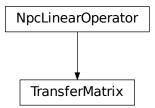
→infinite
```

Example measuring <br/> <br/> bralSzSxlket> on each second site, for inhomogeneous sites:

## **TransferMatrix**

- full name: tenpy.networks.mps.TransferMatrix
- parent module: tenpy.networks.mps
- type: class

## **Inheritance Diagram**



## Methods

TransferMatrixinit(bra, ket[,])	Initialize self.
TransferMatrix.adjoint()	Return the hermitian conjugate of self
TransferMatrix.eigenvectors([num_ev,	Find (dominant) eigenvector(s) of self using scipy.
])	sparse.
TransferMatrix.initial_guess([diag])	Return a diagonal matrix as initial guess for the eigen-
	vector.
TransferMatrix.matvec(vec)	Given <i>vec</i> as an npc.Array, apply the transfer matrix.
TransferMatrix.to_matrix()	Contract <i>self</i> to a matrix.

## **Class Attributes and Properties**

```
TransferMatrix.acts_on
```

Bases: tenpy.linalg.sparse.NpcLinearOperator

Transfer matrix of two MPS (bra & ket).

For an iMPS in the thermodynamic limit, we often need to find the 'dominant RP' (and LP). This mean nothing else than to take the transfer matrix of the unit cell and find the (right/left) eigenvector with the largest (magnitude) eigenvalue, since it will dominate  $(TM)^nRP$  (or  $LP(TM)^n$ ) in the limit  $n \to \infty$  - whatever the initial RP is. This class provides exactly that functionality with eigenvectors ().

Given two MPS, we define the transfer matrix as:

Here the M denotes the matrices of the bra and N the ones of the ket, respectively. To view it as a matrix, we combine the left and right indices to pipes:

Note that we keep all M and N as copies.

Deprecated since version 0.6.0: The default for *shift\_ket* was the value of *shift\_bra*, this will be changed to 0.

#### **Parameters**

- **bra** (MPS) The MPS which is to be (complex) conjugated.
- **ket** (MPS) The MPS which is not (complex) conjugated.
- **shift\_bra** (*int*) We start the *N* of the bra at site *shift\_bra* (i.e. the *j* in the above network).
- **shift\_ket** (*int* / *None*) We start the *M* of the ket at site *shift\_ket* (i.e. the *i* in the above network). None is deprecated, default will be changed to 0 in the future.
- transpose (bool) Wheter self.matvec acts on RP (False) or LP (True).

- **charge\_sector** (None | charges | 0) Selects the charge sector of the vector onto which the Linear operator acts. None stands for *all* sectors, 0 stands for the zero-charge sector. Defaults to 0, i.e., **assumes** the dominant eigenvector is in charge sector 0.
- **form** ('B' | 'A' | 'C' | 'G' | 'Th' | None | tuple(float, float)) In which canonical form we take the *M* and *N* matrices.

L

Number of physical sites involved in the transfer matrix, i.e. the least common multiple of bra.L and ket.L.

```
Type int
```

#### shift bra

We start the *N* of the bra at site *shift\_bra*.

Type int

## shift\_ket

We start the *M* of the ket at site *shift\_ket*. None defaults to *shift\_bra*.

**Type** int | None

#### transpose

Wheter self.matvec acts on RP (True) or LP (False).

Type bool

## qtotal

Total charge of the transfer matrix (which is gauged away in matvec).

Type charges

#### form

In which canonical form (all of) the M and N matrices are.

Type tuple(float, float) | None

## flat\_linop

Class lifting matvec() to ndarrays in order to use speigs().

Type FlatLinearOperator

#### pipe

Pipe corresponding to '(vL.vL\*)' for transpose=False or to '(vR.vR\*)' for transpose=True.

Type LegPipe

## label\_split

```
['vL', 'vL*'] if tranpose=False or ['vR', 'vR*'] if transpose=True.
```

#### \_bra\_N

Complex conjugated matrices of the bra, transposed for fast *matvec*.

Type list of npc.Array

### \_ket\_M

The matrices of the ket, transposed for fast *matvec*.

**Type** list of npc.Array

### matvec(vec)

Given *vec* as an npc.Array, apply the transfer matrix.

**Parameters vec** (Array) – Vector to act on with the transfermatrix. If not *transposed*, *vec* is the right part RP of an environment, with legs ' (vL.vL\*)' in a pipe or splitted. If *transposed*, the left part LP of an environment with legs ' (vR\*.vR)'.

**Returns mat\_vec** – The tranfer matrix acted on *vec*, in the same form as given.

**Return type** Array

## initial\_guess(diag=1.0)

Return a diagonal matrix as initial guess for the eigenvector.

**Parameters diag** (float | 1D ndarray) – Should be 1. for the identity or some singular values squared.

**Returns mat** – A 2D array with *diag* on the diagonal such that *matvec()* can act on it.

Return type Array

**eigenvectors** (num\_ev=1, max\_num\_ev=None, max\_tol=1e-12, which='LM', v0=None, \*\*kwargs) Find (dominant) eigenvector(s) of self using scipy.sparse.

If no charge\_sector was selected, we look in *all* charge sectors.

#### **Parameters**

- num\_ev (int) Number of eigenvalues/vectors to look for.
- max\_num\_ev (int) scipy.sparse.linalg.speigs() somtimes raises a No-ConvergenceError for small num\_ev, which might be avoided by increasing num\_ev. As a work-around, we try it again in the case of an error, just with larger num\_ev up to max\_num\_ev. None defaults to num\_ev + 2.
- max\_tol (float) After the first NoConvergenceError we increase the tol argument to that value.
- which (str) Which eigenvalues to look for, see *scipy.sparse.linalg.speigs*.
- \*\*kwargs Further keyword arguments given to speigs ().

#### **Returns**

- eta (1D ndarray) The eigenvalues, sorted according to which.
- w (list of Array) The eigenvectors corresponding to eta, as npc. Array with LegPipe.

## adjoint()

Return the hermitian conjugate of self

If self is hermitian, subclasses can choose to implement this to define the adjoint operator of self.

#### to matrix()

Contract *self* to a matrix.

If *self* represents an operator with very small shape, e.g. because the MPS bond dimension is very small, an algorithm might choose to contract *self* to a single tensor.

**Returns matrix** – Contraction of the represented operator.

Return type Array

### **Functions**

```
build_initial_state(size, states, filling[, ...]) Build an "initial state" list.
```

## build initial state

- full name: tenpy.networks.mps.build\_initial\_state
- parent module: tenpy.networks.mps
- type: function

tenpy.networks.mps.build\_initial\_state (size, states, filling, mode='random', seed=None)
Build an "initial state" list.

Uses two iterables ('states' and 'filling') to determine how to fill the state. The two lists should have the same length as every element in 'filling' gives the filling fraction for the corresponding state in 'states'.

## **Example**

size = 6, states = [0, 1, 2], filling = [1./3, 2./3, 0.] n\_states = size \* filling = [2, 4, 0] ==> Two sites will get state 0, 4 sites will get state 1, 0 sites will get state 2.

**Todo:** Make more general: it should be possible to specify states as strings.

## **Parameters**

- size (int) length of state
- states (iterable) Containing the possible local states
- **filling** (*iterable*) Fraction of the total number of sites to get a certain state. If infinite fractions (e.g. 1/3) are needed, one should supply a fraction (1./3.)
- mode (str / None) State filling pattern. Only 'random' is implemented
- seed (int / None) Seed for random number generators

## Returns initial\_state (list)

Return type the initial state

#### **Raises**

- ValueError If fractonal fillings are incommensurate with system size.
- **AssertionError** If the total filling is not equal to 1, or the length of *filling* does not equal the length of *states*.

## **Module description**

This module contains a base class for a Matrix Product State (MPS).

An MPS looks roughly like this:

```
| -- B[0] -- B[1] -- B[2] -- ...
```

We use the following label convention for the *B* (where arrows indicate *qconj*):

We store one 3-leg tensor  $\_B[i]$  with labels 'vL', 'vR', 'p' for each of the L sites  $0 \le i \le L$ . Additionally, we store L+1 singular value arrays  $\_S[ib]$  on each bond  $0 \le ib \le L$ , independent of the boundary conditions.  $\_S[ib]$  gives the singlur values on the bond i-1, i. However, be aware that e.g. chi returns only the dimensions of the nontrivial\_bonds depending on the boundary conditions.

The matrices and singular values always represent a normalized state (i.e. np.linalg.norm(psi.\_S[ib]) == 1 up to roundoff errors), but (for finite MPS) we keep track of the norm in norm (which is respected by overlap(), ...).

Valid MPS boundary conditions (not to confuse with  $bc\_coupling$  of tenpy.models.model.CouplingModel) are the following:

bc	description	
'fi-	Finite MPS, G0 s1 G1 s{L-1} G{1-1}. This is acchieved by using a trivial left and right bond	
nite'	s[0] = s[-1] = np.array([1.]).	
'seg-	Generalization of 'finite', describes an MPS embedded in left and right environments. The left environment	
ment	is described by chi[0] orthonormal states which are weighted by the singular values s[0]. Similar,	
	s[L] weight some right orthonormal states. You can think of the left and right states to be generated by	
	additional MPS, such that the overall structure is something like s L s L [s0 G0 s1 G1	
	$s\{L-1\}$ $G\{L-1\}$ $s\{L\}$ ] R s R s R (where we save the part in the brackets [ ] ).	
'in-	infinite MPS (iMPS): we save a 'MPS unit cell' [s0 G0 s1 G1 s{L-1} G{L-1}] which is	
fi-	repeated periodically, identifying all indices modulo self.L. In particular, the last bond L is identified	
nite'	with 0. (The MPS unit cell can differ from a lattice unit cell). bond is identified with the first one.	

An MPS can be in different 'canonical forms' (see [Vidal2004], [Schollwoeck2011]). To take care of the different canonical forms, algorithms should use functions like  $get\_theta()$ ,  $get\_B()$  and  $set\_B()$  instead of accessing them directly, as they return the B in the desired form (which can be chosen as an argument). The values of the tuples for the form correspond to the exponent of the singular values on the left and right. To keep track of a "mixed" canonical form A A A B B, we save the tuples for each site of the MPS in MPS. form.

form	tuple	description	
'B'	(0, 1)	right canonical: _B[i] = Gamma[i] s[i+1] The default form, which algorithms	
		asssume.	
'C'	(0.5,	symmetric form: _B[i] = s[i] **0.5 Gamma[i] s[i+1] **0.5	
	0.5)		
'A'	(1, 0)	left canonical: _B[i] = s[i] Gamma[i]	
'G'	(0, 0)	Save only _B[i] = Gamma[i]	
'Th'	(1, 1)	Form of a local wave function <i>theta</i> with singular value on both sides. psi.get_B(i, 'Th')	
		is equivalent to ``psi.get_theta(i, n=1).	
None	None	General non-canoncial form. Valid form for initialization, but you need to call	
		canonical_form() (or similar) before using algorithms.	

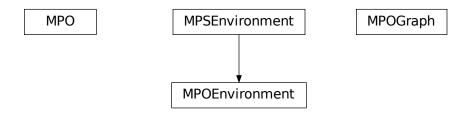
# 7.10.3 mpo

• full name: tenpy.networks.mpo

• parent module: tenpy.networks

• type: module

## **Classes**



MPO(sites, Ws[, bc, IdL, IdR, max_range,])	Matrix product operator, finite (MPO) or infinite (iMPO).
MPOEnvironment(bra, H, ket[, init_LP,])	Stores partial contractions of $< bra H ket >$ for an MPO $H$ .
<pre>MPOGraph(sites[, bc, max_range])</pre>	Representation of an MPO by a graph, based on a 'finite state machine'.

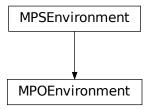
## **MPOEnvironment**

• full name: tenpy.networks.mpo.MPOEnvironment

• parent module: tenpy.networks.mpo

• type: class

## **Inheritance Diagram**



## **Methods**

MPOEnvironmentinit(bra, H, ket[,])	Initialize self.
MPOEnvironment.del_LP(i)	Delete stored part strictly to the left of site <i>i</i> .
MPOEnvironment.del_RP(i)	Delete storde part scrictly to the right of site <i>i</i> .
MPOEnvironment.expectation_value(ops[,	Expectation value <pre>bra ops ket&gt; of (n-site) opera-</pre>
])	tor(s).
MPOEnvironment.full_contraction(i0)	Calculate the energy by a full contraction of the net-
	work.
MPOEnvironment.get_LP(i[, store])	Calculate LP at given site from nearest available one (in-
	cluding <i>i</i> ).
MPOEnvironment.get_LP_age(i)	Return number of physical sites in the contractions of
	get_LP(i).
MPOEnvironment.get_RP(i[, store])	Calculate RP at given site from nearest available one
	(including <i>i</i> ).
MPOEnvironment.get_RP_age(i)	Return number of physical sites in the contractions of
	get_RP(i).
MPOEnvironment.get_initialization_data	()Return data for (re-)initialization.
MPOEnvironment.init_LP(i)	Build initial left part LP.
MPOEnvironment.init_RP(i)	Build initial right part RP for an
	MPS/MPOEnvironment.
MPOEnvironment.set_LP(i, LP, age)	Store part to the left of site <i>i</i> .
MPOEnvironment.set_RP(i, RP, age)	Store part to the right of site <i>i</i> .
MPOEnvironment.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

 $\textbf{class} \texttt{ tenpy.networks.mpo.MPOEnvironment} (\textit{bra}, \textit{H}, \textit{ket}, \textit{init\_LP=None}, \textit{init\_RP=None}, \\ \textit{age\_LP=0}, \textit{age\_RP=0})$ 

Bases: tenpy.networks.mps.MPSEnvironment

Stores partial contractions of < bra|H|ket > for an MPO H.

The network for a contraction < bra|H|ket > of an MPO H bewteen two MPS looks like:

We use the following label convention (where arrows indicate *qconj*):

To avoid recalculations of the whole network e.g. in the DMRG sweeps, we store the contractions up to some site index in this class. For bc='finite', 'segment', the very left and right part LP[0] and RP[-1] are trivial and don't change in the DMRG algorithm, but for iDMRG (bc='infinite') they are also updated (by inserting another unit cell to the left/right).

The MPS *bra* and *ket* have to be in canonical form. All the environments are constructed without the singular values on the open bond. In other words, we contract left-canonical *A* to the left parts *LP* and right-canonical *B* to the right parts *RP*.

## **Parameters**

- **bra** (MPS) The MPS to project on. Should be given in usual 'ket' form; we call *conj()* on the matrices directly.
- **H** (MPO) The MPO sandwiched between *bra* and *ket*. Should have 'IdL' and 'IdR' set on the first and last bond.
- **ket** (MPS) The MPS on which H acts. May be identical with bra.
- init\_LP (None | Array) Initial very left part LP. If None, build trivial one with :meth`init LP`.
- init\_RP (None | Array) Initial very right part RP. If None, build trivial one with init\_RP().
- age\_LP (int) The number of physical sites involved into the contraction yielding firstLP.
- age\_RP (int) The number of physical sites involved into the contraction yielding lastRP.

Н

The MPO sandwiched between bra and ket.

Type MPO

## test\_sanity()

Sanity check, raises ValueErrors, if something is wrong.

### $init_LP(i)$

Build initial left part LP.

**Parameters** i(int) – Build LP left of site i.

**Returns init\_LP** – Identity contractible with the  $\nu L$  leg of .ket.get\_B(i), multiplied with a unit vector nonzero in H.IdL[i], with labels 'vR\*', 'wR', 'vR'.

Return type Array

#### init RP(i)

Build initial right part RP for an MPS/MPOEnvironment.

**Parameters** i (int) – Build RP right of site i.

**Returns init\_RP** – Identity contractible with the  $\nu R$  leg of self.get\_B(i), multiplied with a unit vector nonzero in H.IdR[i], with labels 'vL\*', 'wL', 'vL'.

**Return type** *Array* 

#### get\_LP (i, store=True)

Calculate LP at given site from nearest available one (including *i*).

The returned LP\_i corresponds to the following contraction, where the M's and the N's are in the 'A' form:

#### **Parameters**

- i (int) The returned LP will contain the contraction strictly left of site i.
- **store** (bool) Wheter to store the calculated *LP* in *self* (True) or discard them (False).

**Returns** LP\_i – Contraction of everything left of site *i*, with labels 'vR\*', 'wR', 'vR' for *bra*, H, *ket*.

Return type Array

## get\_RP (i, store=True)

Calculate RP at given site from nearest available one (including *i*).

The returned RP\_i corresponds to the following contraction, where the M's and the N's are in the 'B' form:

#### **Parameters**

- i (int) The returned RP will contain the contraction strictly rigth of site i.
- **store** (bool) Wheter to store the calculated RP in self (True) or discard them (False).

**Returns RP\_i** – Contraction of everything right of site i, with labels 'vL\*', 'wL', 'vL' for bra, H, ket.

## Return type Array

### $full\_contraction(i0)$

Calculate the energy by a full contraction of the network.

The full contraction of the environments gives the value  $\langle bra|H|ket \rangle / (norm(|bra>)*norm(|ket>))$ , i.e. if *bra* is *ket* and normalized, the total energy. For this purpose, this function contracts get\_LP(i0+1, store=False) and get\_RP(i0, store=False).

**Parameters** iO (int) – Site index.

#### $del_LP(i)$

Delete stored part strictly to the left of site *i*.

#### $del_RP(i)$

Delete storde part scrictly to the right of site i.

## expectation\_value (ops, sites=None, axes=None)

Expectation value <br/> <br/>bra | ops | ket > of (n-site) operator(s).

Calculates n-site expectation values of operators sandwiched between bra and ket. For examples the contraction for a two-site operator on site *i* would look like:

Here, the B are taken from ket, the  $B^*$  from bra. The call structure is the same as for MPS. expectation\_value().

#### **Parameters**

- **ops** ((list of) { *Array* | str }) The operators, for wich the expectation value should be taken, All operators should all have the same number of legs (namely 2 n). If less than len(sites) operators are given, we repeat them periodically. Strings (like 'Id', 'Sz') are translated into single-site operators defined by sites.
- **sites** (*list*) List of site indices. Expectation values are evaluated there. If None (default), the entire chain is taken (clipping for finite b.c.)
- **axes** (None | (list of str, list of str)) Two lists of each n leg labels giving the physical legs of the operator used for contraction. The first n legs are contracted with conjugated B, the second n legs with the non-conjugated B. None defaults to (['p'], ['p\*']) for single site (n=1), or (['p0', 'p1', ... 'p{n-1}'], ['p0\*', 'p1\*', ... 'p{n-1}\*']) for n > 1.

**Returns exp\_vals** – Expectation values,  $\exp_vals[i] = \langle bra|ops[i]|ket \rangle$ , where ops[i] acts on site(s) j, j+1, ..., j+{n-1} with j=sites[i].

Return type 1D ndarray

#### **Examples**

One site examples (n=1):

```
>>> env.expectation_value('Sz')
[Sz0, Sz1, ..., Sz{L-1}]
>>> env.expectation_value(['Sz', 'Sx'])
[Sz0, Sx1, Sz2, Sx3, ...]
>>> env.expectation_value('Sz', sites=[0, 3, 4])
[Sz0, Sz3, Sz4]
```

Two site example (n=2), assuming homogeneous sites:

Example measuring <br/>
stalSzSxlket> on each second site, for inhomogeneous sites:

## $get_LP_age(i)$

Return number of physical sites in the contractions of get\_LP(i).

Might be None.

## $get_RP_age(i)$

Return number of physical sites in the contractions of get\_RP(i).

Might be None.

## get\_initialization\_data()

Return data for (re-)initialization.

The returned parameters are collected in a dictionary with the following names.

#### **Returns**

- init\_LP, init\_RP (Array) LP on the left of site 0 and RP on the right of site L-1, which can be used as  $init_LP$  and  $init_LP$  for the initialization of a new environment.
- age\_LP, age\_RP (*int*) The number of physical sites involved into the contraction yielding *init\_LP* and *init\_RP*, respectively.

### set LP (i, LP, age)

Store part to the left of site i.

#### set RP (i, RP, age)

Store part to the right of site i.

## **MPOGraph**

• full name: tenpy.networks.mpo.MPOGraph

• parent module: tenpy.networks.mpo

• type: class

## **Inheritance Diagram**

MPOGraph

#### **Methods**

<pre>MPOGraphinit(sites[, bc, max_range])</pre>	Initialize self.
MPOGraph.add(i, keyL, keyR, opname, strength)	Insert an edge into the graph.
MPOGraph.add_missing_IdL_IdR([insert_all_id]	Add missing identity ('Id') edges connecting
	'IdL' -> 'IdL' and ``' $IdR' -> 'IdR'$ .
MPOGraph.add_string(i, j, key[, opname,])	Insert a bunch of edges for an 'operator string' into the
	graph.
MPOGraph.build_MPO([Ws_qtotal])	Build the MPO represented by the graph ( <i>self</i> ).
<pre>MPOGraph.from_term_list(term_list, sites, bc)</pre>	Initialize form a list of operator terms and prefactors.
<pre>MPOGraph.from_terms(onsite_terms,[,])</pre>	Initialize an MPOGraph from OnsiteTerms and Cou-
	plingTerms.
MPOGraph.has_edge(i, keyL, keyR)	True if there is an edge from keyL on bond (i-1, i) to
	keyR on bond (i, i+1).
MPOGraph.test_sanity()	Sanity check, raises ValueErrors, if something is wrong.

## **Class Attributes and Properties**

MPOGraph.L	Number of physical sites; for infinite boundaries the
	length of the unit cell.

 $\textbf{class} \texttt{ tenpy.networks.mpo.MPOGraph} (\textit{sites}, \textit{bc='finite'}, \textit{max\_range=None})$ 

Bases: object

Representation of an MPO by a graph, based on a 'finite state machine'.

This representation is used for building H\_MPO from the interactions. The idea is to view the MPO as a kind of 'finite state machine'. The **states** or **keys** of this finite state machine life on the MPO bonds *between* the Ws. They label the indices of the virtul bonds of the MPOs, i.e., the indices on legs wL and wR. They can be anything hash-able like a str, int or a tuple of them.

The edges of the graph are the entries W[keyL, keyR], which itself are onsite operators on the local Hilbert

space. The indices keyL and keyR correspond to the legs 'wL', 'wR' of the MPO. The entry W[keyL, keyR] connects the state keyL on bond (i-1, i) with the state keyR on bond (i, i+1).

The keys 'IdR' (for 'idenity left') and 'IdR' (for 'identity right') are reserved to represent only 'Id' (=identity) operators to the left and right of the bond, respectively.

**Todo:** might be useful to add a "cleanup" function which removes operators cancelling each other and/or unused states. Or better use a 'compress' of the MPO?

#### **Parameters**

- **sites** (list of Site) Local sites of the Hilbert space.
- **bc** ({ 'finite', 'infinite'}) MPO boundary conditions.
- max\_range (int / np.inf / None) Maximum range of hopping/interactions (in unit of sites) of the MPO. None for unknown.

#### sites

Defines the local Hilbert space for each site.

Type list of Site

#### chinfo

The nature of the charge.

Type ChargeInfo

bc

MPO boundary conditions.

**Type** {'finite', 'infinite'}

## max\_range

Maximum range of hopping/interactions (in unit of sites) of the MPO. None for unknown.

**Type** int | np.inf | None

#### states

states [i] gives the possible keys at the virtual bond (i-1, i) of the MPO. L+1 enries.

**Type** list of set of keys

## graph

For each site i a dictionary {keyL: {keyR: [(opname, strength)]}} with keyL in states[i] and keyR in states[i+1].

**Type** list of dict of dict of list of tuples

## \_grid\_legs

The charges for the MPO

Type None | list of LegCharge

**classmethod from\_terms** (*onsite\_terms*, *coupling\_terms*, *sites*, *bc*, *insert\_all\_id=True*) Initialize an MPOGraph from OnsiteTerms and CouplingTerms.

### **Parameters**

- onsite\_terms (OnsiteTerms) Onsite terms to be added to the new MPOGraph.
- coupling\_terms (CouplingTerms | MultiCouplingTerms) Coupling terms to be added to the new MPOGraph.

- **sites** (list of Site) Local sites of the Hilbert space.
- **bc** ('finite' | 'infinite') MPO boundary conditions.
- insert\_all\_id(bool) Whether to insert identities such that *IdL* and *IdR* are defined on each bond. See add\_missing\_IdL\_IdR().

**Returns** graph – Initialized with the given terms.

```
Return type MPOGraph
```

#### See also:

from\_term\_list() equivalent for other representation terms.

classmethod from\_term\_list (term\_list, sites, bc, insert\_all\_id=True)

Initialize form a list of operator terms and prefactors.

#### **Parameters**

- term\_list (TermList) Terms to be added to the MPOGraph.
- **sites** (list of *Site*) Local sites of the Hilbert space.
- **bc**('finite' | 'infinite') MPO boundary conditions.
- insert\_all\_id(bool) Whether to insert identities such that *IdL* and *IdR* are defined on each bond. See add\_missing\_IdL\_IdR().

**Returns** graph – Initialized with the given terms.

```
Return type MPOGraph
```

#### See also:

**from\_terms()** equivalent for other representation of terms.

#### test sanity()

Sanity check, raises ValueErrors, if something is wrong.

### property L

Number of physical sites; for infinite boundaries the length of the unit cell.

add (i, keyL, keyR, opname, strength, check\_op=True, skip\_existing=False)
Insert an edge into the graph.

#### **Parameters**

- i (int) Site index at which the edge of the graph is to be inserted.
- **keyL** (hashable) The state at bond (i-1, i) to connect from.
- **keyR** (hashable) The state at bond (i, i+1) to connect to.
- **opname** (*str*) Name of the operator.
- **strength** (*str*) Prefactor of the operator to be inserted.
- **check\_op** (bool) Whether to check that 'opname' exists on the given *site*.
- **skip\_existing** (bool) If True, skip adding the graph node if it exists (with same keys and *opname*).

 $\verb"add_string" (i, j, key, opname='Id', check\_op=True, skip\_existing=True)$ 

Insert a bunch of edges for an 'operator string' into the graph.

Terms like  $S_i^z S_j^z$  actually stand for  $S_i^z \otimes \prod_{i < k < j} \mathbb{1}_k \otimes S_j^z$ . This function adds the  $\mathbb{1}_k$  terms to the graph.

#### **Parameters**

- $\mathbf{j}(i, j)$  An edge is inserted on all bonds between i and j, i < j. j can be larger than L, in which case the operators are supposed to act on different MPS unit cells.
- **key** (hashable) The state at bond (i-1, i) to connect from and on bond (j-1, j) to connect to. Also used for the intermediate states. No operator is inserted on a site i < k < j if has\_edge (k, key, key).
- **opname** (str) Name of the operator to be used for the string. Useful for the Jordan-Wigner transformation to fermions.
- **skip\_existing** (bool) Whether existing graph nodes should be skipped.

**Returns label\_j** – The *key* on the left of site j to connect to. Usually the same as the parameter *key*, except if j - i > self.L, in which case we use the additional labels (key, 1), (key, 2),... to generate couplings over multiple unit cells.

**Return type** hashable

## add\_missing\_IdL\_IdR(insert\_all\_id=True)

Add missing identity ('ld') edges connecting 'IdL'->'IdL' and ``'IdR'->'IdR'.

This function should be called *after* all other operators have been inserted.

Parameters insert\_all\_id (bool) - If True, insert 'Id' edges on *all* bonds. If False and boundary conditions are finite, only insert 'IdL'->'IdL' to the left of the rightmost existing 'IdL' and 'IdR'->'IdR' to the right of the leftmost existing 'IdR'. The latter avoid "dead ends" in the MPO, but some functions (like *make\_WI*) expect 'IdL'/'IdR' to exist on all bonds.

## $has\_edge(i, keyL, keyR)$

True if there is an edge from keyL on bond (i-1, i) to keyR on bond (i, i+1).

## build\_MPO(Ws\_qtotal=None)

Build the MPO represented by the graph (*self*).

**Parameters Ws\_qtotal** (None | (list of) charges) - The qtotal for each of the Ws to be generated, default (None) means 0 charge. A single qtotal holds for each site.

**Returns** mpo – the MPO which self represents.

Return type MPO

#### **Functions**

grid_insert_ops(site, grid)	Replaces entries representing operators in a grid of
	W[i] with npc.Arrays.
$make_{V_II(t, A, B, C, D)}$	W_II approx to exp(t H) from MPO parts (A, B, C, D).

## grid\_insert\_ops

```
• full name: tenpy.networks.mpo.grid_insert_ops
```

• parent module: tenpy.networks.mpo

• type: function

```
tenpy.networks.mpo.grid_insert_ops(site, grid)
```

Replaces entries representing operators in a grid of W[i] with npc.Arrays.

#### **Parameters**

- **site** (*site*) The site on which the grid acts.
- grid (list of list of entries) Represents a single matrix W of an MPO, i.e. the lists correspond to the legs 'vL', 'vR', and entries to onsite operators acting on the given site. entries may be None, Array, a single string or of the form [('opname', strength), ...], where 'opname' labels an operator in the site.

Returns grid - Copy of grid with entries [('opname', strength), ...] replaced by sum([strength\*site.get\_op('opname') for opname, strength in entry]) and entries 'opname' replaced by site.get\_op('opname').

**Return type** list of list of {None | Array}

## make\_W\_II

- full name: tenpy.networks.mpo.make\_W\_II
- parent module: tenpy.networks.mpo
- · type: function

```
tenpy.networks.mpo.make_W_II (t, A, B, C, D) W_II approx to exp(t H) from MPO parts (A, B, C, D).
```

Get the W\_II approximation of [Zaletel2015].

In the paper, we have two formal parameter "phi\_{r/c}" which satisfies  $\phi_r^2 = phi_c^2 = 0$ . To implement this, we temporarily extend the virtual Hilbert space with two hard-core bosons "br, bl". The components of Eqn (11) can be computed for each index of the virtual row/column independently The matrix exponential is done in the hard-core extended Hilbert space

#### **Parameters**

- **t** (*float*) The time step per application of the propagator. Should be imaginary for real time evolution!
- B, C, D(A,) Blocks of the MPO tensor to be exponentiated, as defined in [Zaletel2015]. Legs 'wL', 'wR', 'p', 'p\*'; legs projected to a single IdL/IdR can be dropped.

## **Module description**

Matrix product operator (MPO).

An MPO is the generalization of an MPS to operators. Graphically:

So each 'matrix' has two physical legs p, p\* instead of just one, i.e. the entries of the 'matrices' are local operators. Valid boundary conditions of an MPO are the same as for an MPS (i.e. 'finite' | 'segment' | 'infinite'). (In general, you can view the MPO as an MPS with larger physical space and bring it into canoncial form. However, unlike for an MPS, this doesn't simplify calculations. Thus, an MPO has no *form*.)

We use the following label convention for the W (where arrows indicate qconj):

If an MPO describes a sum of local terms (e.g. most Hamiltonians), some bond indices correspond to 'only identities to the left/right'. We store these indices in *IdL* and *IdR* (if there are such indices).

Similar as for the MPS, a bond index i is *left* of site i, i.e. between sites i-1 and i.

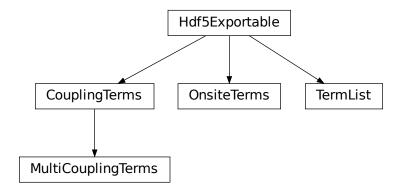
## 7.10.4 terms

• full name: tenpy.networks.terms

• parent module: tenpy.networks

· type: module

## Classes



CouplingTerms(L)	Operator names, site indices and strengths representing two-site coupling terms.
MultiCouplingTerms(L)	Operator names, site indices and strengths representing
	general <i>M</i> -site coupling terms.
OnsiteTerms(L)	Operator names, site indices and strengths representing
	onsite terms.
TermList(terms, strength)	A list of terms (=operator names and sites they act on)
	and associated strengths.

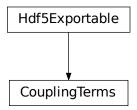
# CouplingTerms

• full name: tenpy.networks.terms.CouplingTerms

• parent module: tenpy.networks.terms

• type: class

# **Inheritance Diagram**



## Methods

CouplingTermsinit(L)	Initialize self.
CouplingTerms.add_coupling_term(strength,	Add a two-site coupling term on given MPS sites.
i,)	
CouplingTerms.add_to_graph(graph)	Add terms from coupling_terms to an MPOGraph.
CouplingTerms.coupling_term_handle_JW(.	.Helping function to call before
])	<pre>add_multi_coupling_term().</pre>
CouplingTerms.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
CouplingTerms.max_range()	Determine the maximal range in coupling_terms.
CouplingTerms.plot_coupling_terms(ax,	"Plot coupling terms into a given lattice.
lat[,])	
CouplingTerms.remove_zeros([tol_zero])	Remove entries close to 0 from coupling_terms.
CouplingTerms.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	
CouplingTerms.to_TermList()	Convert onsite_terms into a TermList.
CouplingTerms.to_nn_bond_Arrays(sites)	Convert the coupling_terms into Arrays on nearest
	neighbor bonds.

```
\textbf{class} \texttt{ tenpy.networks.terms.CouplingTerms} \ (L)
```

Bases: tenpy.tools.hdf5\_io.Hdf5Exportable

Operator names, site indices and strengths representing two-site coupling terms.

**Parameters** L (int) – Number of sites.

L

Number of sites.

Type int

#### coupling\_terms

Filled by  $add\_coupling\_term()$ . Nested dictionaries of the form {i: {('opname\_i', 'opname\_string'): {j: {'opname\_j': strength}}}. Note that always i < j, but entries with j >= L are allowed for bc\_MPS == 'infinite', in which case they indicate couplings between different iMPS unit cells.

**Type** dict of dict

## ${\tt max\_range}\,(\,)$

Determine the maximal range in <code>coupling\_terms</code>.

**Returns max\_range** – The maximum of j - i for the i, j occurring in a term of coupling\_terms.

Return type int

 $\verb"add_coupling_term" (strength, i, j, op_i, op_j, op_string='Id')$ 

Add a two-site coupling term on given MPS sites.

## **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}(i,)$  The MPS indices of the two sites on which the operator acts. We require  $0 <= i < N_sites$  and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_sites$ , it indicates couplings between unit cells of an infinite MPS.

- op2 (op1,) Names of the involved operators.
- op string (str) The operator to be inserted between i and j.

## coupling\_term\_handle\_JW (strength, term, sites, op\_string=None)

Helping function to call before add\_multi\_coupling\_term().

#### **Parameters**

- **strength** (float) The strength of the coupling term.
- **term**([(str, int), (str, int)]) List of two tuples (op, i) where *i* is the MPS index of the site the operator named *op* acts on.
- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings.
- op\_string (None / str) Operator name to be used as operator string between the operators, or None if the Jordan Wigner string should be figured out.

**Returns** Arguments for MultiCouplingTerms.add\_multi\_coupling\_term() such that the added term corresponds to the parameters of this function.

**Return type** strength, i, j, op\_i, op\_j, op\_string

"Plot coupling terms into a given lattice.

This function plots the coupling\_terms

#### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- lat (Lattice) The lattice for plotting the couplings, most probably the M.lat of the corresponding model M, see lat.
- **style\_map** (function | None) Function which get's called with arguments i, j, op\_i, op\_string, op\_j, strength for each two-site coupling and should return a keyword-dictionary with the desired plot-style for this coupling. By default (None), the *linewidth* is given by the absolute value of *strength*, and the linecolor depends on the phase of *strength* (using the *hsv* colormap).
- **common\_style** (*dict*) Common style, which overwrites values of the dictionary returned by style\_map. A 'label' is only used for the first plotted line.
- text (format\_string | None) If not None, we add text labeling the couplings in the plot. Available keywords are i, j, op\_i, op\_string, op\_j, strength as well as strength\_abs, strength\_angle, strength\_real.
- **text\_pos** (float) Specify where to put the text on the line between i (0.0) and j (1.0), e.g. 0.5 is exactly in the middle between i and j.

## See also:

tenpy.models.lattice.Lattice.plot\_sites() plot the sites of the lattice.

#### add\_to\_graph (graph)

Add terms from coupling\_terms to an MPOGraph.

**Parameters** graph (MPOGraph) – The graph into which the terms from coupling\_terms should be added.

## to\_nn\_bond\_Arrays (sites)

Convert the coupling\_terms into Arrays on nearest neighbor bonds.

**Parameters sites** (list of *Site*) – Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.

Returns H\_bond - The coupling\_terms rewritten as sum\_i H\_bond[i] for MPS indices i. H\_bond[i] acts on sites (i-1, i), None represents 0. Legs of each H\_bond[i] are ['p0', 'p0\*', 'p1', 'p1\*'].

**Return type** list of {*Array* | None}

## remove\_zeros (tol\_zero=1e-15)

Remove entries close to 0 from coupling\_terms.

**Parameters tol\_zero** (float) - Entries in coupling\_terms with strength < tol\_zero are considered to be zero and removed.

#### to\_TermList()

Convert onsite\_terms into a TermList.

**Returns** term\_list – Representation of the terms as a list of terms.

Return type TermList

## classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save\_hdf5().

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

#### save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of  $\_dict\_$  with  $save\_dict\_$  content (), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

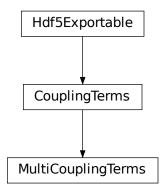
# MultiCouplingTerms

 $\bullet \ \ full \ name: tenpy.networks.terms. MultiCoupling Terms$ 

• parent module: tenpy.networks.terms

• type: class

## **Inheritance Diagram**



## **Methods**

MultiCouplingTerms. $\_$ init $\_$ ( $ m L)$	Initialize self.
MultiCouplingTerms.	Add a two-site coupling term on given MPS sites.
$ extit{add\_coupling\_term}([,])$	
MultiCouplingTerms.	Add a multi-site coupling term.
add_multi_coupling_term()	
MultiCouplingTerms.add_to_graph(graph[,	Add terms from coupling_terms to an MPOGraph.
_i, ])	
MultiCouplingTerms.	Helping function to call before
coupling_term_handle_JW $(\dots)$	<pre>add_multi_coupling_term().</pre>
MultiCouplingTerms.from_hdf5(hdf5_loader,	Load instance from a HDF5 file.
)	
MultiCouplingTerms.max_range()	Determine the maximal range in coupling_terms.
MultiCouplingTerms.	Helping function to call before
multi_coupling_term_handle_J $\mathbb{W}(\ldots)$	<pre>add_multi_coupling_term().</pre>
MultiCouplingTerms.	"Plot coupling terms into a given lattice.
<pre>plot_coupling_terms(ax, lat)</pre>	
MultiCouplingTerms.	Remove entries close to 0 from coupling_terms.
remove_zeros([tol_zero,_d0])	
MultiCouplingTerms.save_hdf5(hdf5_saver,	Export self into a HDF5 file.
)	

continues on next page

Table 151 – continued from previous page

MultiCouplingTerms.to_TermList()	Convert onsite_terms into a TermList.
MultiCouplingTerms.	Convert the coupling_terms into Arrays on nearest
to_nn_bond_Arrays(sites)	neighbor bonds.

## class tenpy.networks.terms.MultiCouplingTerms (L)

Bases: tenpy.networks.terms.CouplingTerms

Operator names, site indices and strengths representing general M-site coupling terms.

Generalizes the <code>coupling\_terms</code> of <code>CouplingTerms</code> to <code>M</code>-site couplings. The structure of the nested dictionary <code>coupling\_terms</code> is similar, but we allow an arbitrary recursion depth of the dictionary.

**Parameters** L (int) – Number of sites.

L

Number of sites.

Type int

## coupling\_terms

Nested dictionaries of the following form:

For a M-site coupling, this involves a nesting depth of 2\*M dictionaries. Note that always  $i < j < k < \ldots < 1$ , but entries with j,k,l >= L are allowed for the case of bc\_MPS == 'infinite', when they indicate couplings between different iMPS unit cells.

Type dict of dict

add\_multi\_coupling\_term (strength, ijkl, ops\_ijkl, op\_string='Id')

Add a multi-site coupling term.

### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- ijkl (list of int) The MPS indices of the sites on which the operators acts. With i, j, k, ... = ijkl, we require that they are ordered ascending, i < j < k < ... and that 0 <= i < N\_sites. Inidees >= N\_sites indicate couplings between different unit cells of an infinite MPS.
- ops\_ijkl (list of str) Names of the involved operators on sites i, j, k, ....
- op\_string ((list of) str) Names of the operator to be inserted between the operators, e.g., op\_string[0] is inserted between i and j. A single name holds for all inbetween segments.

multi\_coupling\_term\_handle\_JW (strength, term, sites, op\_string=None)
Helping function to call before add\_multi\_coupling\_term().

Handle/figure out Jordan-Wigner strings if needed.

#### **Parameters**

- **strength** (*float*) The strength of the term.
- **term**(*list* of (*str*, *int*))—List of tuples (op\_i, i) where *i* is the MPS index of the site the operator named *op\_i* acts on. We **require** the operators to be sorted (strictly ascending) by sites. If necessary, call *order\_combine\_term*() beforehand.
- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings.
- op\_string (None / str) Operator name to be used as operator string between the operators, or None if the Jordan Wigner string should be figured out.

**Returns** Arguments for MultiCouplingTerms.add\_multi\_coupling\_term() such that the added term corresponds to the parameters of this function.

Return type strength, ijkl, ops\_ijkl, op\_string

### max\_range()

Determine the maximal range in coupling\_terms.

**Returns max\_range** – The maximum of j - i for the i, j occurring in a term of coupling\_terms.

## Return type int

```
add_to_graph (graph, _i=None, _d1=None, _label_left=None)
```

Add terms from coupling\_terms to an MPOGraph.

## **Parameters**

- **graph** (MPOGraph) The graph into which the terms from <code>coupling\_terms</code> should be added.
- \_d1, \_label\_left  $(_i,)$  Should not be given; only needed for recursion.

## remove\_zeros (tol\_zero=1e-15, \_d0=None)

Remove entries close to 0 from coupling\_terms.

#### **Parameters**

- tol\_zero (float) Entries in coupling\_terms with strength < tol\_zero are considered to be zero and removed.
- **d0** (*None*) Should not be given; only needed for recursion.

#### to TermList()

Convert onsite terms into a TermList.

**Returns term\_list** – Representation of the terms as a list of terms.

Return type TermList

 $\verb"add_coupling_term" (strength, i, j, op\_i, op\_j, op\_string='Id')$ 

Add a two-site coupling term on given MPS sites.

#### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- $\mathbf{j}$  (i,) The MPS indices of the two sites on which the operator acts. We require 0 <= i < N\_sites and i < j, i.e.,  $op_i$  acts "left" of  $op_j$ . If  $j >= N_s$ ites, it indicates couplings between unit cells of an infinite MPS.

- op2 (op1,) Names of the involved operators.
- op\_string (str) The operator to be inserted between i and j.

# coupling\_term\_handle\_JW (strength, term, sites, op\_string=None)

Helping function to call before add multi coupling term().

#### **Parameters**

- **strength** (*float*) The strength of the coupling term.
- **term**([(str, int), (str, int)]) List of two tuples (op, i) where i is the MPS index of the site the operator named op acts on.
- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings.
- op\_string (None / str) Operator name to be used as operator string between the operators, or None if the Jordan Wigner string should be figured out.

**Returns** Arguments for MultiCouplingTerms.add\_multi\_coupling\_term() such that the added term corresponds to the parameters of this function.

**Return type** strength, i, j, op\_i, op\_j, op\_string

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

"Plot coupling terms into a given lattice.

This function plots the coupling\_terms

### **Parameters**

- ax (matplotlib.axes.Axes) The axes on which we should plot.
- lat (Lattice) The lattice for plotting the couplings, most probably the M.lat of the corresponding model M, see lat.
- **style\_map** (function | None) Function which get's called with arguments i, j, op\_i, op\_string, op\_j, strength for each two-site coupling and should return a keyword-dictionary with the desired plot-style for this coupling. By default (None), the *linewidth* is given by the absolute value of *strength*, and the linecolor depends on the phase of *strength* (using the *hsv* colormap).
- **common\_style** (dict) Common style, which overwrites values of the dictionary returned by style\_map. A 'label' is only used for the first plotted line.
- text (format\_string | None) If not None, we add text labeling the couplings in the plot. Available keywords are i, j, op\_i, op\_string, op\_j, strength as well as strength\_abs, strength\_angle, strength\_real.

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• text\_pos (float) – Specify where to put the text on the line between i (0.0) and j (1.0), e.g. 0.5 is exactly in the middle between i and j.

#### See also:

tenpy.models.lattice.Lattice.plot\_sites() plot the sites of the lattice.

### save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

### to\_nn\_bond\_Arrays (sites)

Convert the <code>coupling\_terms</code> into Arrays on nearest neighbor bonds.

**Parameters sites** (list of *Site*) – Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.

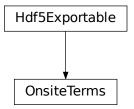
Returns H\_bond - The coupling\_terms rewritten as sum\_i H\_bond[i] for MPS indices i. H\_bond[i] acts on sites (i-1, i), None represents 0. Legs of each H\_bond[i] are ['p0', 'p0\*', 'p1', 'p1\*'].

**Return type** list of {*Array* | None}

## **OnsiteTerms**

- full name: tenpy.networks.terms.OnsiteTerms
- parent module: tenpy.networks.terms
- type: class

# **Inheritance Diagram**



# Methods

OnsiteTermsinit(L)	Initialize self.
OnsiteTerms.add_onsite_term(strength, i,	Add a onsite term on a given MPS site.
op)	
OnsiteTerms.add_to_graph(graph)	Add terms from onsite_terms to an MPOGraph.
OnsiteTerms.add_to_nn_bond_Arrays(H_bon	dAdd self.onsite_terms into nearest-neighbor
)	bond arrays.
OnsiteTerms.from_hdf5(hdf5_loader, h5gr, sub-	Load instance from a HDF5 file.
path)	
OnsiteTerms.remove_zeros([tol_zero])	Remove entries close to 0 from onsite_terms.
OnsiteTerms.save_hdf5(hdf5_saver, h5gr, sub-	Export self into a HDF5 file.
path)	
OnsiteTerms.to_Arrays(sites)	Convert the onsite_terms into a list of
	np_conserved Arrays.
OnsiteTerms.to_TermList()	Convert onsite_terms into a TermList.

# class tenpy.networks.terms.OnsiteTerms(L)

Bases: tenpy.tools.hdf5\_io.Hdf5Exportable

Operator names, site indices and strengths representing onsite terms.

Represents a sum of onsite terms where the operators are only given by their name (in the form of a string). What the operator represents is later given by a list of Site with  $get\_op()$ .

**Parameters** L (int) – Number of sites.

L

Number of sites.

Type int

### onsite\_terms

Filled by  $meth:add\_onsite\_term$ . For each index i a dictionary { 'opname': strength} defining the onsite terms.

Type list of dict

### add\_onsite\_term(strength, i, op)

Add a onsite term on a given MPS site.

## **Parameters**

- **strength** (*float*) The strength of the term.
- i (int) The MPS index of the site on which the operator acts. We require 0 <= i < L.
- op (str) Name of the involved operator.

# add\_to\_graph (graph)

Add terms from onsite\_terms to an MPOGraph.

**Parameters graph** (MPOGraph) - The graph into which the terms from onsite\_terms should be added.

### to\_Arrays (sites)

Convert the <code>onsite\_terms</code> into a list of np\_conserved Arrays.

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**Parameters sites** (list of *Site*) – Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.

**Returns onsite\_arrays** – Onsite terms represented by self. Entry i of the list lives on sites[i].

**Return type** list of *Array* 

### remove\_zeros (tol\_zero=1e-15)

Remove entries close to 0 from onsite terms.

**Parameters tol\_zero** (float) - Entries in onsite\_terms with strength < tol\_zero are considered to be zero and removed.

add\_to\_nn\_bond\_Arrays (H\_bond, sites, finite, distribute=0.5, 0.5)

Add self.onsite\_terms into nearest-neighbor bond arrays.

### **Parameters**

- H\_bond (list of {Array | None}) The coupling\_terms rewritten as sum\_i H\_bond[i] for MPS indices i. H\_bond[i] acts on sites (i-1, i), None represents 0. Legs of each H\_bond[i] are ['p0', 'p0\*', 'p1', 'p1\*']. Modified in place.
- **sites** (list of *Site*) Defines the local Hilbert space for each site. Used to translate the operator names into *Array*.
- **distribute**((float, float)) How to split the onsite terms (in the bulk) into the bond terms to the left (distribute[0]) and right (distribute[1]).
- **finite** (bool) Boundary conditions of the MPS, MPS.finite. If finite, we distribute the onsite term of the

## to\_TermList()

Convert onsite\_terms into a TermList.

**Returns** term\_list – Representation of the terms as a list of terms.

Return type TermList

classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export *self* into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with <code>save\_dict\_content()</code>, storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

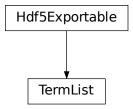
### **TermList**

• full name: tenpy.networks.terms.TermList

• parent module: tenpy.networks.terms

• type: class

## **Inheritance Diagram**



### Methods

TermListinit(terms, strength)	Initialize self.
TermList.from_hdf5(hdf5_loader, h5gr, subpath)	Load instance from a HDF5 file.
TermList.order_combine(sites)	Order and combine operators in each term.
TermList.save_hdf5(hdf5_saver, h5gr, subpath)	Export self into a HDF5 file.
TermList.to_OnsiteTerms_CouplingTerms(s	sites)nvert to OnsiteTerms and CouplingTerms

```
class tenpy.networks.terms.TermList(terms, strength)
    Bases: tenpy.tools.hdf5_io.Hdf5Exportable
```

A list of terms (=operator names and sites they act on) and associated strengths.

A representation of terms, similar as OnsiteTerms, CouplingTerms and MultiCouplingTerms.

This class does not store operator strings between the sites. Jordan-Wigner strings of fermions are added during conversion to (Multi)CouplingTerms.

#### **Parameters**

- **terms** (*list* of *list* of (*str*, *int*)) List of terms where each *term* is a list of tuples (opname, i) of an operator name and a site *i* it acts on. For Fermions, the order is the order in the mathematic sense, i.e., the right-most/last operator in the list acts last.
- strengths (list of float/complex) For each term in terms an associated pref-

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actor or strength (e.g. expectation value).

#### terms

List of terms where each term is a tuple (opname, i) of an operator name and a site i it acts on.

```
Type list of list of (str, int)
```

#### strengths

For each term in *terms* an associated prefactor or strength (e.g. expectation value).

Type 1D ndarray

### to\_OnsiteTerms\_CouplingTerms (sites)

Convert to OnsiteTerms and CouplingTerms

Performs Jordan-Wigner transformation for fermionic operators.

**Parameters sites** (list of Site) – Defines the local Hilbert space for each site. Used to check whether the operators need Jordan-Wigner strings. The length is used as L for the *onsite\_terms* and *coupling\_terms*.

#### Returns

- **onsite\_terms** (OnsiteTerms) Onsite terms.
- coupling\_terms (CouplingTerms | MultiCouplingTerms) Coupling terms. If self contains terms involving more than two operators, a MultiCouplingTerms instance, otherwise just CouplingTerms.

### order combine (sites)

Order and combine operators in each term.

**Parameters** sites (list of Site) – Defines the local Hilbert space for each site. Used to check whether the operators anticommute (= whether they need Jordan-Wigner strings) and for multiplication rules.

### See also:

order\_and\_combine\_term() does it for a single term.

### classmethod from\_hdf5 (hdf5\_loader, h5gr, subpath)

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with <code>save\_hdf5()</code>.

#### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- **h5gr** (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns obj** – Newly generated class instance containing the required data.

Return type cls

### save\_hdf5 (hdf5\_saver, h5gr, subpath)

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct self with from\_hdf5().

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class `Group`) HDF5 group which is supposed to represent self.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

#### **Functions**

order\_combine\_term(term, sites)

Combine operators in a term to one terms per site.

## order combine term

• full name: tenpy.networks.terms.order\_combine\_term

• parent module: tenpy.networks.terms

• type: function

tenpy.networks.terms.order\_combine\_term(term, sites)

Combine operators in a term to one terms per site.

Takes in a term of operators and sites they acts on, commutes operators to order them by site and combines operators acting on the same site with multiply\_op\_names().

#### **Parameters**

- **term** (a list of (opname\_i, i) tuples) Represents a product of onsite operators with site indices *i* they act on. Needs not to be ordered and can have multiple entries acting on the same site.
- **sites** (list of Site) Defines the local Hilbert space for each site. Used to check whether the operators anticommute (= whether they need Jordan-Wigner strings) and for multiplication rules.

### Returns

- combined\_term Equivalent to term but with at most one operator per site.
- **overall\_sign**  $(+1 \mid -1 \mid 0)$  Comes from the (anti-)commutation relations. When the operators in *term* are multiplied from left to right, and then multiplied by *overall\_sign*, the result is the same operator as the product of *combined\_term* from left to right.

## **Module description**

Classes to store a collection of operator names and sites they act on, together with prefactors.

This modules collects classes which are not strictly speaking tensor networks but represent "terms" acting on them. Each term is given by a collection of (onsite) operator names and indices of the sites it acts on. Moreover, we associate a *strength* to each term, which corresponds to the prefactor when specifying e.g. a Hamiltonian.

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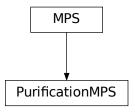
# 7.10.5 purification mps

• full name: tenpy.networks.purification\_mps

• parent module: tenpy.networks

• type: module

## **Classes**



PurificationMPS(sites, Bs, SVs[, bc, form, norm]) An MPS representing a finite-temperature ensemble using purification.

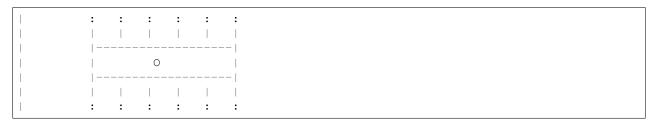
# Module description

This module contains an MPS class representing an density matrix by purification.

Usually, an MPS represents a pure state, i.e. the density matrix is  $\rho = |\psi\rangle < \psi|$ , describing observables as  $< O > = Tr(O|\psi\rangle < \psi|) = <\psi|O|\psi\rangle$ . Clearly, if  $|\psi\rangle$  is the ground state of a Hamiltonian, this is the density matrix at T=0.

At finite temperatures T>0, we want to describe a non-pure density matrix  $\rho=\exp(-H/T)$ . This can be accieved by the so-called purification: in addition to the physical space P, we introduce a second 'auxiliar' space Q and define the density matrix of the physical system as  $\rho=Tr_Q(|\phi><\phi|)$ , where  $|\phi>$  is a pure state in the combined physical and auxiliar system.

For  $T=\infty$ , the density matrix  $\rho_{\infty}$  is the identity matrix. In other words, expectation values are sums over all possible states  $< O >= Tr_P(\rho_{\infty}O) = Tr_P(O)$ . Saying that each: on top is to be connected with the corresponding: on the bottom, the trace is simply a contraction:



Clearly, we get the same result, if we insert an identity operator, written as MPO, on the top and bottom:

We use the following label convention:

You can view the MPO as an MPS by combining the p and q leg and defining every physical operator to act trivial on the q leg. In expectation values, you would then sum over over the q legs, which is exactly what we need. In other words, the choice  $B = \delta_{p,q}$  with trivial (length-1) virtual bonds yields infinite temperature expectation values for operators action only on the p legs!

Now, you go a step further and also apply imaginary time evolution (acting only on p legs) to the initial infinite temperature state. For example, the normalized state  $|\psi> \propto \exp(-\beta/2H)|\phi>$  yields expectation values

$$< O > = Tr(\exp(-\beta H)O)/Tr(\exp(-\beta H)) \propto < \phi |\exp(-\beta/2H)O\exp(-\beta/2H)|\phi > .$$

An additional real-time evolution allows to calculate time correlation functions:

$$\langle A(t)B(0) \rangle \propto \langle \phi | \exp(-\beta H/2) \exp(+iHt)A \exp(-iHt)B \exp(-\beta H/2) | \phi \rangle$$

Time evolution algorithms (TEBD and MPO application) are adjusted in the module purification.

See also [Karrasch2013] for additional tricks! On of their crucial observations is, that one can apply arbitrary unitaries on the auxiliar space (i.e. the q) without changing the result. This can actually be used to reduce the necessary virtual bond dimensions: From the definition, it is easy to see that if we apply exp(-iHt) to the p legs of  $|\phi>$ , and exp(+iHt) to the q legs, they just cancel out! (They commute with  $exp(-\beta H/2)...$ ) If the state is modified (e.g. by applying A or B to calculate correlation functions), this is not true any more. However, we still can find unitaries, which are 'optimal' in the sense of reducing the entanglement of the MPS/MPO to the minimal value. For a discussion of P is P is P in the sense of P is P and P is P in the sense of reducing the entanglement of the MPS/MPO to the minimal value.

**Note:** The classes MPSEnvironment and TransferMatrix should also work for the PurificationMPS defined here. For example, you can use <code>expectation\_value()</code> for the expectation value of operators between different PurificationMPS. However, this makes only sense if the *same* disentangler was applied to the *bra* and *ket* PurificationMPS.

**Note:** The literature (e.g. section 7.2 of [Schollwoeck2011] or [Karrasch2013]) suggests to use a *singlet* as a maximally entangled state. Here, we use instead the identity  $\delta_{p,q}$ , since it is easier to generalize for p running over more than two indices, and allows a simple use of charge conservation with the above qconj convention. Moreover, we don't split the physical and auxiliar space into separate sites, which makes TEBD as costly as  $O(d^6\chi^3)$ .

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**Todo:** One can also look at the canonical ensembles by defining the conserved quantities differently, see Barthel (2016), arXiv:1607.01696 for details. Idea: usual charges on p, trivial charges on q; fix total charge to desired value. I think it should suffice to implement another  $from\_infiniteT$ .

# **7.11 tools**

full name: tenpy.toolsparent module: tenpy

• type: module

# **Module description**

A collection of tools: mostly short yet quite useful functions.

Some functions are explicitly imported in other parts of the library, others might just be useful when using the library. Common to all tools is that they are not just useful for a single algorithm but fairly general.

## **Submodules**

hdf5_io	Tools to save and load data (from TeNPy) to disk.
params	Tools to handle config options/paramters for algorithms.
misc	Miscellaneous tools, somewhat random mix yet often
	helpful.
math	Different math functions needed at some point in the
	library.
fit	tools to fit to an algebraic decay.
string	Tools for handling strings.
process	Tools to read out total memory usage and get/set the
	number of threads.
optimization	Optimization options for this library.

# 7.11.1 hdf5\_io

• full name: tenpy.tools.hdf5\_io

• parent module: tenpy.tools

• type: module

# Classes



Hdf5Exportable	Interface specification for a class to be exportable to our
	HDF5 format.
Hdf5Ignored([name])	Placeholder for a dataset/group to be ignored during
	both loading and saving.
Hdf5Loader(h5group[,ignore_unknown])	Class to load and import object from a HDF5 file.
Hdf5Saver(h5group[, format_selection])	Engine to save simple enough objects into a HDF5 file.

# Hdf5Exportable

• full name: tenpy.tools.hdf5\_io.Hdf5Exportable

• parent module: tenpy.tools.hdf5\_io

• type: class

# **Inheritance Diagram**

Hdf5Exportable

# **Methods**

Hdf5Exportableinit	Initialize self.
Hdf5Exportable.from_hdf5(hdf5_loader, h5gr,	Load instance from a HDF5 file.
)	
Hdf5Exportable.save_hdf5(hdf5_saver, h5gr,	Export self into a HDF5 file.
)	

class tenpy.tools.hdf5\_io.Hdf5Exportable
 Bases: object

Interface specification for a class to be exportable to our HDF5 format.

To allow a class to be exported to HDF5 with <code>save\_to\_hdf5()</code>, it only needs to implement the <code>save\_hdf5()</code> method as documented below. To allow import, a class should implement the classmethod <code>from\_hdf5()</code>. During the import, the class already needs to be defined; loading can only initialize instances, not define classes.

The implementation given works for sufficiently simple (sub-)classes, for which all data is stored in \_\_dict\_\_. In particular, this works for python-defined classes which simply store data using self.data = data in their methods.

```
save_hdf5 (hdf5_saver, h5gr, subpath)
```

Export self into a HDF5 file.

This method saves all the data it needs to reconstruct *self* with *from\_hdf5()*.

This implementation saves the content of \_\_dict\_\_ with save\_dict\_content(), storing the format under the attribute 'format'.

#### **Parameters**

- hdf5\_saver (Hdf5Saver) Instance of the saving engine.
- h5gr (:class`Group`) HDF5 group which is supposed to represent self.
- **subpath** (str) The *name* of h5gr with a '/' in the end.

```
classmethod from_hdf5 (hdf5_loader, h5gr, subpath)
```

Load instance from a HDF5 file.

This method reconstructs a class instance from the data saved with save hdf5().

### **Parameters**

- hdf5\_loader (Hdf5Loader) Instance of the loading engine.
- h5gr (Group) HDF5 group which is represent the object to be constructed.
- **subpath** (*str*) The *name* of *h5gr* with a '/' in the end.

**Returns** obj – Newly generated class instance containing the required data.

Return type cls

# Hdf5lgnored

• full name: tenpy.tools.hdf5\_io.Hdf5Ignored

• parent module: tenpy.tools.hdf5\_io

• type: class

# **Inheritance Diagram**

Hdf5Ignored

### **Methods**

Hdf5Ignored.\_\_init\_\_([name])

Initialize self.

```
class tenpy.tools.hdf5_io.Hdf5Ignored(name='unknown')
    Bases: object
```

Placeholder for a dataset/group to be ignored during both loading and saving.

Objects of this type are not saved. Moreover, if a saved dataset/group has the *type* attribute matching *REPR\_IGNORED*, instance of this class are returned instead of loading the data.

**Parameters** name (str) – The name of the dataset during loading; just for reference.

#### name

See above.

Type str

# Hdf5Loader

• full name: tenpy.tools.hdf5\_io.Hdf5Loader

• parent module: tenpy.tools.hdf5\_io

• type: class

# **Inheritance Diagram**

Hdf5Loader

# Methods

Hdf5Loaderinit(h5group[, ig-	Initialize self.
nore_unknown])	
Hdf5Loader.find_class(module, classname)	Get the class of the qualified classname in a given
	python module.
Hdf5Loader.get_attr(h5gr, attr_name)	Return attribute h5gr.attrs[attr_name], if ex-
	istent.
Hdf5Loader.load([path])	Load a Python object from the dataset.
Hdf5Loader.load_dataset(h5gr, type_info,	Load a h5py Dataset and convert it into the desired
subpath)	type.
<pre>Hdf5Loader.load_dict(h5gr, type_info, sub-</pre>	Load a dictionary in the format according to <i>type_info</i> .
path)	
Hdf5Loader.load_dtype(h5gr, type_info, sub-	Load a numpy.dtype.
path)	
<pre>Hdf5Loader.load_general_dict(h5gr,)</pre>	Load a dictionary with general keys.
<pre>Hdf5Loader.load_hdf5exportable(h5gr,)</pre>	Load an instance of a userdefined class.
Hdf5Loader.load_ignored(h5gr, type_info,	Ignore the group to be loaded.
subpath)	
<pre>Hdf5Loader.load_list(h5gr, type_info, sub-</pre>	Load a list.
path)	
	Load the None object from a dataset.
path)	
Hdf5Loader.load_range(h5gr, type_info, sub-	Load a range.
path)	
<pre>Hdf5Loader.load_set(h5gr, type_info, subpath)</pre>	Load a set.
${\it Hdf5Loader.load\_simple\_dict(h5gr,}$	Load a dictionary with simple keys.
type_info,)	
Hdf5Loader.load_tuple(h5gr, type_info, sub-	Load a tuple.
path)	
Hdf5Loader.memorize_load(h5gr,obj)	Store objects already loaded in the memo_load.

# **Class Attributes and Properties**

Hdf5Loader.dispatch\_load

class tenpy.tools.hdf5\_io.Hdf5Loader(h5group, ignore\_unknown=True)

Bases: object

Class to load and import object from a HDF5 file.

The intended use of this class is through <code>load\_from\_hdf5()</code>, which is simply an alias for <code>Hdf5Loader(h5group).load(path)</code>.

It can load data exported with <code>save\_to\_hdf5()</code> or the <code>Hdf5Saver</code>, respectively.

The basic structure of this class is similar as the *Unpickler* from pickle.

See Saving to disk: input/output for a specification of what can be saved and what the resulting datastructure is.

### **Parameters**

• h5group (Group) – The HDF5 group (or file) where to save the data.

• ignore\_unknown (bool) - Whether to just warn (True) or raise an Error (False) if a class to be loaded is not found.

## h5group

The HDF5 group (or HDF5 File) where to save the data.

```
Type Group
```

### ignore\_unknown

Whether to just warn (True) or raise an Error (False) if a class to be loaded is not found.

Type bool

# dispatch\_load

Mapping from one of the global REPR\_\* variables to (unbound) methods f of this class. The method is called as f(self, h5gr, type\_info, subpath). The call to f should load and return an object obj from the h5py Group or Dataset h5gr; and memorize the loaded obj with  $memorize_load()$ . subpath is just the name of h5gr with a guaranteed '/' in the end.  $type_info$  is often the REPR\_\* variable of the type or some other information about the type, which allows to use a single dispatch\_load function for different datatypes.

Type dict

### memo load

A dictionary to remember all the objects which we already loaded from h5group. The dictionary key is a h5py group- or dataset id; the value is the loaded object. See memorize\_load().

Type dict

### load (path=None)

Load a Python object from the dataset.

See load\_from\_hdf5() for more details.

**Parameters** path (None | str | Reference) – Path within h5group to be used for loading. Defaults to the name of h5group itself.

**Returns obj** – The Python object loaded from *h5group* (specified by *path*).

Return type object

### $memorize\_load(h5gr, obj)$

Store objects already loaded in the memo\_load.

This allows to avoid copies, if the same dataset appears multiple times in the hdf5 group of *obj*. Examples can be shared *LegCharge* objects or even shared *Array*.

To handle cyclic references correctly, this function should be called *before* loading data from subgroups with new calls of <code>load()</code>.

### static get\_attr(h5gr, attr\_name)

Return attribute h5gr.attrs[attr\_name], if existent.

Raises Hdf5ImportError – If the attribute does not exist.

### static find\_class(module, classname)

Get the class of the qualified *classname* in a given python *module*.

Imports the module.

### load\_none (h5gr, type\_info, subpath)

Load the None object from a dataset.

## load\_dataset (h5gr, type\_info, subpath)

Load a h5py Dataset and convert it into the desired type.

```
load_list (h5gr, type_info, subpath)
     Load a list.
load_set (h5gr, type_info, subpath)
     Load a set.
load_tuple (h5gr, type_info, subpath)
    Load a tuple.
load_dict (h5gr, type_info, subpath)
     Load a dictionary in the format according to type_info.
load_general_dict (h5gr, type_info, subpath)
     Load a dictionary with general keys.
load_simple_dict (h5gr, type_info, subpath)
     Load a dictionary with simple keys.
load_range (h5gr, type_info, subpath)
     Load a range.
load_dtype (h5gr, type_info, subpath)
     Load a numpy.dtype.
load_hdf5exportable (h5gr, type_info, subpath)
     Load an instance of a userdefined class.
load_ignored (h5gr, type_info, subpath)
     Ignore the group to be loaded.
```

## Hdf5Saver

- full name: tenpy.tools.hdf5\_io.Hdf5Saver
- parent module: tenpy.tools.hdf5\_io
- type: class

# **Inheritance Diagram**

Hdf5Saver

### Methods

Hdf5Saverinit(h5group[, for-	Initialize self.
mat_selection])	
Hdf5Saver.create_group_for_obj(path, obj)	Create an HDF5 group self.h5group[path] to
	store <i>obj</i> .
Hdf5Saver.memorize_save(h5gr,obj)	Store objects already saved in the memo_save.
Hdf5Saver.save(obj[, path])	Save obj in self.h5group[path].
<pre>Hdf5Saver.save_dataset(obj, path, type_repr)</pre>	Save <i>obj</i> as a hdf5 dataset; in dispatch table.
<pre>Hdf5Saver.save_dict(obj, path, type_repr)</pre>	Save the dictionary <i>obj</i> ; in dispatch table.
Hdf5Saver.save_dict_content(obj, h5gr,	Save contents of a dictionary $obj$ in the existing $h5gr$ .
subpath)	
<pre>Hdf5Saver.save_dtype(obj, path, type_repr)</pre>	Save a dtype object; in dispatch table.
<pre>Hdf5Saver.save_ignored(obj, path, type_repr)</pre>	Don't save the Hdf5Ignored object; just return None.
Hdf5Saver.save_iterable(obj, path, type_repr)	Save an iterable <i>obj</i> like a list, tuple or set; in dispatch
	table.
Hdf5Saver.save_iterable_content(obj,	Save contents of an iterable $obj$ in the existing $h5gr$ .
h5gr,)	
Hdf5Saver.save_none(obj, path, type_repr)	Save the None object as a string (dataset); in dispatch
	table.
<pre>Hdf5Saver.save_range(obj, path, type_repr)</pre>	Save a range object; in dispatch table.

# **Class Attributes and Properties**

Hdf5Saver.dispatch\_save

class tenpy.tools.hdf5\_io.Hdf5Saver(h5group, format\_selection=None)

Bases: object

Engine to save simple enough objects into a HDF5 file.

The intended use of this class is through <code>save\_to\_hdf5()</code>, which is simply an alias for Hdf5Saver(h5group).save(obj, path).

It exports python objects to a HDF5 file such that they can be loaded with the <code>Hdf5Loader</code>, or a call to <code>load\_from\_hdf5()</code>, respectively.

The basic structure of this class is similar as the *Pickler* from pickle.

See Saving to disk: input/output for a specification of what can be saved and what the resulting datastructure is.

### **Parameters**

- h5group (Group) The HDF5 group (or HDF5 File) where to save the data.
- **format\_selection** (dict) This dictionary allows to set a output format selection for user-defined <code>Hdf5Exportable.save\_hdf5()</code> implementations. For example, LegCharge checks it for the key "LegCharge".

#### h5group

The HDF5 group (or HDF5 File) where to save the data.

Type Group

# dispatch\_save

Mapping from a type keytype to methods f of this class. The method is called as f(self, obj,

path, type\_repr). The call to f should save the object obj in self.h5group[path], call  $memorize\_save()$ , and set h5gr.attr[ATTR\_TYPE] = type\_repr to a string  $type\_repr$  in order to allow loading with the dispatcher in Hdf5Loader.dispatch\_save[type\_repr].

### Type dict

#### memo save

A dictionary to remember all the objects which we already stored to *h5group*. The dictionary key is the object id; the value is a two-tuple of the hdf5 group or dataset where an object was stored, and the object itself. See *memorize* save().

### Type dict

### format\_selection

This dictionary allows to set a output format selection for user-defined <code>Hdf5Exportable.save\_hdf5()</code> implementations. For example, <code>LegCharge</code> checks it for the key "<code>LegCharge</code>".

### Type dict

### save (obj, path='/')

Save obj in self.h5group[path].

#### **Parameters**

- **obj** (object) The object (=data) to be saved.
- **path** (str) Path within h5group under which the obj should be saved. To avoid unwanted overwriting of important data, the group/object should not yet exist, except if path is the default '/'.

**Returns** h5gr – The h5py group or dataset in which *obj* was saved.

Return type Group | Dataset

## create\_group\_for\_obj (path, obj)

Create an HDF5 group self.h5group[path] to store obj.

Also handle ending of path with '/', and memorize obj in memo\_save.

#### **Parameters**

- path (str) Path within h5group under which the obj should be saved. To avoid unwanted overwriting of important data, the group/object should not yet exist, except if path is the default '/'.
- **obj** (object) The object (=data) to be saved.

#### Returns

- h5group (Group) Newly created h5py (sub)group self.h5group[path], unless path is '/', in which case it is simply the existing self.h5group['/'].
- **subpath** (*str*) The *group.name* ending with '/', such that other names can be appended to get the path for subgroups or datasets in the group.

:raises ValueError : if self.h5group[path]` already existed and path is not '/'.:

### memorize\_save(h5gr, obj)

Store objects already saved in the memo\_save.

This allows to avoid copies, if the same python object appears multiple times in the data of *obj*. Examples can be shared *LegCharge* objects or even shared *Array*. Using the memo also avoids crashes from cyclic references, e.g., when a list contains a reference to itself.

#### **Parameters**

- h5gr (Group | Dataset) The h5py group or dataset in which obj was saved.
- **obj** (object) The object saved.

# save\_none (obj, path, type\_repr)

Save the None object as a string (dataset); in dispatch table.

#### **save** dataset (*obj*, *path*, *type repr*)

Save *obj* as a hdf5 dataset; in dispatch table.

### save\_iterable (obj, path, type\_repr)

Save an iterable *obj* like a list, tuple or set; in dispatch table.

### save\_iterable\_content (obj, h5gr, subpath)

Save contents of an iterable *obj* in the existing h5gr.

#### **Parameters**

- **obj** (dict) The data to be saved
- h5gr (Group) h5py Group under which the keys and values of *obj* should be saved.
- **subpath** (*str*) Name of h5gr with '/' in the end.

### save\_dict (obj, path, type\_repr)

Save the dictionary *obj*; in dispatch table.

# save\_dict\_content (obj, h5gr, subpath)

Save contents of a dictionary *obj* in the existing *h5gr*.

The format depends on whether the dictionary *obj* has simple keys valid for hdf5 path components (see <code>valid\_hdf5\_path\_component()</code>) or not. For simple keys: directly use the keys as path. For non-simple keys: save list of keys und "keys" and list of values und "values".

## **Parameters**

- obj (dict) The data to be saved
- **h5gr** (Group) h5py Group under which the keys and values of *obj* should be saved.
- **subpath** (str) Name of h5gr with '/' in the end.

**Returns type\_repr** – Indicates whether the data was saved in the format for a dictionary with simple keys or general keys, see comment above.

Return type REPR DICT SIMPLE | REPR DICT GENERAL

# $\verb"save_range" (obj, path, type_repr")$

Save a range object; in dispatch table.

### save\_dtype (obj, path, type\_repr)

Save a dtype object; in dispatch table.

## save\_ignored (obj, path, type\_repr)

Don't save the Hdf5Ignored object; just return None.

# **Exceptions**

Hdf5ExportError	This exception is raised when something went wrong
	during export to hdf5.
Hdf5FormatError	Common base class for errors regarding our HDF5 for-
	mat.
Hdf5ImportError	This exception is raised when something went wrong
	during import from hdf5.

# Hdf5ExportError

• full name: tenpy.tools.hdf5\_io.Hdf5ExportError

• parent module: tenpy.tools.hdf5\_io

• type: exception

# exception tenpy.tools.hdf5\_io.Hdf5ExportError

This exception is raised when something went wrong during export to hdf5.

### Hdf5FormatError

• full name: tenpy.tools.hdf5\_io.Hdf5FormatError

• parent module: tenpy.tools.hdf5\_io

· type: exception

### exception tenpy.tools.hdf5\_io.Hdf5FormatError

Common base class for errors regarding our HDF5 format.

# Hdf5ImportError

• full name: tenpy.tools.hdf5\_io.Hdf5ImportError

• parent module: tenpy.tools.hdf5\_io

• type: exception

# exception tenpy.tools.hdf5\_io.Hdf5ImportError

This exception is raised when something went wrong during import from hdf5.

# **Functions**

load(filename)			Load data from file with given <i>filename</i> .
load_from_hdf5(h5group[,	path,	ig-	Load an object from hdf5 file or group.
nore_unknown])			
save(data, filename[, mode])			Save data to file with given filename.
save_to_hdf5(h5group, obj[, pa	ıth])		Save an object <i>obj</i> into a hdf5 file or group.
valid_hdf5_path_componer	t(name)		Determine if <i>name</i> is a valid HDF5 path component.

### load

```
full name: tenpy.tools.hdf5_io.loadparent module: tenpy.tools.hdf5_io
```

tenpy.tools.hdf5\_io.load(filename)

Load data from file with given filename.

Guess the type of the file from the filename ending, see <code>save()</code> for possible endings.

**Parameters filename** (str) – The name of the file to load.

**Returns data** – The object loaded from the file.

Return type obj

## load\_from\_hdf5

• type: function

• full name: tenpy.tools.hdf5\_io.load\_from\_hdf5

• parent module: tenpy.tools.hdf5\_io

• type: function

tenpy.tools.hdf5\_io.load\_from\_hdf5 (h5group, path=None, ignore\_unknown=True) Load an object from hdf5 file or group.

Roughly equivalent to obj = h5group[path][...], but handle more complicated objects saved as hdf5 groups and/or datasets with <code>save\_to\_hdf5()</code>. For example, dictionaries are handled recursively. See <code>Saving to disk: input/output</code> for a specification of what can be saved/loaded and what the corresponding datastructure is,

### **Parameters**

- h5group (Group) The HDF5 group (or h5py File) to be loaded.
- **path** (None | str | Reference) Path within *h5group* to be used for loading. Defaults to the *h5group* itself specified.
- **ignore\_unknown** (bool) Whether to just warn (True) or raise an Error (False) if a class to be loaded is not found.

**Returns obj** – The Python object loaded from *h5group* (specified by *path*).

Return type object

## save

• full name: tenpy.tools.hdf5 io.save

• parent module: tenpy.tools.hdf5\_io

• type: function

tenpy.tools.hdf5\_io.save(data, filename, mode='w')

Save *data* to file with given *filename*.

This function guesses the type of the file from the filename ending. Supported endings:

ending	description
.pkl	Pickle without compression
.pklz	Pickle with gzip compression.
.hdf5	Hdf5 file (using <i>h5py</i> ).

#### **Parameters**

- **filename** (str) The name of the file where to save the data.
- mode (str) File mode for opening the file. 'w' for write (discard existing file), 'a' for append (add data to exisiting file). See open () for more details.

## save\_to\_hdf5

• full name: tenpy.tools.hdf5\_io.save\_to\_hdf5

• parent module: tenpy.tools.hdf5\_io

• type: function

tenpy.tools.hdf5\_io.save\_to\_hdf5(h5group, obj, path='/')

Save an object *obj* into a hdf5 file or group.

Roughly equivalent to h5group [path] = obj, but handle different types of *obj*. For example, dictionaries are handled recursively. See *Saving to disk: input/output* for a specification of what can be saved and what the resulting datastructure is.

#### **Parameters**

- h5group (Group) The HDF5 group (or h5py File) to which *obj* should be saved.
- **obj** (object) The object (=data) to be saved.
- path (str) Path within h5group under which the obj should be saved. To avoid unwanted overwriting of important data, the group/object should not yet exist, except if path is the default '/'.

**Returns** h5obj – The h5py group or dataset under which *obj* was saved.

Return type Group | Dataset

# valid\_hdf5\_path\_component

• full name: tenpy.tools.hdf5\_io.valid\_hdf5\_path\_component

• parent module: tenpy.tools.hdf5\_io

· type: function

tenpy.tools.hdf5\_io.valid\_hdf5\_path\_component(name)

Determine if *name* is a valid HDF5 path component.

Conditions: String, no '/', and overall name != '.'.

# **Module description**

Tools to save and load data (from TeNPy) to disk.

**Note:** This module is maintained in the repository https://github.com/tenpy/hdf5\_io.git

See *Saving to disk: input/output* for a motivation and specification of the HDF5 format implemented below. .. online at https://tenpy.readthedocs.io/en/latest/intro/input\_output.html

The functions save() and load() are convenience functions for saving and loading quite general python objects (like dictionaries) to/from files, guessing the file type (and hence protocol for reading/writing) from the file ending.

On top of that, this function provides support for saving python objects to [HDF5] files with the Hdf5Saver and Hdf5Loader classes and the wrapper functions save\_to\_hdf5(), load\_from\_hdf5().

**Note:** To use the export/import features to HDF5, you need to install the h5py python package (and hence some version of the HDF5 library).

### Global module constants used for our HDF5 format

Names of HDF5 attributes:

```
tenpy.tools.hdf5_io.ATTR_TYPE = 'type'
     Attribute name for type of the saved object, should be one of the REPR_*
tenpy.tools.hdf5_io.ATTR_CLASS = 'class'
     Attribute name for the class name of an HDF5Exportable
tenpy.tools.hdf5_io.ATTR_MODULE = 'module'
     Attribute name for the module where ATTR_CLASS can be retrieved
tenpy.tools.hdf5_io.ATTR_LEN = 'len'
     Attribute name for the length of iterables, e.g., list, tuple
tenpy.tools.hdf5 io.ATTR FORMAT = 'format'
     indicates the ATTR_TYPE format used by Hdf5Exportable
Names for the ATTR TYPE attribute:
tenpy.tools.hdf5 io.REPR HDF5EXPORTABLE = 'instance'
     saved object is instance of a user-defined class following the Hdf5Exportable style.
tenpy.tools.hdf5_io.REPR_ARRAY = 'array'
     saved object represents a numpy array
tenpy.tools.hdf5_io.REPR_INT = 'int'
     saved object represents a (python) int
tenpy.tools.hdf5_io.REPR_FLOAT = 'float'
     saved object represents a (python) float
tenpy.tools.hdf5_io.REPR_STR = 'str'
     saved object represents a (python unicode) string
tenpy.tools.hdf5_io.REPR_COMPLEX = 'complex'
     saved object represents a complex number
```

```
tenpy.tools.hdf5_io.REPR_INT64 = 'np.int64'
    saved object represents a np.int64
tenpy.tools.hdf5_io.REPR_FLOAT64 = 'np.float64'
    saved object represents a np.float64
tenpy.tools.hdf5 io.REPR INT32 = 'np.int32'
    saved object represents a np.int32
tenpy.tools.hdf5_io.REPR_FLOAT32 = 'np.float32'
    saved object represents a np.float32
tenpy.tools.hdf5_io.REPR_BOOL = 'bool'
    saved object represents a boolean
tenpy.tools.hdf5_io.REPR_NONE = 'None'
    saved object is None
tenpy.tools.hdf5_io.REPR_RANGE = 'range'
    saved object is a range
tenpy.tools.hdf5 io.REPR LIST = 'list'
    saved object represents a list
tenpy.tools.hdf5_io.REPR_TUPLE = 'tuple'
    saved object represents a tuple
tenpy.tools.hdf5 io.REPR SET = 'set'
    saved object represents a set
tenpy.tools.hdf5_io.REPR_DICT_GENERAL = 'dict'
    saved object represents a dict with complicated keys
tenpy.tools.hdf5_io.REPR_DICT_SIMPLE = 'simple_dict'
    saved object represents a dict with simple keys
tenpy.tools.hdf5_io.REPR_DTYPE = 'dtype'
    saved object represents a np.dtype
tenpy.tools.hdf5_io.REPR_IGNORED = 'ignore'
    ignore the object/dataset during loading and saving
tenpy.tools.hdf5_io.TYPES_FOR_HDF5_DATASETS = ((<class 'numpy.ndarray'>, 'array'), (<class
    tuple of (type, type_repr) which h5py can save as datasets; one entry for each type.
```

**Todo:** For memory caching with big MPO environments, we need a Hdf5Cacher clearing the memo's every now and then (triggered by what?).

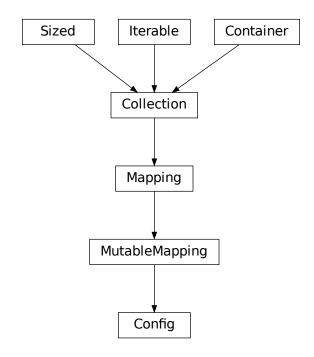
# 7.11.2 params

• full name: tenpy.tools.params

• parent module: tenpy.tools

• type: module

# Classes



Config(config, name)	Dict-like wrapper class for parameter/configuration dic-
	tionaries.

# **Functions**

asConfig(config, name)	Convert a dict-like config to a Config.
<pre>get_parameter(params, key, default, descr[,])</pre>	Read out a parameter from the dictionary and/or provide
	default values.
unused_parameters(params[, warn])	Returns a set of the parameters which have not been read
	out with get_parameters.

# asConfig

```
• full name: tenpy.tools.params.asConfig
```

• parent module: tenpy.tools.params

· type: function

```
tenpy.tools.params.asConfig(config, name)
```

Convert a dict-like *config* to a Config.

#### **Parameters**

- config (dict | Config) If this is a Config, just return it. Otherwise, create a Config
  from it and return that.
- name (str) Name to be used for the Config.

**Returns config** – Either directly *config* or Config (config, name).

Return type Config

# get\_parameter

• full name: tenpy.tools.params.get\_parameter

• parent module: tenpy.tools.params

· type: function

tenpy.tools.params.get\_parameter(params, key, default, descr, asarray=False)

Read out a parameter from the dictionary and/or provide default values.

This function provides a similar functionality as params.get(key, default). *Unlike dict.get* this function writes the default value into the dictionary (i.e. in other words it's more similar to params. setdefault(key, default)).

This allows the user to save the modified dictionary as meta-data, which gives a concrete record of the actually used parameters and simplifies reproducing the results and restarting simulations.

Moreover, a special entry with the key 'verbose' in the params can trigger this function to also print the used value. A higer *verbose* level implies more output. If verbose >= 100, it is printed every time it's used. If verbose >= 2, its printed for the first time time its used. and for verbose >= 1, non-default values are printed the first time they are used. otherwise only for the first use.

Internally, whether a parameter was used is saved in the set params ['\_used\_param']. This is used in unused\_parameters() to print a warning if the key wasn't used at the end of the algorithm, to detect mis-spelled parameters.

# **Parameters**

- params (dict) A dicionary of the parameters as provided by the user. If key is not a valid key, params [key] is set to default.
- **key** (string) The key for the parameter which should be read out from the dictionary.
- **default** The default value for the parameter.
- **descr** (str) A short description for verbose output, like 'TEBD', 'XXZ\_model', 'truncation'.
- **asarray** (bool) If True, convert the result to a numpy array with np.asarray (...) before returning.

**Returns** params [key] if the key is in params, otherwise *default*. Converted to a numpy array, if *asarray*.

Return type value

## **Examples**

In the algorithm Engine gets a dictionary of parameters. Beside doing other stuff, it calls tenpy.models.model.NearestNeighborModel.calc\_U\_bond() with the dictionary as argument, which looks similar like:

```
>>> def model_calc_U(U_param):
>>> dt = get_parameter(U_param, 'dt', 0.01, 'TEBD')
>>> # ... calculate exp(-i * dt* H) ....
```

Then, when you call *time\_evolution* without any parameters, it just uses the default value:

```
>>> tenpy.algorithms.tebd.time_evolution(..., dict()) # uses dt=0.01
```

If you provide the special keyword 'verbose' you can triger this function to print the used parameter values:

```
>>> tenpy.algorithms.tebd.time_evolution(..., dict(verbose=1))
parameter 'dt'=0.01 (default) for TEBD
```

Of course you can also provide the parameter to use a non-default value:

```
>>> tenpy.algorithms.tebd.time_evolution(..., dict(dt=0.1, verbose=1)) parameter 'dt'=0.1 for TEBD
```

# unused\_parameters

- full name: tenpy.tools.params.unused\_parameters
- parent module: tenpy.tools.params
- · type: function

tenpy.tools.params.unused\_parameters(params, warn=None)

Returns a set of the parameters which have not been read out with *get\_parameters*.

This function might be useful to check for typos in the parameter keys.

### **Parameters**

- params (dict) A dictionary of parameters which was given to (functions using) get\_parameter()
- warn (None / str) If given, print a warning "unused parameter for {warn!s}: {unused\_keys!s}".

Returns unused\_keys - The set of keys of the params which was not used

Return type set

# **Module description**

Tools to handle config options/paramters for algorithms.

See the doc-string of Config for details.

# 7.11.3 misc

• full name: tenpy.tools.misc

ullet parent module: tenpy.tools

• type: module

# **Functions**

add_with_None_0(a, b)	Return a + b, treating <i>None</i> as zero.
any_nonzero(params, keys[, verbose_msg])	Check for any non-zero or non-equal entries in some
	parameters.
anynan(a)	check whether any entry of a ndarray a is 'NaN'.
argsort(a[, sort])	wrapper around np.argsort to allow sorting ascend-
	ing/descending and by magnitude.
atleast_2d_pad(a[, pad_item])	Transform a into a 2D array, filling missing places with
	pad_item.
<pre>build_initial_state(size, states, filling[,])</pre>	
chi_list(chi_max[, dchi, nsweeps, verbose])	
inverse_permutation(perm)	reverse sorting indices.
lexsort(a[, axis])	wrapper around np.lexsort: allow for trivial case
	a.shape[0] = 0 without sorting
list_to_dict_list(l)	Given a list <i>l</i> of objects, construct a lookup table.
pad(a[, w_l, v_l, w_r, v_r, axis])	Pad an array along a given axis.
<pre>setup_executable(mod, run_defaults[,])</pre>	Read command line arguments and turn into useable
	dicts.
to_array(a[, shape])	Convert a to an numpy array and tile to matching di-
	mension/shape.
to_iterable(a)	If a is a not iterable or a string, return [a], else return
	a.
$ exttt{to\_iterable\_of\_len(a, L)}$	If a is a non-string iterable of length $L$ , return $a$ , other-
	wise return [a]*L.
transpose_list_list(D[, pad])	Returns a list of lists T, such that T[i][j] =
	D[j][i].
zero_if_close(a[, tol])	set real and/or imaginary part to 0 if their absolute value
	is smaller than tol.

# add\_with\_None\_0

```
• full name: tenpy.tools.misc.add_with_None_0
```

• parent module: tenpy.tools.misc

· type: function

```
tenpy.tools.misc.add_with_None_0 (a, b)
```

Return a + b, treating *None* as zero.

**Parameters**  $b(a_1)$  – The two things to be added, or None.

**Returns** a + b, except if a or b is *None*, in which case the other variable is returned.

Return type sum

## any\_nonzero

• full name: tenpy.tools.misc.any\_nonzero

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.any\_nonzero(params, keys, verbose\_msg=None)

Check for any non-zero or non-equal entries in some parameters.

#### **Parameters**

- params (dict / Config) A dictionary of parameters, or a Config instance.
- **keys** (list of {key | tuple of keys}) For a single key, check params [key] for non-zero entries. For a tuple of keys, all the params [key] have to be equal (as numpy arrays).
- **verbose\_msg** (*None | str*) If params['verbose'] >= 1, we print *verbose\_msg* before checking, and a short notice with the *key*, if a non-zero entry is found.

**Returns** match – False, if all params[key] are zero or *None* and True, if any of the params[key] for single *key* in *keys*, or if any of the entries for a tuple of *keys* 

Return type bool

### anynan

• full name: tenpy.tools.misc.anynan

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.anynan(a)

check whether any entry of a ndarray a is 'NaN'.

# argsort

• full name: tenpy.tools.misc.argsort

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.argsort(a, sort=None, \*\*kwargs)

wrapper around np.argsort to allow sorting ascending/descending and by magnitude.

### **Parameters**

- a (array\_like) The array to sort.
- sort ('m>', 'm<', '>', '<', None) Specify how the arguments should be sorted.

sort	order
'm>', 'LM'	Largest magnitude first
'm<', 'SM'	Smallest magnitude first
'>', 'LR', 'LA'	Largest real part first
'<', 'SR', 'SA'	Smallest real part first
'LI'	Largest imaginary part first
'Si'	Smallest imaginary part first
None	numpy default: same as '<'

• \*\*kwargs - Further keyword arguments given directly to numpy.argsort().

**Returns index\_array** – Same shape as *a*, such that a [index\_array] is sorted in the specified way.

Return type ndarray, int

# atleast\_2d\_pad

• full name: tenpy.tools.misc.atleast\_2d\_pad

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.atleast\_2d\_pad(a, pad\_item=0)

Transform a into a 2D array, filling missing places with pad\_item.

Given a list of lists, turn it to a 2D array (pad with 0), or turn a 1D list to 2D.

**Parameters a** (list of lists) – to be converted into ad 2D array.

**Returns a\_2D** – a converted into a numpy array.

Return type 2D ndarray

### **Examples**

```
>>> atleast_2d_pad([3, 4, 0])
array([[3, 4, 0]])
```

# build\_initial\_state

• full name: tenpy.tools.misc.build\_initial\_state

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.build\_initial\_state(size, states, filling, mode='random', seed=None)

### chi list

• full name: tenpy.tools.misc.chi\_list

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.chi\_list(chi\_max, dchi=20, nsweeps=20, verbose=0)

# inverse\_permutation

• full name: tenpy.tools.misc.inverse\_permutation

• parent module: tenpy.tools.misc

• type: function

tenpy.tools.misc.inverse\_permutation(perm) reverse sorting indices.

Sort functions (as LegCharge.sort()) return a (1D) permutation perm array, such that sorted\_array = old\_array[perm]. This function inverts the permutation perm, such that old\_array = sorted\_array[inverse\_permutation(perm)].

Parameters perm (1D array\_like) - The permutation to be reversed.

Assumes that it is a permutation with unique indices. If it is, inverse\_permutation(inverse\_permutation(perm)) == perm.

**Returns inv\_perm** – The inverse permutation of *perm* such that inv\_perm[perm[j]] = j = perm[inv\_perm[j]].

**Return type** 1D array (int)

### lexsort

```
full name: tenpy.tools.misc.lexsort
parent module: tenpy.tools.misc
type: function
tenpy.tools.misc.lexsort (a, axis=-1)
wrapper around np.lexsort: allow for trivial case a.shape[0] = 0 without sorting
```

## list\_to\_dict\_list

- full name: tenpy.tools.misc.list\_to\_dict\_list
- parent module: tenpy.tools.misc
- type: function

```
\texttt{tenpy.tools.misc.list\_to\_dict\_list}\,(l)
```

Given a list l of objects, construct a lookup table.

This function will handle duplicate entries in l.

**Parameters 1** (iterable of iterabele of immutable) – A list of objects that can be converted to tuples to be used as keys for a dictionary.

```
Returns lookup – A dictionary with (key, value) pairs (key):[i1,i2,...] where i1, i2, ... are the indices where key is found in l: i.e. key == tuple(l[i1]) == tuple(l[i2]) == ...
```

Return type dict

### pad

- · full name: tenpy.tools.misc.pad
- parent module: tenpy.tools.misc
- · type: function

```
\texttt{tenpy.tools.misc.pad} \ (a, w\_l=0, v\_l=0, w\_r=0, v\_r=0, axis=0)
```

Pad an array along a given axis.

### **Parameters**

- a (ndarray) the array to be padded
- w\_l (int) the width to be padded in the front
- **v\_1** (*dtype*) the value to be inserted before *a*
- w\_r (int) the width to be padded after the last index
- **v\_r** (dtype) the value to be inserted after a
- axis (int) the axis along which to pad

**Returns** padded – a copy of a with enlarged axis, padded with the given values.

Return type ndarray

### setup executable

• full name: tenpy.tools.misc.setup\_executable

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.setup\_executable (mod, run\_defaults, identifier\_list=None)

Read command line arguments and turn into useable dicts.

Uses default values defined at: - model class for model\_par - here for sim\_par - executable file for run\_par Alternatively, a model\_defaults dictionary and identifier\_list can be supplied without the model

### NB: for setup\_executable to work with a model class, the model class needs to define two things:

- defaults, a static (class level) dictionary with (key, value) pairs that have the name of the parameter (as string) as key, and the default value as value.
- identifier, a static (class level) list or other iterable with the names of the parameters to be used in filename identifiers.

#### **Parameters**

- mod (model / dict) Model class (or instance) OR a dictionary containing model defaults
- run\_defaults (dict) default values for executable file parameters
- identifier\_list (ieterable, optional) variables

**Returns** containing all parameters. args | namespace with raw arguments for some backwards compatibility with executables.

**Return type** model\_par, sim\_par, run\_par (dicts)

### to\_array

• full name: tenpy.tools.misc.to\_array

• parent module: tenpy.tools.misc

· type: function

tenpy.tools.misc.to\_array(a, shape=None)

Convert a to an numpy array and tile to matching dimension/shape.

This function provides similar functionality as numpys broadcast, but not quite the same: Only scalars are broadcasted to higher dimensions, for a non-scalar, we require the number of dimension to match. If the shape does not match, we repeat periodically, e.g. we tile  $(3, 4) \rightarrow (6, 16)$ , but  $(4, 4) \rightarrow (6, 16)$  will raise an error.

#### **Parameters**

- **a** (scalar | array\_like) The input to be converted to an array. A scalar is reshaped to the desired dimension.
- **shape** (tuple of {None | int}) The desired shape of the array. An entry None indicates arbitrary len >=1. For int entries, tile the array periodically to fit the len.

**Returns** a\_array – A copy of a converted to a numpy ndarray of desired dimension and shape.

Return type ndarray

## to iterable

# to\_iterable\_of\_len

- full name: tenpy.tools.misc.to\_iterable\_of\_len
- parent module: tenpy.tools.misc
- type: function

```
tenpy.tools.misc.to_iterable_of_len(a, L)
```

If a is a non-string iterable of length L, return a, otherwise return [a]\*L.

Raises ValueError if a is already an iterable of different length.

# transpose\_list\_list

- full name: tenpy.tools.misc.transpose\_list\_list
- parent module: tenpy.tools.misc
- type: function

```
tenpy.tools.misc.transpose_list_list (D, pad=None)
Returns a list of lists T, such that T[i][j] = D[j][i].
```

#### **Parameters**

- D (list of list) to be transposed
- pad Used to fill missing places, if D is not rectangular.

```
Returns T – transposed, rectangular version of D. constructed such that T[i][j] = D[j][i] if i < len(D[j]) else pad
```

Return type list of lists

### zero if close

- full name: tenpy.tools.misc.zero\_if\_close
- parent module: tenpy.tools.misc
- type: function

```
tenpy.tools.misc.zero_if_close(a, tol=1e-15)
```

set real and/or imaginary part to 0 if their absolute value is smaller than tol.

#### **Parameters**

- a (ndarray) numpy array to be rounded
- tol (float) the threashold which values to consider as '0'.

# **Module description**

Miscellaneous tools, somewhat random mix yet often helpful.

# 7.11.4 math

• full name: tenpy.tools.math

• parent module: tenpy.tools

• type: module

### **Functions**

entropy(p[, n])	Calculate the entropy of a distribution.
gcd(a, b)	Computes the greatest common divisor (GCD) of two
	numbers.
gcd_array(a)	Return the greatest common divisor of all of entries in a
1 cm(a, b)	Returns the least common multiple (LCM) of two posi-
	tive numbers.
matvec_to_array(H)	transform an linear operator with a <i>matvec</i> method into
	a dense numpy array.
perm_sign(p)	Given a permutation <i>p</i> of numbers, returns its sign.
qr_li(A[, cutoff])	QR decomposition with cutoff to discard nearly linear
	dependent columns in $Q$ .
rq_li(A[, cutoff])	RQ decomposition with cutoff to discard nearly linear
	dependent columns in $Q$ .
speigs(A, k, *args, **kwargs)	Wrapper around scipy.sparse.linalg.
	eigs(), lifting the restriction $k < rank(A)-1$ .
speigsh(A, k, *args, **kwargs)	Wrapper around scipy.sparse.linalg.
	eigsh(), lifting the restriction $k < rank(A) -1$ .

# entropy

• full name: tenpy.tools.math.entropy

• parent module: tenpy.tools.math

• type: function

tenpy.tools.math.entropy(p, n=1)

Calculate the entropy of a distribution.

Assumes that p is a normalized distribution (np.sum (p) ==1.).

# **Parameters**

- p (1D array) A normalized distribution.
- n(1 | float | np.inf) Selects the entropy, see below.

**Returns entropy** – Shannon-entropy  $-\sum_i p_i \log(p_i)$  (n=1) or Renyi-entropy  $\frac{1}{1-n} \log(\sum_i p_i^n)$  (n != 1) of the distribution p.

Return type float

# gcd

```
full name: tenpy.tools.math.gcd
parent module: tenpy.tools.math
type: function
tenpy.tools.math.gcd(a, b)
```

Computes the greatest common divisor (GCD) of two numbers.

Return 0 if both a, b are zero, otherwise always return a non-negative number.

# gcd\_array

```
full name: tenpy.tools.math.gcd_array
parent module: tenpy.tools.math
type: function
```

```
tenpy.tools.math.gcd_array(a)
```

Return the greatest common divisor of all of entries in a

### **Icm**

```
• full name: tenpy.tools.math.lcm
```

```
• parent module: tenpy.tools.math
```

· type: function

```
tenpy.tools.math.lcm (a, b)
```

Returns the least common multiple (LCM) of two positive numbers.

# matvec\_to\_array

```
• full name: tenpy.tools.math.matvec_to_array
```

```
• parent module: tenpy.tools.math
```

• type: function

```
tenpy.tools.math.matvec_to_array(H)
```

transform an linear operator with a matvec method into a dense numpy array.

Parameters H (linear operator) – should have shape, dtype attributes and a matvec method.

**Returns**  $H_{dense}$  – a dense array version of H.

Return type ndarray, shape (H.dim, H.dim)

## perm\_sign

```
full name: tenpy.tools.math.perm_sign
parent module: tenpy.tools.math
type: function
tenpy.tools.math.perm_sign(p)
Given a permutation p of numbers, returns its sign. (+1 or -1)
```

Assumes that all the elements are distinct, if not, you get crap.

#### **Examples**

#### qr li

- full name: tenpy.tools.math.qr\_li
- parent module: tenpy.tools.math
- · type: function

```
tenpy.tools.math.qr_li(A, cutoff=1e-15)
```

QR decomposition with cutoff to discard nearly linear dependent columns in Q.

Perform a QR decomposition with pivoting, discard columns where R[i,i] < cuttoff, reverse the permututation from pivoting and perform another QR decomposition to ensure that R is upper right.

```
Parameters A (numpy.ndarray) – Matrix to be decomposed as A = Q.R
```

**Returns** Q, R – Decomposition of A into isometry  $Q^{A}Q = 1$  and upper right R with diagonal entries larger than *cutoff*.

Return type numpy.ndarray

#### rq\_li

- full name: tenpy.tools.math.rq\_li
- parent module: tenpy.tools.math
- type: function

```
tenpy.tools.math.rq_li(A, cutoff=1e-15)
```

RQ decomposition with cutoff to discard nearly linear dependent columns in Q.

Uses  $qr_1i$  () on transpose of A. Note that R is nonzero in the lowest left corner; R has entries below the diagonal for non-square R.

```
Parameters A (numpy.ndarray) - Matrix to be decomposed as A = Q.R
```

**Returns R, Q** – Decomposition of A into isometry Q  $Q^{\wedge}d = 1$  and upper right R with diagonal entries larger than *cutoff*. If M, N = A. shape, then R. shape = M, K and Q. shape = K, N with  $K \leq \min(M, N)$ .

Return type numpy.ndarray

## speigs

- full name: tenpy.tools.math.speigs
- parent module: tenpy.tools.math
- type: function

```
tenpy.tools.math.speigs(A, k, *args, **kwargs)
```

Wrapper around scipy.sparse.linalg.eigs(), lifting the restriction k < rank(A) - 1.

#### **Parameters**

- **A** (MxM ndarray or like scipy.sparse.linalg.LinearOperator) the (square) linear operator for which the eigenvalues should be computed.
- **k** (*int*) the number of eigenvalues to be computed.
- \*args Further arguments directly given to scipy.sparse.linalg.eigs()
- \*\*kwargs Further keyword arguments directly given to scipy.sparse.linalg. eigs()

## Returns

- w (*ndarray*) array of min(k, A.shape[0]) eigenvalues
- v (ndarray) array of min(k, A.shape[0]) eigenvectors, v[:, i] is the i-th eigenvector.

  Only returned if kwargs['return\_eigenvectors'] == True.

## speigsh

- full name: tenpy.tools.math.speigsh
- parent module: tenpy.tools.math
- · type: function

```
tenpy.tools.math.speigsh(A, k, *args, **kwargs)
```

Wrapper around scipy.sparse.linalg.eigsh(), lifting the restriction k < rank(A) - 1.

#### **Parameters**

- **A** (MxM ndarray or like scipy.sparse.linalg.LinearOperator) The (square) hermitian linear operator for which the eigenvalues should be computed.
- $\mathbf{k}$  (int) The number of eigenvalues to be computed.
- \*args Further arguments directly given to scipy.sparse.linalg.eigsh().
- \*\*kwargs Further keyword arguments directly given to scipy.sparse.linalg. eigsh().

#### Returns

• w (ndarray) – Array of min(k, A.shape[0]) eigenvalues.

• v (ndarray) – Array of min(k, A.shape[0]) eigenvectors, v[:, i] is the i-th eigenvector. Only returned if kwargs['return\_eigenvectors'] == True.

## **Module description**

Different math functions needed at some point in the library.

tenpy.tools.math.LeviCivita3 = array([[[ 0, 0, 0], [ 0, 0, 1], [ 0, -1, 0]], [[ 0, 0, -1], Levi-Civita Symbol of int type

## 7.11.5 fit

• full name: tenpy.tools.fit

• parent module: tenpy.tools

• type: module

#### **Functions**

alg_decay(x, a, b, c)	define the algebraic decay.
<pre>alg_decay_fit(x, y[, npts, power_range,])</pre>	Fit y to the form $a*x**(-b) + c$ .
alg_decay_fit_res(log_b, x, y)	Returns the residue of an algebraic decay fit of the form
	$x**(-np.exp(log_b)).$
<pre>alg_decay_fits(x, ys[, npts, power_range,])</pre>	Fit arrays of y's to the form $a * x**(-b) + c$ .
$fit\_with\_sum\_of\_exp(f, n[, N])$	Approximate a decaying function $f$ with a sum of expo-
	nentials.
lin_fit_res(x, y)	Returns the least-square residue of a linear fit y vs x.
linear_fit(x, y)	Perform a linear fit of y to ax + b.
<pre>plot_alg_decay_fit(plot_module, x, y, fit_par)</pre>	Given x, y, and fit_par (output from alg_decay_fit), pro-
	duces a plot of the algebraic decay fit.
<pre>sum_of_exp(lambdas, prefactors, x)</pre>	Evaluate sum_i prefactor[i] *
	lambda[i] **x for different $x$ .

## alg\_decay

• full name: tenpy.tools.fit.alg\_decay

• parent module: tenpy.tools.fit

• type: function

tenpy.tools.fit.alg\_decay (x, a, b, c) define the algebraic decay.

## alg\_decay\_fit

```
full name: tenpy.tools.fit.alg_decay_fit
parent module: tenpy.tools.fit
type: function
tenpy.tools.fit.alg_decay_fit (x, y, npts=5, power_range=0.01, 4.0, power_mesh=[60, 10])
Fit y to the form a*x**(-b) + c.
Returns a triplet [a, b, c].
```

npts specifies the maximum number of points to fit. If npts < len(x), then alg\_decay\_fit() will only fit to the last npts points. power\_range is a tuple that gives that restricts the possible ranges for b. power\_mesh is a list of numbers, which specifies how fine to search for the optimal b. E.g., if power\_mesh = [60,10], then it'll first divide the power\_range into 60 intervals, and then divide those intervals by 10.

## alg\_decay\_fit\_res

```
• full name: tenpy.tools.fit.alg_decay_fit_res
```

```
• parent module: tenpy.tools.fit
```

• type: function

```
tenpy.tools.fit.alg_decay_fit_res(log_b, x, y)
```

Returns the residue of an algebraic decay fit of the form  $x**(-np.exp(log_b))$ .

#### alg\_decay\_fits

```
• full name: tenpy.tools.fit.alg_decay_fits
```

```
• parent module: tenpy.tools.fit
```

· type: function

```
tenpy.tools.fit.alg_decay_fits (x, ys, npts=5, power\_range=0.01, 4.0, power\_mesh=[60, 10]) Fit arrays of y's to the form a * x**(-b) + c.
```

Returns arrays of [a, b, c].

## fit\_with\_sum\_of\_exp

```
• full name: tenpy.tools.fit.fit_with_sum_of_exp
```

```
• parent module: tenpy.tools.fit
```

· type: function

```
tenpy.tools.fit.fit_with_sum_of_exp (f, n, N=50)
```

Approximate a decaying function f with a sum of exponentials.

MPOs can naturally represent long-range interactions with an exponential decay. A common technique for other (e.g. powerlaw) long-range interactions is to approximate them by sums of exponentials and to include them into the MPOs. This function allows to do that.

The algorithm/implementation follows the appendix of [Murg2010].

#### **Parameters**

- f(function) Decaying function to be approximated. Needs to accept a 1D numpy array
- **n** (*int*) Number of exponentials to be used.
- N(int) Number of points at which to evaluate/fit f; we evaluate and fit f at the points x = np.arange(1, N+1).

**Returns lambdas, prefactors** – Such that  $f(k) \approx \sum_i x_i \lambda_i^k$  for (integer)  $1 \le k \le N$ . The function  $sum_of_exp()$  evaluates this for given x.

Return type 1D arrays

### lin\_fit\_res

• full name: tenpy.tools.fit.lin\_fit\_res

• parent module: tenpy.tools.fit

· type: function

```
tenpy.tools.fit.lin_fit_res(x, y)
```

Returns the least-square residue of a linear fit y vs x.

## linear\_fit

• full name: tenpy.tools.fit.linear\_fit

• parent module: tenpy.tools.fit

• type: function

tenpy.tools.fit.linear\_fit (x, y)

Perform a linear fit of y to ax + b.

Returns a, b, res.

#### plot alg decay fit

• full name: tenpy.tools.fit.plot\_alg\_decay\_fit

• parent module: tenpy.tools.fit

· type: function

Given x, y, and fit\_par (output from alg\_decay\_fit), produces a plot of the algebraic decay fit.

plot\_module is matplotlib.pyplot, or a subplot. x, y are the data (real, 1-dimensional np.ndarray) fit\_par is a triplet of numbers [a, b, c] that describes and algebraic decay (see alg\_decay()). xfunc is an optional parameter that scales the x-axis in the resulting plot. kwargs is a dictionary, whoses key/items are passed to the plot function. plot\_fit\_args is a dictionary that controls how the fit is shown.

## sum\_of\_exp

```
full name: tenpy.tools.fit.sum_of_exp
parent module: tenpy.tools.fit
type: function
tenpy.tools.fit.sum_of_exp(lambdas, prefactors, x)
Evaluate sum_i prefactor[i] * lambda[i]**x for different x.
See fit_sum_of_exp() for more details.
```

## **Module description**

tools to fit to an algebraic decay.

# 7.11.6 string

full name: tenpy.tools.stringparent module: tenpy.tools

• type: module

## **Functions**

is_non_string_iterable(x)	Check if x is a non-string iterable, (e.g., list, tuple, dic-
	tionary, np.ndarray)
to_mathematica_lists(a)	convert nested a to string readable by mathematica us-
	ing curly brackets '{}'.
<pre>vert_join(strlist[, valign, halign, delim])</pre>	Join strings with multilines vertically such that they ap-
	pear next to each other.

#### is\_non\_string\_iterable

• full name: tenpy.tools.string.is\_non\_string\_iterable

• parent module: tenpy.tools.string

· type: function

```
tenpy.tools.string.is_non_string_iterable(x)
```

Check if x is a non-string iterable, (e.g., list, tuple, dictionary, np.ndarray)

## to\_mathematica\_lists

```
• full name: tenpy.tools.string.to_mathematica_lists
```

```
• parent module: tenpy.tools.string
```

• type: function

```
tenpy.tools.string.to_mathematica_lists(a) convert nested a to string readable by mathematica using curly brackets '\{...\}'.
```

## vert\_join

- full name: tenpy.tools.string.vert\_join
- parent module: tenpy.tools.string
- type: function

```
tenpy.tools.string.vert_join (strlist, valign='t', halign='l', delim='')

Join strings with multilines vertically such that they appear next to each other.
```

#### **Parameters**

- **strlist** (list of str) the strings to be joined vertically
- valing ('t', 'c', 'b') vertical alignment of the strings: top, center, or bottom
- halign ('l', 'c', 'r') horizontal alignment of the strings: left, center, or right
- **delim** (str) field separator between the strings

**Returns** joined – a string where the strings of strlist are aligned vertically

Return type str

#### **Examples**

## **Module description**

Tools for handling strings.

## **7.11.7 process**

full name: tenpy.tools.processparent module: tenpy.tools

• type: module

#### **Functions**

load_omp_library([libs, verbose])	Tries to load openMP library.
memory_usage()	Return memory usage of the running python process.
mkl_get_nthreads()	wrapper around MKL get_max_threads.
mkl_set_nthreads(n)	wrapper around MKL set_num_threads.
<pre>omp_get_nthreads()</pre>	wrapper around OpenMP get_max_threads.
omp_set_nthreads(n)	wrapper around OpenMP set_nthreads.

## load\_omp\_library

• full name: tenpy.tools.process.load\_omp\_library

• parent module: tenpy.tools.process

· type: function

tenpy.tools.process.load\_omp\_library(libs=['libiomp5.so', None, 'libgomp.so.1'], verbose=True)

Tries to load openMP library.

# Parameters

- **libs** list of possible library names we should try to load (with ctypes.CDLL).
- **verbose** (bool) wheter to print the name of the loaded library.

**Returns omp** – OpenMP shared libary if found, otherwise None. Once it was successfully imported, no re-imports are tried.

Return type CDLL | None

## memory\_usage

• full name: tenpy.tools.process.memory\_usage

 $\bullet \ \ parent \ module: \ \textit{tenpy.tools.process}$ 

· type: function

tenpy.tools.process.memory\_usage()

Return memory usage of the running python process.

You can pip install psutilif you get only -1..

**Returns** mem – Currently used memory in megabytes. –1. if no way to read out.

Return type float

#### mkl get nthreads

```
• full name: tenpy.tools.process.mkl_get_nthreads
   • parent module: tenpy.tools.process
   • type: function
tenpy.tools.process.mkl_get_nthreads()
     wrapper around MKL get_max_threads.
          Returns max_threads - The maximum number of threads used by MKL. -1 if unable to read out.
          Return type int
mkl_set_nthreads
   • full name: tenpy.tools.process.mkl set nthreads
   • parent module: tenpy.tools.process
   • type: function
tenpy.tools.process.mkl_set_nthreads(n)
     wrapper around MKL set num threads.
          Parameters n(int) – the number of threads to use
          Returns success – whether the shared library was found and set.
          Return type bool
omp_get_nthreads
   • full name: tenpy.tools.process.omp_get_nthreads
   • parent module: tenpy.tools.process
   · type: function
tenpy.tools.process.omp_get_nthreads()
     wrapper around OpenMP get_max_threads.
          Returns max threads – The maximum number of threads used by OpenMP (and thus MKL). –1
             if unable to read out.
          Return type int
omp_set_nthreads
   • full name: tenpy.tools.process.omp set nthreads
   • parent module: tenpy.tools.process
```

**Parameters** n(int) – the number of threads to use

tenpy.tools.process.omp\_set\_nthreads(n)
 wrapper around OpenMP set\_nthreads.

• type: function

**Returns** success – whether the shared library was found and set.

### Return type bool

## **Module description**

Tools to read out total memory usage and get/set the number of threads.

If your python is compiled against MKL (e.g. if you use *anaconda* as recommended in INSTALL), it will by default use as many threads as CPU cores are available. If you run a job on a cluster, you should limit this to the number of cores you reserved – otherwise your colleagues might get angry... A simple way to achieve this is to set a suitable enviornment variable before calling your python program, e.g. on the linux bash <code>export OMP\_NUM\_THREADS=4</code> for 4 threads. (MKL used OpenMP and thus respects its settings.)

Alternatively, this module provides <code>omp\_get\_nthreads()</code> and <code>omp\_set\_nthreads()</code>, which give their best to get and set the number of threads at runtime, while still being failsave if the shared OpenMP library is not found. In the latter case, you might also try the equivalent <code>mkl\_get\_nthreads()</code> and <code>mkl\_set\_nthreads()</code>.

# 7.11.8 optimization

• full name: tenpy.tools.optimization

• parent module: tenpy.tools

• type: module

#### **Classes**



OptimizationFlag	Options for the global 'optimization level' used for dynamical optimizations.
temporary_level(temporary_level)	Context manager to temporarily set the optimization level to a different value.

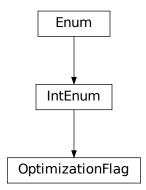
## **OptimizationFlag**

• full name: tenpy.tools.optimization.OptimizationFlag

• parent module: tenpy.tools.optimization

• type: class

## **Inheritance Diagram**



## **Class Attributes and Properties**

OptimizationFlag.default	
OptimizationFlag.none	
OptimizationFlag.safe	
OptimizationFlag.skip_arg_checks	

## class tenpy.tools.optimization.OptimizationFlag

Bases: enum.IntEnum

Options for the global 'optimization level' used for dynamical optimizations.

Whether we optimize dynamically is decided by comparison of the global "optimization level" with one of the following flags. A higher level *includes* all the previous optimizations.

Leve	l Flag	Description
0	none	Don't do any optimizations, i.e., run many sanity checks. Used for testing.
1	default	Skip really unnecessary sanity checks, but also don't try any optional optimizations if
		they might give an overhead.
2	safe	Activate safe optimizations in algorithms, even if they might give a small overhead. Ex-
		ample: Try to compress the MPO representing the hamiltonian.
3	skip_arg_cl	neClassafe! Skip (some) class sanity tests and (function) argument checks.

**Warning:** When unsafe optimizations are enabled, errors will not be detected that easily, debugging is much harder, and you might even get segmentation faults in the compiled parts. Use this kind of optimization only for code which you successfully ran before with (very) similar parmeters and disabled optimizations! Enable this optimization only during the parts of the code where it is really necessary. Check whether it actually helps - if it doesn't, keep the optimization disabled!

## temporary\_level

- full name: tenpy.tools.optimization.temporary\_level
- parent module: tenpy.tools.optimization
- type: class

### **Inheritance Diagram**

temporary\_level

#### **Methods**

```
class tenpy.tools.optimization.temporary_level (temporary_level)
    Bases: object
```

Context manager to temporarily set the optimization level to a different value.

**Parameters temporary\_level** (int | OptimizationFlag | str | None) - The optimization level to be set during the context. *None* defaults to the current value of the optimization level.

#### temporary\_level

The optimization level to be set during the context.

**Type** None | OptimizationFlag

## \_old\_level

Optimization level to be restored at the end of the context manager.

Type OptimizationFlag

#### **Examples**

It is recommended to use this context manager in a with statement:

```
# optimization level default
with temporary_level(OptimizationFlag.safe):
    do_some_stuff() # temporarily have Optimization level `safe`
    # you can even change the optimization level to something else:
    set_level(OptimizationFlag.skip_args_check)
    do_some_really_heavy_stuff()
# here we are back to the optimization level as before the ``with ...`` statement
```

#### **Functions**

get_level()	Return the global optimization level.
<pre>optimize([level_compare])</pre>	Called by algorithms to check whether it should (try to)
	do some optimizations.
set_level([level])	Set the global optimization level.
to_OptimizationFlag(level)	Convert strings and int to a valid OptimizationFlag.
<pre>use_cython([func, replacement, check_doc])</pre>	Decorator to replace a function with a Cython-
	equivalent from _npc_helper.pyx.

### get level

• full name: tenpy.tools.optimization.get\_level

• parent module: tenpy.tools.optimization

· type: function

tenpy.tools.optimization.get\_level()
Return the global optimization level.

#### optimize

• full name: tenpy.tools.optimization.optimize

• parent module: tenpy.tools.optimization

• type: function

tenpy.tools.optimization.optimize(level\_compare=<OptimizationFlag.default: 1>)
Called by algorithms to check whether it should (try to) do some optimizations.

**Parameters** level\_compare (OptimizationFlag) - At which level to start optimization, i.e., how safe the suggested optimization is.

**Returns optimize** – True if the algorithms should try to optimize, i.e., whether the global "optimization level" is equal or higher than the level to compare to.

Return type bool

#### set level

```
• full name: tenpy.tools.optimization.set_level
```

```
• parent module: tenpy.tools.optimization
```

• type: function

```
tenpy.tools.optimization.set_level(level=1) Set the global optimization level.
```

**Parameters level** (int | OptimizationFlag | str | None) - The new global optimization level to be set. None defaults to keeping the current level.

## to\_OptimizationFlag

• full name: tenpy.tools.optimization.to\_OptimizationFlag

```
• parent module: tenpy.tools.optimization
```

· type: function

```
tenpy.tools.optimization.to_OptimizationFlag(level)
```

Convert strings and int to a valid OptimizationFlag.

None defaults to the current level.

## use cython

- full name: tenpy.tools.optimization.use\_cython
- parent module: tenpy.tools.optimization
- · type: function

tenpy.tools.optimization.use\_cython (func=None, replacement=None, check\_doc=True)

Decorator to replace a function with a Cython-equivalent from \_npc\_helper.pyx.

This is a decorator, which is supposed to be used in front of function definitions with an @ sign, for example:

```
@use_cython
def my_slow_function(a):
    "some example function with slow python loops"
    result = 0.
    for i in range(a.shape[0]):
        for j in range(a.shape[1]):
            #... heavy calculations ...
            result += np.cos(a[i, j]**2) * (i + j)
    return result
```

This decorator indicates that there is a Cython implementation in the file tenpy/linalg/\_npc\_helper. pyx, which should have the same signature (i.e. same arguments and return values) as the decorated function, and can be used as a replacement for the decorated function. However, if the cython code could not be compiled on your system (or if the environment variable TENPY\_OPTIMIZE is set to negative values, or the environment variable TENPY\_NO\_CYTHON is "true"), we just pass the previous function.

**Note:** In case that the decorator is used for a class method, the corresponding Cython version needs to have an @cython.binding(True).

#### **Parameters**

- func (function) The defined function
- **replacement** (*string* / *None*) The name of the function defined in tenpy/ linalg/\_npc\_helper.pyx which should replace the decorated function. None defaults to the name of the decorated function, e.g., in the above example *my\_slow\_function*.
- **check\_doc** (bool) If True, we check that the cython version of the function has the exact same doc string (up to a possible first line containing the function signature) to exclude typos and inconsistent versions.

**Returns** replacement\_func – The function replacing the decorated function *func*. If the cython code can not be loaded, this is just *func*, otherwise it's the cython version specified by *replacement*.

Return type function

## **Module description**

Optimization options for this library.

Let me start with a quote of "Micheal Jackson" (a programmer, not the musician):

```
First rule of optimization: "Don't do it."
Second rule of optimization (for experts only): "Don't do it yet."
Third rule of optimization: "Profile before optimizing."
```

Luckily, following the third optimization rule, namely profiling code, is fairly simple in python, see the documentation. If you have a python skript running your code, you can simply call it with python -m "cProfile" -s "tottime" your\_skript.py. Alternatively, save the profiling statistics with python -m "cProfile" -o "profile\_data.stat" your\_skript.py and run these few lines of python code:

```
import pstats
p = pstats.Pstats("profile_data.stat")
p.sort_stats('cumtime') # sort by 'cumtime' column
p.print_stats(30) # prints first 30 entries
```

That being said, I actually did profile and optimize (parts of) the library; and there are a few knobs you can turn to tweak the most out of this library, explained in the following.

- 1) Simply install the 'bottleneck' python package, which allows to optimize slow parts of numpy, most notably 'NaN' checking.
- 2) Figure out which numpy/scipy/python you are using. As explained in *Installation instructions*, we recommend to use the Python distributed provided by Intel or Anaconda. They ship with numpy and scipy which use Intels MKL library, such that e.g. np.tensordot is parallelized to use multiple cores.
- 3) In case you didn't do that yet: some parts of the library are written in both python and Cython with the same interface, so you can simply compile the Cython code, as explained in *Installation instructions*. Then everything should work the same way from a user perspective, while internally the faster, pre-compiled cython code from tenpy/linalg/\_npc\_helper.pyx is used. This should also be a safe thing to do. The replacement of the optimized functions is done by the decorator use\_cython().
- 4) One of the great things about python is its dynamical nature anything can be done at runtime. In that spirit, this module allows to set a global "optimization level" which can be changed *dynamically* (i.e., during runtime) with set\_level(). The library will then try some extra optimiztion, most notably skip sanity checks of arguments. The possible choices for this global level are given by the OptimizationFlag. The default initial value for the global optimization level can be adjusted by the environment variable TENPY\_OPTIMIZE.

**Warning:** When this optimizing is enabled, we skip (some) sanity checks. Thus, errors will not be detected that easily, and debugging is much harder! We recommend to use this kind of optimization only for code which you successfully have run before with (very) similar parmeters! Enable this optimization only during the parts of the code where it is really necessary. The context manager <code>temporary\_level</code> can help with that. Check whether it actually helps - if it doesn't, keep the optimization disabled! Some parts of the library already do that as well (e.g. DMRG after the first sweep).

5) You might want to try some different compile time options for the cython code, set in the *setup.py* in the top directory of the repository. Since the *setup.py* reads out the *TENPY\_OPTIMIZE* environment variable, you can simple use an export TENPY\_OPTIMIZE=3 (in your bash/terminal) right before compilation. An export TENPY\_OPTIMIZE=0 activates profiling hooks instead.

**Warning:** This increases the probability of getting segmentation faults and anyway might not help that much; in the crucial parts of the cython code, these optimizations are already applied. We do *not* recommend using this!

```
tenpy.tools.optimization.bottleneck = None
tenpy.tools.optimization.have_cython_functions = False
   bool whether the import of the cython file tenpy/linalg/_npc_helper.pyx succeeded.
   The value is set in the first call of use_cython().
```

## 7.12 version

full name: tenpy.version
parent module: tenpy
type: module

#### Module description

Access to version of this library.

The version is provided in the standard python format major.minor.revision as string. Use pkg\_resources.parse\_version before comparing versions.

```
tenpy.version.version = '0.7.0'
    current release version as a string

tenpy.version.released = True
    whether this is a released version or modified

tenpy.version.short_version = 'v0.7.0'
    same as version, but with 'v' in front

tenpy.version.git_revision = '1f681357618c45b328b90f53b02730ebb675b981'
    the hash of the last git commit (if available)

tenpy.version.full_version = '0.7.0'
    if not released additional info with part of git revision
```

tenpy.version.version\_summary = 'tenpy 0.7.0 (not compiled), \ngit revision 1f681357618c45b2 summary of the tenpy, python, numpy and scipy versions used

7.12. version 589

# **CHAPTER**

# **EIGHT**

# **INDICES AND TABLES**

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- [conda] "conda package manger", https://docs.conda.io/en/latest/ A package and environment management system that allows to easily install (multiple version of) various software, and in particular python packages like TeNPy.
- [pip] "pip the Python Package installer", https://pip.pypa.io/en/stable/ Traditional way to handle installed python packages with pip install ... and pip uninstall ... on the command line.
- [matplotlib] "Matplotlib", https://matplotlib.org/ A Python 2D plotting library. Some TeNPy functions expect matplotlib.axes.Axes as arguments to plot into.
- [HDF5] "Hierarchical Data Format 5 (R)", https://portal.hdfgroup.org/display/HDF5/HDF5 A file format and library for saving data (including metadata). We use it through the python interface of the h5py <a href="https://docs.h5py.org/en/stable/">https://docs.h5py.org/en/stable/</a>>, see Saving to disk: input/output.

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